5

Markov models — practical exercises

5.1 Revision of beta+binomial conjugate Bayesian inference

This question can be omitted if you are confident about the material on conjugate Bayesian inference from lecture 2. However, it may help in understanding later questions on Markov models constructed from Dirichlet+Multinomial models, which are a generalisation of Beta+Binomial models.

a. Write down the likelihood function for $r = 15$ people falling asleep during the lecture hour, given $n = 40$ were awake at the start. (Assume these are independent events and everyone has the same probability, π, of falling asleep).
b. Assuming a Beta(1, 1) prior distribution for π, write down the posterior distribution, given the numbers in each state in the lecture hour.
c. Simulate a vector of 1000 samples from this posterior distribution using Monte Carlo simulation in `R`
d. Assuming a time-homogeneous 2-state Markov chain with sleep as an absorbing state (people don't wake up again), obtain a vector of 1000 samples from the predictive distribution for the number awake after two hours.

5.2 Dirichlet / Multinomial conjugate Bayesian inference

In the asthma example from the lecture, suppose that the model was simplified so that the two "exacerbation" states (Hex and Pex, hospital or primary-care managed exacerbations) are merged and considered as a single state (Ex: exacerbation).

a. Using data from the SFC treatment arm, write down the likelihood function for the transition probabilities out of state STW, given that $\boldsymbol{r} = (210, 60, 1, 1)$ people end up in states (STW, UTW, Ex, and TF) one week after being in state STW.
b. Assume a Dirichlet(1, 1, 1, 1) prior distribution for $\boldsymbol{\pi}_1$, the vector of probabilities for moving from state STW to the other states in one week. Write down the posterior distribution of $\boldsymbol{\pi}_1$, given data.
c. Simulate a vector of 1000 samples from this posterior distribution using Monte Carlo simulation in `R`

5.3 Markov modelling in `R`

In this section, we will construct a Markov model from the asthma data given in the lecture, using the simplified four-state representation (STW, UTW, Ex, and TF).

The required `R` code gets more complex as this question goes on. If you would like to practice constructing this code from scratch, then use the code given in the lecture notes as the basis for the answer. Otherwise, just step through the `R` code given in the solutions to make sure it makes sense. The solutions with explanations are in `Solutions.pdf`, with code that you can copy and run in `Solutions.R`.

a, Suppose that before this study, we have observed people visiting state Ex on about 100 occasions, but only once did somebody stay in that state for a period of more than one week. Also suppose we are unsure about what happens to people in the week following a period in the Ex state. Construct a Dirichlet prior for the transition probabilities out of state Ex based on this information.

b. Suppose we observed the following transition count data

	to state			
from state	STW	UTW	Ex	TF
STW	210	60	1	1
UTW	88	641	4	13
Ex	1	0	0	1

Extend the code from section 5.2 to generate a sample from the posterior distribution of the full transition probability matrix, using the prior from part (a) for the transition probabilities from state Ex, and uniform Dirichlet priors for the transition probabilities out of STW and UTW, and noting that TF is an absorbing state. Thus estimate the posterior mean transition probability matrix.

c. Given these transition probabilities, generate a sample from the posterior distribution of the probabilities of occupying each state for each of cycles 1 to 12 of a Markov model, assuming everyone starts in state STW.

d. Extend the code to draw from the posterior distribution of the (undiscounted) expected costs and health effects over 12 weeks, assuming costs of 7.96, 7.96, and 1000 for STW, UTW and Ex respectively, and a utility of 1 in STW and 0 otherwise.

e. Plot the joint distribution of costs and effects for the SFC group as a scatterplot in the cost-effectiveness plane.

f. If you have time, repeat the above analysis for the FP group. Thus compute and plot the incremental costs and effects, and compare with the cost-effectiveness scatterplot given in the lecture. Use costs of 2.38 for STW, UTW, and 1000 for Ex, and the same utilities as before.