Stochastic SIR models

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From deterministic to stochastic

- Deterministic models don’t capture small sample variability
- Capturing uncertainty important for many real-life scenarios

- Main differences:
- Continuous variables -> discrete numbers
- Process rates -> process probabilities

(Relative) Probabilities for SIR

Process	Probability
Host birth	$a_1 = m(S + I + R)$
Death of susceptible host	$a_2 = mS$
Death of infected host	$a_3 = (m + v)I$
Death of recovered host	$a_4 = mR$
Infection	$a_5 = bSI$
Recovery	$a_6 = rI$

Simulation

- Computers like discrete variables
- Could take a *brute force* approach
  - For “small” time steps i.e. only at most one event can happen, determine state of system
  - Avoid multiple events simultaneously
- This is inefficient because in most steps nothing happens
- Can we do better…?

Gillespie algorithm

- Key idea: First determine *when* something happens
- Suppose current time *t*
- *t + e* at which something happens next is exponentially distributed random number
  - Scaled by the sum of all process rates
- In R we write
  - `tau <- rexp(1, rate=sum(a))`

Gillespie, Daniel T. (1977). "Exact Stochastic Simulation of Coupled Chemical Reactions". *The Journal of Physical Chemistry*. **81** (25): 2340–2361

- Then *what* happened next
- Draw randomly from all possible processed according to their respective probabilities
- Practically can sample an index
- In R
  - `sample(length(a),1,prob=a)`
- We can then update the state of the system and iterate along.