Ejemplos de prueba de los métodos de la primera revisión

1. Busquedas Incrementales

```
irb(main):023> Methods::NonLinearEquations::IncrementalSearch.exec('ln((sin(x)^2)+1)-1.0/2.0', -3, 0.5)
=>
{:iterations=>
  [{:x0=>-2.5, :x1=>-2.0},
   {:x0=>-1.0, :x1=>-0.5},
   {:x0=>0.5, :x1=>1.0},
   {:x0=>2.0, :x1=>2.5},
   {:x0=>4.0, :x1=>4.5},
   {:x0=>5.0, :x1=>5.5},
   {:x0=>7.0, :x1=>7.5},
   {:x0=>8.0, :x1=>8.5},
   {:x0=>10.0, :x1=>10.5},
   {:x0=>11.5, :x1=>12.0},
   {:x0=>13.5, :x1=>14.0},
   {:x0=>14.5, :x1=>15.0},
   {:x0=>16.5, :x1=>17.0},
   {:x0=>17.5, :x1=>18.0},
   {:x0=>19.5, :x1=>20.0},
   {:x0=>21.0, :x1=>21.5},
   {:x0=>22.5, :x1=>23.0},
   {:x0=>24.0, :x1=>24.5},
   {:x0=>26.0, :x1=>26.5},
   {:x0=>27.0, :x1=>27.5},
   {:x0=>29.0, :x1=>29.5},
   {:x0=>30.0, :x1=>30.5},
   {:x0=>32.0, :x1=>32.5},
   {:x0=>33.5, :x1=>34.0},
   {:x0=>35.0, :x1=>35.5},
   {:x0=>36.5, :x1=>37.0},
   {:x0=>38.5, :x1=>39.0},
   {:x0=>39.5, :x1=>40.0},
   {:x0=>41.5, :x1=>42.0},
   {:x0=>43.0, :x1=>43.5},
   {:x0=>44.5, :x1=>45.0},
   {:x0=>46.0, :x1=>46.5}],
 :errors=>[]}
```

2. Bisección

```
irb(main):021> Methods::NonLinearEquations::Bisection.exec('ln((sin(x)^2)+1)-1.0/2.0', 0, 1)
=>
{:conclution=>{:message=>"root_aproximation", :value=>0.9364045262336731, :iteration=>24},
 :iterations=>
  [{:i=>1, :a=>0.0, :m=>0.5, :b=>1.0, :fa=>0.03536607938024017, :fm=>-0.2931087267313766, :fb=>-0.5, :error=>Infinity},
   {:i=>2, :a=>0.5, :m=>0.75, :b=>1.0, :fa=>0.03536607938024017, :fm=>-0.11839639385347844, :fb=>-0.2931087267313766, :error=>0.25},
   {:i=>3, :a=>0.75, :m=>0.875, :b=>1.0, :fa=>0.03536607938024017, :fm=>-0.036817690757380395, :fb=>-0.11839639385347844, :error=>0.125},
   {:i=>4, :a=>0.875, :m=>0.9375, :b=>1.0, :fa=>0.03536607938024017, :fm=>0.0006339161592386899, :fb=>-0.036817690757380395, :error=>0.0625},
   {:i=>5, :a=>0.875, :m=>0.90625, :b=>0.9375, :fa=>0.0006339161592386899, :fm=>-0.017772289226861138, :fb=>-0.036817690757380395, :error=>0.03125},
   {:i=>6, :a=>0.90625, :m=>0.921875, :b=>0.9375, :fa=>0.0006339161592386899, :fm=>-0.008486582211768012, :fb=>-0.017772289226861138, :error=>0.015625},
   {:i=>7, :a=>0.921875, :m=>0.9296875, :b=>0.9375, :fa=>0.0006339161592386899, :fm=>-0.0039053586270640928, :fb=>-0.008486582211768012, :error=>0.0078125},
   {:i=>8, :a=>0.9296875, :m=>0.93359375, :b=>0.9375, :fa=>0.0006339161592386899, :fm=>-0.0016304381170096915, :fb=>-0.0039053586270640928, :error=>0.00390625},
   {:i=>9, :a=>0.93359375, :m=>0.935546875, :b=>0.9375, :fa=>0.0006339161592386899, :fm=>-0.0004969353153195244, :fb=>-0.0016304381170096915, :error=>0.001953125},
   {:i=>10, :a=>0.935546875, :m=>0.9365234375, :b=>0.9375, :fa=>0.0006339161592386899, :fm=>6.882244496264622e-05, :fb=>-0.0004969353153195244, :error=>0.0009765625},
   {:i=>11, :a=>0.935546875, :m=>0.93603515625, :b=>0.9365234375, :fa=>6.882244496264622e-05, :fm=>-0.00021397350516394464, :fb=>-0.0004969353153195244, :error=>0.00048828125},
   {:i=>12, :a=>0.93603515625, :m=>0.936279296875, :b=>0.9365234375, :fa=>6.882244496264622e-05, :fm=>-7.255478812057126e-05, :fb=>-0.00021397350516394464, :error=>0.000244140625},
   {:i=>13, :a=>0.936279296875, :m=>0.9364013671875, :b=>0.9365234375, :fa=>6.882244496264622e-05, :fm=>-1.8609849000705836e-06, :fb=>-7.255478812057126e-05, :error=>0.0001220703125},
   {:i=>14, :a=>0.9364013671875, :m=>0.936462402343750, :b=>0.9365234375, :fa=>6.882244496264622e-05, :fm=>3.3482026848830006e-05, :fb=>-1.8609849000705836e-06, :error=>6.103515625e-05},
   {:i=>15, :a=>0.9364013671875, :m=>0.9364318847656250, :b=>0.936462402343750, :fa=>3.3482026848830006e-05, :fm=>1.5810845160335596e-05, :fb=>-1.8609849000705836e-06, :error=>3.0517578125e-05},
   {:i=>16, :a=>0.9364013671875, :m=>0.9364166259765625, :b=>0.9364318847656250, :fa=>1.5810845160335596e-05, :fm=>6.975011174192858e-06, :fb=>-1.8609849000705836e-06, :error=>1.52587890625e-05},
   {:i=>17, :a=>0.9364013671875, :m=>0.9364089965820312, :b=>0.9364166259765625, :fa=>6.975011174192858e-06, :fm=>2.5570333977986692e-06, :fb=>-1.8609849000705836e-06, :error=>7.62939453125e-06},
   {:i=>18, :a=>0.9364013671875, :m=>0.9364051818847656, :b=>0.9364089965820312, :fa=>2.5570333977986692e-06, :fm=>3.4802931392352576e-07, :fb=>-1.8609849000705836e-06, :error=>3.814697265625e-06},
   {:i=>19, :a=>0.9364013671875, :m=>0.9364032745361328, :b=>0.9364051818847656, :fa=>3.4802931392352576e-07, :fm=>-7.56476526753147e-07, :fb=>-1.8609849000705836e-06, :error=>1.9073486328125e-06},
   {:i=>20, :a=>0.9364032745361328, :m=>0.9364042282104492, :b=>0.9364051818847656, :fa=>3.4802931392352576e-07, :fm=>-2.042232898902263e-07, :fb=>-7.56476526753147e-07, :error=>9.536743164062e-07},
   {:i=>21, :a=>0.9364042282104492, :m=>0.9364047050476074, :b=>0.9364051818847656, :fa=>3.4802931392352576e-07, :fm=>7.190309125881811e-08, :fb=>-2.042232898902263e-07, :error=>4.76837158203125e-07},
   {:i=>22, :a=>0.9364042282104492, :m=>0.9364044666290283, :b=>0.9364047050476074, :fa=>7.190309125881811e-08, :fm=>-6.616007947046754e-08, :fb=>-2.042232898902263e-07, :error=>2.384185791015625e-07},
   {:i=>23, :a=>0.9364044666290283, :m=>0.9364045858383179, :b=>0.9364047050476074, :fa=>7.190309125881811e-08, :fm=>2.8715108069121698e-09, :fb=>-6.616007947046754e-08, :error=>1.1920928955078125e-07},
   {:i=>24, :a=>0.9364044666290283, :m=>0.9364045262336731, :b=>0.9364045858383179, :fa=>2.8715108069121698e-09, :fm=>-3.1644283082776784e-08, :fb=>-6.616007947046754e-08, :error=>5.96046447753906e-08}],
 :errors=>[]}
```

3. Regla falsa

```
irb(main):022> Methods::NonLinearEquations::FalsePosition.exec('ln((sin(x)^2)+1)-1.0/2.0', 0, 1)
=>
{:conclution=>{:message=>"root_aproximation", :value=>0.936404580879889, :iteration=>5},
 :iterations=>
  [{:i=>1, :a=>0.0, :m=>0.9339403807182157, :b=>1.0, :fa=>-0.5, :fm=>-0.001429076703685472 3, :fb=>0.03536607938024017, :error=>Infinity},
   {:i=>2, :a=>0.9339403807182157, :m=>0.9365060516656253, :b=>1.0, :fa=>-0.001429076703685472 3, :fm=>5.875600835814065 4e-05, :fb=>0.03536607938024017, :error=>0.002656709474095596},
   {:i=>3, :a=>0.9339403807182157, :m=>0.9364047307426412, :b=>0.9365060516656253, :fa=>-0.001429076703685472 3, :fm=>8.678254082017389e-08, :fb=>5.875600835814065 4e-05, :error=>0.0001013209229840939},
   {:i=>4, :a=>0.9339403807182157, :m=>0.9364045811008692, :b=>0.9364047307426412, :fa=>-0.001429076703685472 3, :fm=>1.2815426497780 89e-10, :fb=>8.678254082017389e-08, :error=>1.4964177197374084e-07},
   {:i=>5, :a=>0.9339403807182157, :m=>0.936404580879889, :b=>0.9364045811008692, :fa=>-0.001429076703685472 3, :fm=>1.8907098109366 416e-13, :fb=>1.2815426497780 89e-10, :error=>2.2098023411132317e-10}],
 :errors=>[]}
```

4. Newton

```
irb(main):008> Methods::NonLinearEquations::Newton.exec('ln((sin(x)^2)+1)-1.0/2.0', '2*(((sin(x)^2)+1)^-1)*sin(x)*cos(x)', 0.5)
=>
{:conclution=>{:message=>"root_aproximation", :value=>0.9364045808795621, :iteration=>4},
 :iterations=>
  [{:i=>0, :x=>0.5, :fx=>-0.2931087267313766, :f_prime=>0.6842068330717285, :error=>Infinity},
   {:i=>1, :x=>0.9283919899125719, :fx=>-0.2931087267313766, :f_prime=>0.6842068330717285, :error=>0.4283919899125719},
   {:i=>2, :x=>0.9363667412673313, :fx=>-0.004662157097372055, :f_prime=>0.5846147284064961, :error=>0.007974751354759446},
   {:i=>3, :x=>0.9364045800189902, :fx=>-2.1912619882713535e-05, :f_prime=>0.5791052537949999, :error=>3.783875165885853e-05},
   {:i=>4, :x=>0.9364045808795621, :fx=>-4.98339092214195e-10, :f_prime=>0.5790789133390186, :error=>8.605719470367035e-10}],
 :errors=>[]}
```

5. Punto fijo

```
irb(main):012> Methods::NonLinearEquations::FixedPoint.exec('ln((sin(x)^2)+1)-1.0/2.0-x', 'ln((sin(x)^2)+1)-1.0/2.0', -0.5)
=>
{:conclution=>{:message=>"root_aproximation", :value=>-0.37444505296105535, :iteration=>30},
 :iterations=>
  [{:i=>0, :x=>-0.5, :fx=>0.2068912732686234, :gx=>-0.2931087267313766, :error=>Infinity},
   {:i=>1, :x=>-0.2931087267313766, :fx=>-0.12671281687488073, :gx=>-0.5, :error=>0.2068912732686234},
   {:i=>2, :x=>-0.41982154360625734, :fx=>0.07351702442859226, :gx=>-0.2931087267313766, :error=>0.12671281687488073},
   {:i=>3, :x=>-0.3463045191776651, :fx=>-0.0446539373646444, :gx=>-0.41982154360625734, :error=>0.07351702442859226},
   {:i=>4, :x=>-0.3909584565423095, :fx=>0.02655342164817026, :gx=>-0.3463045191776651, :error=>0.0446539373646444},
   {:i=>5, :x=>-0.3644050348941392, :fx=>-0.016021268273817058, :gx=>-0.3909584565423095, :error=>0.02655342164817026},
   {:i=>6, :x=>-0.3804263031679563, :fx=>0.009589507887747428, :gx=>-0.3644050348941392, :error=>0.016021268273817058},
   {:i=>7, :x=>-0.37083679528020885, :fx=>-0.005768850083372357, :gx=>-0.3804263031679563, :error=>0.009589507887747428},
   {:i=>8, :x=>-0.3766056453635812, :fx=>0.003460227756392209, :gx=>-0.37083679528020885, :error=>0.005768850083372357},
   {:i=>9, :x=>-0.373145417607189, :fx=>-0.002079223579867173, :gx=>-0.3766056453635812, :error=>0.003460227756392209},
   {:i=>10, :x=>-0.3752246411870562, :fx=>0.00124805513874654, :gx=>-0.373145417607189, :error=>0.002079223579867173},
   {:i=>11, :x=>-0.37397658604830963, :fx=>-0.0007496296601224861, :gx=>-0.3752246411870562, :error=>0.00124805513874654},
   {:i=>12, :x=>-0.3747262157084321, :fx=>0.0004500823979797816874, :gx=>-0.37397658604830963, :error=>0.0007496296601224861},
   {:i=>13, :x=>-0.37427613331045395, :fx=>-0.0002702951476383219 6, :gx=>-0.3747262157084321, :error=>0.0004500823979797816874},
   {:i=>14, :x=>-0.3745464284580923, :fx=>0.0001623020232475736, :gx=>-0.37427613331045395, :error=>0.0002702951476383219 6},
   {:i=>15, :x=>-0.3743841264348447, :fx=>-9.746439711039168e-05, :gx=>-0.3745464284580923, :error=>0.0001623020232475736},
   {:i=>16, :x=>-0.3744815908319551, :fx=>5.8525648058027624e-05, :gx=>-0.3743841264348447, :error=>9.746439711039168e-05},
   {:i=>17, :x=>-0.37442306518389706, :fx=>-3.514467880877392e-05, :gx=>-0.3744815908319551, :error=>5.8525648058027624e-05},
   {:i=>18, :x=>-0.37445820986270584, :fx=>2.110401325022826e-05, :gx=>-0.37442306518389706, :error=>3.514467880877392e-05},
   {:i=>19, :x=>-0.3744371058494556, :fx=>-1.2672877957420337e-05, :gx=>-0.37445820986270584, :error=>2.110401325022826e-05},
   {:i=>20, :x=>-0.37444977872741303, :fx=>7.609964212673681e-06, :gx=>-0.3744371058494556, :error=>1.2672877957420337e-05},
   {:i=>21, :x=>-0.37444216876320036, :fx=>-4.5697420043566694e-06, :gx=>-0.37444977872741303, :error=>7.609964212673681e-06},
   {:i=>22, :x=>-0.3744467385052047, :fx=>2.744098679452467e-06, :gx=>-0.37444216876320036, :error=>4.5697420043566694e-06},
   {:i=>23, :x=>-0.37444399440652526, :fx=>-1.647814738270359e-06, :gx=>-0.3744467385052047, :error=>2.744098679452467e-06},
   {:i=>24, :x=>-0.37444564222126353, :fx=>9.895019896788426e-07, :gx=>-0.37444399440652526, :error=>1.647814738270359e-06},
   {:i=>25, :x=>-0.37444465271927385, :fx=>-5.941897863737111e-07, :gx=>-0.37444564222126353, :error=>9.895019896788426e-07},
   {:i=>26, :x=>-0.3744452469090602, :fx=>3.568071592630062e-07, :gx=>-0.37444465271927385, :error=>5.941897863737111e-07},
   {:i=>27, :x=>-0.37444489010190096, :fx=>-2.1426045238026603e-07, :gx=>-0.3744452469090602, :error=>3.568071592630062e-07},
   {:i=>28, :x=>-0.37444510436235334, :fx=>1.28662038245686e-07, :gx=>-0.37444489010190096, :error=>2.1426045238026603e-07},
   {:i=>29, :x=>-0.3744449757003151, :fx=>-7.726074024994034e-08, :gx=>-0.37444510436235334, :error=>1.28662038245686e-07},
   {:i=>30, :x=>-0.37444505296105535, :fx=>4.63945839523916e-08, :gx=>-0.3744449757003151, :error=>7.726074024994034e-08}],
 :errors=>[]}
```

6. Secante

```
irb(main):017> Methods::NonLinearEquations::Secant.exec('ln((sin(x)^2)+1)-1.0/2.0-x', 0.5, 1)
=>
{:conclution=>{:message=>"root_aproximation", :value=>-0.37444502397338 44, :iteration=>9},
 :iterations=>
  [{:i=>0, :x=>0.5, :fx=>-0.7931087267313766, :error=>Infinity},
   {:i=>1, :x=>1.0, :fx=>-0.9646339206197598, :error=>Infinity},
   {:i=>2, :x=>-1.8119307104455946, :fx=>-0.9646339206197598, :error=>2.8119307104455946},
   {:i=>3, :x=>0.0776323330079951, :fx=>1.976149720001667, :error=>1.8895630434535897},
   {:i=>4, :x=>-0.3463208134896999, :fx=>-0.5716356675029044, :error=>0.423953146497695},
   {:i=>5, :x=>-0.38222223612415746, :fx=>-0.04462831310055826, :error=>0.03590142263445756},
   {:i=>6, :x=>-0.37437897529720754, :fx=>0.012475211395381047, :error=>0.007843260826949916},
   {:i=>7, :x=>-0.37444487659847514, :fx=>-0.00010570845565638365, :error=>6.590130126760 13e-05},
   {:i=>8, :x=>-0.3744450239762172, :fx=>-2.35872598008946e-07, :error=>1.4737774206574628e-07},
   {:i=>9, :x=>-0.37444502397338 44, :fx=>4.533873276812983e-12, :error=>2.832789558482318e-12}],
 :errors=>[]}
```

7. Raíces multiples

```
irb(main):016> Methods::NonLinearEquations::MultipleRoots.exec('e(x)-x-1', 'e(x)-1', 'e(x)', 1)
=>
{:conclution=>{:message=>"root_found", :value=>-4.218590698935789e-11, :iteration=>5},
 :iterations=>
  [{:i=>0, :x=>1, :fx=>0.7182818284590451, :error=>Infinity},
   {:i=>1, :x=>-0.23421061355351425, :fx=>0.7182818284590451, :error=>1.2342106135535142},
   {:i=>2, :x=>-0.008458279910761O9, :fx=>0.025405775475345838, :error=>0.22575233364275316},
   {:i=>3, :x=>-1.1890183808588653e-05, :fx=>3.567060801401567e-05, :error=>0.008446389726952502},
   {:i=>4, :x=>-4.218590698935789e-11, :fx=>7.068789997788372e-11, :error=>1.1890141622681664e-05},
   {:i=>5, :x=>-4.218590698935789e-11, :fx=>0.0, :error=>0.0}],
 :errors=>[]}
```

8. Eliminación gaussiana simple

```
irb(main):005> Methods::LinearEquations::GaussEliminationSimple.exec([[2,-1,0,3],[1,0.5,3,8],[0,13,-2,11],[14,5,-2,3]], [1,1,1,1])
=>
{:result=>[0.03849518810148722, -0.18022747156605434, -0.30971128608923887, 0.24759405074365706],
 :iterations=>
  [{:step=>0, :matrix=>[[2, -1, 0, 3], [1, 0.5, 3, 8], [0, 13, -2, 11], [14, 5, -2, 3]], :vector=>[1, 1, 1, 1]},
   {:step=>1, :matrix=>[[2, -1, 0, 3], [0.0, 1.0, 3.0, 6.5], [0.0, 13.0, -2.0, 11.0], [0.0, 12.0, -2.0, -18.0]], :vector=>[1, 0.5, 1.0, -6.0]},
   {:step=>2, :matrix=>[[2, -1, 0, 3], [0.0, 1.0, 3.0, 6.5], [0.0, 0.0, -41.0, -73.5], [0.0, 0.0, -38.0, -96.0]], :vector=>[1, 0.5, -5.5, -12.0]},
   {:step=>3, :matrix=>[[2, -1, 0, 3], [0.0, 1.0, 3.0, 6.5], [0.0, 0.0, -41.0, -73.5], [0.0, 0.0, 0.0, -27.878048780487802]], :vector=>[1, 0.5, -5.5, -6.902439024390244]}],
 :errors=>[]}
```

9. Eliminación gaussiana con pivoteo parcial

```
irb(main):004> Methods::LinearEquations::GaussEliminationPartial.exec([[2,-1,0,3],[1,0.5,3,8],[0,13,-2,11],[14,5,-2,3]], [1,1,1,1])
=>
{:result=>Vector[0.03849518810148731, -0.18022747156605426, -0.30971128608923887, 0.24759405074365706],
 :iterations=>
  [{:step=>0, :matrix=>[[2, -1, 0, 3], [1, 0.5, 3, 8], [0, 13, -2, 11], [14, 5, -2, 3]], :vector=>[1, 1, 1, 1]},
   {:step=>1,
    :matrix=>[[14, 5, -2, 3], [0.0, 0.1428571428571429, 3.1428571428571143, 7.785714285714286], [0.0, 13.0, -2.0, 11.0], [0.0, -1.7142857142857142, 0.2857142857142857, 2.5714285714285716]],
    :vector=>[1, 0.9285714285714286, 1.0, 0.8571428571428572]},
   {:step=>2,
    :matrix=>[[14, 5, -2, 3], [0.0, 13.0, -2.0, 11.0], [0.0, 0.0, 3.1648351648351647, 7.664835164835164], [0.0, 2.220446049250313e-16, 0.021978021978021955, 4.021978021978022]],
    :vector=>[1, 1.0, 0.9175824175824177, 0.989010989010989]},
   {:step=>3,
    :matrix=>[[14, 5, -2, 3], [0.0, 13.0, -2.0, 11.0], [0.0, 0.0, 3.1648351648351647, 7.664835164835164], [0.0, 2.220446049250313e-16, 0.0, 3.96875]],
    :vector=>[1, 1.0, 0.9175824175824177, 0.982638888888889]}],
 :errors=>[]}
```

10. Eliminación gaussiana con pivoteo total

```
irb(main):003> Methods::LinearEquations::GaussEliminationTotal.exec([[2,-1,0,3],[1,0.5,3,8],[0,13,-2,11],[14,5,-2,3]], [1,1,1,1])
=>
{:result=>[0.03849518810148732, -0.18022747156605423, -0.3097112860892388, 0.24759405074365703],
 :iterations=>
  [{:step=>0, :matrix=>[[2, -1, 0, 3], [1, 0.5, 3, 8], [0, 13, -2, 11], [14, 5, -2, 3]], :vector=>[1, 1, 1, 1]},
   {:step=>1,
    :matrix=>[[14, 5, -2, 3], [0.0, 0.1428571428571429, 3.1428571428571143, 7.785714285714286], [0.0, 13.0, -2.0, 11.0], [0.0, -1.7142857142857142, 0.2857142857142857, 2.5714285714285716]],
    :vector=>[1, 0.9285714285714286, 1.0, 0.8571428571428572]},
   {:step=>2,
    :matrix=>[[14, 5, -2, 3], [0.0, 13.0, -2.0, 11.0], [0.0, 0.0, 3.1648351648351647, 7.664835164835164], [0.0, 2.220446049250313e-16, 0.021978021978021955, 4.021978021978022]],
    :vector=>[1, 1.0, 0.9175824175824177, 0.989010989010989]},
   {:step=>3,
    :matrix=>[[14, 5, 3, -2], [0.0, 13.0, 11.0, -2.0], [0.0, 0.0, 7.664835164835164, 3.1648351648351647], [0.0, 2.220446049250313e-16, 0.0, -1.638709677419355]],
    :vector=>[1, 1.0, 0.9175824175824177, 0.5075268817204301]}],
 :errors=>[]}
```