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Sample Complexity Bounds for Linear System Identification from a Finite Set

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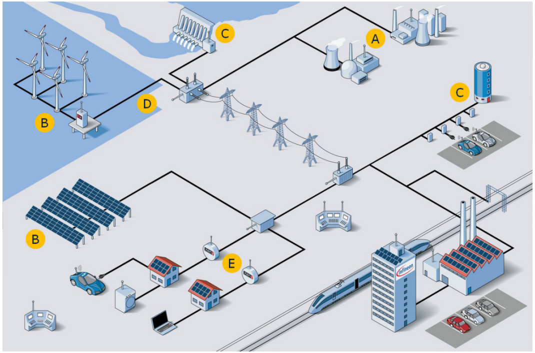
Motivation

Example: Active mode detection

- Plant has different modes with different dynamics
- **Goal:** Identify dynamics of active mode



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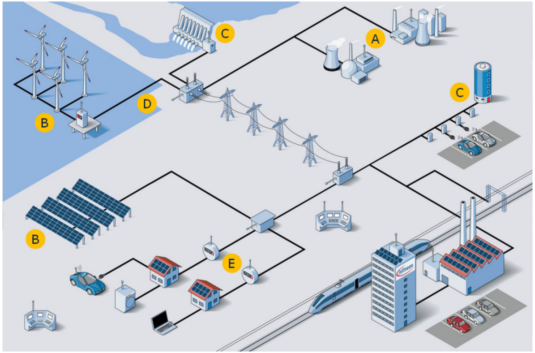

How does a finite hypothesis class influence identification?

Problem setup

- Discrete-time LTI plant $x(t+1) = A_* x(t) + B_* u(t) + w(t)$
- Finite set of candidate systems

$$\theta_* = (A_*, B_*) \in \mathcal{S} := \{(A_1, B_1), \dots, (A_N, B_N)\}$$

- Gaussian noise and control input: $w(t) \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \Sigma_w)$, $u(t) \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \Sigma_u)$
- Data $\{x(\tau)\}_{t=0}^{T}$, $\{u(\tau)\}_{t=0}^{T-1}$ collected from a finite trajectory (non-i.i.d.)

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Estimator

- Weighted prediction error: $\ell_{\theta_i}(x(t), u(t)) = \|x(t+1) - A_i x(t) - B_i u(t)\|^2_{\Sigma_w^{-1}}$
- Maximum Likelihood Estimator (MLE):

$$\hat{\theta}_T \in \arg\min_{\theta \in \mathcal{S}} \frac{1}{T} \sum_{t=0}^{T-1} \ell_\theta(x(t), u(t))$$

Goals

Key Contributions

Leverage tools from *high-dimensional statistics* to understand

- when identification of θ_* can be guaranteed with high-probability
- how the problem instance influences the hardness of identification
- the effects of finite vs. infinite hypothesis class

Sample complexity upper bound - Theorem

Theorem 1: High-probability identification (informal)

Define $\Delta A_i := A_* - A_i$, $\Delta B_i := B_* - B_i$ and

$$\Delta\Lambda_i^u(t) := \Delta A_i \sum_{s=0}^{t} A_*^s B_* \Sigma_u^{1/2} \text{ and } \Delta\Lambda_i^w(t) := \Delta A_i \sum_{s=0}^{t} A_*^s \Sigma_w^{1/2}.$$

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Fix a failure probability $\delta \in [0,1]$. Then the MLE yields the true system with probability at least $1-\delta$, if $T/k \gtrsim \log(n_x N/\delta)$ and

$$\underbrace{\|\Sigma_w^{-1/2}\Delta B_i \Sigma_u^{1/2}\|_F^2 + \|\Sigma_w^{-1/2}\Delta\Lambda_i^u(k/2)\|_F^2}_{\text{excitation due to } \boldsymbol{u}} + \underbrace{\|\Sigma_w^{-1/2}\Delta\Lambda_i^w(k/2)\|_F^2}_{\text{excitation due to } \boldsymbol{w}} \gtrsim n_x,$$

for some blocksize $k \in [1, T]$.

Sample complexity upper bound - Takeaways

- High-probability identification guarantee
- No stability of A_* required

Proof Idea

Show concentration and anti-concentration of empirical prediction error using BMSB property

$$\mathbb{P}[\frac{1}{T}\sum_{t=1}^{T} \ell_{\theta_i}(x(t), u(t)) \geq c] \geq 1 - \delta_i, \quad \forall \theta_i \neq \theta_* \qquad \text{(anti-concentration)}$$

$$\mathbb{P}[\frac{1}{T}\sum_{t=1}^{T} \ell_{\theta_*}(x(t), u(t)) < c] \geq 1 - \delta_* \qquad \text{(concentration)}$$

Theorem 2: Sample complexity lower bound

Fix $\delta \in (0,1)$. For any *reasonable* algorithm the sample complexity T is at least as large as the smallest $\bar{T}$ satisfying

$$\underbrace{\bar{T}\left\|\Sigma_w^{-\frac{1}{2}}\Delta B_i \Sigma_u^{1/2}\right\|_{\mathrm{F}}^2 + \sum_{s=0}^{\bar{T}-1}\left\|\Sigma_w^{-1/2}\Delta\Lambda_i^u(s)\right\|_{\mathrm{F}}^2}_{\text{excitation due to } \boldsymbol{u}} + \underbrace{\left\|\Sigma_w^{-1/2}\Delta\Lambda_i^w(s)\right\|_{\mathrm{F}}^2}_{\text{excitation due to } \boldsymbol{w}} \geq 2\log\left(\frac{1}{2.4\delta}\right) \quad \forall i \in [1, N].$$

Sample complexity lower bounds

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- Specifies when learning is hard
- Fundamental limits of identification

Conclusion

- Sample complexity bounds for SysID from finite hypothesis class
- Hardness of learning specified through system theoretic quantities
- No stability assumption necessary

Outlook

- Less conservative upper bound
- Experiment design

Sample Complexity Bounds for Linear System Identification from a Finite Set, NC and Andrea Iannelli, IEEE Control Systems Letters, 2024

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