Exam Seat No. _____

THAKUR COLLEGE OF SCIENCE & COMMERCE

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CERTIFICATE

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Semester 2:

Practical No. 1.

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① $\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$ ② $\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

③ $\lim_{x \to \pi/6} \left[\frac{\cos x - \sqrt{3}\sin x}{\pi - 6x} \right]$ ④ $\lim_{x \to \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$

⑤ Examine the continuity of the foll. fn:

(i) $f(x) = \frac{\sin 2x}{\sqrt{1-\cos x}}, \quad 0 < x \le \pi/2$

$= \frac{\cos x}{\pi - 2x}, \quad \pi/2 < x < \pi$

at $x = \pi/2$

(ii) $f(x) = \frac{x^2-9}{x-3}, \quad 0 < x < 3 = \frac{x^2-9}{x+3} \quad 6 \le x < 9$

$= x+3, \quad 3 \le x < 6$

at $x = 3$ & $x = 6$

⑥ Find value of K, so that f(x) is cont at indicated:

(i) $f(x) = \frac{1-\cos 4x}{x^2}, \quad x < 0$

$= K, \quad x = 0$

at $x = 0$

(ii) $f(x) = (\sec^2 x)^{\cot^2 x}, \quad x \ne 0$

$= K, \quad x = 0$

at $x = 0$

(iii) $f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x}, \quad x \ne \pi/3$

$= K, \quad x = \pi/3$

at $x = \pi/3$

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(7) Discuss the continuity of the foll. fn which of the has removable discontinuity. Redefine fn so as to remove discontinuity :-

(1) $f(x) = \dfrac{1-\cos^3 x}{x + a^n x}$, $x \neq 0$

$= 9$, $x = 0$ } at $x=0$

(2) $f(x) = \dfrac{(e^{3x} - 1)\sin x^0}{x^2}$, $x \neq 0$

$= \pi/60$, $x = 0$ } at $x=0$

(8) If $f(x) = \dfrac{e^{x^2} - \cos x}{x^2}$ for $x \neq 0$ is cont at $x=0$ find $f(0)$.

(9) If $f(x) = \dfrac{\sqrt{x} - \sqrt{1-\sin x}}{\cos 2x}$ for $x \neq \pi/2$, find $f(\pi/2)$

Answers :-

1) $\lim_{x \to 0}$ [

$\lim_{x \to 0}$ [

[

$\therefore$ fn

2) $\lim_{y \to \infty}$ [

$\Rightarrow \lim_{y \to \infty}$ [

$\Rightarrow \dfrac{1}{9+0+}$

3) $\lim_{x \to \pi/6}$ [

$\therefore \lim_{x \to \pi/6}$ [

$\Rightarrow$ [cosh

[cosh - √3

Answers :-

① $\lim_{x\to 0} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \right]$ 022

$\lim_{x\to 0} \left[\frac{(\sqrt{a+2x})^2 - (\sqrt{3x})^2}{(\sqrt{3a+x} - 2\sqrt{x})(\sqrt{a+2x} + \sqrt{3x}} \right]$

$\left[\frac{x-a}{-} \right] \rightarrow$ since : $x = a \longrightarrow x - a = 0$

$\therefore$ fn is dis continuous.

② $\lim_{y\to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right] \rightarrow \lim_{y\to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}} \right]$

$\Rightarrow \lim_{y\to 0} \left[\frac{a+y-a}{y(a+y) - ya\sqrt{y}} \right] \rightarrow \lim_{y\to 0} \left[\frac{1}{(a+y) + a\sqrt{y}} \right]$

$\Rightarrow \frac{1}{a+0 + a(\sqrt{0})} = \frac{1}{2a}$

③ $\lim_{x\to \pi/6} \left[\frac{\cos x - \sqrt{3}\sin x}{\pi - 6x} \right]$: $\pi/6$ $x - \pi/6 = h$ $\therefore (x = h + \pi/6)$

$\therefore \lim_{x\to \pi/6} \left[\frac{\cos(h+\pi/6) - \sqrt{3}\sin(h+\pi/6)}{\pi - 6(h+\pi/6)} \right]$

$\Rightarrow \left[\frac{\cos h + \cos\pi/6 - \sqrt{3}\sin h - \sqrt{3}\sin\pi/6}{\pi - 6h + (-\pi)} \right] \rightarrow \left[\frac{\cos h - \sqrt{3}\sin h + \frac{2}{\sqrt{3}} - 2}{-6h} \right]$

$\left[\frac{\cos h - \sqrt{3}\sin h + (-1)}{-6h} \right] \cdot \frac{2}{6} \lim_{h\to 0} \frac{\sin h}{h} \cdot \quad 1/3 \cdot 1 = 1/3$ Ans.

(4) $\lim_{x\to\infty}\left[\dfrac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$

$\therefore \lim_{x\to\infty}\left[\dfrac{(\sqrt{x^2+5}-\sqrt{x^2-3})}{(\sqrt{x^2+3}-\sqrt{x^2+1})}\times\dfrac{(\sqrt{x^2+3}+\sqrt{x^2+1})}{(\sqrt{x^2+3}+\sqrt{x^2+1})}\right]$

$\therefore \lim_{x\to\infty}\left[\dfrac{(\sqrt{x^2+5}-\sqrt{x^2-3})\times(\sqrt{x^2+3}+\sqrt{x^2+1})}{(\sqrt{x^2+3})^2-(\sqrt{x^2+1})^2)}\right]$

$\lim_{x\to\infty}\left[\dfrac{(\sqrt{x^2+5}-\sqrt{x^2-3})(\sqrt{x^2+3}+\sqrt{x^2+1})}{x^2+3-x^2-1}\right]$

$\lim_{x\to\infty}\left[\dfrac{((\sqrt{x^2+5})^2-(\sqrt{x^2-3})^2)(\sqrt{x^2+3}+\sqrt{x^2+1})}{2(\sqrt{x^2+5}-\sqrt{x^2-3})}\right]$

$\lim_{x\to\infty}\left[\dfrac{x^2+5-x^2+3}{\quad}\right] = \lim_{x\to\infty}\left[\dfrac{8(\sqrt{x^2+3}+\sqrt{x^2+1})}{2(\sqrt{x^2+5}-\sqrt{x^2-3})}\right]$

$= 4\lim_{x\to\infty}\left[\dfrac{\quad}{\quad}\right] = 4$

(5) $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$ for $0<x\le \pi/2$

$= \dfrac{\cos x}{\pi-2x}$, $\dfrac{\pi}{2}<x<\pi$ } at $x=\pi/2$

$\therefore f(\pi/2) = \dfrac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$ $\therefore f(\pi/2)=0$

$x=\dfrac{\pi}{2}$, defined.

$$\lim_{x \to \pi/2^+} f(x) = \lim_{x \to \frac{\pi}{2}^+} \frac{\cos x}{\pi - 2x}$$

$x - \frac{\pi}{2} = h \quad \therefore \quad x = h + \pi/2$ where, $h \to 0$

$$\lim_{h \to 0} \frac{\cos(h + \frac{\pi}{2})}{\pi - 2(h + \pi/2)} \to \lim_{h \to 0} \frac{\cos(\pi + \pi/2)}{\pi - 2h - \pi}$$

$$\therefore \frac{\cos h \cdot \cos\frac{\pi}{2} - \sin h \sin\frac{\pi}{2}}{-2h} \to \frac{\cos h \cdot 0 - \sin h}{-2h}$$

$$\therefore \frac{-\sin h}{-2h} \quad \therefore \quad 1/2$$

$$\lim_{x \to \pi/2} f(x) = \lim_{x \to \pi/2^+} \frac{\sin 2x}{\sqrt{1 - \cos 2x}}, \quad \lim_{x \to \pi/2} \frac{2\sin x \cos x}{\sqrt{2\sin^2 x}}$$

$$\lim_{x \to \frac{\pi}{2}^-} \frac{2\cancel{\sin x}\cos x}{\sqrt{2}\cancel{\sin x}} = \frac{2}{\sqrt{2}} \lim_{x \to \pi/2^-} \cos x$$

LHL $\neq$ RHL. $\therefore$ f is not cont at $\pi/2$

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6 (ii) $f(x) = \begin{cases} \dfrac{x^2-a}{x-3} & 0 < x < 3 \\ x+3 & 3 \le x < 6 \\ \dfrac{x^2-a}{x+3} & 6 \le x < 9 \end{cases}$ $x=3$ & $x=6$

at $x=3$

(1) $f(3) = \dfrac{x^2-9}{x-3} = 0$

(2) $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+}$ f at $x=3$ is defined.

(1) $f(3) = x+3 = 6$ $\therefore f$ is defined at $x=3$

(2) $\lim_{x\to 3^+} = \lim_{x\to 3^+} (x+3) = 6$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \dfrac{x^2-9}{x-3} = \dfrac{(x-3)(x+3)}{(x-3)}$

$\therefore$ LHL = RHL $\therefore f$ is cont at $x=3$

for $x=6$

(1) $f(6) = \dfrac{x^2-9}{x+3} = \dfrac{36-9}{6+3} = 3$

(2) $\lim_{x\to 6^+} = \dfrac{x^2-9}{x+3} = \dfrac{(x+3)(x-3)}{(x+3)} = 3$

$\therefore$ LHL $\neq$ RHL :

$\therefore f$ is not cont. at $x=6$.

(i) $f(x) = \dfrac{1-\cos 4x}{x^2}$, $x<0$ } at $x=0$

$= k$, $x=0$

f is cts at $x=0$

$\lim_{x\to 0} f(x) = f(0) \Rightarrow \lim_{x\to 0} \dfrac{1-\cos 4x}{x^2} = K$

$2\lim_{x\to 0}\left(\dfrac{\sin^2 2x}{x}\right) = k \quad \therefore 2(2)^2 = K \quad \therefore K = 8/1.$

(ii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $x \neq 0$ } at $x=0$

$= K$ $x=0$

(2) $\lim_{x\to 0}(1+\tan^2 x)^{1/\tan^2 x} = K = e$

(3) $\therefore K = e$

$\cancel{\dfrac{\sqrt{3}\tan - 3\tan h - \sqrt{3} - \tan h}{\dfrac{1-\sqrt{3}\tan h}{-3h}}}$

$\cancel{\therefore \dfrac{4}{3}\left(\dfrac{1}{1-0}\right) = 4/3}$

$\dfrac{-4\tan h}{3h(1-\sqrt{3}\tan h)}$

Sol^n:- (1) $\lim_{x\to 0} f(x) = \lim_{x\to 0}(\sec^2 x)^{\cot^2 x}$

$= \lim_{x\to 0}$

(iii) $f(x) = \dfrac{\sqrt{3}-\tan x}{\pi - 3x}$ $x \neq \pi/3$ } at $x = \pi/3$

$= k$ $x = \pi/3$

$x - \pi/3 = h \qquad x = h + \pi/3 \qquad \therefore \tan(A+B).$

$h \to 0$

$f(\pi/3 + h) = \dfrac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$

$\dfrac{-4\tan h}{-3h(1-\sqrt{3}\tan h)} = \dfrac{4}{3}\lim_{h\to 0}\dfrac{\tan h}{h}\lim_{h\to 0}\dfrac{1}{1-\sqrt{3}\tan h}$

$\dfrac{4}{3}\dfrac{1}{1-3(0)} \quad \therefore 4/3$

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(7)(b) $f(x) = \dfrac{1-\cos 3x}{x\tan x}$ $\quad x \neq 0$ $\Big\}$ at $x=0$

$= 9 \quad x=0$

$$f(x) = \frac{1-\cos 3x}{x\tan x} = \frac{\dfrac{2\sin^2 3x/2}{x^2} \times x^2}{\dfrac{x\tan x}{x^2} \times x^2}$$

$$\therefore 2 \lim_{x\to 0} \frac{\left(\frac{3}{2}\right)^2}{1} = 9/2$$

$\lim_{x\to 0} f(x) = 9/2 \neq f(0) \quad \therefore f$ is not cont.

$$f(x) = \begin{cases} \dfrac{1-\cos 3x}{x\tan x} & x \neq 0 \\ 9/2 & x=0 \end{cases}$$

Now $\lim_{x\to 0} f(x) = f(0) \quad \therefore f$ has removable discont. at $x=0$

$$f(x) = \left.\begin{cases} \dfrac{[e^x-1]\sin x^0}{x^2} & , x \neq 0 \\ \pi/6 & , x=0 \end{cases}\right] \text{ at } x=0$$

$3 \lim_{x \to 0} \frac{e^{3x}-1}{3x}$ $\lim_{x \to 0} \frac{\sin\left(\frac{\pi x}{100}\right)}{x}$

$3 \log e \frac{\pi}{100} = \pi/60 = f(0)$

$= \frac{e^{x^2} - \cos x}{x^2} = f(0)$

$\frac{e^{x^2} - \cos x - 1 + 1}{x^2} = \frac{(e^{x^2}-1) + (1-\cos x)}{x^2}$

$\frac{e^{x^2}-1}{x^2} + \lim_{x \to 0} \frac{2\sin^2 x/2}{x^2} = \log e + 2\left(\frac{\sin x/2}{x}\right)^2$

$\therefore \frac{3}{2} = f(0)$

(ii) $f(x) = (\sec^2 x)^{\cot 2x}$ $x \neq 0$ } at $x=0$

$= K$ $x = 0$

$\left(1 + \tan^2 x\right)^{\frac{1}{\tan^2 x}} = e = K$ $= K = e$

(1) $f(x) = \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos 2x}$, $x \neq \pi/2$

$f(0)$ is cts at $x = \pi/2$

$\therefore \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos 2x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$

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RSO:

$$= \frac{2-1+\sin x}{\cos^2 x\,(\sqrt{2}+\sqrt{1+\sin x})} = \frac{1}{(1-\sin x)(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \frac{1}{2(\sqrt{2}+\sqrt{2})} = \frac{1}{4\sqrt{2}} \quad \therefore f(\pi/2) = \frac{1}{4\sqrt{2}}$$

(2) $f(x) = e^{x^2} - \cos x$ for $x \neq 0$ is cont at $x=0$ find $f(0)$

Given, f is cont at $x=0$

$$\lim_{x\to 0} f(x) = f(0)$$

$$\lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2}$$

$$\lim_{x\to 0} \frac{(e^{x^2}-1) + (1-\cos x)}{x^2}$$

$$\log e + \lim_{x\to 0} \frac{2\sin^2 x/2}{x^2}$$

$$\log e + 2\lim_{x\to 0} \frac{\sin^2 x/2}{x^2}$$

$$1 + 2\times 1/4 = 3/2 \quad \therefore f(0) = 3/2$$

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Practical 2.

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Topic: Derivation.

① Show that the following function defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable:-

(i) $\cot x$ (ii) $\operatorname{cosec} x$ (iii) $\sec x$

② If $f(x) = 4x+1, \quad x \leq 2$
$= x^2+5, \quad x > 0$, at $x = 2$, find f is differentiable or not.

③ If $f(x) = 4x+7, \quad x < 3$
$= x^2+3x+1, \quad x \geq 3$ at $x = 3$, then find f is differentiable or not?

④ If $f(x) = 8x-5, \quad x \leq 2$
$= 3x^2-4x+7, \quad x > 2$ at $x = 2$ then find f is differentiable or not.

*Answers:

① (i) $\cot x$ $\quad f(x) = \cot x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x-a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x-a}$$

$$= \lim_{x \to a} \frac{\tan a - \tan x}{(x-a)\tan x \tan a}$$

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put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a+h-a) * \tan(a+h) \cdot \tan a}$$

$$\therefore \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h * \tan(a+h) \tan a}$$

formula : $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

$\tan A - \tan B = \tan(A-B)(\tan A \tan B + 1)$

$$\Rightarrow \lim_{h \to 0} \frac{\tan(a-a-h) - (1 + \tan a \tan(a+h))}{h \times \tan(a+h) \tan a}$$

$$\Rightarrow \lim_{h \to 0} -\frac{\tan h}{h} \times \frac{1 * \tan a \tan(a+h)}{\tan(a+h) \tan a}$$

$$\Rightarrow -1 \times \frac{1 + \tan^2 a}{\tan^2 a} = \frac{-\sec^2 a}{\tan^2 a} = \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

$\Rightarrow -\operatorname{cosec}^2 a$ $\quad \therefore Df(a) = -\cos^2 a$ $\quad \therefore f$ is differentiable

$\forall a \in R$

(ii) cosec x

$f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\sin x} - \frac{1}{\sin a}}{x - a}$$

$$= \lim_{x \to a} \frac{\sin a - \sin x}{(x-a) \sin a \sin x}$$

put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a) \sin a \cdot \sin(a+h)}$$

formula:

$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right) \cdot \sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \cdot \sin(a+h)}$$

$$= \lim_{h \to 0} \frac{-\sin h/2}{h/2} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$= -\frac{1}{2} \times \frac{2\cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)} = \frac{-\cos a}{\sin 2a} = -\cot a \operatorname{cosec} a.$$

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(iii) $\sec x$

$f(x) = \sec x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\sec x - \sec a}{x - a}$$

$$= \lim_{x \to a} \frac{1/\cos x - 1/\cos a}{x - a}$$

$$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a)\cos a \cos x}$$

Put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{x \to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cos(a+h)}$$

Formula: $-2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$$\Rightarrow \lim_{h \to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(a - \frac{a}{2} - h\right)}{h \times \cos a \cos(a+h)}$$

$$\Rightarrow \frac{-1}{2} \times \frac{-2\sin\left(\frac{2a+0}{2}\right)}{\cos a \cos(a+0)}$$

$= \tan a \sec a$

Q2. If $f(x) = 4x+1 \quad , \quad x \le 2$
$\quad\quad = x^2+5 \quad , \quad x > 0$

LHD: $Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$

$= \lim_{x \to 2^-} \frac{4x+1 - (4 \times 2 + 1)}{x-2}$

$= \lim_{x \to 2^-} \frac{4x+1-9}{x-2}$

$= \lim_{x \to 2^-} \frac{4x-8}{x-2}$

$= \lim_{x \to 2^-} \frac{4(x-2)}{(x-2)} = 4$

$Df(2^-) = 4$ RHD: $Df(2^+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$

$= \lim_{x \to 2^+} \frac{(x+2)(x-2)}{(x-2)}$

$= 4$

$Df(2^+) = 4$ RHD = LHD

f is differentiable at $x = 2$

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Q3) If $f(x) = 4x+7, \quad x<3$
$= x^2+3x+1, \quad x \geq 3$

RHD $Df(3^+) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+3x+1 - (3^2+3+3+1)}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+3x-18}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+6x-3x-18}{x-3} = \lim_{x \to 3} \frac{(x+6)(x-3)}{(x-3)}$

$\Rightarrow 3+6 \Rightarrow 9$

$Df(3^+) = 9$

LHD $= Df(3^-)$

$= \lim_{x \to 3^-} \frac{f(x) - f(3)}{x-3}$

$= \lim_{x \to 3^-} \frac{4x+7-19}{x-3}$

$= \lim_{x \to 3^-} \frac{4(x-3)}{(x-3)} = Df(3^+) = 4$

RHD $\neq$ LHD

f is not differentiable at $x=3$

Q4 If $f(x) = 8x - 5 \quad , x \leq 2$
$= 3x^2 - 4x + 7 \quad , x > 2$

$f(2) = 8 \times 2 - 5 = 11$

RHD $\rightarrow Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2}$

$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x - 2}$

$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x - 2}$

$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x - 2}$

$= \lim_{x \to 2^+} \frac{(3x + 2)\cancel{(x-2)}}{\cancel{(x-2)}}$

$= 3 \times 2 + 2 = 8$

$Df(2^+) = 8$

LHD: $Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$

$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x - 2}$

$= \lim_{x \to 2^-} \frac{8x - 16}{x - 2}$

$= \lim_{x \to 2^-} \frac{8(x-2)}{(x-2)}$

$= 8$

$Df(2^-) = 8 \qquad$ LHD = RHD

f is differentiable at $x = 3$.

9/12/19

RSD

Practical 3.

: Application of Derivative :

Q1. Find the intervals in which $f(x)$ is increasing or decreasing:-

(i) $f(x) = x^3 - 5x - 11$

(ii) $f(x) = x^2 - 4x$

(iii) $f(x) = 2x^3 + x^2 - 20x + 4$

(iv) $f(x) = x^3 - 27x + 5$

(v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

Q2. Find the intervals in which function is concave upwards & concave downwards:-

(i) $y = 3x^2 - 2x^3$

(ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

(iii) $y = x^3 - 27x + 5$

(iv) $y = 69 - 24x - 9x^2 + 2x^3$

(v) $y = 2x^3 + x^2 - 20x + 4$

Q1. Answer:

(i) $f(x) = x^3 - 5x - 11$

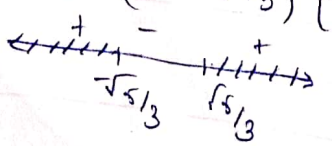
1) $\Rightarrow f'(x) = 3x^2 - 5$

f is increasing iff $f(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) > 0$

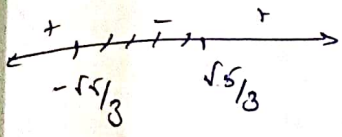


$x \in (-\infty, -\sqrt{5/3}) \cup (\sqrt{5/3}, \infty)$

for f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$\therefore (x - \sqrt{5}/3)(x + \sqrt{5}/3) \leq 0$



$x \in (-\sqrt{5}/3, \sqrt{5}/3)$

② $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$ $\quad \because f(x)$ is increasing iff $f'(x) > 0$

$\therefore 2x - 4 > 0$

$2(x-2) > 0$

$x - 2 > 0$

$x \in (2, \infty)$

& f is decreasing iff $f'(x) < 0$.

$\therefore 2x - 4 < 0$ $\quad x \in (-\infty, 2)$

$\therefore 2(x-2) < 0$

$x - 2 < 0$

③ $f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$\therefore$ f is increasing iff $f'(x) > 0$

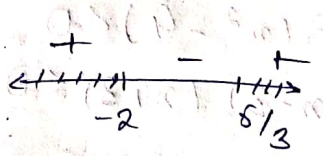
$\therefore 6x^2 + 2x - 20 > 0$

$2(3x^2 + x - 10) > 0$

$\therefore 3x^2 + x - 10 > 0$

$3x^2 + 6x - 5x - 10 > 0$

$(x+2)(3x-5) > 0$



$x \in (-\infty, -2) \cup (5/3, \infty$

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& f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 + 2x - 20 < 0$

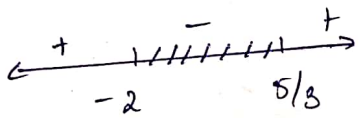
$\therefore 2(3x^2 + x - 10) < 0$

$3x^2 + x - 10 < 0$

$3x^2 + 6x - 5x - 10 < 0$

$3x(x+2) - 5(x+2) < 0$

$(x+2)(3x-5) < 0$



$x \in (-2, 5/3)$

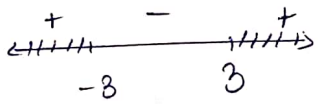
(4) $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$\therefore$ f is increasing iff $f'(x) > 0$

$\therefore 3(x^2 - 9) > 0$

$(x-3)(x+3) > 0$



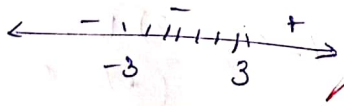
$x \in (-\infty, -3) \cup (3, \infty)$

& f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 27 < 0$

$\therefore 3(x^2 - 9) < 0$

$(x-3)(x+3) < 0$



$\therefore x \in (-3, 3)$

(1) $f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$\therefore \quad 6(x^2 - 3x - 4) > 0$

$\therefore \quad x^2 - 4x + x - 4 > 0$

$(x-4)(x+1) > 0$

$x \in (-\infty, -1) \cup (4, \infty)$

and f is decreasing iff $f'(x) < 0$

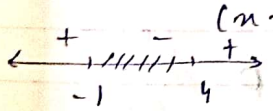
$\therefore \quad 6x^2 - 18x - 24 < 0$

$6(x^2 - 3x - 4) < 0$

$x^2 - 4x + x - 4 < 0$

$x(x-4) + 1(x-4) < 0$

$(x-4)(x+1) < 0$



$x \in (-1, 4)$

Q2.

i) $y = 3x^2 - 2x^3$

$\therefore \quad f(x) = 3x^2 - 2x^3$

$\therefore \quad f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

f is concave upwards if $f''(x) > 0$

$\therefore \quad (6 - 12x) > 0$

$12(6/12 - x) > 0$

$x - 1/2 > 0 \qquad x > 1/2$

$\therefore \quad f''(x) > 0 \qquad x \in (1/2, \infty)$

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2 $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

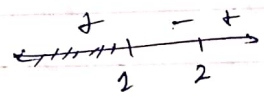
f is concave iff $f''(x) > 0$

$\therefore 12x^2 - 36x + 24 > 0$

$12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 2x - x + 2 > 0$

$\therefore (x-2)(x-1) > 0$



$x \in (-\infty, 1) \cup (2, \infty)$

(8) $y = x^3 - 27x + 6$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upwards iff $f''(x) > 0$

$\therefore 6x > 0$

$x > 0$

$x \in (0, \infty)$

(9) $y = 69 - 24x - 9x^2 + 2x^3$

$f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

f is concave upwards iff $f''(x) > 0$

$12x - 18 > 0$

$12(x - 18/12) > 0$

$x - 3/2 > 0$ $\therefore x > 3/2$

$\therefore x \in (3/2, \infty)$

6. $y = 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$\therefore f''(x) > 0$

$12x + 2 > 0$

$12(x + 2/12) > 0$

$x + 1/6 > 0$

$x < -1/6$

$\therefore f''(x) \cancel{>} 0$

$\therefore$ There exists no inflection.

$x \in (-1/6, \infty)$

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Practical 4:

SEO Application of Derivative & Newton's Method.

Q1 Find maximum & minimum value of following functions

(i) $f(x) = x^2 + \frac{16}{x^2}$

(ii) $f(x) = 3 - 5x^3 + 3x^5$

(iii) $f(x) = x^3 - 3x^2 + 1$ in $[-1/2, 4]$

(iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$ in $[-2, 3]$

Q2 Find the roots of following equation by Newton's Method (Take 4 iterations only) Correct upto 4 decimal.

1) $f(x) = x^3 - 3x^2 - 55x + 9.5$ (take $x_0 = 0$)

2) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

(iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$

Answers: →

Q1)

(i) $f(x) = x^2 + 16/x^2$ Now Consider, $f'(x) = 0$

$f'(x) = 2x - 32/x^3$ $\quad 2x - 32/x^3 = 0$ $\quad f''(x) = 2 + 96/x^4$

$x^4 = 32/2$ $\quad \therefore \boxed{x = \pm 2}$ $\quad 2x = 32/x^3$ $\quad f''(2) = 2 + 6 = 8 > 0$

$\therefore f$ has minimum value at $x = 2$ $\quad \therefore f(2) = 2^2 + 16/2^2 = 8$

$f''(-2) = 2 + 96/-2^4 = 8 > 0$
$= 2 + 96/16$

$\therefore f$ has minimum value at $x = 2$

$\therefore$ function reaches minimum value at $x = 2$ & ...

(ii) $f(x) = 3 - 5x^3 + 3x^5$ $\therefore f'(x) = -15x^2 + 15x^4$

Consider, $f'(x) = 0$ $\therefore -15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$ $\therefore x^2 = 1$ $\therefore x = \pm 1$

$f''(x) = -30x + 60x^3$

$f''(1) = -30 + 60$ $\therefore 30 > 0$

$\therefore f$ has minimum value at $x = 1$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5 = 6 - 5 = 1$

$f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$ $\therefore -30 < 0$

$\therefore f$ has maximum value at $x = -1$

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5 = 5$

$\therefore f$ has the maximum value 5 at $x = -1$

& f has minimum value 1 at $x = 1$

(iii) $f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x$

Consider, $f'(x) = 0$ $\therefore 3x^2 - 6x = 0$ $\therefore 3x(x-2) = 0$

$\therefore 3x = 0$ or $x - 2 = 0$ $\therefore f''(x) = 6x - 6$

$\therefore f''(0) = 6(0) - 6 = -6 < 0$

$\therefore f$ has maximum value at $x = 0$

$\therefore f(0) = (0)^3 - 3(0)^2 + 1 = 1$

$\therefore f''(2) = 6(2) - 6$ $\therefore 6 > 0$

$\therefore f$ has minimum value at $x = 2$

$\therefore f(2) = (2)^3 - 3(2)^2 + 1 = -3$

$\therefore f$ has maximum value 1 at $x = 0$ &

f has minimum value -3 at $x = 2$.

(iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$f'(x) = 6x^2 - 6x - 12$

Consider, $f'(x) = 0 \therefore 6x^2 - 6x - 12 = 0$

$\therefore 6(x^2 - x - 2) = 0 \therefore x^2 + x - 2x - 2 = 0$

$\therefore x(x+1) - 2(x+1) = 0 \therefore (x-2)(x+1) = 0$

$\therefore x = 2$ or $x = -1$ $\therefore f''(x) = 12x - 6$

$f''(2) = 12(2) - 6 \therefore 24 - 6 \therefore 18 > 0$

f has minimum value at $x = 2$

$f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 16 - 12 - 24 + 1$

$= -19$

$f''(-1) = 12(-1) - 6$

$= -12 - 6 = -18 < 0$

f has maximum value at $x = -1$

$f(-1) = 2(-1)^3 - 3(-1)^2 - 12(1) + 1 = 8$

$\therefore f$ has maximum value 8 at $x = -1$ &

f has minimum value -19 at $x = 2$

Q2:

(i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ $\qquad x_0 = 0.34$ → Given

$f'(x) = 3x^2 - 6x - 55$

By Newton's Method.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \qquad \therefore x_1 = 0 + 9.5/55$$

$$\therefore x_1 = 0.1727 \qquad \therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$$

$$= -0.0829$$

$$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$$

$$= 0.0895 - 1.0362 - 55$$

$$= -55.9467.$$

$$\therefore x_2 = x_1 - f(x_1)/f'(x_1) = 0.1727 - 0.0829/55.9467$$

$$= 0.1712$$

(ii) $f(x) = x^3 - 4x - 9$ $\qquad [2, 3]$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9 = 8 - 8 - 9 = -9$

$f(3) = 3^3 - 4(3) - 9 = 27 - 12 - 9 = 6$

Let $x_0 = 3$ be the initial approximation,

$\therefore$ By newton's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad \therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 3 - 6/23$$

$$= 2.7392$$

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$$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$$
$$= 20.5528 - 10.9568 - 9 = 0.596$$

$$f'(x_1) = 23(2.7392)^2 - 4$$
$$= 18.5096$$

$$x_2 = x_1 - f(x_1)/f'(x_1) = 2.7392 - \frac{0.596}{18.5096} = 2.7071$$

$$f(x_2) = (2.7071)^3 - 4(2.7071) - 9$$
$$= 0.0102$$

$$f'(x_2) = 3(2.7071)^2 - 4$$
$$= 17.9851$$

$$\therefore x_3 = x_2 - f(x_2)/f'(x_2)$$
$$= 2.7071 - \frac{0.0102}{17.9851} = 2.7071 - 0.0056 = 2.7015$$

$$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$$
$$= -0.0901$$

$$f'(x_3) = 3(2.7015)^2 - 4 = 17.8943$$
$$= 17.8943$$

$$x_4 = 2.7015 + \frac{0.0901}{17.8943}$$
$$= 2.7015 + 0.0050 = 2.7065$$

(iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ $[1, 2]$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$

$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$

$= 8 - 7.2 - 20 + 17 = -2.2$

Let $x_0 = 2$ be initial approximation

By Newton's Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_2)}{f'(x_2)} = 2 - \frac{2.2}{5.2} = 2 - 0.4230$$

$= 1.577$

$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$

$= 7.4608 - 5.6772 - 10 = -8.2164$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.577 + \frac{0.6755}{8.2164}$$

$= 1.577 + 0.0822 = 1.6592$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$

$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$

$= 8.2588 - 5.97312 - 10$

$= -7.7143$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.6592 + \frac{0.0204}{0.7143} = 1.6618$$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$

$= 4.5892 - 4.9708 - 16.618 + 17$

$= 0.0004$

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$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$

$= 8.2847 - 5.9824 - 10$

$= -7.6977$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$

$= 8.2847 - 5.9824 - 10$

$= -7.6977$

$x_4 = x_3 - f(x_3)/f'(x_3)$

$= 1.6618 + \frac{0.0004}{7.6977} = 1.6618$

$\therefore$ root of equation 1.6618.

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:Practical 5:

: Integration:

Q. Solve the following integration:

(i) $\int \frac{dx}{\sqrt{x^2+2x-3}}$ (ii) $\int (4e^{3x}+1)\,dx$ (iii) $\int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$

(iv) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$ (v) $\int t^7 \sin(2t^4)\,dt$

(vi) $\int \sqrt{x}\,(x^2-1)\,dx$ (vii) $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

(viii) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\,dx$ (ix) $\int e^{\cos^2 x} \sin 2x\,dx$

(x) $\int \left(\frac{x^2-2x}{(x^3-3x^2+1)}\right) dx$

Answer :-

Q.

(i) $\int \frac{dx}{\sqrt{x^2+2x-3}} = \int \frac{1}{\sqrt{x^2+2x+1-4}}\,dx$

$\Rightarrow \int \frac{1}{\sqrt{(x+1)^2-4}}\,dx$ $[\because a^2+2ab+b^2 = (a+b)^2]$

$\therefore$ Put $x+1 = t$

$\therefore dx = \frac{1}{t}\,dt$ where $t = x+1$ & $r = 1$

$= \int \frac{1}{\sqrt{t^2-4}}\, dt$ $\therefore$ by using formula ;

$\int \frac{1}{\sqrt{x^2-a^2}}\, dx = \ln\left(\left|x - \sqrt{x^2-a^2}\right|\right)$

$= \ln\left(\left|t + \sqrt{t^2-4}\right|\right)$

Return $t = x+1$

$= \ln\left(\left|x+1 + \sqrt{(x+1)^2-4}\right|\right)$

$= \ln\left(\left|x+1 + \sqrt{x^2+2x-3}\right|\right) + c$

2) $\int (4e^{3x}+1)\, dx$

$\Rightarrow \int 4e^{3x} dx + \int 1 dx$ # $\int e^{ax} dx = \frac{1}{a} \times e^{ax}$

$\Rightarrow \frac{4e^{3x}}{3} + x$ $\Rightarrow \frac{4e^{3x}}{3} + x + c$

3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\, dx$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2}\, dx$ # $\sqrt[n]{a^m} = a^{m/n}$

$= \int 2x^2\, dx - \int 3\sin(x)\, dx + \int 5x^{1/2}\, dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10 \times \sqrt{x}}{3} + c$

$= \frac{2x^3 + 10x\sqrt{x}}{3} + 3\cos x + c$

4) $\int \frac{x^3 + 3x + 4}{\sqrt{x}} \, dx = \int \frac{x^3 + 3x + 4}{x^{1/2}} \, dx$

; splitting the denominator as.

$\int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}} \, dx$

$= \int x^{5/2} + 3x^{1/2} + 4/x^{1/2} \, dx$

$= \frac{2x^3 \sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + c$

5) $\int t^7 \times \sin(2t^4) \, dt$ put $u = 2t^4$ $du = 2 \times 4t^3$

$= \int t^7 \sin(2t^4) \times 1/2 \times 4t^3 \, du$

$= \int t^4 \sin(2t^4 \times \frac{1}{8} \, du = t^4 \times \sin \frac{(2t^4)}{8} \, du$

Substitute t^4 with $u/2$ $= \int u/2 \times \frac{\sin(u)}{2} / 8 \, du$

$= \int u \times \frac{(\sin(u))}{16} \, du = \frac{1}{16} \int u \times \sin(u) - du$

$\int u dv = uv - \int v du$ where $v = u$

$= 1/16 \, (u \times (-\cos(u)) - \int -\cos(u) \, du$

$= 1/16 \times [u \times (-\cos u)) + \int \cos(u) du$

$\int \cos 2x dx = \sin 2x$

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Return the substitution $u = 2t^4$

$1/16 = (2t^4 \times (-\cos(2t^4)) + \sin(2t^4))$

$\therefore -t^4 \times \frac{\cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + C$

(6) $\int \sqrt{x}(x^2-1)\,dx = \int \sqrt{x}x^2 - \sqrt{x}\,dx$

$\int x^{5/2} - x^{1/2}\,dx$

$\therefore I_2 = \frac{x^{5/2}+1}{5/2+1} = \frac{x^{7/2}}{7/2} = \frac{2x^3\sqrt{x}}{7}$

$I_2 = \frac{x^{1/2}+1}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2\sqrt{x}\,3}{3}$

$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x}\,3}{3} + C$

(8) $\int \frac{\cos x}{\sqrt[3]{\sin(x)}^2}\,dx = \int \frac{\cos x}{\sin x^{2/3}}$

Put $t = \sin(x)$

$t = \cos x$

$= \int \frac{\cos(x)}{\sin(x)^{2/3}} \times \frac{1}{\cos(x)}\,dt$

$= \frac{1}{\sin x^{2/3}}\,dt = \frac{1}{t^{2/3}}\,dt$

$$I = \int \frac{1}{t^{2/3}}\, dt = \frac{-1}{(2/3)^{-1} t^{2/3-1}}$$

$$\frac{-1}{-1/3\, t^{2/3-1}} = \frac{t^{1/3}}{t^{1/3}} = 3t^{1/3} = 3\sqrt[3]{t}$$

Return substitution $t = \sin(x)$

$$= 3\sqrt[3]{\sin(x)} + C$$

(x) $$\int \frac{x^2 - 2x}{x^3 - 3x^2 + 1}\, dx$$

Put $x^3 - 3x^2 + 1 = dt$

$$I = \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 3 \times 2x}\, dt$$

$$= \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 6x}\, dt$$

$$= \int \frac{1}{x^3 - 3x^2 + 1} \cancel{\times 1/3\, dt}$$

$$= \int \left(\frac{1}{3(x^3 - 3x^2 + 1)}\right) dt = \int 1/3t\, dt$$

$$= 1/3 \int 1/t\, dt \quad \int 1/x\, dx = \ln|x|$$

$$= 1/3 \times \ln|t| + C = 1/3 \times \ln(|x^3 - 3x^2 + 1|) + C$$

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(7) $\int \frac{1}{x^3} \sin(1/x^2)\, dx$

$I = \int 1/x^3 \sin(1/x^2)\, dx$

let $1/x^2 = t$ $\quad x^{-2} = t$ $\quad -2/x^3\, dx = dt$

$I = -1/2 \int -2/x^3 \sin(1/x^2)\, dx$

$= -1/2 \int \sin t \quad = -1/2(-\cos t) + c$

$= 1/2 \cos t + c$

Resubstitution $t = 1/x^2$

$I = 1/2 \cos(1/x^2) + c$

(9) $\int e \cos^2 x \cdot \sin 2x\, dx$

$I = \int e^{\cos^2 x} = t$

$-2\cos x \cdot \sin x\, dx = dt$

$-2\sin 2x\, dx = dt$

$I = \int -2\sin 2x \cos 2x\, dx$

$= -\int e^t\, dt$

$= e^t + c$

Resbst. $t = \cos^2 x$

$I = -e^{\cos 2x} + c$

Alz
06/01/2026

: Practical No. 6 :

Application of Integration & Numerical Integration:

① Find length of the following curve:

1. $x = t\sin t$, $y = 1-\cos t$, $t \in [0, 2\pi]$
2. $y = \sqrt{4-x^2}$, $x \in [-2, 2]$

② $y = x^{3/2}$ in $[0, 4]$

4. $x = 3\sin t$, $y = 3\cos t$, $t \in [0, 2\pi]$
5. $x = 1/6\, y^3 + 1/2\, y$, $y \in [1, 2]$

② Using Simpson's rule, solve the following:-

1. $\int_0^2 e^{x^2} dx$ with $n = 4$.
2. $\int_0^4 x^2 dx$ with $n = 4$
3. $\int_0^{\pi/3} \sqrt{\sin x}\, dx$ with $n = 6$

Answers:

①

1. $x = t\sin t$, $y = 1-\cos t$, $t \in [0, 2\pi]$ $\quad L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$x = t - \sin t \qquad dx/dt = t - \cos t \qquad dy/dt = \sin t$

$L = \int_0^{2\pi} \sqrt{(t - \cos t + \sin t)^2 + (\sin t)^2}\, dt \qquad \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt$

$\int_0^{2\pi} 2\left|\sin t/2\right| dt \quad \therefore \sin^2 t/2 = \frac{1-\cos t}{2}$

$$\left(-4\cos\left[t/2\right]\right)_0^{\pi} = (-4\cos\pi) - (-4\cos 0)$$

$$= 4 - 4 = 8$$

Q2. $\frac{dy}{dx} = \frac{-2x}{2\sqrt{4-x^2}} = \frac{-x}{\sqrt{4-x^2}}$ $\quad \therefore L = 2\int_0^2 1 + \left[\frac{-x}{\sqrt{4-x^2}}\right]^2 dx$

$$2\int_0^2 1 + \frac{x^2}{4-x^2}\,dx = 4\int_0^2 \frac{1}{\sqrt{4-x}}\,dx = 4\left[\sin^{-1}\left[x/2\right]\right]$$

(3) $f'(x) = 3/2\, x^{1/2} = \left[f'(x)\right]^2 = 9/4\, x$

$$L = \int_0^b \sqrt{1 + \left[f'(x)\right]^2}\,dx$$

$$= \int_0^2 \sqrt{1 + 9/4\, x}\,dx \quad \text{put } u = 1 + 9/4\, x \quad du = 9/4\, dx$$

$$= L = \int_1^{1+9/4\,x} \frac{4}{9}\sqrt{u}\,du = \left[4/9 \cdot 2/3\left(u^{3/2}\right)\right]^{1+9/4\,x}$$

$$= 8/27\left[\left(1 + 9x/4\right) - 1\right]$$

(4) $x = 3\sin t$, $y = 3\cos t$, this is a circle radius 040
r centered at the origin

$\frac{dx}{dt} = 3\cos t$, $dy/dt = 3\sin t$

length $L = \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt = \int_0^{2\pi} 3\sqrt{\sin^2 t + \cos^2 t}\, dt$

$= 3\int_0^{2\pi} dt = 6\pi$

(5) $\frac{dx}{dy} = y^2/2 - 1/2y^2 = \frac{dx}{dy} = \frac{y^x - 1}{2y^2}$

$L = \int_1^2 \sqrt{1 + (dx/dy)^2}\, dy$

$= \int_1^2 \sqrt{\frac{(y^x + 1)^2}{(2y)^2}}\, dy = \int_1^2 \frac{y^x + 1}{2y^2}\, dy$

$= 1/2 \int_1^2 y^2\, dy + 1/2 \int_1^2 y^{-2}\, dy$

$= 1/2 \left[y^3/3 - y^{-2}/1 \right]_1^2 = 1/2 \left[8/3 - 1/2 - 1/3 + 1 \right]$

$= 1/2 \left[7/3 - 1/2 \right] = 17/12$ units.

Q2

① $\int_0^2 e^{x^2} dx = 16.04526$

In each case the width of the sub int be $\Delta x = \frac{2-0}{4} = 1/2$

and so the sub int will be $[0, 0.5]$ $[0.5, 1]$ $[1, 1.5]$ $[1.5, 2]$

By simpson rule

$$\int_0^2 e^{x^2} dx = \frac{1/2}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right]$$

$$\approx 1/2/3\left[e^{0^2} + 4e^{(0.5)^2} + 2e^{(1)^2} + 4e^{(1.5)^2} + e^{(2)^4}\right]$$

$$\approx 17.3536\ 2645$$

② $\Delta x = \frac{4-0}{4} = 1$

$$\int_a^b f(x)dx = \frac{\Delta x}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right]$$

$$\approx 1/3\left[y(0) + 4f(1) + 2f(2) + 4f(3) + f(4)\right]$$

$$\approx 1/3\left[0^2 + (4)(1)^2 + 2(2)^2 + 4(3)^2 + 4^2\right]$$

$$\approx 64/3 \approx 21.333$$

(3) $\Delta x = \frac{b-a}{n} = \frac{\pi/3 - 0}{6} = \pi/18$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$	$6\pi/18 = \pi/3$
$y = \sqrt{\sin x}$	0	0.4167	0.584	0.707	0.801	0.875	0.930
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$$\int_0^{\pi/3} \sqrt{\sin x}\,dx \approx \frac{\Delta x}{3}\left(y_0 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4) + y_6\right)$$

$$\approx \frac{\pi/18}{3}\left(0 + 4(0.4167 + 0.707 + 0.875) + 2(0.584 + 0.801) + 0.930\right)$$

$$\approx 0.681.$$

1A0 : Practical 7 :

: Differential Equation :

Q1. Solve the following differential equs

① $x\,dy/dx + y = e^x$

② $e^x\,dy/dx + 2e^x y = 1$

③ $x\,dy/dx = \cos x/x - 2y$

④ $x\,dy/dx + 3y = \sin x/x^2$

⑤ $e^{2x}\,dy/dx + 2e^{2x} y = 2x$

⑥ $\sec^2 x \tan y\,dx + \sec^2 y \tan x\,dy = 0$

⑦ $dy/dx = \sin^2(x-y+1)$

⑧ $dy/dx = 2x+3y-1/6x+9y+6$

Answers:

① $\frac{x\,dy}{dx} + 1/x\,y = e^x/x$ $\quad p(x) = 1/x \quad Q(x) = e^x/x$

If $e^{\int 1/x\,dx}$

$y(IF) = \int Q(x)\,(IF)\,dx + C$

$= \int e^x/x \cdot x \cdot dx + C$

$= \int e^x\,dx + c$

$xy = e^x + c$

(2) $e^x \, dy/dx + 2e^x y = 1$

$dy/dx + 2y = 1/e^x \qquad dy/dx + 2y = e^{-x}$

$P(x) = 2 \qquad Q(x) = e^{-x}$

$\int P(x) = dx \qquad If = e^{\int 2dx}$

$= e^{2x}$

$y(IF) = \int Q(x)(IF)\,dx + c$

$= \int e^x \, dx + c$

$= y \cdot e^{2x}$

$= e^x + c$

(3) $x \, dy/dx = \frac{\cos x}{x} - 2y$

$dy/dx + 2y/x = \cos x/x^2$

$P(x) = 2(x) \qquad Q(x) = \frac{\cos x}{x^2}$

$If = e^{\int P(x)\,dx}$

$= e^{\int 2x\,dx} \qquad y(IF) = \int Q(x)(IF)\,dx + c$

$= \int \frac{\cos x}{x^2} - x^2 dx + c$

$\therefore x^2 y = \sin x + c \quad \leftarrow$

$= \int \cos x + c$

(4) $dy/dx + 3y/x = \frac{\sin x}{x^3}$

$P(x) = 3/x \qquad Q(x) = \sin x/x^3$

$P(x) = \int 3/x \, dx = x^3$

$If = e^{\int P(x)\,dx} = x^3$

$y(IF) = \int Q(x)(IF)\,dx + c$

$= \int \frac{\sin x}{x^3} \cdot x^3 \, dx + c = \int \sin x + c$

$\therefore x^3 y = -\cos x + c$

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(5) $\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$P(x) = 2 \quad Q(x) = 2x/e = 2xe^{-2x}$

$(IF) = e^{\int P(x)dx}$

$= e^{\int 2dx} = e^{2x}$

$y(IF) = \int Q(x)\,(IF)\,dx + c$

$= \int 2x\, e^{-2x}\, e^{2x} + c$

$y e^{2x} = \int 2x + c = x^2 + c$

(6) $\sec^2 x \cdot \tan y\, dx + \sec^2 y \tan x\, dy = 0$

$\frac{\sec^2 x}{\tan x} dx = \frac{-\sec^2 x}{\tan y} dy$

$\int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 x}{\tan y} dy$

$\log|\tan x| = -\log|\tan y| + c$

$\log|\tan x + \tan y| = c$

$\tan x + \tan y = e^c$

(7) $\frac{dy}{dx} = \sin^2(x-y+1)$

put $x-y+1 = v$

$1 - dy/dx = dv/dx \Rightarrow \frac{1-dv}{dx} = dy/dx$

$\frac{dv}{dx} = 1 - \sin^2 v \qquad dv/dx = \cos^2 x$

$\frac{dv}{\cos^2 x} = dx \qquad \int \sec^2 x\, dv = \int dx$

$\tan v = x + c$

$\tan(x-y+1) = x + c$

(8) $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

put $2x+3y = U$

$2 + \frac{3dy}{dx} = \frac{du}{dx}$

$\frac{dy}{dx} = 1/3\left(\frac{du}{dx} - 2\right) = 1/3\left(\frac{dv}{dx} - 2\right) = 1/3\left(\frac{v-1}{v+2}\right)$

$\frac{dv}{dx} = \frac{v-1}{v+2} + 2 = \frac{dv}{dx} = \frac{v-1+2v+v}{v+2}$

$= \frac{3v+3}{v+2} = \frac{3(v+1)}{(v+2)}$

$= \int \frac{v+2}{v+1} dv = 3dx$

$= \int \frac{v+1}{v} dx + \int \frac{1}{v+1} dv = \int 3dx$

$v \log|v| = 3x+c$

$2x + 3y + \log|2x+3y+1| = 3x+c$

$3y = x - \log|2x+3y+1| + c$

EDO

: Practical 8 :

Euler's Method.

① $\frac{dy}{dx} = y + e^{x} - 2$ $y(0) = 2$ $h = 0.5$ find $y(2) = ?$

Soln $f(x) = y + e^{x} - 2$ $x_0 = 0$ $y(0) = 2$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205	8.2021	9.8215
4	2	9.8215		

$\therefore y(2) = 9.8215$

② $\frac{dy}{dx} = 1 + y^2$, $y(0) = 1$, $h = 0.2$ find $y(1) = ?$

Soln $x_0 = 0$; $y_0 = 0$; $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.6412
3	0.6	0.6412	1.4111	0.9284
4	0.8	0.9284	1.8526	1.2939
5	1	1.2939		

$y(1) = 1.2939$

③ $\frac{dy}{dx} = \sqrt{x/y}$ $y(0) = 1$ $h = 0.2$ find $y(1) = ?$

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n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0.0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6059	1.2105
3	0.6	1.2105	0.7040	1.3513
4	0.8	1.3513	0.7696	1.5051
5	2	1.5051		

$\therefore y(1) = \underline{1.5051}$

④

(i) $\frac{dy}{dx} = 3x^2 + 1$ $y(1) = 2$ find $y(2)$ $h = 0.5$, $h = 0.25$

$y_0 = 2$ $x_0 = 1$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	7.475	7.875
2	2	7.895		

$\underline{y(2) = 7.875}$

(ii) $y_0 = 2$ $x_0 = 1$ $h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.5875	4.4218
2	1.5	4.4218	19.6569	19.3360
3	1.75	19.3360	1122.6426	299.9960
4	2	299.9960		

$y(2) = \underline{299.9960}$

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IPD

(5) $\frac{dy}{dx} = \sqrt{xy} + 2 \quad y(1) = 1 \quad h = 0.2$

$x_0 = 1 \quad y_0 = 1 \quad h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$y(1.2) = 3.6$

20/01/2020

: Practical No. 9 :

Limits & Partial Order

Questions :

① (i) $\lim_{(x,y)\to(-4,-1)} \dfrac{x^3 - 3y + y^2 - 1}{xy + 5}$ (Evaluate)

(ii) $\lim_{(x,y)\to(2,0)} \dfrac{(x+1)(x^2 + y^2 - 4x)}{x + 3y}$

(iii) $\lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2 - y^2 * z^2}{x^3 - x^2yz}$

② Find f_x, f_y for each of the following :

(i) $f(x,y) = xy\, e^{x^2+y^2}$

(ii) $f(x,y) = e^x \cos y$.

③ Using defination find values of f_x, f_y at $(0,0)$

$$f(x,y) = \frac{2x}{1+y^2}$$

④ Find all second order partial derivative of f. Also verify whether $f_{xy} = f_{yx}$

(i) $f(x,y) = y^2 - \dfrac{xy}{x^2}$

(ii) $f(x,y) = x^3 + 3x^2y^2 = \log(x^2+1)$

(iii) $f(x,y) = \sin(xy) + e^{xy}$

⑤ Find linearization of $f(x,y)$

(i) $f(x,y) = \sqrt{x^2 + y^2}$ at $(1,1)$

(ii) $f(x,y) = 1 - x + y\sin x$ at $(\pi/2, 0)$

(iii) $f(x,y) = \log x + \log y$ at $(1,1)$

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FAO

Answer:

① ① $x = -4$
$y = -1$

$$\frac{x^3 - 3y + y^2 - 1}{xy + 5}$$

$$\frac{(-4)^3 - 3(-1) + (-1)^2 - 1}{(-4)(-1) + 5}$$

$$\frac{-64 + 3 + 1 - 1}{4 + 5} = -\frac{61}{9}$$

② $x = 2$
$y = 0$

$$\frac{(x+1)(x^2 + y^2 - 4x)}{x + 3y}$$

$$= \frac{(2+1)((2)^2 + (0)^2 - 4(2))}{2 + 3(0)}$$

$$= \frac{(3)(4 + 0 - 8)}{2 + 0} = \frac{(3)(-4)}{2} = \frac{-12}{2} = -6$$

③ $x = 1$
$y = 1$
$z = 1$

$$\frac{x^2 - y^2 z^2}{x^3 - x^2 y z} = \frac{(1)^3 - (1)^2 (1)^2}{(1)^3 - (1)^2 (1)(1)} = \frac{1-1}{1-1}$$

②

① $f(x,y) = xye^{x^2+y^2}$

$$\therefore f_x = \frac{\partial}{\partial x}(f(x,y))$$

$$= \frac{\partial}{\partial x}(xye^{x^2+y^2})$$

$$= ye^{x^2+y^2}(2x)$$

$\therefore f_x = 2xye^{x^2+y^2}$

$f_y = \partial/\partial y\,(f(x,y))$

$= \frac{\partial}{\partial y}\,(xye^{x^2+y^2})$

$= xe^{x^2+y^2}(2y)$

$\therefore f_y = 2y\,xe^{x^2+y^2}$

(ii) $f(x,y) = e^x \cos y$

$f_x = \partial/\partial x\,(f(x,y))$

$= \partial/\partial x\,(e^x \cos y)$

$\therefore f_x = e^x \cos y$

$f_y = \partial/\partial y\,(f(x,y))$

$= \partial/\partial y\,(e^x \cos y)$

$f_y = -e^x \sin y$

(iii) $f(x,y) = x^3 + y^2 - 3x^2 + y^3 + 1$

$f(x) = \partial/\partial x\,(f(x,y))$

$= \partial/\partial x\,(x^3 y^2 - 3x^2 y + y^3 + 1)$

$f_x = 3x^2 y^2 - 6xy$

$f_y = \partial/\partial y\,(f(x,y)) = \partial/\partial y\,(x^3 y^2 - 3x^2 y + y^3 + 1)$

$f_y = 2x^3 y - 3x^2 + 3y^2$

$= -6$

$\frac{1-1}{1-1} = 0$

③ SAO

(i) $f(x,y) = 2x/1+y^2$

$f_x = \partial/\partial x \left(\frac{2x}{1+y^2}\right)$

$= \frac{1+y^2 \, \partial/\partial x (2x) - 2x \, \partial/\partial x (1+y^2)}{(1+y^2)^2}$

$= \frac{2+2y^2 - 0}{(1+y^2)^2} = \frac{2}{1+y^2}$

At $(0,0)$ $= \frac{2}{1} = 2$

$f_y = \frac{\partial}{\partial y}\left(\frac{2x}{1+y^2}\right) = \frac{1+y^2 \, \partial/\partial x (2x) - 2x \, \partial/\partial x (1+y^2)}{(1+y^2)^2}$

$= \frac{1+y^2 (0) - 2x(2y)}{(1+y^2)^2}$

$= \frac{-4xy}{(1+y^2)^2}$

At $(0,0)$ $= \frac{-4(0)(0)}{(1+0)^2} = 0$

(4) $f(x,y) = \frac{y^2 - xy}{x^2}$

(1) $\therefore f_x = x^2 \, \partial/\partial x \, (y^2 - xy) - (y^2 - xy) \frac{d}{dx}(x^2)$

$= \frac{x^2 (-y) - (y^2 - xy)(2x)}{x^4}$

$= \frac{-x^2 y - 2x(y^2 - xy)}{x^4}$

$f_y = \frac{2y - x}{x^2}$

$f_{xx} = \frac{d}{dx}\left(\frac{-x^2 y - 2x(xy^2 - xy)}{x^4}\right)$

$= \frac{x^4 \left(d/dx\left(-x^2 y - 2xy^2 + 2x^2 y\right)\right) - \left(-x^2 y - 2xy + 2x^2 y\right) d/dx(x^4)}{(x^4)^2}$

$= \frac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^2 y - 2xy + 2x^2 y)}{x^6}$

$f_{yy} = \frac{d}{dy}\left(\frac{2y - x}{x^2}\right) = \frac{2 - 0}{x^2} = \frac{2}{x^2}$

$f_{xy} = \frac{d}{dx}\left(\frac{-x^2 y - 2xy^2 + 2x^2 y}{x^4}\right)$

$f_{yx} = \frac{d}{dx}\left(\frac{2y - x}{x^2}\right)$

$= \frac{x^2 \frac{d}{dx}(2y - x) - (2y - x)\frac{d}{dx}(x^2)}{(x^2)^2}$

$= \frac{-x^2 - 4xy + 2x^2}{x^4}$ $\quad \therefore$ From (3) & (4)

$\therefore f_{xy} = f_{yx}$

(2) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$f_x = \frac{d}{dx}\left(x^3 + 3x^2y^2 - \log(x^2+1)\right)$

$= 3x^2 + 6xy^2 - 2x/x^2+1$

$f_y = \frac{d}{dy}\left(x^3 + 3x^2y^2 - \log(x^2+1)\right)$

$= 0 + 6x^2y - 0 = 6x^2y$

$f_{xx} = 6x + 6y^2 - \left(x^2 + 1\,\frac{d(2x)}{dx} - 2x\,\frac{d(x^2+1)}{dx}\right)$

$(x^2+1)^2$

$= 6x + 6y^2 - \left(\frac{2(x^2+1) - 4x^2}{(x^2+1)^2}\right)$ — (1)

$f_{xy} = \frac{d}{dy}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$ — (2)

$= 0 + 12xy + 0 = 12xy$

$f_{yx} = \frac{d}{dx}\left(6x^2y\right) = 12xy$

$\therefore f_{xy} = f_{yx}$.

③ $f(x,y) = \sin(xy) + e^{x+y}$

$\rightarrow f_x = y\cos(xy) + e^{x+y}(1)$

$= y\cos(xy) + e^{x+y}$

$f_{xx} = \frac{d}{dx}(y\cos(xy) + e^{x+y})$

$= -y\sin(xy)\cdot(y) + e^{x+y}(1)$

$= -y^2\sin(xy) + e^{x+y}$ — ①

$f_y = x\cos(xy) + e^{x+y}(1)$

$= x\cos(xy) + e^{x+y}$

$f_{yy} = \frac{d}{dy}(x\cos(xy) + e^{x+y})$

$= -x\sin(xy)(x) + e^{x+y}(1)$

$= -x^2\sin(xy) + e^{x+y}$ — ②

$f_{xy} = \frac{d}{dy}(y\cos(xy) + e^{x+y})$

$= -y^2\sin(xy) + \cos(xy) + e^{x+y}$ — ③

$f_{yx} = \frac{d}{dx}(x\cos(xy) + e^{x+y})$

$= -x^2\sin(xy) + \cos(xy) + e^{x+y}$ — ④

$\therefore$ from ③ & ④

$f_{xy} \neq f_{yx}$.

Q6

① $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$f(1,1) = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$

$= \frac{x}{\sqrt{x^2+y^2}}$

$f(x)$ at $(1,1)$ $= \frac{1}{\sqrt{2}}$

$f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$

$= \frac{y}{\sqrt{x^2+y^2}}$

$f(y)$ at $(1,1)$ $= \frac{1}{\sqrt{2}}$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x+y-2)$

$= \frac{x+y}{\sqrt{2}}$

(2) $f(x,y) = 1 - x + y\sin x$ at $(\pi/2, 0)$

$f(\frac{\pi}{2}, 0) = 1 - \frac{\pi}{2} + 0 = 1 - \pi/2$

$f_x = 0 - 1 + y\cos x$ $\qquad$ $f_y = 0 - 0 + \sin x$

f_x at $(\frac{\pi}{2}, 0) = -1 + 0 = -1$ $\qquad$ f_y at $(\frac{\pi}{2}, 0) = \sin\frac{\pi}{2} = 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 1 - \frac{\pi}{2} + (-1)(x - \frac{\pi}{2}) + 1(y-0)$

$= 1 - \frac{\pi}{2} - x + \frac{\pi}{2} + y$

$= 1 - x + y$

(3) $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1) = 0$

$f_x = \frac{1}{x} + 0$ $\qquad$ $f_y = 0 + 1/y$

f_x at $(1,1) = 1$ $\qquad$ f_y at $(1,1) = 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= x + y - 2$

CAO : Practical 10 :

: Directional Derivative, Gradient Vector, & Maxima, Minima, Tangent & Normal Curve :

Q1. Find directional derivative of the foll func at given points & in the direction of given vector:

(1) $f(x,y) = x+2y-3$, $a = (1,-1)$, $u = 3i - j$

(2) $f(x,y) = y^2 - 4x + 1$, $a = (3,4)$, $u = i + 5j$

(3) $f(x,y) = 2x + 3y$, $a = (1,2)$, $u = 3i + 4j$

Q2. Find gradient vector for the following fn at given point

(i) $f(x,y) = x^y + y^x$, $a = (1,1)$

(ii) $f(x,y) = (\tan^{-1} x) \cdot y^2$ $a = (1,-1)$

(iii) $f(x,y,z) = xyz - e^{xyz}$, $a = (1,-1,0)$

Q3. Find the eqn of the tangent & normal to each of the following curves at given points:

(i) $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

(ii) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2,-2)$

Q4. Find the equation of tangent & normal to each of the following curves at given point:

(i) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

(ii) $3xyz - x - y + z = -4$ at $(1,-1,2)$

Q5. Find the local maxima & minima for the foll fn:

(i) $f(x,y) = 8x^2 + y^2 - 8xy + 6x - 4y$

(ii) $f(x,y) = 2x^4 + 3x^2 y - y^2$

(iii) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

Answers

Q1 (1) Here,

$|\bar{u}|$

Unit

$f(a)$

$f(a)$

(2) Here

$|\bar{u}|$

Unit

$f(a) =$

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Answers:

Q1 ① Here, $u = 3i - j$ is not a unit vector.

$|\bar{u}| = \sqrt{(3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = (1) + 2(-1) - 3 = -4$

$f\left(1 + \frac{3}{\sqrt{10}}\right), \left(-1 - \frac{h}{\sqrt{10}}\right)$

$f(a+hu) = \left(1 + \frac{3}{\sqrt{10}}\right) + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3$

$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$ $\qquad D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h \to 0} \frac{-4 + h/\sqrt{10} + 4}{h}$ $\qquad D_u f(a) = \frac{1}{\sqrt{10}}$

② Here $u = i + 5j$ is not a unit vector

$|\bar{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1,5)$

$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$

$f(a+hu) = f(3,4) + h\left(1/\sqrt{26},\ 5/\sqrt{26}\right)$

$= f\left(3 + h/\sqrt{26},\ 4 + 5h/\sqrt{26}\right)$

$f_{x+y}(a+hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$

$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$

$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

$D_u f(a) = \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{h}$

$\frac{\cancel{h}\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{\cancel{h}}$ $\quad \therefore D_u f(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$

(ii) $\because 2x + 8y \quad a = (1,2) \quad u = (3i + 4j)$

Here $u = 3i + 4j$ is not unit vector

$|\bar{u}| = \sqrt{(3)^2 - (4)^2} = \sqrt{25} = 5$

Unit vector along u is $\frac{u}{|u|} = 1/5\ (3,4)$

$= (3/5,\ 4/5)$

$f(a) = f(1,2) = 2(1) + 3(2) = 8$

$f(a+hu) = f(1,2) + h\,(3/5,\ 4/5)$

$= f\left(1 + 3h/5,\ 2 + 4/5\right)$

$f(a+hu) = 2\left(1+\frac{3h}{5}\right)+3\left(2+\frac{4h}{5}\right)$

$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5} = \frac{18h}{5} + 8$

$D_u f(a) = \lim_{h\to 0} \frac{18h/5 + 8 - 8}{h} = \frac{18}{5}$

Q2.

(i) $f_x = y \cdot x^{y-1} + y^x \log y$

$f_y = x^y \log x + xy^{x-1}$

$\nabla f(x,y) = (f_x, f_y)$

$= (yx^{y-1} + y^2 \log y,\ x^y \log x + xy^{x-1})$

$f(1,1) = (1+0,\ 1+0) = (1,1)$

(ii) $f(x,y) = (\tan^{-1}x)\cdot y^2 \quad a = (1,-1)$

$f_x = \frac{1}{1+x^2}\cdot y^2$

$f_y = 2y \tan^{-1}x$

$\nabla f(x,y) = (f_x, f_y) = \left(y^2/1+x^2,\ 2y\tan^{-1}x\right)$

$f(1,-1) = \left(1/2,\ \tan^{-1}(1)(-2)\right)$

$= \left(1/2,\ \frac{\pi(-2)}{4}\right) = \left(\frac{1}{2}, \frac{-\pi}{2}\right)$

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(iii) $f(x,y,z) = xyz - e^{x+y+z}$, $a = (1,-1,0)$

$f(x) = yz - e^{x+y+z}$

$f(y) = xz - e^{x+y+z}$

$f(z) = xy - e^{x+y+z}$

$\nabla f(x,y,z) = f_x , f_y , f_z$

$= yz - e^{x+y+z} , xz - e^{x+y+z} , xy - e^{x+y+z}$

$f(1,-1,0) = ((-1)(0) - e^{1+(-1)+0} , (1)(0) - e^{1+(-1)+0} ,$

$= (0 - e^0 , 0 - e^0 , -1 - e^0)$

$= (-1, -1, -2$

Q8.

(1) $f_x = \cos y\, 2x + e^{xy}\, y$

$f_y = x^2(-\sin y) + e^{xy} . x$

$(x_0, y_0) = (1,0)$ $\therefore$ $x_0 = 1$, $y_0 = 0$

eqn of tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$f_x(x_0, y_0) = \cos 0\, 2(1) + e^0\, 0$

$= 1(2) + 0$

$f_y(x_0, y_0) = (1)^2 = 2$

$(-\sin 0) + e^0 \cdot 1$

$= 0 + 1 \cdot 1$

$= 1$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$; Reqd. eqn by tangent.

eqn of Normal

$= ax + by + c = 0$

$= bx + ay + d = 0$

$1(1) + 2(y) + d = 0$

$1 + 2y + d = 0$ at $(1, 0)$

$= 1 + 2(0) + d = 0$ $\therefore d + 1 = 0$ $\therefore d = -1$

(ii) $x^2 + y^2 = 2x + 3y + 2 = 0$ at $(2, -2)$

$f_x = 2x + 0 - 2 + 0 + 0 = 2x - 2$

$f_y = 0 + 2y - 0 + 3 + 0 = 2y + 3$

$(x_0, y_0) = (2, -2)$ $\therefore x_0 = 2, y_0 = -2$

$f_x(x_0, y_0) = 2(2) - 2 = 2$

$f_y(x_0, y_0) = 2(-2) + 3 = -1$

eqn of tangent :

$f_x(x - x_0) + f_y(y - y_0) = 0$

$\therefore 2(x - 2) + (-1(y + 2)) = 0$

$2x - 2 - y - 2 = 0$

$\therefore 2x - y - 4 = 0$ $\therefore$ Reqd eqn of tangent.

Eqn of Normal

$= ax + by + c = 0$

$bx + ay + d = 0$

$= -1(x) + 2(y) + d = 0$

$-x + 2y + d = 0$ at $(2, -2)$

$-2 + 2(-2) + d = 0$

$-2 - 4 + d = 0$

$\therefore d = 6$

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Qu

① $f_x = 2x - 0 + 0 + z$

$f_x = 2x + z$

$f_y = 0 - 2z + 3 + 0$

$= 2z + 3$

$f_z = 0 - 2y + 0 + x$

$= -2y + x$

$(x_0, y_0, z_0) = (2, 1, 0) \therefore x_0 = 2, y_0 = 1, z_0 = 0$

$f_x(x_0, y_0, z_0) = 2(2) + 0 = 4$

$f_y(x_0, y_0, z_0) = 2(0) + 3 = 3$

$f_z(x_0, y_0, z_0) = -2(1) + 2 = 0$

$\therefore$ Eqn of TANGENT:

$f_x(x_0 - x_0) + f_y(y_0 - y_0) + f_z(z_0 - z_0) = 0$

$= 4(x-2) + 3(y-1) + 0(z-0) = 0$

$= 4x - 8 + 3y - 3 = 0$

$\therefore 4x + 3y - 11 = 0$ → Reqd tangent.

Eqn of Normal at $(4, 3, -11)$

$\frac{(x - x_0)}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$

$= \frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}$

(11) $3xyz - x - y + z = -4$

$f_x = 3yz - 1 - 0 + 0 + 0$
$= 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0$
$= 3xz - 1$

$f_z = 3yx - 0 - 0 + 1 + 0$
$= 3xy + 1$

$(x_0, y_0, z_0) = (1, -1, 2)$ $\therefore x_0 = 1,\ y_0 = -1,\ z_0 = 2$

$f_x(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$
$f_y(\ \text{"}\ ) = 3(+1)(2) - 1 = 5$
$f_z(\ \text{"}\ ) = 3(1)(-1) + 1 = -2$

Eqn of tangent

$-7(x-1) + 5(y+1) - 2(z-2) = 0$
$-7x + 7 + 5y + 5 - 2z + 4 = 0$
$-7x + 5y - 2z + 16 = 0$ $\therefore$ Reqd by tangent.

Eqn of Normal at $(-7, 5, -2)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

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Q:

(i) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$

$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4$

$= 2y - 3x - 4$

$f_x = 0$

$6x - 3y + 6 = 0$

$3(2x - y + 2) = 0$

$2x - y = -2 \quad -(1)$

$f_y = 0 \qquad 2y - 3x - 4 = 0$

$2y - 3x = 4 \quad -(2)$

Multiply eqn (1) with (2)

$4x - 2y = -4$

$x = 0$

Substitute value of x in eqn (1)

$2(0) - y = -2$

$-y = -2 \qquad \therefore y = 2$

$\therefore$ Critical points are $(0, 2)$

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

$\therefore A > 0$

$: rt - s^2$

$= 6(2) - (-3)^2$

$= 12 - 9$

$= 3 > 0$

$\therefore f$ has maximum at $(0,2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0,2)$

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$= 0 + 4 - 0 + 0 - 8 = -4$

(ii) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$ $\quad \therefore 8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$\therefore 4x^2 + 3y = 0$ — (1)

$f_y = 0$ $\quad 3x^2 - 2y = 0$ → (2)

Multiply eqn (1) with 3

(2) with (4)

$12x^2 + 9y = 0$

$-12x^2 - 8y = 0$

$\therefore y = 0$

Substituting $4x^2 + 3(0) = 0$

$\therefore x = 0$ $\quad \therefore$ Critical Point is $(0,0)$

$r: f_{xx} = 24x^2 + 6x$

$t: f_{yy} = 0 - 2 = -2$

$s: f_{xy} = 6x - 0 = 6(0) = 0$ $\quad \therefore r$ at $(0,0)$

$= 24(0) + 6(0) = 0$

$\therefore r = 0$

$rt - s^2 = 0(-2) - (0)^2 = 0$

Inconclusive

(ii) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x+2$

$f_y = -2y + 8$

$f_x = 0 \quad \therefore 2x+2 = 0$

$\therefore x = -1$

$f_y = 0 \quad \therefore -2y+8 = 0$

$\therefore y = 4$

$\therefore$ Critical Point is $(-1, 4)$

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0 \quad \therefore rt > 0$

$\therefore rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0 \quad \therefore -4 < 0$

$f(x,y)$ at $(-1, 4)$

$= (-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$= 1 + 16 - 2 + 32 - 70$

$= 17 + 30 - 70$

$= 37 - 70 = 33$

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