Fiscal Multipliers in Small Open Economies With Heterogeneous Households*

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Abstract

We study fiscal multipliers in a small open economy Heterogeneous Agent New Keynesian (SOE-HANK) model. We provide a set of equivalence results under which the fiscal multiplier in our SOE-HANK model is the same—at any horizon—as in a corresponding representative-agent (RANK) model. Under more general assumptions, the fiscal multipliers in the two models are not equivalent, but remain relatively similar. Yet, we show that the underlying channels driving the fiscal multipliers differ substantially. In particular, consumption increases while net exports tend to decline in the HANK model, whereas the opposite is true in the RANK model.

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1 Introduction

In this paper, we study fiscal multipliers in the context of a small open economy model with nominal rigidities, incomplete financial markets, and heterogeneous households, i.e. a SOE-HANK model. Previous literature has established that, in a closed-economy setting, fiscal multipliers are in general substantially higher when financial markets are incomplete and a fraction of households have limited access to credit markets, as compared to the case of a standard complete-markets model with a representative agent (RANK); see, e.g., Galí et al. (2007), Hagedorn et al. (2019), Bilbiie (2020), or Auclert et al. (2023). In a nutshell, the presence of a considerable fraction of households with (realistically) high marginal propensities to consume (MPCs) generates a "Keynesian multiplier effect" on aggregate demand. In an open economy, there are two reasons why this logic may not hold up, or at least have a smaller quantitative impact: *First*, the Keynesian multiplier exerts a weaker effect on aggregate domestic demand, since a portion of the additional income will be spent on imported goods. *Second*, to the extent that aggregate domestic demand increases, this will tend to appreciate the real exchange rate, thereby reducing net exports. On the other hand, any appreciation raises the real purchasing power of domestic households and thus works in the opposite direction (the "real income channel" discussed by Auclert et al., 2021b). Against this backdrop, our aim is to compare fiscal multipliers in the SOE-HANK model to those obtained in a corresponding RANK model.

We start by decomposing the output response into six distinct channels in stylized versions of the models. The first four of these channels are also present in a closed-economy setting, the last two only operate in an open economy. The output response to a change in government spending is composed of (i) the direct effect of the spending increase, (ii) the effect from higher tax burdens on spending, (iii) intertemporal substitution effects via interest rate changes, (iv) a Keynesian multiplier effect, (v) expenditure switching effects from changes in the real exchange rate, and (vi) real income effects from changes in the real exchange rate. The relative magnitudes of these is not unambiguous. The drag from taxes tends to be larger in HANK, the drag from intertemporal substitution larger in RANK; Keynesian multiplier and real income effects tends to boost the multiplier in HANK compared to RANK, while the relative effect of expenditure switching is unclear a priori.

Guided by the decomposition, we then show that, under some restrictions on parameters, the multipliers in the two frameworks are equivalent.

First, we show that, in the limiting case of complete openness, the fiscal multiplier in

HANK and RANK is identical. Complete openness implies that the home consumption basket contains only foreign goods. As a result, household income does not affect spending on domestically produced goods, so the Keynesian multiplier is shut down, and the financing and intertemporal substitution channels have no effect on domestic output. If the real interest rate and thus the real exchange rate stay constant, output rises by exactly the size of the fiscal stimulus. If the real interest rises to stabilize output, then the real exchange rate appreciates and expenditure switching exactly crowds out the fiscal stimulus (reminiscent of the standard Mundell–Fleming result).

Second, a comparable but slightly less general equivalence result obtains when we consider the degree of substitutability between foreign and domestic goods. In the limiting case of infinite substitutability between home and domestic goods, we can show that fiscal multipliers coincide in HANK and RANK, and equal zero provided monetary policy is active. Fiscal spending is fully crowded out in this case via taxes in the HANK model and expenditure switching in both as the real rate rises and the real exchange rate appreciates.

Third, if the elasticity is unity and the fiscal stimulus is financed with a balanced-budget increase in taxes, we can show that the multipliers in both models are identical at all horizons and non-negative. In this case, income and substitution effects from real exchange rate changes offset each other so net exports are unchanged, and the economy behaves almost as a closed economy.

Numerically, we then show that, in the stylized model, for every value of the trade elasticity there is a degree of financing that equalizes the multipliers across models. The multiplier in HANK is larger compared to RANK the less of the stimulus is financed with taxes and the smaller the trade elasticity. A lower trade elasticity makes expenditure switching weaker which raises multipliers in both models. It does so more in HANK than RANK because of the additional Keynesian multiplier effect.

Overall, our results highlight how in an open economy the effectiveness of fiscal spending depends not just on MPCs, but importantly on openness, trade elasticities, and their interaction with financing of the stimulus. It is not clear, in particular, that an open economy HANK framework features higher multipliers than the corresponding RANK model: If enough of the stimulus is financed with taxes, or if preferences are such that a substantial part of the stimulus is spent abroad, then these effects can dominate higher marginal propensities to consume and lead to smaller multipliers in HANK compared to RANK.

We then proceed to compare fiscal multipliers in quantitative versions of the models. The stylized HANK model was parameterized to replicate empirically observed MPCs since that is central to our question of fiscal multipliers. In the quantitative model, we also incorporate a standard monetary policy reaction function, debt stationarity, and introduce a number of standard features that allow us to match additional relevant moments (debt levels, wealth levels and markups, time-varying trade elasticities).

In these quantitative models, we show that multipliers across HANK and RANK are similar. The impact multiplier is 1.1 in the HANK model and 0.91 in the RANK model. The present value multipliers are smaller in our setting since output displays comparatively little persistence.

Even though multipliers are of a similar magnitude, the underlying dynamics are quite different across the two models. In the HANK model, a fiscal expansion is accompanied by a consumption boom which is partly offset by a drop in net exports. The overall result is a multiplier that is comparable to the RANK model, where consumption falls and net exports rise.

These results differ markedly from the closed-economy literature. For example, according to Auclert et al. (2023), RANK multipliers are typically below 0.5, whereas the HANK multiplier tends to be well above 1, depending on assumptions about the financing of the increase in government spending. We confirm that the similarity of HANK and RANK multipliers is robust to a range of modifications of our baseline model environment, including the introduction of capital, various assumptions about financing, a fixed nominal exchange rate, and alternative paths of the real exchange rate.

1.1 Related Literature

Our paper contributes to a large literature on fiscal multipliers, see Ramey (2019) for a recent survey. In a closed economy, Keynesian theories predict large output effects from increased government spending due to Keynesian multiplier effects. Neoclassical models on the other hand emphasize downward pressure on private consumption due to negative wealth effects from higher (future) taxation, and thus tend to predict small multipliers, as for example in Baxter and King (1993). New Keynesian models with representative agents can generate sizeable fiscal multipliers, with the magnitude depending on the source and extent of nominal rigidities, as well as the response of monetary policy, see for example Woodford (2011). In open economies, Erceg and Linde (2012), Corsetti et al. (2013), and Nakamura and Steinsson (2014) show that fiscal multipliers are generally low in flexible exchange-rate open economies, which is consistent with the traditional Mundell–Fleming view that fiscal policy is more effective in fixed exchange-rate regimes.

More recently, studies of fiscal spending have turned to the empirically realistic case of heterogeneous agents. In a closed-economy setting, fiscal multipliers are found to be substantially higher under incomplete financial markets, as compared to the case of a standard complete-markets model with a representative agent; see, e.g., Galí et al. (2007), Bilbiie (2020), Hagedorn et al. (2019) or Auclert et al. (2023).[1]

A number of recent papers have extended the HANK framework to an international setting, including De Ferra et al. (2020), Auclert et al. (2021b), Bellifemine et al. (2023), Guo et al. (2023), Oskolkov (2023), and Bayer et al. (2024). In our own previous work, we contribute to—and offer a short survey of—this emerging strand of the literature; see Druedahl et al. (2022). To the best of our knowledge, fiscal multipliers have not previously been systematically studied in this framework.

Our contribution in this paper is to quantify the relationship between fiscal multipliers in open-economy HANK versus open-economy RANK models. In closed economies, the literature has shown that allowing for heterogeneous agents magnifies estimates of fiscal multipliers; we ask whether this remains true when instead looking at an open economy. Our results indicate that multipliers tend to be similar in open economies across HANK and RANK frameworks, and can indeed be identical, but that the implied dynamics are quite different. In particular, in the HANK model multipliers work via a consumption boom and a trade deficit, whereas it is the reverse in the RANK setting.

These results are consistent with Farhi and Werning (2016) who study, among other things, fiscal policy in open economies with hand-to-mouth agents. They show that trade deficits emerge and that therefore fiscal stabilization is more effective when regions are less open to trade. Aggarwal et al. (2023) study fiscal transfers in a multicountry HANK setting and show likewise that these lead to persistent trade deficits.

Our results also speak to the empirical debate on the effects of fiscal policy on open economy variables such as the real exchange rate and the trade balance. Existing empirical studies have found evidence of an increase in private consumption along with a worsening of the trade balance in response to a fiscal expansion, in line with

1. In this respect, the study by Broer et al. (2023) represents a notable exception, as they present several cases in which the HANK and RANK multipliers coincide. Their model differs from most of the HANK literature, since only "capitalists"—who receive profits, but no labor income—display a high MPC, thus effectively shutting down the intertemporal Keynesian labor-income multiplier.

the implications of our HANK model, see, e.g., Corsetti and Müller (2006), Monacelli and Perotti (2010), and Ravn et al. (2012). Lambertini and Proebsting (2023) find that fiscal policy primarily affects imports rather than exports both in terms of prices and quantities, and that real exchange rate responses are driven by relative changes in the price of nontraded goods instead (a fiscal expansion raises the relative price of services, not exports). This is consistent with our theoretical results.

1.2 Structure

We start by presenting a stylized model in Section 2. We use this model to derive and discuss analytical results about fiscal multipliers in Section 3. In Section 4, we provide numerical results on fiscal multipliers in a larger model. Section 5 concludes.

2 A Stylized Model of Fiscal Multipliers

In this section, we study fiscal policy in a small open economy (SOE) with a general household problem. The core of the model is similar to the one considered by Auclert et al. (2021b)—which in turn is an incomplete-markets version of the canonical SOE model of Galí and Monacelli (2005)—extended with fiscal policy. We keep the model simple in order to highlight some special cases where the fiscal multipliers obtained in the HANK and RANK models coincide. Studying these cases helps us understand the channels at work in each of the two models.

2.1 Model Description

We consider the sequence-space representation of the model following Auclert et al. (2021a). In this framework, the key objects are the infinite-dimensional vectors of perturbations of variables from steady state, $d\boldsymbol{X} = (dX_0, dX_1, \dots)'$, where $dX_t \approx X_t - X_{ss}$ is the deviation from steady state for any variable X. We restrict our attention to perfect-foresight shocks.

2.1.1 Households

We consider two household structures. The first case is a heterogeneous-agents (HA) model featuring a continuum of households. A household with assets a_{t-1} and idiosyncratic earnings e_t chooses consumption c_t and end-of-period assets a_t to solve

$$
\begin{aligned}
V_t(a_{t-1}, e_t) = \max_{c_t, a_t} \log c_t + \beta \mathbb{E}_t \left[V_{t+1}(a_t, e_{t+1}) \right], \\
\text{s.t.} \\
c_t + a_t = (1 + r_t^a) a_{t-1} + Z_t e_t, \\
a_t \geq 0, \\
\ln e_t = \rho_e \ln e_{t-1} + \epsilon_t^e, \quad \epsilon_t^e \sim \mathcal{N}\left(0, \sigma_e^2\right),
\end{aligned}
$$

where r_t^a denotes real asset returns. $Z_t = (1 - \tau_t) w_t N_t$ is real labor income, where w_t is the real wage rate, N_t is labor supply, and τ_t is the tax rate. Idiosyncratic income, e_t, follows an AR(1) process in logs with i.i.d. normal innovations. The household has access to three types of assets: domestic government bonds (which pay the real interest rate r_t), foreign bonds (which pay a fixed real rate r^*), and domestic equity (which pays real after-tax dividends D_t). With perfect international capital mobility, this gives rise to the following two no-arbitrage conditions:

$$1 + r_t = \frac{D_{t+1} + p_{t+1}^D}{p_t^D}, \tag{1}$$

$$1 + r_t = (1 + r^*) \frac{Q_{t+1}}{Q_t}, \tag{2}$$

where $r_t = \mathbb{E}_t r_{t+1}^a$ is the ex-ante real return, and p_t^D is the price of firm equity.[2] $Q_t \equiv E_t \frac{P^*}{P_t}$ is the real exchange rate, with E_t denoting the nominal exchange rate (the price of foreign currency in terms of domestic currency), and where we normalize $P^* = 1$. It then follows that (1) equates the real return on domestic stocks and bonds, while (2) is a real UIP condition, stating that the expected real return on foreign and domestic assets must be equalized.

As an alternative to the HA structure, we also consider a standard representative-agent (RA) model, where the household acts according to a standard Euler equation for all $t = 0, 1, \ldots$:

$$C_t^{-1} = \beta (1 + r_{t+1}^a) C_{t+1}^{-1},$$

2. Along the perfect foresight transition path, the ex-post and ex-ante returns are equalized; $r_t^a = r_{t-1}$ for $t = 1, 2, \ldots$. Ex-post returns in period zero are given by $r_0^a = \frac{p_0^D + D_0}{p_{ss}^D}$, since we assume that both foreign and domestic bond holdings are zero in steady state, i.e., $B_{ss} = B_{ss}^* = 0$.

with $C_\infty = C_{ss}$. This version of the model is similar to the one considered by Galí and Monacelli (2005), with the exception that we do not allow for international risk sharing, whereas their setup features perfect risk sharing across countries.

For a given level of domestic consumption, C_t, domestic households split consumption between home and foreign goods with home bias $(1-\alpha) \in (0,1)$ as follows:

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t, \quad \text{and} \quad C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t, \tag{3}$$

where $C_{H,t}$ and $C_{F,t}$ denote domestic consumption of domestic and foreign goods, respectively, and $P_{H,t}$ and $P_{F,t}$ are the prices of these. η is the elasticity of substitution between domestic and foreign goods, and P_t is the consumer price index (CPI),

$$P_t = \left[(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{4}$$

2.1.2 Firms

Production is linear in labor, $Y_t = N_t$. The price of home goods in domestic currency is set as a markup over the nominal wage, $P_{H,t} = \mu W_t$, where $W_t = w_t P_t$. The price in foreign currency is then pinned down through the nominal exchange rate, E_t, and the law of one price:

$$P^*_{H,t} = \frac{P_{H,t}}{E_t}. \tag{5}$$

Real dividends net of taxes are:

$$D_t = (1-\tau_t)\frac{P_{H,t}Y_t - W_t N_t}{P_t}. \tag{6}$$

The source of nominal rigidities in our model is sticky wages, implying a standard new-Keynesian wage Phillips curve (NKWPC), following Auclert et al. (2021b):

$$\pi_{W,t} = \kappa\left(\frac{\psi N_t^{\varphi}}{(1-\tau_t)w_t U'_{c_t}/\mu} - 1\right) + \beta\pi_{W,t+1}, \tag{7}$$

where $\pi_{W,t} \equiv W_t/W_{t-1} - 1$ is nominal wage growth and U'_{c_t} denotes productivity-weighted aggregate marginal utility of consumption.[3] This is consistent with a micro-foundation where a union sets nominal wages to maximize average household welfare, given the same labor supply N_t for all households.

2.1.3 Government

The government has a standard budget constraint:

$$B_t = (1 + r_{t-1})B_{t-1} + \frac{P_{H,t}}{P_t} G_t - T_t, \tag{8}$$

where real tax receipts are given by:

$$T_t = \tau_t \left(w_t N_t + \frac{P_{H,t} Y_t - W_t N_t}{P_t} \right) = \tau_t \frac{P_{H,t}}{P_t} Y_t.$$

We assume that government consumption, G_t, displays a home bias of 1. Furthermore, G_t is assumed to be exogenous, following an AR(1) process with persistence ρ_G. The tax rate, τ_t, adjusts to ensure that tax receipts satisfy

$$T_t - T_{ss} = \phi_G (G_t - G_{ss}). \tag{9}$$

For analytical simplicity, we restrict our attention to a steady state characterized by a balanced government budget and no government debt; $B_{ss} = 0$. Additionally, the steady state we consider is in the vicinity of $r_{ss} = 0$, so that the steady-state version of (8) implies $G_{ss} = T_{ss}$. We proceed by setting both of these to zero, but slightly modified versions of the equivalence results presented below hold for any level of tax revenues and government spending satisfying $G_{ss} = T_{ss}$. We relax these assumptions when we turn to numerical analyses in Section 4. As for monetary policy, we assume that it is governed by the following real interest-rate rule:

$$r_t = r_{ss} + \phi_Y \left(\frac{Y_t}{Y_{ss}} - 1 \right), \tag{10}$$

with $\phi_Y \geq 0$. The absence of a monetary policy response to inflation simplifies the analytical solution of the model, as it implies that the Phillips curve (7) does not

3. In HANK this is given by $U'_{c_t} = \int e_t c_t (a_{t-1}, e_t) \, d\mathcal{D}_t (a_{t-1}, e_t)$ where $\mathcal{D}_t$ is the distribution of households over states. In RANK it is simply $U'_{c_t} = C_t^{-1}$.

enter in the determination of real variables (though it matters for nominal variables). However, given that the responses of output and inflation to a government spending shock usually have the same sign in our model, this choice is not crucial from a qualitative viewpoint. We consider a more general interest-rate rule in Section 4.

2.1.4 The Foreign Economy

The consumption of domestically produced goods by foreign consumers is given by:

$$C^*_{H,t} = \alpha \left(P^*_{H,t}\right)^{-\eta} C^*_{ss}, \tag{11}$$

with $C^*_{ss} = C_{ss}$ denoting the fixed level of foreign consumption. With $P^* = 1$, the law of one price implies that $E_t = P_{F,t}$.

2.1.5 Goods Market Clearing

Lastly, goods market clearing for tradeable goods is:

$$Y_t = C_{H,t} + C^*_{H,t} + G_t. \tag{12}$$

Defining net exports as $NX_t = \frac{P_{H,t}}{P_t} C^*_{H,t} - \frac{P_{F,t}}{P_t} C_{F,t}$, the above expression can also be expressed as $\frac{P_{H,t}}{P_t} Y_t = C_t + \frac{P_{H,t}}{P_t} G_t + NX_t$.

2.2 Equilibrium and Solution

We define the overall equilibrium of model as:

Definition 1 (Equilibrium)**.** *Given sequences for G_t and T_t, an initial household distribution over assets and earnings $\mathcal{D}_0(a, e)$, and an initial portfolio allocation between foreign and domestic assets, a competitive equilibrium in the domestic economy is a path of household policies $\{c_t(a_{t-1}, e_t), a_t(a_{t-1}, e_t)\}$, distributions $\mathcal{D}_t(a, e)$, prices:*

$$\left\{E_t, Q_t, P_t, P_{H,t}, P_{F,t}, W_t, p^D_t, r_t, r^a_t\right\},$$

and quantities:

$$\{C_t, C_{H,t}, C_{F,t}, Y_t, N_t, D_t, \tau_t, B_t\},$$

such that all households and firms optimize, monetary and fiscal policy follow their rules, and the goods market (12) *clears.*

In the numerical solution of the model we solve the households' problem using the endogenous grid method of Carroll (2006). We then rely on the "fake news algorithm" of Auclert et al. (2021a) to compute the Jacobian of the household problem around the deterministic steady state. The full non-linear transition paths are solved for using Broyden's method.[4]

3 The Fiscal Multiplier in HANK and RANK

Building on the model established above, we now characterize the fiscal multiplier under each of the two household structures: The HANK and the RANK model.

3.1 The Output Response to a Fiscal Policy Shock

Consider the equilibrium following a fiscal policy shock, i.e. paths of government consumption, $(G_t)_{t=0}^{\infty}$, and taxes, $(T_t)_{t=0}^{\infty}$.

Proposition 1. *An equilibrium of the model with household structure $j \in \{RA, HA\}$ following a fiscal policy shock satisfies*

$$\begin{aligned} d\boldsymbol{Y}^j = \; & \underbrace{d\boldsymbol{G}}_{\text{1. Gov. consumption}} - \underbrace{(1-\alpha)\boldsymbol{M}^j d\boldsymbol{T}}_{\text{2. Taxes}} + \underbrace{(1-\alpha)\boldsymbol{R}^j d\boldsymbol{r}^j}_{\text{3. Interest rate}} \\ & + \underbrace{(1-\alpha)\boldsymbol{M}^j d\boldsymbol{Y}^j}_{\text{4. Multiplier}} + \underbrace{\frac{2-\alpha}{1-\alpha}\alpha\eta d\boldsymbol{Q}^j}_{\text{5. Exp. switching}} - \underbrace{\alpha\boldsymbol{M}^j d\boldsymbol{Q}^j}_{\text{6. Real income}}, \end{aligned}$$

where $\boldsymbol{M}^j$ is the matrix of intertemporal marginal propensities to consume, and $\boldsymbol{R}^j$ is the matrix of intertemporal effects on consumption of real interest rate changes.

Proof. See Appendix A.1.

Proposition 1 decomposes the response of output across different models into six channels. Conceptually, the first four of these channels are also operative in a closed economy, while the last two only appear in an open economy. We now consider

4. The code is written in Python and based on the GEModelTools package.

each of the channels separately, and discuss how they differ across models. While Proposition 1 holds for any set of paths $(G_t, T_t)_{t=0}^{\infty}$ and any monetary policy rule, we now consider an increase in government consumption, $dG_t > 0$ partly financed by higher taxes, $dT_t \geq 0$, with monetary policy following the rule given by (10).

1. **Government consumption.** Higher government consumption increases output directly via goods market equilibrium. This channel is independent of household behavior, and thus equivalent in the two models.

2. **Taxes.** Higher taxes reduce spending. The strength of this channel depends on the MPC, which is governed by household behavior. In the RANK model the MPC is low, so this channel is weak. In the HANK model the MPC is higher, so the channel is potentially strong. In an open economy, this and the next two channels are scaled by the factor $(1-\alpha)$, reflecting that only a fraction $(1-\alpha)$ of income is spent at home, with the rest flowing abroad.

3. **Interest rate.** The real interest rate is likely to change in response to fiscal policy. If the real interest rate increases—which will typically be the case given the interest rate rule we have assumed—households increase savings and reduce current consumption (intertemporal substitution). This channel is likely to be stronger in the RANK model than in the HANK model (see Kaplan et al., 2018 or Druedahl et al., 2022).

4. **Multiplier.** Higher output means more labor income, which in turn implies higher consumption, and thus higher output. This is the intertemporal Keynesian multiplier (Auclert et al., 2023).

5. **Expenditure switching.** Given a positive response of the domestic real interest rate, the real exchange rate appreciates through the UIP (i.e., $dQ_t < 0$).[5] This makes domestic goods more expensive, inducing both domestic and foreign consumers to substitute away from them, reducing output. This channel depends on the magnitude of the appreciation, which can differ across models depending on household behavior, as well as the trade elasticity, η.

5. Solving the UIP condition (2) forward yields $Q_t = \prod_{k=0}^{\infty} \frac{1+r^*}{1+r_{t+k}}$, with terminal condition $Q_\infty = 1$.

6. **Real income.** As the real exchange rate appreciates, this stimulates the real purchasing power of domestic households, who then consume more, boosting output. How much spending is increased depends on the MPC, so this channel is stronger in the HANK model than in the RANK model, as argued by Auclert et al. (2021b).

In Table 1, we summarize the signs of the channels in the two models. This table is helpful in order to determine which of the two models displays the largest fiscal multiplier, and how this may be affected when going from a closed to an open economy. First, to the extent that after-tax labor income increases in response to a government spending shock (i.e., that the multiplier channel dominates the tax channel), the closed-economy fiscal multiplier in HANK exceeds the one in RANK—and more so as the real interest rate is increased in response to the shock, since this exerts a larger drag on economic activity in the RANK model. In an open economy, all of these effects are scaled down by the factor $(1 - \alpha)$, since the fraction α of the additional income is spent on imported goods. All else equal, this tends to reduce the gap between the HANK and RANK multipliers, as compared to a closed economy.

In addition, the open-economy multiplier is affected by the expenditure switching and real income channels. The former effect—which unambiguously reduces the multiplier relative to the closed-economy case—may be stronger or weaker in the HANK or the RANK model, as its magnitude depends on the size of the domestic boom: A larger boom induces a stronger monetary policy tightening and hence a larger exchange-rate appreciation, which makes households substitute away from domestic goods on a larger scale. In contrast, the real income channel—which works in favor of a higher fiscal multiplier—is stronger in the HANK than in the RANK model, as it is scaled by the MPC.

In summary, it is difficult to make general statements about the relative size of the open-economy fiscal multiplier in the HANK and the RANK model. We now proceed by considering some special cases in which the fiscal multipliers in the two models coincide exactly. Using these as starting points, we then consider the relationship between the multipliers in the two models as we depart from these special cases.

3.2 Equivalence and the Degree of Openness

We begin by considering the degree of openness, measured by α. We provide an equivalence result, according to which fiscal multipliers coincide in the HANK and RANK models in this case.

	RANK		HANK
1. Government consumption	$+$	$=$	$+$
2. Taxes	≈ 0	$>$	$-$
3. Interest rate	$-$	$<$	$-$
4. Multiplier	≈ 0	$<$	$+$
5. Expenditure switching	$-$	$\lesseqgtr$	$-$
6. Real income	≈ 0	$<$	$+$

Table 1: Signs of channels in the fiscal multiplier for the RANK and HANK models

Note: The table shows the signs of the contributions of each of the channels from Proposition 1 in the RANK and HANK models. The signs do not indicate whether the channel itself is stronger or weaker, but whether it contributes to a larger or a smaller fiscal multiplier. For example, the interest rate channel is stronger in the RANK model, but since it exerts a negative impact (in both models), it contributes to a larger fiscal multiplier in the HANK model.

Proposition 2. *Consider a government spending shock. It then holds that the entire path of output is identical in the RANK and HANK models, i.e.* $dY_t^{RA} = dY_t^{HA}, \forall t$, *when*

$$\alpha \to 1.$$

Since the entire path of output is the same, the fiscal multiplier is also the same. In particular,

$$d\boldsymbol{Y}^{RA} = d\boldsymbol{Y}^{HA} = \begin{cases} \boldsymbol{0} & \text{if } \phi_Y > 0 \\ d\boldsymbol{G} & \text{if } \phi_Y = 0 \end{cases}.$$

Proof. See Appendix A.2.

Proposition 2 establishes our first equivalence result regarding the fiscal multiplier in HANK and RANK. It highlights that, as the degree of openness tends to its upper bound of 1 (the opposite of home bias, as the domestic economy consumes only foreign goods), the various forces at play in the two models exactly cancel out. Since the entire path of output is identical in the two models, an implication is that our equivalence result extends to any measure of the fiscal multiplier, such as the peak multiplier (as studied by Blanchard and Perotti, 2002) or the present-value cumulative multiplier (as proposed by Ramey, 2016). The result is sufficiently general to not require any assumptions about the financing of the increase in government spending. While our equivalence result also holds for any value of the monetary policy response parameter, ϕ_Y, the *level* of the multiplier changes with this parameter.

The intuition behind the result is that when no domestically produced goods are consumed, the Keynesian multiplier—a defining feature of the HANK model—is shut down completely, since all of domestic households' spending goes abroad. If the central bank responds to the shock by raising the real interest rate ($\phi_Y > 0$), the resulting appreciation of the real exchange rate implies that the expenditure switching channel exactly offsets the increase in government spending, i.e., both models feature a fiscal multiplier of zero. If instead the central bank keeps the real interest rate fixed ($\phi_Y = 0$), the real exchange rate also remains constant, implying a multiplier of unity in both models (i.e., $d\boldsymbol{Y} = d\boldsymbol{G}$).

Figure 1 reports the decomposition of $d\boldsymbol{Y}$ proposed in Proposition 1 for the case of $\alpha \to 1$ studied in Proposition 2. These responses are based on a standard calibration of the remaining model parameters, as discussed in the next section.[6] The figure confirms that in this case, the increase in government spending is fully crowded out by net exports through the expenditure switching channel, while all other channels are muted, implying a multiplier of zero in both models.[7] We end this subsection by pointing out that while the limiting case of $\alpha \to 1$ may appear less relevant from an empirical viewpoint, as no country purchases zero domestically produced goods, the HANK and RANK multipliers are very similar even for more conventional values of α, e.g., around 0.5, as shown in Figure A.2 in Appendix A.2.

3.3 Equivalence and the Trade Elasticity

We turn now to consider variations in the trade elasticity, η, instead of the openness parameter, α. While the latter measures the magnitude of the reduction in the intertemporal Keynesian multiplier, as discussed above, the former determines the strength of the expenditure switching channel. This allows us to establish additional equivalence results.

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6. The calibration employed in this section differs from the one described in Section 4.2 in a few cases: First, while we target the same MPC throughout the paper, the implied discount factor in our stylized model is $\beta = 0.988$, whereas the model in the next section features discount factor heterogeneity. Second, our assumption of $r_{ss} \approx 0$ implies $\mu \approx 1$ in order to ensure a finite price of equity. Finally, we set the monetary policy response to output gap deviations to $\phi_Y = 0.25$, and the tax response parameter to $\phi_G = 0.1$.

7. In the limiting case where the output response approaches zero, the equilibrium responses of the real interest rate and the real exchange rate also approach zero. Thus, in terms of the decomposition in Proposition 1, the real income channel is shut off. However, the expenditure switching channel is scaled by the factor $\frac{2-\alpha}{1-\alpha}$, which tends to ∞ as $\alpha \to 1$, so that the entire term converges to -1, as confirmed in Figure 1.

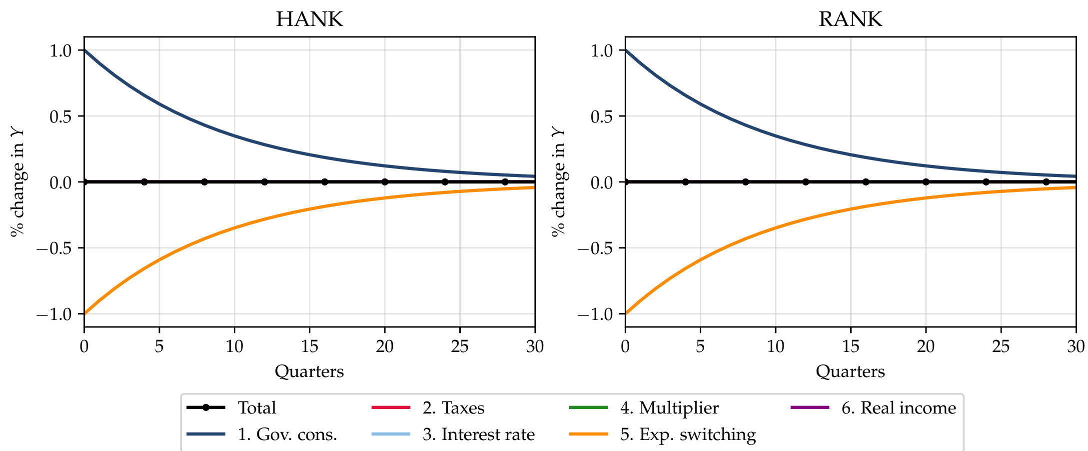

Figure 1: Decomposition of output ($d\boldsymbol{Y}$) when $\alpha \to 1$ and $\phi_Y = 0.25$

Note: The figure shows the decomposition of the response of output from Proposition 1 for the HANK and RANK models when $\alpha \to 1$ and $\phi_Y = 0.25$.

Proposition 3. *Consider a government spending shock, and assume that $\phi_Y > 0$. It then holds that the entire path of output is identical in the RANK and HANK models, i.e. $dY_t^{RA} = dY_t^{HA}, \forall t$, when*

$$\eta \to \infty.$$

Since the entire path of output is the same, the fiscal multiplier is also the same. In particular,

$$d\boldsymbol{Y}^{RA} = d\boldsymbol{Y}^{HA} = \boldsymbol{0}.$$

Proof. See Appendix A.3.

Proposition 3 establishes another case where the fiscal multiplier is identical in the two models, at all horizons. As the trade elasticity approaches infinity, there is full crowding out of the increase in government spending, since net exports respond very strongly to any movement in the real exchange rate. Thus, in equilibrium, the response of the real exchange rate approaches zero as $\eta \to \infty$, as does the real interest rate, since output does not move. Figure 2 summarizes these effects in terms of the decomposition studied above. The figure confirms that output does not respond at any horizon. In the RANK model, the increase in demand from the government is exactly cancelled out by the drop in net exports arising through the expenditure switching channel. In the HANK model, the increase in government spending is counteracted by a small increase in taxes, thus dampening the boom in the domestic economy. The expenditure switching channel is therefore slightly weaker than in the

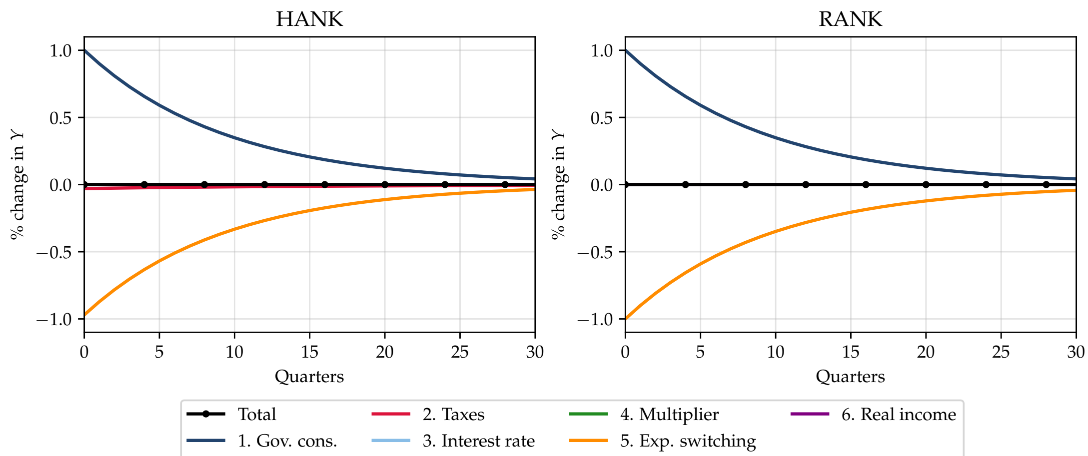

Figure 2: Decomposition of output ($d\boldsymbol{Y}$) when $\eta \to \infty$

Note: The figure shows the decomposition of the response of output from Proposition 1 for the HANK and RANK models when $\eta \to \infty$.

RANK model.

Unlike Proposition 2, the result in Proposition 3 requires an active monetary policy response, i.e., $\phi_Y > 0$. Without such a response, the real interest rate would remain constant, leaving also the real exchange rate unaffected. Thus, the expenditure switching channel would not be active, irrespective of the value of η.

3.3.1 A Special Case: Period-By-Period Financing

While the previous results hold for any assumption regarding the financing of the increase in government spending, it turns out that another equivalence result between the HANK and RANK multipliers can be obtained in the special case in which the increase in government spending is fully financed on a period-by-period basis, so as to maintain a balanced government budget, i.e., we set $\phi_G = 1$ in (9).

Proposition 4. *Consider a government spending shock that is financed period-by-period, i.e. $dG_t = dT_t, \forall t$. In this case, the entire path of output is identical in the RANK and HANK models, i.e. $dY_t^{RA} = dY_t^{HA}, \forall t$, when*

$$\eta = 1.$$

Since the entire path of output is the same, the fiscal multiplier is also the same. In particular,

$$d\boldsymbol{Y}^{RA} = d\boldsymbol{Y}^{HA} = d\boldsymbol{G} - \frac{1}{1-\alpha}\boldsymbol{U}d\boldsymbol{r}, \tag{13}$$

where $\boldsymbol{U}$ *is an upper triangular matrix of unit entries.*

Proof. See Appendix A.4.

Proposition 4 establishes that, given a balanced government budget, a unitary value of the trade elasticity, η, implies that the various forces at play in the two models exactly cancel out.[8] Why do we obtain equivalence for exactly $\eta = 1$ and not some other value? Because a unitary trade elasticity is the "Cole-Obstfeld calibration" (since we have log utility, see Cole and Obstfeld, 1991). In this parametrization, the income and substitution effects from changes in the real exchange rate exactly offset each other, so the economy thus behaves almost like a closed economy. In the closed economy, equivalence between HANK and RANK then follows from Werning (2015). We provide more details on this intuition in Appendix A.5.

To shed further light on the mechanics behind this result, Figure 3 shows the decomposition of $d\boldsymbol{Y}$ proposed in Proposition 1 under the assumptions stated in Proposition 4, i.e., $dG_t = dT_t, \forall t$, and $\eta = 1$. The figure shows how the different channels exactly cancel out in this case, such that the fiscal multiplier is the same in both models. The additional drag from higher taxes in the HANK model, as compared to the RANK model, is cancelled out by a smaller drag on output due to intertemporal substitution, along with positive contributions from the real income channel and the multiplier channel. Since the magnitude of the domestic boom is the same in the two models, the strength of the expenditure switching channel is equivalent across the two models. As for the magnitude of the multiplier, it is 0.19 in both models, both on impact and when measured by the present-value cumulative multiplier (the two coincide in this case, but not in general). While this number may seem low, it is not inconsistent with empirical estimates for open economies with flexible exchange rates (e.g., Corsetti et al., 2012; Ilzetzki et al., 2013).

3.3.2 The General Case: Partial Financing

Having considered the limiting case of a balanced budget, we now turn to the more general case where some degree of debt financing is allowed. Tax revenues are assumed to follow (9)—which in linear form reads $dT_t = \phi_G dG_t$—with $\phi_G \in [0, 1]$.

8. A unitary trade elasticity is consistent with the existing empirical literature. For example, recent estimates by Boehm et al. (2023) suggest a short-run trade elasticity of 0.76, increasing to 1 after three-four years, and around 2 after a decade.

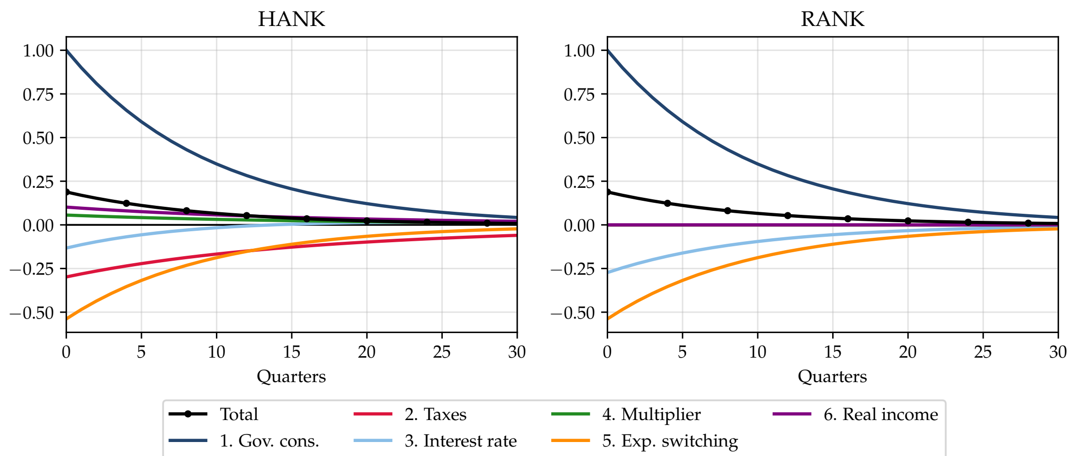

Figure 3: Decomposition of output ($d\boldsymbol{Y}$) when $\eta = 1$

Note: The figure shows the decomposition of the response of output from Proposition 1 for the HANK and RANK models when $\eta = 1$ and the government maintains a balanced budget.

We now explore the potential existence of values of η that establish equivalence between multipliers across HANK and RANK models for arbitrary values of ϕ_G. To this end, we resort to a numerical analysis, in which we focus on the present-value cumulative fiscal multiplier up to some horizon T, defined as:

$$\mathcal{M}^j \equiv \frac{\sum_{t=0}^{T} \frac{dY_t^j}{(1+r)^t}}{\sum_{t=0}^{T} \frac{dG_t}{(1+r)^t}}. \tag{14}$$

In practice, we compute the multiplier over 5 years (i.e. $T = 20$) for a range of values of ϕ_G and η for each of the two models. The results are reported in Figure 4, which shows the difference between the multipliers in the two models, i.e., $\mathcal{M}^{\text{HA}} - \mathcal{M}^{\text{RA}}$.[9] The green area indicates tuples (ϕ_G, η) for which the HANK fiscal multiplier is larger than the RANK fiscal multiplier, i.e. $\mathcal{M}^{\text{HA}} > \mathcal{M}^{\text{RA}}$. The red area, instead, indicates tuples (ϕ_G, η) where the RANK fiscal multiplier is larger, i.e. $\mathcal{M}^{\text{RA}} > \mathcal{M}^{\text{HA}}$. The black line indicates the border between these to areas, i.e. where $\mathcal{M}^{\text{RA}} = \mathcal{M}^{\text{HA}}$. Note that the black line crosses through the limiting case studied in Proposition 4, i.e., $(\phi_G, \eta) = (1, 1)$, whereas it converges to the case from Proposition 3, i.e., $\eta \to \infty$.

9. We report the multiplier for each model separately in Figure A.5. While Propositions 2, 3, and 4 provided equivalence results for the response of output at all points in time, i.e. $dY_t^{\text{HA}} = dY_t^{\text{RA}}, \forall t$, Figure 4 only establishes equivalence for the present-value cumulative multiplier. This implies that the paths of output might differ across models, even if the multiplier is the same. We have verified that the figure looks very similar if we focus on the impact or the peak multiplier instead of the present-value cumulative multiplier.

By "connecting the dots", the black line shows that equivalence between the two multipliers can be established numerically for intermediate values of ϕ_G and η.

To understand the results in Figure 4, consider first variations in the degree of financing, ϕ_G. A reduction in the degree of financing, i.e., a drop in ϕ_G, implies a smaller increase in taxes. This entails that the negative impact of the tax channel on consumption in the HANK model is weakened, so the HANK multiplier is more likely to exceed the RANK multiplier, all else equal, as indicated by moving left towards the green area (or further away from the red area), for a given value of η.

Next, consider variations in the trade elasticity, η, holding ϕ_G fixed. As the trade elasticity increases, expenditure switching becomes more important in both models, which reduces the fiscal multiplier in both cases. Furthermore, the smaller increase in output weakens the intertemporal Keynesian multiplier present in the HANK model.[10] As a result, an increase in η exerts a stronger negative effect on the fiscal multiplier in the HANK model than in the RANK model, consistent with moving towards the red area (or further away from the green area).

Lastly, it is worth noting that the differences between the fiscal multipliers in the HANK and RANK models in Figure 4 are small for any calibration. In particular, the largest difference is around 0.1 for pure debt-financing with values even closer to 0 for any other calibration.

3.4 Summary

To sum up this section, we have documented that the same six channels are at play in response to fiscal policy shocks in open-economy HANK and RANK models. However, their magnitude—and potentially their sign—has been shown to vary across models. This paves the way for equivalence results, where the channels cancel each other out, leading to equivalence of output and fiscal multipliers in the two models. We have provided a set of such results, where the entire paths of output, and hence the fiscal multipliers, are the same in the HANK and RANK models.

10. Analytically, this can be seen by collecting the $d\boldsymbol{Y}^j$-terms in the expression in Proposition 1 and then solving for $d\boldsymbol{Y}^j$, which entails that the expenditure switching term gets premultiplied by $[\boldsymbol{I} - (1-\alpha)\boldsymbol{M}^j]^{-1}$, where $\boldsymbol{I}$ is the identity matrix. Thus, the effect on output of a change in η is seen to scale with the MPCs, as reflected by the matrix $\boldsymbol{M}^j$.

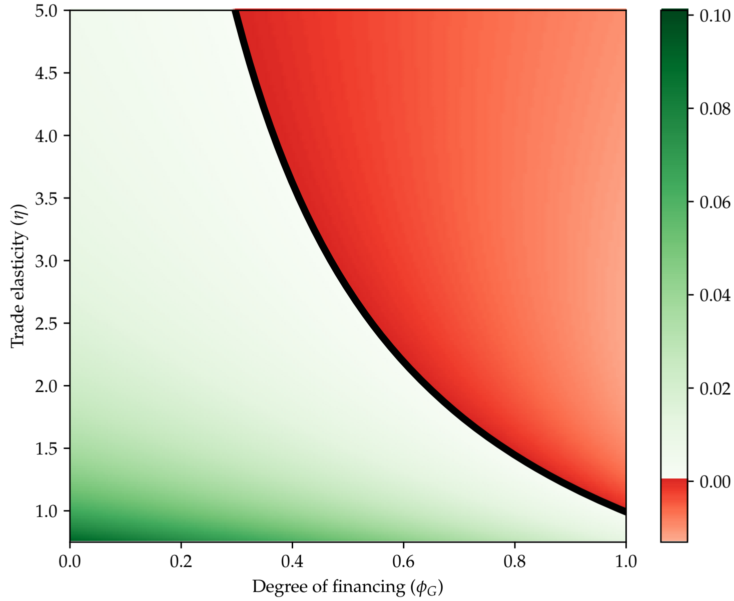

Figure 4: Difference between fiscal multipliers in HANK and RANK

Note: The figure shows the difference in cumulative fiscal multipliers between the HANK and RANK models in the stylized setting, i.e. $\mathcal{M}^{\text{HA}} - \mathcal{M}^{\text{RA}}$, for a range of values of the trade elasticity (η) and the degree of financing (ϕ_G). The cumulative fiscal multiplier is computed over 5 years, i.e. $T = 20$ in (14).

4 Numerical Results

Motivated by the equivalence results established in the previous section, we now turn to a quantitative assessment of the fiscal multiplier in the HANK and RANK models, with the aim of exploring whether these remain relatively similar for realistic calibrations.

4.1 Quantitative Model Elements

We extend the stylized model described in Section 2 with a number of elements to make it more realistic from a quantitative viewpoint. In particular, we consider four new model elements: Permanent discount factor heterogeneity, time-varying trade elasticities, a more elaborate government sector, and a fixed cost in production.

4.1.1 Permanent Discount Factor Heterogeneity

We introduce permanent heterogeneity in household discount factors: Half of the households are impatient with discount factor, β^{low}, while the remaining households are patient with discount factor, β^{high}. This allows us to simultaneously match the

MPC (as in the stylized model) along with a realistic level of government debt (in contrast to the stylized model). Households of both types solve the same problem as in the stylized model, but with (permanently) different discount factors.[11]

4.1.2 Time-Varying Trade Elasticities

Following recent advances in the trade literature (see Drozd et al., 2021 and Boehm et al., 2023), we allow for a dynamic trade elasticity, which is low in the short run but high in the long run. Specifically, we assume that the domestic CES demand for imports and the Armington demand for exports are given by:

$$C_{F,t} = \alpha \left(\hat{p}_t\right)^{-\eta} C_t, \quad \hat{p}_t = \left(\hat{p}_{t-1}\right)^{\rho_\eta} \left(\frac{P_{F,t}}{P_t}\right)^{1-\rho_\eta}, \tag{15}$$

$$C^*_{H,t} = \alpha^* \left(\hat{p}^*_t\right)^{-\eta} C^*_t, \quad \hat{p}^*_t = \left(\hat{p}^*_{t-1}\right)^{\rho_\eta} \left(\frac{P^*_{H,t}}{P^*_t}\right)^{1-\rho_\eta}, \tag{16}$$

where $\rho_\eta \in [0, 1)$ captures the smoothness embedded in the process. We retrieve a constant trade elasticity when $\rho_\eta = 0$.

4.1.3 Government

We expand on the reaction of the monetary policy authority. In particular, we now consider a Taylor rule featuring an inflation response and interest rate smoothing:

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\left(i_{ss} + \phi_\pi \pi_t + \phi_Y \left[\frac{Y_t}{Y_{ss}} - 1\right]\right). \tag{17}$$

Furthermore, we consider a tax rule reacting to the lagged level of debt:

$$T_t = T_{ss} + \phi_B (B_{t-1} - B_{ss}), \tag{18}$$

reminiscent of Galí (2020). Unlike the rule employed in Subsection 3.3.2, this rule ensures that government debt returns to its original steady state.

11. In the NKWPC (7), we use the average discount factor: $(\beta^{\text{low}} + \beta^{\text{high}})/2$.

4.1.4 Fixed Cost

Finally, we introduce a fixed cost in production in order to be able to simultaneously obtain a realistic markup and a level of aggregate wealth to GDP consistent with the data. In particular, firm profits are now:

$$D_t = (1-\tau_t)\frac{P_{H,t}Y_t - W_t N_t}{P_t} - F, \tag{19}$$

where F is the fixed cost. Goods market clearing then becomes

$$Y_t = C_{H,t} + C^*_{H,t} + G_t + \frac{P_t}{P_{H,t}}F. \tag{20}$$

With no fixed cost, a realistic markup would imply an implausibly high wealth level.

4.2 Calibration

Our calibration follows the one of Druedahl et al. (2022) on most dimensions, but adapted for a one-sector model. Thus, the calibration targets the average small open economy in a sample of OECD countries. The baseline calibration of the model is summarized in Table 2.

Regarding household preferences, we set the inverse of the Frisch elasticity to $\varphi = 2$. We set the discount factor of patient households to match an annual real interest rate of 2%, as discussed in Druedahl et al. (2022). The discount factor of impatient households is set to match an annual MPC of 0.51, following Fagereng et al. (2021). We take the parameters governing the idiosyncratic component of income from Druedahl et al. (2022), who use standard values from Floden and Lindé (2001). The calibration of the representative agent model differs regarding the discount factor, which is given by $1/(1+r)$.

On the firm side, we opt for a markup of 20%. We then set the fixed cost to ensure that the total supply of assets yields an aggregate ratio of assets to output (A/Y) of 10 (2.5 annually) as in Druedahl et al. (2022). The slope of the wage Philips curve is set to 0.05, as in Druedahl et al. (2022).

	Description	Value	Target (Source)
	Households		
β^{RA}	RANK discount factor	0.995	Interest rate
β^{high}	High discount factor	0.991	Interest rate
β^{low}	Low discount factor	0.878	MPC $=$ 0.51 (F21)
ρ_e	Persistence of idiosyncratic income	0.966	(D22, FL01)
σ_e	Std. of idiosyncratic income	0.13	(D22, FL01)
$1/\varphi$	Frisch elasticity	0.5	(D22, C11)
	Firms and Philips-curve		
μ	Markup	1.2	(D22)
κ	Slope of Philips-curve	0.05	(D22)
F	Fixed cost	0.116	$A/Y = 10$ (D22, OECD)
	Government and monetary policy		
G_{ss}	Public consumption to GDP	0.20	$G/Y = 20\%$ (D22, OECD)
B_{ss}	Government debt to GDP	2.32	$B/Y = 232\%$ (D22, OECD)
ϕ_π	Taylor rule coefficient on π	1.5	(D22)
ϕ_Y	Taylor rule coefficient on Y	0	(D22)
ρ_i	Interest rate smoothing	0.9	(D22)
ϕ_B	Debt financing	0.02	(G20)
ρ_G	Gov. cons. AR(1) coefficient	0.9	(HMM19)
	Trade		
i^*	Foreign interest rate (annual)	2%	(D22)
α	C_F share	0.614	$M/Y = 42\%$ (D22, OECD)
η	Trade elasticity	2	(D22, B23)
ρ_η	Trade rigidity	0.9	(D22, B23)

Table 2: Calibration

Note: The table summarizes the baseline parameter values. B23 is Boehm et al. (2023). C11 is Chetty et al. (2011). D22 is Druedahl et al. (2022). F21 is Fagereng et al. (2021). FL01 is Floden and Lindé (2001). G20 is Galí (2020). HMM19 is Hagedorn et al. (2019). OECD refers to data from the OECD for the sample in Druedahl et al. (2022).

For the government, we set the steady state values of government consumption and debt to match the OECD data, giving us $G_{ss} = 0.2$ and $B_{ss} = 2.32$ (implying a debt-to-GDP ratio of 58% annually), as steady-state output is normalized to 1. The steady-state level of tax revenues then follows residually. Regarding the financing of spending shocks, we set $\phi_B = 0.02$ following Galí (2020), implying that increases in government spending are financed mostly by higher public debt in the short run, with taxes responding very slowly. We assume that the government spending shock follows an AR(1) process with persistence $\rho_G = 0.9$, following Hagedorn et al. (2019). Monetary policy is assumed to respond to consumer price inflation with a coefficient of $\phi_\pi = 1.5$, whereas we set the output-gap response to zero, $\phi_Y = 0$, as a baseline. The interest-rate smoothing parameter is set to 0.9.

Finally, regarding the foreign economy, we assume a net foreign asset position of zero in the steady state, such that trade is initially balanced. We calibrate the foreign share of consumption (α) to match an import-to-GDP ratio of 42%, corresponding to the average across OECD countries. As in Druedahl et al. (2022), we calibrate the (long-run) trade elasticity, η, and the rigidity in substitution, ρ_η, to match the evidence from Boehm et al. (2023). This yields $\eta = 2$ and $\rho_\eta = 0.9$.

4.3 Baseline Results

The impulse-response functions (IRFs) to a government spending shock in both the HANK and the RANK model are shown in Figure 5. As the figure shows, the response of GDP is very similar in the two models, though slightly larger—and more short-lived—in the HANK model. The impact multiplier is 1.1 in the HANK model and 0.91 in the RANK model, whereas the present-value cumulative multiplier is 0.36 and 0.39, respectively, when considering the first 20 quarters after the shock (see Table 3). Overall, the main insight from the previous section is therefore confirmed: unlike in closed economies, the open-economy fiscal multiplier is relatively similar across HANK and RANK models.

The underlying dynamics, however, are quite different. The HANK model displays a large increase in domestic consumption, reflecting the high MPC displayed by households in this economy. Part of the increase in consumption is directed towards foreign goods, leading to a notable increase in imports and, in turn, a decline in net exports. In the RANK model, in contrast, domestic consumption declines, since the MPC in this economy is low, and households instead postpone consumption in response to the increase in the real interest rate (except for the first few quarters). This spills over into a drop in imports, and therefore an increase in net exports in the RANK economy. In the aggregate, the different responses of consumption and net exports in the two models roughly cancel out, explaining the relatively similar dynamics of output discussed above. The real interest rate and the real exchange rate display relatively similar patterns in the two models: the real interest rises, except for an initial decline due to nominal interest-rate smoothing, while the real exchange rate

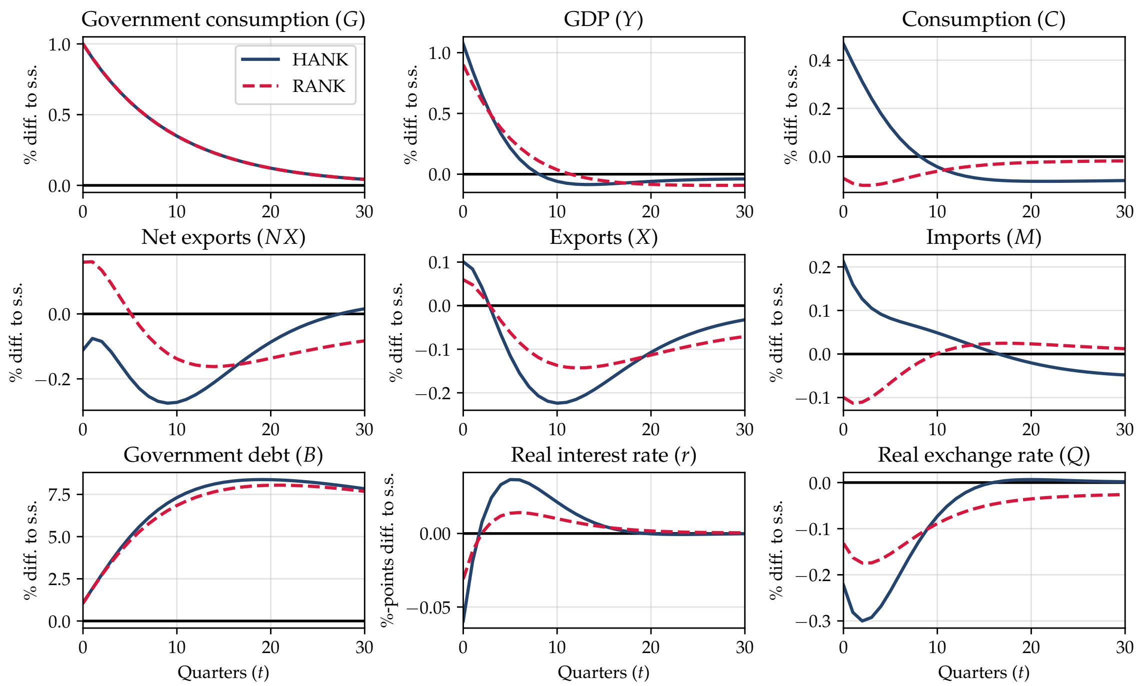

Figure 5: IRFs to a government consumption shock

Note: The figure shows impulse response functions to a government consumption shock in the baseline model.

appreciates. Finally, the pattern of government debt is almost identical in the two models.

4.4 Alternative Model Specifications

We now consider several extensions of the baseline framework and their implications for fiscal multipliers.

4.4.1 Model with Capital

We begin by introducing capital formation into the model. The production function of domestic firms is then given by $Y_t = N_t^{1-\alpha_K} K_{t-1}^{\alpha_K}$. The capital stock is owned by capital firms, and evolves according to a standard law of motion. Capital firms are subject to quadratic investment adjustment costs. The details are presented in Appendix B.1. We set the capital share (α_K) to a standard value of 0.33, while the depreciation rate of capital is 1.25% per quarter. The IRFs to a government spending shock in the models with capital are shown in Figure 6. The introduction of capital reduces the fiscal multiplier, as investment displays a clear decline in both models (while the leaving the dynamics of consumption and net exports largely unaffected, relative

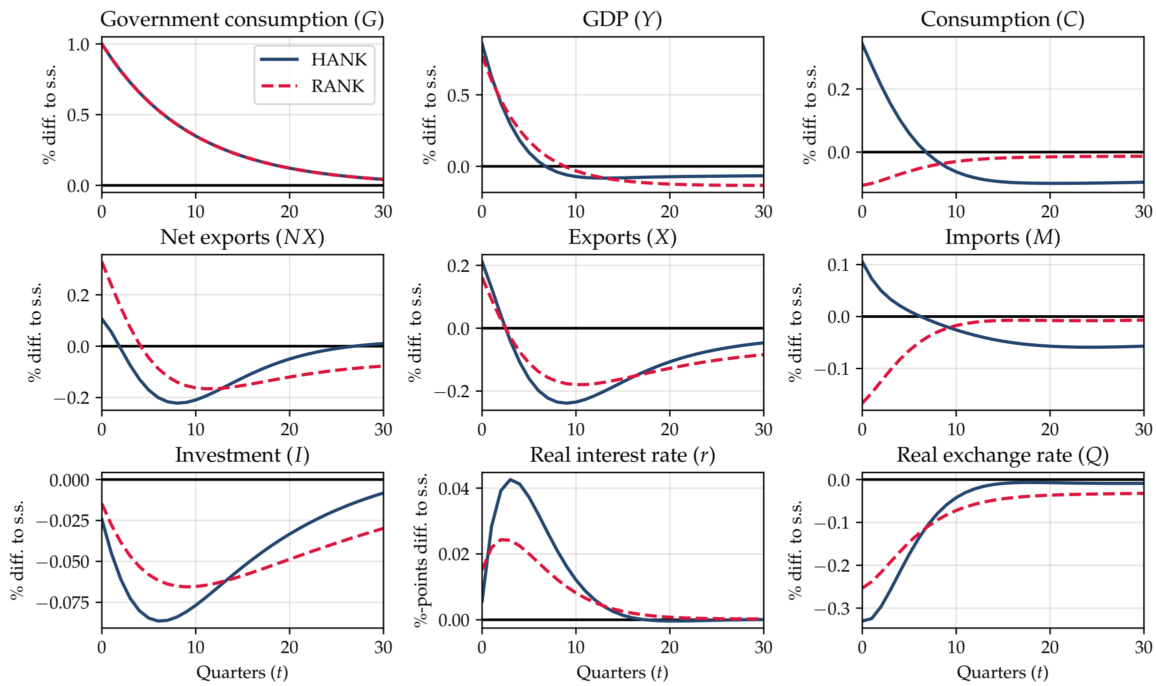

Figure 6: IRFs to a government consumption shock with capital

Note: The figure shows impulse response functions to a government consumption shock in the model with capital.

to the baseline case). However, the dynamics of output remains very similar across the HANK and RANK economies, with present-value cumulative multiplier of 0.07 and 0.10, respectively. In other words, the introduction of capital does not materially affect our main message.

4.4.2 The Response of the Real Exchange Rate

In our model, the real exchange rate is determined through the UIP condition, given a path for monetary policy. As seen above, our baseline model features an appreciation of the real exchange in response to a government spending shock, consistent with the idea that a fiscal expansion drives up the relative price of domestic goods. In the existing empirical literature, however, this response is at the centre of a long-standing debate, with several authors reporting evidence of a—counterintuitive—depreciation of the real exchange rate (see, e.g., Kim and Roubini, 2008, Monacelli and Perotti, 2010, and Ravn et al., 2012). More recent contributions have challenged or modified this result somewhat (see, e.g., Ferrara et al., 2021 or Born et al., 2024, who also provide surveys of this literature). While we do not wish to take a stand on this question, we find it important to document that our theoretical insight is not sensitive to the conditional response of the real exchange rate. To this end, we allow for UIP deviations in the form of a UIP shock, $\varepsilon_t^{\text{UIP}}$. This implies that the real UIP condition (2) is modified to:[12]

$$1 + r_t = (1 + r^*)\frac{Q_{t+1}}{Q_t} + \varepsilon_t^{\text{UIP}}.$$

We can then back out the path for $\varepsilon_t^{\text{UIP}}$ required to obtain any desired response of the real exchange rate.[13] We consider two cases: One where the UIP shock ensures that the real exchange rate remains constant, $dQ_t = 0$ for all t, and another where the UIP deviation is such that the real exchange rate moves exactly opposite the baseline case, i.e., it depreciates by the same amount that it appreciates in the baseline.

The implied IRFs are reported in Figure 7. As compared to the baseline case of an appreciation from Figure 5, a constant real exchange rate implies a boost to net exports through expenditure switching, so that net exports display a clear increase in both models (top row). This effect is even stronger in the case of a depreciation (bottom row). However, in both cases, the increase in net exports is counteracted by a decline in domestic consumption. In the HANK model, this reflects a reversal of the powerful real income channel, as domestic households are poorer in real terms. In the RANK model, the drop in consumption is due to the increase in the domestic real interest rate resulting from the UIP shock. In other words, both models display a negative comovement between domestic consumption and net exports. As seen from the figure, the response of output is very similar in the HANK and RANK models, irrespective of the response of the real exchange rate.[14]

4.4.3 Alternative Financing Assumptions

The previous literature has established that, in the context of HANK models, the timing of the increase in taxes required to finance the additional government spending plays a crucial role for the size of fiscal multipliers (see, e.g., Hagedorn et al., 2019 and

12. We also adjust the asset returns, r_t^a, to reflect that returns are not equalized even after period 0 due to the UIP deviation.

13. We have opted for this reduced-form approach to obtain a depreciation of the real exchange rate. Structural mechanisms proposed in the literature to obtain such a response include spending reversals (Corsetti et al., 2010) and deep habits (Ravn et al., 2012).

14. Furthermore, as seen from Table 3, the impact multipliers from each model are rather insensitive to movements in the real exchange rate. The cumulative multipliers are significantly larger when the UIP shock is active, as compared to the baseline model, since the increase in output becomes more persistent in these cases.

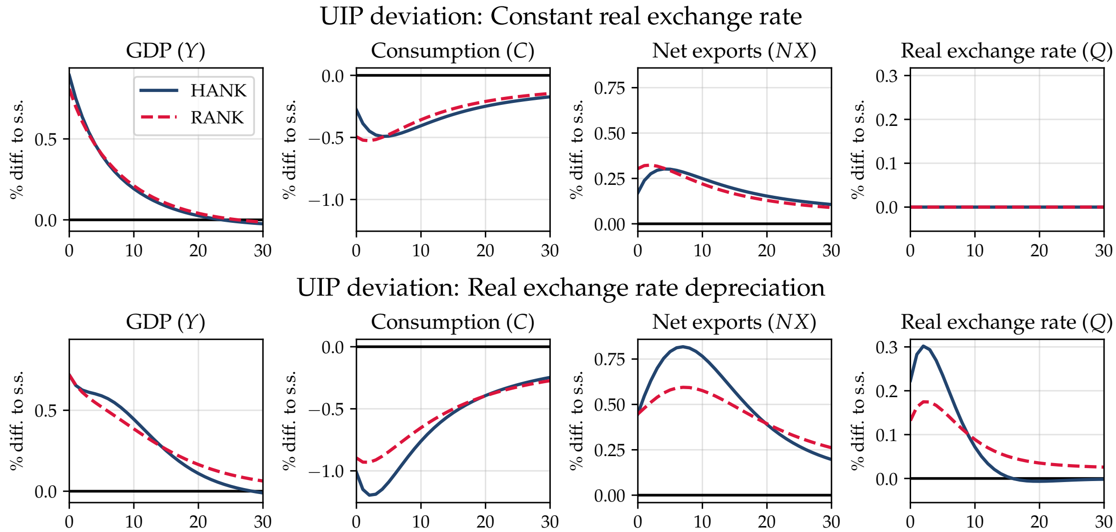

Figure 7: IRFs to a government consumption shock with UIP deviations

Note: The figure shows impulse response functions to a government consumption shock in the model with UIP deviations. In the top row, the UIP deviation is such that the real exchange rate remains constant. In the bottom row, the UIP deviation is such that the real exchange rate response is the exact opposite of that observed in Figure 5.

Auclert et al., 2023). This is in contrast to the literature based on representative-agent models, where some form of Ricardian equivalence typically applies. Thus, one may suspect that alternative financing assumptions could break the similarity between HANK and RANK multipliers established above. To explore this aspect, we now consider a variety of financing rules proposed in the existing literature, and their implications for fiscal multipliers in a SOE. Specifically, we employ the following rules: i) Following Galí et al. (2007), we assume that taxes adjust to contemporaneous debt and public spending according to

$$dT_t = \phi_B dB_t + \phi_G dG_t,$$

with $\phi_B = 0.33$ and $\phi_G = 0.1$. ii) Following Auclert et al. (2023), we assume that taxes adjust such that public debt satisfies

$$dB_t = \phi_B (dB_{t-1} + dG_t),$$

with $\phi_B = 0.93$. iii) Following Hagedorn et al. (2019), we assume that taxes are fixed initially, and then phased in after a while in order to stabilize the debt level:

$$T_t = \begin{cases} T_{ss} & \text{if } t < t_B, \\ (1-\omega)\, T_{ss} + \omega \tilde{T}_t & \text{if } t \in [t_B, t_B + \Delta_B], \\ \tilde{T}_t & \text{if } t > t_B + \Delta_B, \end{cases}$$

where $\tilde{T}_t = T_{ss}\left(\frac{B_{t-1}}{B_{ss}}\right)$, and where $t_B = 50$ and $\Delta_B = 20$.[15] iv) Finally, we consider a balanced budget rule, such that $B_t = B_{ss}$ for all t.

The IRFs to a government spending shock under these financing schemes are reported in Figure 8, where the different cases have been ordered according to the magnitude of the implied increase in government debt, as seen from the right column. The various fiscal rules have notably different implications for the path of government debt: In the first three rows, the increase in government debt is smaller than in our baseline model, implying a larger increase in taxes, which dampens the increase in consumption in the HANK model. An extreme case of this is seen under a balanced budget, where a drop in consumption can be observed (top row). In contrast, the bottom row shows a rule where government debt displays a larger increase, boosting the response of consumption. The key insight from this exercise, however, is that the response of output displays only very small changes as we vary the financing rule. As seen from Table 3, the impact multipliers are barely affected by this choice, and remain very similar in the HANK and RANK models, whereas the cumulative multipliers are somewhat more responsive, though without altering the main picture. The explanation is that the response of net exports tends to mirror that of private consumption; declining by more when consumption increases by more, and vice versa, partly because a fraction of the increase in consumption is directed towards foreign goods, and partly due to expenditure switching resulting from the appreciation of the real exchange rate.

4.4.4 Additional Sensitivity Checks

In order to subject our main insights to a final round of sensitivity checks, we first explore alternative assumptions regarding monetary policy. We begin by treating the

15. In the practical implementation, ω is a function of $x = \left(\frac{t-t_B}{\Delta_B}\right)$, with $\omega(x) = 3x^2 - 2x^3$.

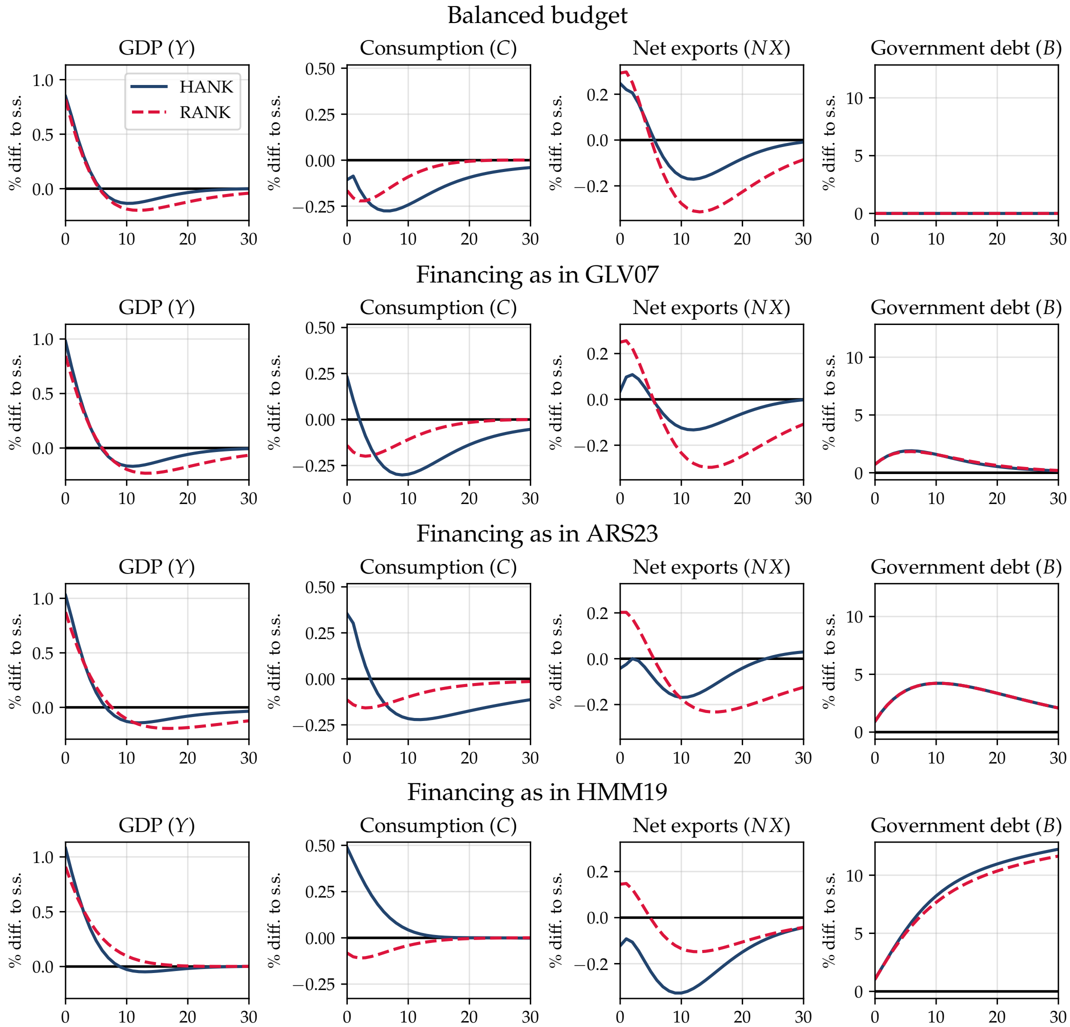

Figure 8: IRFs to a government consumption shock with different financing schemes

Note: The figure shows impulse response functions to a government consumption shock under a range of different assumptions regarding the financing of the increase in spending. GLV07 is Galí et al. (2007), ARS23 is Auclert et al. (2023), and HMM19 is Hagedorn et al. (2019).

case of a fixed exchange rate, i.e., where the Taylor rule (17) is replaced by a rule that keeps the *nominal* exchange rate fixed at all time, i.e. $E_t = E_{ss}$. The IRFs are reported in Figure A.7 in Appendix B.2. The responses of all variables are very similar to those observed under a floating exchange rate, implying that the fiscal multipliers are also barely affected, as seen from Table 3.[16]

16. Observe that the response of the real interest rate is quite similar across exchange-rate regimes. Under a currency peg, the central bank keeps the nominal interest rate fixed. The pattern of the real interest rate is then determined by the dynamics of consumer price inflation, which first increases in response to the increase in domestic producer prices, but then declines as a reflection of expenditure switching towards relatively cheaper foreign goods.

We have also considered the case of a more aggressive response of monetary policy, implemented by setting $\phi_\pi = 2$ and $\phi_Y = 1$. A stronger increase in the real interest rate implies a larger appreciation of the real exchange rate. In the HANK model, this strengthens both the real income channel (leading to a stronger increase in domestic consumption) and the expenditure switching channel (leading to larger drop in net exports). Again, the negative comovement between these two variables leads to a muted effect on output, which therefore remains fairly similar in the two models, as seen from Figure A.6 in Appendix B.2. Another alternative is a "passive" monetary policy rule, according to which the central bank keeps the real interest rate fixed at all times, i.e. $r_t = r_{ss}$. Such a rule implies a somewhat higher multiplier than our baseline model. In fact, the cumulative multipliers in the HANK and RANK models are identical and equal to 1 over the entire horizon, i.e., as $T \to \infty$ in (14), as established in Sundram (2024).

Given the considerable uncertainty surrounding estimates of the trade elasticity in the existing literature, and the central role of this parameter for the transmission of shocks to consumption in HANK and RANK models established by Auclert et al. (2021b), we also consider variations of η. In particular, we first consider a higher value of the long-run elasticity, with $\eta = 4$, and then a trade elasticity that is not dynamic, i.e. we set $\rho_\eta = 0$ (so that the value of η is constant at 2). The resulting IRFs are shown in Figure A.6 in Appendix B.2. A higher trade elasticity has a very modest impact on our results, reducing the fiscal multiplier slightly in both models due to a stronger expenditure switching effect. A static trade elasticity works in the same direction, but exerts a more powerful effect, reflecting that changes in the short-run trade elasticity are more powerful than long-run changes. However, since the HANK and RANK multipliers are reduced in parallel, they remain very similar.

Lastly, while our assumption of sticky wages is consistent with most of the existing HANK literature (e.g., Hagedorn et al., 2019 and Auclert et al., 2023), models in the RANK tradition have typically focused on sticky prices (e.g., Galí and Monacelli, 2005). Broer et al. (2023) show that this choice is not innocuous for fiscal multipliers. We therefore add sticky prices, that is, we replace the pricing equation, $P_{H,t} = \mu W_t$, with the following New-Keynesian Phillips Curve (NKPC),

$$\pi_{H,t} = \kappa_H \left(\frac{W_t}{P_{H,t}} - \frac{1}{\mu} \right) + \beta \pi_{H,t+1}.$$

We set $\kappa_H = 0.15$ following Druedahl et al. (2022). We first consider the case with sticky prices and flexible wages, i.e. $\kappa \to \infty$. As seen from Figure A.6 in Appendix

B.2 (row five), this widens the gap between HANK and RANK multipliers somewhat, at least on impact (see also Table 3). Under this specification, firm profits become countercyclical, and real wages display a stronger increase, boosting the consumption response in HANK. At the same time, since prices are rigid, the real exchange rate is less responsive. Thus, in this case, the drop in net exports is not as large as the increase in consumption, implying a larger output response in HANK. In the RANK model, instead, the increase in real wages cancels out with the drop in profits, from the viewpoint of the representative household, so the RANK response is very similar to the baseline. In the empirically plausible case where wages are more sticky than prices—and profits are procyclical—fiscal multipliers are instead very close to the baseline (Figure A.6, bottom row).

5 Conclusion

We have studied fiscal multipliers in a small open economy Heterogeneous Agent New-Keynesian model, and compared the fiscal multipliers in this model to those from a representative-agent model. Our general takeaway is that fiscal multipliers are similar in both types of models. This claim is based on two types of arguments. First, we have established analytical equivalence results in a stylized setting, and second, we have shown that fiscal multipliers are generally quite similar in a quantitative model. These results are in contrast with those for a closed economy, where the introduction of households with sizeable marginal propensities to spend have been shown to boost consumption, and thus output. The difference arises from a negative effect through net exports that cancels out the positive effect through domestic consumption.

Our quantitative analysis of fiscal multipliers in the previous section led to various insights. First, we showed that multipliers are generally quite similar in HANK and RANK environments, though typically slightly higher in a HANK setup. Second, the absolute size of fiscal multipliers displayed limited sensitivity to the various model perturbations we considered. In general, the fiscal multiplier is close to 1 on impact in the models we have considered, while the cumulative multiplier is between zero and 0.5, with few exceptions. This is consistent with a boom-bust type of pattern in consumption and output: Output initially rises following higher government spending, but after a while consumption drops to pay back debt to foreigners.

The focus of our analysis has been mostly on the *relative* magnitude of multipliers in HANK and RANK models, and less on the absolute size of these. Likewise, the fact that impact multipliers are generally higher than cumulative multipliers in the models we have considered reflects—at least partly—the absence of model ingredients aimed at obtaining the hump-shaped impulse responses often observed in applied work. Thus, we leave for future research a more detailed assessment of the ability of the models considered above to provide a quantitative account of the empirical effects of fiscal policy in small open economies.

Model Specification	Multiplier	HANK	RANK	Difference
Baseline model	Impact	1.07	0.90	0.17
	Cumulative	0.35	0.42	−0.06
With capital	Impact	0.86	0.77	0.08
	Cumulative	0.20	0.23	−0.03
Flat real exchange rate	Impact	0.89	0.81	0.08
	Cumulative	0.67	0.67	−0.01
Opposite real exchange rate	Impact	0.72	0.72	−0.00
	Cumulative	0.98	0.93	0.06
Financing as in GLV07	Impact	0.98	0.84	0.13
	Cumulative	0.15	0.03	0.13
Financing as in ARS23	Impact	1.03	0.87	0.15
	Cumulative	0.23	0.17	0.06
Financing as in HMM19	Impact	1.08	0.91	0.17
	Cumulative	0.42	0.54	−0.12
Balanced budget	Impact	0.85	0.82	0.03
	Cumulative	0.15	0.03	0.12
Fixed exchange rate	Impact	1.12	0.95	0.17
	Cumulative	0.40	0.47	−0.07
Aggressive monetary policy	Impact	0.95	0.75	0.20
	Cumulative	0.17	0.19	−0.01
Passive monetary policy	Impact	1.19	1.00	0.19
	Cumulative	1.17	1.00	0.17
High trade elasticity	Impact	1.03	0.89	0.14
	Cumulative	0.26	0.26	−0.01
Constant trade elasticity	Impact	0.66	0.66	0.00
	Cumulative	0.23	0.30	−0.07
Sticky prices	Impact	1.37	0.87	0.50
	Cumulative	0.25	0.37	−0.12
Sticky prices and wages	Impact	1.13	0.93	0.20
	Cumulative	0.41	0.46	−0.05

Table 3: Fiscal multipliers in different models

Note: The table reports impact and cumulative fiscal multipliers from the HANK and RANK models (and the difference between these) for a range of different modeling assumptions. The cumulative fiscal multiplier is computed over 5 years, i.e. $T = 20$ in (14).

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Appendix

A Stylized Model Appendix

In the following, we present the proofs of each of the propositions in Section 3 in the main text.

A.1 Proof of Proposition 1

We begin by linearizing some key equations that we use to prove the proposition.

A.1.1 Domestic Consumption of Home Goods

Linearizing the CPI in (4) yields

$$dP_t = \alpha dP_{F,t} + (1-\alpha) dP_{H,t}. \tag{21}$$

Next, linearizing the nominal exchange rate yields

$$dE_t = dP_{F,t}. \tag{22}$$

Similarly, linearizing the definition of the real exchange rate implies that

$$dQ_t = dE_t + dP_t^* - dP_t. \tag{23}$$

Inserting (22) into (23) yields

$$dQ_t = dP_{F,t} - dP_t^* + dP_t^* - dP_t = dP_{F,t} - dP_t.$$

Solving for $dP_{F,t}$, we find that

$$dP_{F,t} = dQ_t + dP_t. \tag{24}$$

Inserting this into the linearized CPI (21) and solving for $dP_{H,t} - dP_t$, we obtain

$$dP_{H,t} - dP_t = -\frac{\alpha}{1-\alpha} dQ_t. \tag{25}$$

Linearizing domestic consumption of home goods (3) gives

$$dC_{H,t} = (1-\alpha)\left[dC_t - \eta(dP_{H,t} - dP_t)\right].$$

Inserting (25) yields

$$dC_{H,t} = (1-\alpha)dC_t + \eta\alpha dQ_t. \tag{26}$$

A.1.2 Foreign Consumption of Home Goods

Linearizing $P^*_{H,t}$ in (5) gives

$$dP^*_{H,t} = dP_{H,t} - dE_t.$$

Insert now (22) to get that

$$dP^*_{H,t} = dP_{H,t} - dP_{F,t}.$$

Inserting (24) yields

$$dP^*_{H,t} = dP_{H,t} - dP_t - dQ_t.$$

Insert finally (25) to obtain

$$dP^*_{H,t} = -\frac{1}{1-\alpha}dQ_t. \tag{27}$$

Linearizing foreign consumption of home goods in (11) gives

$$dC^*_{H,t} = -\eta\alpha dP^*_{H,t}.$$

Insert (27) to get that

$$dC^*_{H,t} = \frac{\eta\alpha}{1-\alpha}dQ_t. \tag{28}$$

A.1.3 The Linearized Consumption Function

The linearized consumption function is

$$d\boldsymbol{C} = \boldsymbol{C}^{Z}d\boldsymbol{Z} + \boldsymbol{C}^{r}d\boldsymbol{r} + \boldsymbol{C}^{r^a}d\boldsymbol{r}^{a}_{0}, \tag{29}$$

with Jacobians

$$\boldsymbol{C}^{Z} \equiv \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{Z}}, \quad \boldsymbol{C}^{r} \equiv \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{r}}, \quad \boldsymbol{C}^{r^a} \equiv \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{r}^{a}_{0}}.$$

Insert now $W_t = P_{H,t}/\mu$ into the definition of labor income to get that

$$Z_t = (1-\tau_t)\frac{1}{\mu}\frac{P_{H,t}}{P_t}Y_t.$$

Taking a first-order approximation of this yields

$$\begin{aligned} dZ_t &= (1-\tau)\frac{1}{\mu}\left(dY_t + dP_{H,t} - dP_t\right) - \frac{1}{\mu}d\tau_t \\ &= (1-\tau)\frac{1}{\mu}\left(dY_t - \frac{\alpha}{1-\alpha}dQ_t\right) - \frac{1}{\mu}d\tau_t, \end{aligned}$$

where we have used (25) in the second line. Inserting this into the linearized consumption function in (29), it follows that

$$d\boldsymbol{C} = (1-\tau)\frac{1}{\mu}\boldsymbol{C}^Z\left(d\boldsymbol{Y} - \frac{\alpha}{1-\alpha}d\boldsymbol{Q}\right) - \frac{1}{\mu}\boldsymbol{C}^Z d\tau + \boldsymbol{C}^r d\boldsymbol{r} + \boldsymbol{C}^{r^a} d\boldsymbol{r}_{\boldsymbol{0}}^{\boldsymbol{a}}. \tag{30}$$

To proceed we use that the valuation effect at time 0 is given by $r_0^a = \frac{p_0^D + D_0}{p_{ss}^D} - 1$. If we iterate forward on the equity pricing condition, we have:

$$p_t^D = \frac{D_{t+1}}{1+r_t} + \frac{D_{t+2}}{(1+r_t)(1+r_{t+1})} + \frac{D_{t+3}}{(1+r_t)(1+r_{t+1})(1+r_{t+2})}\dots,$$

implying that that the equity price is a function of the sequences of dividends and real interest rates; $p_t^D = p_t^D\left(\{D_s, r_s\}_{s=0}^{\infty}\right)$. Since r_0^a is a function of dividends and the equity price, we also have $r_0^a = r_0^a\left(\{D_s, r_s\}_{s=0}^{\infty}\right)$. Linearizing r_0^a w.r.t. inputs, we then obtain:

$$\begin{aligned} dr_0^a = &\frac{r_{ss}}{D_{ss}}\left(dD_0 + \frac{dD_1}{1+r} + \frac{dD_2}{(1+r)^2} + \frac{dD_3}{(1+r)^3}\dots\right) \\ &- \frac{D_{ss}}{p_{ss}^D}\left(dr_0\frac{1}{(1+r)}\sum_{s=0}^{\infty}\frac{1}{(1+r)^{1+s}} + dr_1\frac{1}{(1+r)^2}\sum_{s=0}^{\infty}\frac{1}{(1+r)^{1+s}} + \dots\right). \end{aligned}$$

We can then use $p^D_{ss} = \frac{D_{ss}}{r_{ss}}$ and $\frac{1}{1+r}\sum_{s=0}^{\infty}\left(\frac{1}{1+r}\right)^s = \frac{1}{1+r}\frac{1}{1-\frac{1}{1+r}} = \frac{1}{1+r}\frac{1+r}{r} = \frac{1}{r}$ to obtain

$$
\begin{aligned}
dr_0^a = \frac{r_{ss}}{D_{ss}}&\left(dD_0 + \frac{dD_1}{1+r} + \frac{dD_2}{(1+r)^2} + \frac{dD_3}{(1+r)^3}\dots\right) \\
&- \left(dr_0\frac{1}{(1+r)} + dr_1\frac{1}{(1+r)^2} - \dots\right).
\end{aligned}
$$

We can define the following vectors:

$$
\begin{aligned}
\boldsymbol{J}'_D &= \frac{r_{ss}}{D_{ss}}\left(1, \frac{1}{1+r}, \frac{1}{(1+r)^2}, \dots\right) \\
\boldsymbol{J}'_r &= -\left(\frac{1}{(1+r)}, \frac{1}{(1+r)^2}, \frac{1}{(1+r)^3}, \dots\right),
\end{aligned}
$$

to get

$$
d\boldsymbol{r}^a = \boldsymbol{J}'_r d\boldsymbol{r} + \boldsymbol{J}'_D d\boldsymbol{D}.
$$

Using $P_{H,t} = \mu W_t$ and $Y_t = N_t$, we can write dividends as:

$$
D_t = (1-\tau_t)\frac{P_{H,t}Y_t - \frac{1}{\mu}P_{H,t}Y_t}{P_t} = (1-\tau_t)\left(1-\frac{1}{\mu}\right)\frac{P_{H,t}}{P_t}Y_t,
$$

or in sequence space:

$$
\begin{aligned}
d\boldsymbol{D} &= (1-\tau_{ss})\left(1-\frac{1}{\mu}\right)(d\boldsymbol{P_H} - d\boldsymbol{P} + d\boldsymbol{Y}) - \left(1-\frac{1}{\mu}\right)d\tau \\
&= (1-\tau_{ss})\left(1-\frac{1}{\mu}\right)\left(d\boldsymbol{Y} - \frac{\alpha}{1-\alpha}d\boldsymbol{Q}\right) - \left(1-\frac{1}{\mu}\right)d\tau.
\end{aligned}
$$

where the last line uses (25). Returning to the consumption function, (30), we then get:

$$
\begin{aligned}
d\boldsymbol{C} &= (1-\tau_{ss})\frac{1}{\mu}\boldsymbol{C}^Z\left(d\boldsymbol{Y} - \frac{\alpha}{1-\alpha}d\boldsymbol{Q}\right) - \frac{1}{\mu}\boldsymbol{C}^Z d\tau + \boldsymbol{C}^r d\boldsymbol{r} + \boldsymbol{C}^r d\boldsymbol{r}_0^a \\
&= (1-\tau_{ss})\frac{1}{\mu}\boldsymbol{C}^Z\left(d\boldsymbol{Y} - \frac{\alpha}{1-\alpha}d\boldsymbol{Q}\right) - \frac{1}{\mu}\boldsymbol{C}^Z d\tau + \left(\boldsymbol{C}^r\boldsymbol{J}'_r + \boldsymbol{C}^r\right)d\boldsymbol{r} + \boldsymbol{C}^r\boldsymbol{J}'_D d\boldsymbol{D}.
\end{aligned}
$$

Upon rearranging and defining

$$\boldsymbol{M} = \left(1 - \frac{1}{\mu}\right) \boldsymbol{C}^r \boldsymbol{J}_D' + \frac{1}{\mu} \boldsymbol{C}^Z,$$
$$\boldsymbol{R} = \left(\boldsymbol{C}^r \boldsymbol{J}_r' + \boldsymbol{C}^r\right),$$

we obtain

$$d\boldsymbol{C} = (1 - \tau_{ss})\boldsymbol{M}\left(d\boldsymbol{Y} - \frac{\alpha}{1-\alpha} d\boldsymbol{Q} - \frac{1}{1-\tau_{ss}} d\boldsymbol{\tau}\right) + \boldsymbol{R} d\boldsymbol{r}.$$

Note that $\tau_{ss} = 0$, so $d\boldsymbol{\tau} = d\boldsymbol{T}$, yielding

$$d\boldsymbol{C} = \boldsymbol{M}\left(d\boldsymbol{Y} - \frac{\alpha}{1-\alpha} d\boldsymbol{Q} - d\boldsymbol{T}\right) + \boldsymbol{R} d\boldsymbol{r}. \tag{31}$$

A.1.4 Main Proof

Linearizing goods market clearing gives

$$dY_t = dC_{H,t} + dC^*_{H,t} + dG_t.$$

Inserting equations (25), (26), and (28) gives

$$\begin{aligned} dY_t &= (1-\alpha) dC_t + \eta \alpha dQ_t + \frac{\eta \alpha}{1-\alpha} dQ_t + dG_t \\ &= (1-\alpha) dC_t + \frac{2-\alpha}{1-\alpha} \alpha \eta dQ_t + dG_t. \end{aligned}$$

Writing this in sequence space and inserting for consumption yields:

$$\begin{aligned} d\boldsymbol{Y} &= (1-\alpha)\left[\boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}) - \frac{\alpha}{1-\alpha} \boldsymbol{M} d\boldsymbol{Q} + \boldsymbol{R} d\boldsymbol{r}\right] + \frac{2-\alpha}{1-\alpha} \alpha \eta d\boldsymbol{Q} + d\boldsymbol{G} \\ &= d\boldsymbol{G} - (1-\alpha)\boldsymbol{M} d\boldsymbol{T} + (1-\alpha)\boldsymbol{R} d\boldsymbol{r} + \left[\frac{2-\alpha}{1-\alpha} \alpha \eta - \alpha \boldsymbol{M}\right] d\boldsymbol{Q} + (1-\alpha)\boldsymbol{M} d\boldsymbol{Y}. \end{aligned}$$

This is the expression from Proposition 1. Before proceeding, we find it useful to rewrite this expression by exploiting some properties of the model.

A.1.5 Mapping between Income and Interest-Rate Jacobians

We now use that $\boldsymbol{R} = -(\boldsymbol{I} - \boldsymbol{M})\boldsymbol{U}$ (see Lemma 1 in Auclert et al., 2021b), along with $d\boldsymbol{Q} = -\boldsymbol{U} d\boldsymbol{r}$ (since $r_{ss} \to 0$) from the linearized real UIP condition. This allows us to rewrite the previous expression as:

$$
\begin{aligned}
\left[\boldsymbol{I}-(1-\alpha)\boldsymbol{M}\right]d\boldsymbol{Y} = &-(1-\alpha)(\boldsymbol{I}-\boldsymbol{M})\,\boldsymbol{U}d\boldsymbol{r}-(1-\alpha)\boldsymbol{M}d\boldsymbol{T}\\
&-\left[\frac{2-\alpha}{1-\alpha}\alpha\eta-\alpha\boldsymbol{M}\right]\boldsymbol{U}d\boldsymbol{r}+d\boldsymbol{G},
\end{aligned}
$$

or, after collecting terms:

$$
\left[\boldsymbol{I}-(1-\alpha)\boldsymbol{M}\right]d\boldsymbol{Y} = -\left\{(1-\alpha)\boldsymbol{I}-\boldsymbol{M}+\frac{2-\alpha}{1-\alpha}\alpha\eta\right\}\boldsymbol{U}d\boldsymbol{r}-(1-\alpha)\boldsymbol{M}d\boldsymbol{T}+d\boldsymbol{G}.
$$

A.1.6 Primary Deficits

It is useful to state the Keynesian cross in a term related to primary deficits $d\boldsymbol{G}-d\boldsymbol{T}$. To this end, add and subtract $(1-\alpha)\boldsymbol{M}d\boldsymbol{G}$ from the previous expression, and rewrite:

$$
\begin{aligned}
\left[\boldsymbol{I}-(1-\alpha)\boldsymbol{M}\right]d\boldsymbol{Y} = &-\left\{(1-\alpha)-\boldsymbol{M}+\frac{2-\alpha}{1-\alpha}\alpha\eta\right\}\boldsymbol{U}d\boldsymbol{r}\\
&+\left[\boldsymbol{I}-(1-\alpha)\boldsymbol{M}\right]d\boldsymbol{G}+(1-\alpha)\boldsymbol{M}\left(d\boldsymbol{G}-d\boldsymbol{T}\right).
\end{aligned}
$$

Upon defining $\mathcal{G}=\left[\boldsymbol{I}-(1-\alpha)\boldsymbol{M}\right]^{-1}$, we can write this as:

$$
d\boldsymbol{Y} = -\mathcal{G}\left\{(1-\alpha)-\boldsymbol{M}+\frac{2-\alpha}{1-\alpha}\alpha\eta\right\}\boldsymbol{U}d\boldsymbol{r}+d\boldsymbol{G}+(1-\alpha)\mathcal{G}\boldsymbol{M}\left(d\boldsymbol{G}-d\boldsymbol{T}\right). \tag{32}
$$

This expression holds for a general household problem, i.e. nests both HANK and RANK. Let us now consider the RANK model. Linearize the Euler equation:

$$
dC_{t+1} = dC_t + \frac{dr^a_{t+1}}{1+r_{ss}},
$$

using that $\beta=(1+r_{ss})^{-1}$ and $C_{ss}=1$. Solving for dC_t and substituting recursively then gives

$$
dC_t = -\frac{1}{1+r_{ss}}\sum_{s=1}^{\infty} dr^a_{t+s},
$$

where we used that $C_\infty = C_{ss}$ so $dC_\infty = 0$. Use $r^a_{t+1}=r_t$ for $t=0,1,\dots$ and $r_{ss}\to 0$:

$$
dC_t = -\sum_{s=0}^{\infty} dr_{t+s}.
$$

On sequence-space form:

$$d\boldsymbol{C} = -\boldsymbol{U}d\boldsymbol{r},$$

i.e., $\boldsymbol{M}^{\mathrm{RA}} = \mathbf{0}$, which in turn implies that $\mathcal{G} = \boldsymbol{I}$ in the RANK model. We then obtain the following two HANK and RANK expressions directly from (32):

$$d\boldsymbol{Y}^{\mathrm{HA}} = d\boldsymbol{G} - \mathcal{G}\left\{(1-\alpha) - \boldsymbol{M} + \frac{2-\alpha}{1-\alpha}\alpha\eta\right\}\boldsymbol{U}d\boldsymbol{r}^{\mathrm{HA}} + (1-\alpha)\mathcal{G}\boldsymbol{M}\left(d\boldsymbol{G} - d\boldsymbol{T}\right), \quad (33)$$

$$d\boldsymbol{Y}^{\mathrm{RA}} = d\boldsymbol{G} - \left\{(1-\alpha) + \frac{2-\alpha}{1-\alpha}\alpha\eta\right\}\boldsymbol{U}d\boldsymbol{r}^{\mathrm{RA}}, \quad (34)$$

where we just write “$\boldsymbol{M}$” instead of “$\boldsymbol{M}^{\mathrm{HA}}$” (and similarly for $\mathcal{G}$).

A.2 Proof of Proposition 2

Begin by noting that

$$\mathcal{G} \equiv \left[\boldsymbol{I} - (1-\alpha)\boldsymbol{M}\right]^{-1} = \boldsymbol{I} + (1-\alpha)\boldsymbol{M} + (1-\alpha)^2\boldsymbol{M}^2 + \ldots.$$

Post-multiplying by $(1-\alpha)\boldsymbol{M}$ gives

$$(1-\alpha)\mathcal{G}\boldsymbol{M} = (1-\alpha)\boldsymbol{M} + (1-\alpha)^2\boldsymbol{M}^2 + \cdots = \mathcal{G} - \boldsymbol{I}.$$

Thus,

$$\mathcal{G}\boldsymbol{M} = -\frac{1}{1-\alpha}(\boldsymbol{I} - \mathcal{G}). \quad (35)$$

Inserting this into (32) gives

$$\begin{aligned} d\boldsymbol{Y} &= -\mathcal{G}\left\{(1-\alpha) - \boldsymbol{M} + \frac{2-\alpha}{1-\alpha}\alpha\eta\right\}\boldsymbol{U}d\boldsymbol{r} + d\boldsymbol{G} - (\boldsymbol{I} - \mathcal{G})\left(d\boldsymbol{G} - d\boldsymbol{T}\right) \\ &= -\mathcal{G}\left\{(1-\alpha) - \boldsymbol{M} + \frac{2-\alpha}{1-\alpha}\alpha\eta\right\}\boldsymbol{U}d\boldsymbol{r} + \mathcal{G}d\boldsymbol{G} + (\boldsymbol{I} - \mathcal{G})d\boldsymbol{T}. \end{aligned}$$

Use now the tax rule from (9) to get that

$$d\boldsymbol{Y} = -\mathcal{G}\left\{(1-\alpha) - \boldsymbol{M} + \frac{2-\alpha}{1-\alpha}\alpha\eta\right\}\boldsymbol{U}d\boldsymbol{r} + \left[\mathcal{G} + \phi_G(\boldsymbol{I} - \mathcal{G})\right]d\boldsymbol{G}.$$

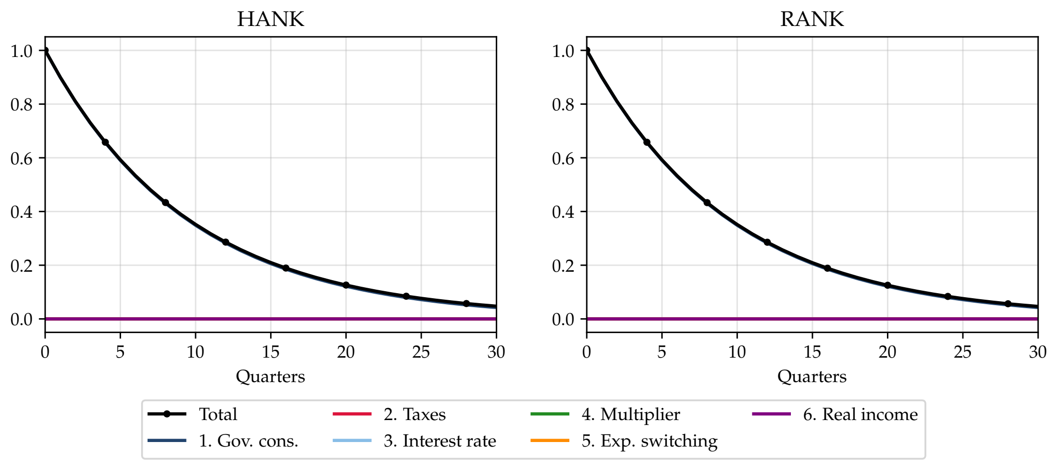


Figure A.1: Decomposition of output ($d\boldsymbol{Y}$) when $\alpha \to 1$ and $\phi_Y = 0$

Note: The figure shows the decomposition of the response of output from Proposition 1 for the HANK and RANK models when $\alpha \to 1$ and $\phi_Y = 0$.

Consider first the case of an active monetary policy, i.e., $\phi_Y > 0$. Use the monetary policy rule in (10) to get that

$$\begin{aligned} d\boldsymbol{Y} &= -\mathcal{G}\left\{(1-\alpha) - \boldsymbol{M} + \frac{2-\alpha}{1-\alpha}\alpha\eta\right\}\boldsymbol{U}\phi_Y d\boldsymbol{Y} + \left[\mathcal{G} + \phi_G(\boldsymbol{I} - \mathcal{G})\right] d\boldsymbol{G} \\ &= \left\{\boldsymbol{I} + \mathcal{G}\left\{(1-\alpha) - \boldsymbol{M} + \frac{2-\alpha}{1-\alpha}\alpha\eta\right\}\boldsymbol{U}\phi_Y\right\}^{-1}\left[\mathcal{G} + \phi_G(\boldsymbol{I} - \mathcal{G})\right] d\boldsymbol{G}. \end{aligned} \tag{36}$$

When $\alpha \to 1$, the term in curly brackets tends to ∞ (provided $\phi_Y > 0$), so that its inverse converges to the null matrix, while the term in the square bracket converges to $\boldsymbol{I}$ because $\mathcal{G}$ converges to $\boldsymbol{I}$. Thus, the entire right-hand side converges to $\boldsymbol{0}$, i.e. $d\boldsymbol{Y} \to \boldsymbol{0}$ as $\alpha \to 1$. In other words, when the real rate responds ($\phi_Y > 0$), there is full crowding out, such that output does not react, $dY_t = 0$ for all t.

Consider now passive monetary policy, i.e. $\phi_Y = 0$. Insert this in (36) to get that

$$d\boldsymbol{Y} = \left[\mathcal{G} + \phi_G(\boldsymbol{I} - \mathcal{G})\right] d\boldsymbol{G}.$$

When $\alpha \to 1$ it follows that $\mathcal{G} \to 1$ by its definition, so $d\boldsymbol{Y} \to d\boldsymbol{G}$. In this case, there is a fiscal multiplier of 1 at all points in time, $dY_t = dG_t$ for all t (see Figure A.1).

Figure A.2 shows how the multipliers of the RANK and HANK models converge as α increases. In the limiting case of $\alpha \to 1$, output converges to 0 at all points in time, i.e. $dY_t^{\text{RA}} = dY_t^{\text{HA}} = 0$ for all t. More generally, it is seen that the two models feature very similar multipliers for values of α significantly smaller than 1, including for realistic

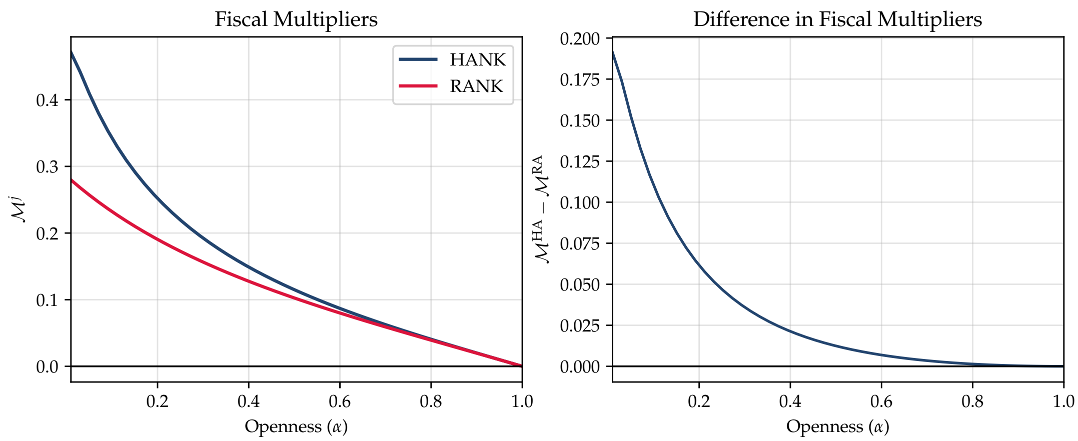


Figure A.2: Fiscal multipliers in HANK and RANK as a function of openness

Note: The figure shows the present-value cumulative multiplier in the HANK and the RANK models (left panel), and the difference between them (right panel), as functions of the openness parameter, α. The figure is based on $\phi_G = 0.1$, $\eta = 2$, and $\phi_Y = 0.25$. The cumulative fiscal multiplier is computed over 5 years, i.e. $T = 20$ in (14).

values of α around 0.5.

A.3 Proof of Proposition 3

Without making any assumptions regarding government financing, we can simply use the linearized monetary policy rule—which reads $d\boldsymbol{r} = \phi_Y d\boldsymbol{Y}$—directly in (33) and (34) to obtain:

$$\begin{aligned} d\boldsymbol{Y}^{\mathrm{HA}} &= \Theta^{\mathrm{HA}} \left\{ \left[\boldsymbol{I} + \mathcal{G}(1-\alpha)\boldsymbol{M} \right] d\boldsymbol{G} - \mathcal{G}(1-\alpha)\boldsymbol{M} d\boldsymbol{T} \right\}, \\ d\boldsymbol{Y}^{\mathrm{RA}} &= \Theta^{\mathrm{RA}} d\boldsymbol{G}, \end{aligned}$$

where

$$\begin{aligned} \Theta^{\mathrm{HA}} &\equiv \left[\boldsymbol{I} + \mathcal{G} \left\{ (1-\alpha) - \boldsymbol{M} + \frac{2-\alpha}{1-\alpha}\alpha\eta \right\} \boldsymbol{U}\phi_Y \right]^{-1}, \\ \Theta^{\mathrm{RA}} &\equiv \left[\boldsymbol{I} + \left\{ (1-\alpha) + \frac{2-\alpha}{1-\alpha}\alpha\eta \right\} \boldsymbol{U}\phi_Y \right]^{-1}. \end{aligned}$$

In the limit $\eta \to \infty$, both of the matrices Θ^{HA} and Θ^{RA} converge to null matrices when $\phi_Y > 0$, so we obtain $d\boldsymbol{Y}^{\mathrm{RA}} = d\boldsymbol{Y}^{\mathrm{HA}} = \boldsymbol{0}$.

A.4 Proof of Proposition 4

When we impose a balanced budget, $d\boldsymbol{T} = d\boldsymbol{G}$, (33) simplifies to

$$d\boldsymbol{Y} = d\boldsymbol{G} - \mathcal{G}\left\{(1-\alpha) - \boldsymbol{M} + \frac{2-\alpha}{1-\alpha}\alpha\eta\right\}\boldsymbol{U}d\boldsymbol{r}.$$

Comparing this expression with (34), it is seen that $d\boldsymbol{Y}^{\text{HA}} = d\boldsymbol{Y}^{\text{RA}}$ if the matrices multiplying $d\boldsymbol{r}$ are equal, since $d\boldsymbol{r}$ is only a function of $d\boldsymbol{Y}$. Define these matrices as

$$\begin{aligned}
\boldsymbol{\Omega}^{\text{HA}} &\equiv -\mathcal{G}\left\{(1-\alpha) - \boldsymbol{M} + \frac{2-\alpha}{1-\alpha}\alpha\eta\right\}\boldsymbol{U}, \\
\boldsymbol{\Omega}^{\text{RA}} &\equiv -\left\{(1-\alpha) + \frac{2-\alpha}{1-\alpha}\alpha\eta\right\}\boldsymbol{U}.
\end{aligned}$$

Subtract the two matrices from each other to get:

$$\begin{aligned}
\boldsymbol{\Omega}^{\text{RA}} - \boldsymbol{\Omega}^{\text{HA}} &= \left\{(1-\alpha) - \mathcal{G}(1-\alpha) + \frac{2-\alpha}{1-\alpha}\alpha\eta - \mathcal{G}\frac{2-\alpha}{1-\alpha}\alpha\eta + \mathcal{G}\boldsymbol{M}\right\}\boldsymbol{U} \\
&= -\left\{[I-\mathcal{G}]\left(1-\alpha+\frac{2-\alpha}{1-\alpha}\alpha\eta\right) + \mathcal{G}\boldsymbol{M}\right\}\boldsymbol{U}. \qquad (37)
\end{aligned}$$

Now use the expression for $\mathcal{G}\boldsymbol{M}$ from (35) to obtain

$$\boldsymbol{\Omega}^{\text{RA}} - \boldsymbol{\Omega}^{\text{HA}} = -[I-\mathcal{G}]\left(1-\alpha+\frac{2-\alpha}{1-\alpha}\alpha\eta - \frac{1}{1-\alpha}\right)\boldsymbol{U}.$$

Clearly, one way to obtain $\boldsymbol{\Omega}^{\text{RA}} = \boldsymbol{\Omega}^{\text{HA}}$ and therefore $d\boldsymbol{Y}^{\text{RA}} = d\boldsymbol{Y}^{\text{HA}}$ is if

$$1-\alpha+\frac{2-\alpha}{1-\alpha}\alpha\eta - \frac{1}{1-\alpha} = 0.$$

If $\alpha \in (0,1)$, the unique solution for η is

$$\eta = 1.$$

This proves that $d\boldsymbol{Y}^{\text{RA}} = d\boldsymbol{Y}^{\text{HA}}$ when $\eta = 1$ and $d\boldsymbol{G} = d\boldsymbol{T}$. One may verify this solution simply by plugging $\eta = 1$ into (33) and (34) to obtain

$$d\boldsymbol{Y}^{\text{RA}} = d\boldsymbol{Y}^{\text{HA}} = d\boldsymbol{G} - \frac{1}{1-\alpha}\boldsymbol{U}d\boldsymbol{r}.$$

A.5 Intuition behind Proposition 4

To build intuition for the result in Proposition 4, it is useful to recall the national accounts identity for GDP, measured in units of the domestic CPI:

$$\frac{P_{H,t}}{P_t}Y_t = C_t + \frac{P_{H,t}}{P_t}G_t + NX_t,$$

with NX_t defined as in Section 2. Linearizing, and writing this in sequence space:

$$d\boldsymbol{Y} = d\boldsymbol{C} + d\boldsymbol{G} + \frac{\alpha}{1-\alpha}d\boldsymbol{Q} + d\boldsymbol{NX}, \tag{38}$$

where we have used that $d\left(\frac{P_{H,t}}{P_t}\right) = -\frac{\alpha}{1-\alpha}dQ_t$. With a unitary trade elasticity, the income and substitution effects from changes in the real exchange rate exactly offset each other, and net exports remain constant, $d\boldsymbol{NX} = 0$. The economy thus behaves almost like a closed economy: Changes in domestic real GDP comes from either private consumer demand, public spending, or movements in relative prices:

$$d\boldsymbol{Y} = d\boldsymbol{C} + d\boldsymbol{G} + \frac{\alpha}{1-\alpha}d\boldsymbol{Q}.$$

Figure A.3 displays the contribution from each of the channels to output in the HANK and RANK model. We know from Proposition 4 that the output responses are equal in the two models. Notably, Figure A.3 shows that the contributions from each channel in (38) are also identical across the models. We can further decompose the response of consumption in the two models into contributions from labor income, Z_t, and the real interest rate, r_t, see Figure A.4. In the HANK model, the total consumption response is driven by both the income and intertemporal substitution effects, although the former dominates, whereas in the RANK model, the entire response is driven by intertemporal substitution. Nonetheless, the aggregate response of consumption is the same in the two models. This is effectively an application of the equivalence result for monetary policy found in Werning (2015), who showed that the aggregate effects of monetary policy in a closed economy are identical in HANK and RANK when i) the intertemporal elasticity of substitution is 1, and ii) individual income in HANK is proportional to aggregate income. Both conditions apply here. To summarize, the equivalence arises due to three different assumptions:

- The assumption of period-by-period financing implies that the multipliers in HANK and RANK would be equal in the *absence* of a monetary policy reaction in a *closed economy*, see Auclert et al. (2023).

- Given that the economy is open, active monetary policy affects the real exchange rate, and therefore net exports. The assumption of a unitary trade elasticity $\eta = 1$ implies that net exports do not move, and the economy resembles a closed economy in this regard (see Cole and Obstfeld, 1991).
- The assumption of a unitary intertemporal elasticity of substitution then implies that we can leverage the result in Werning (2015) to obtain equivalence between HANK and RANK despite having active monetary policy.

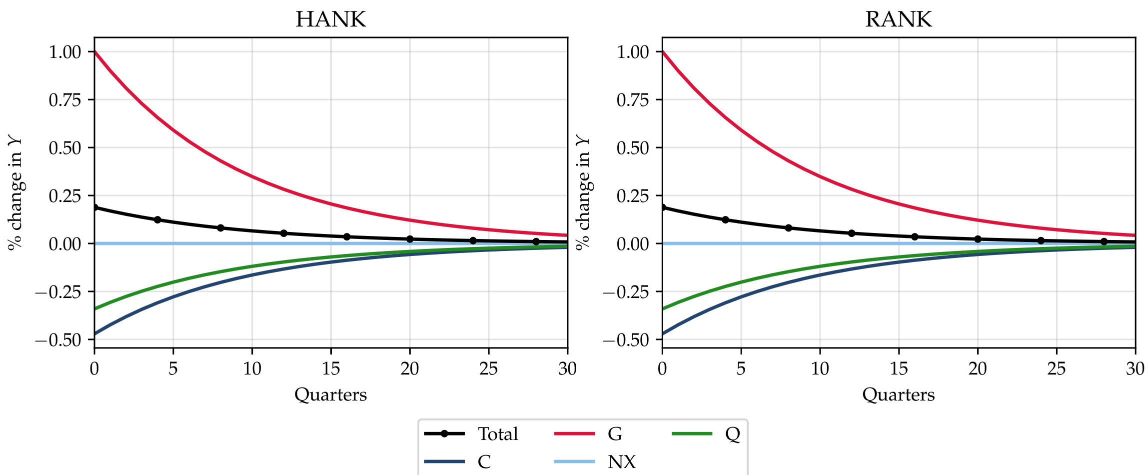


Figure A.3: Decomposition of output using (38) with $\eta = 1$

Note: The figure shows the decomposition of the response of output proposed in (38) for the HANK and RANK models when $\eta = 1$ and the government maintains a balanced budget.

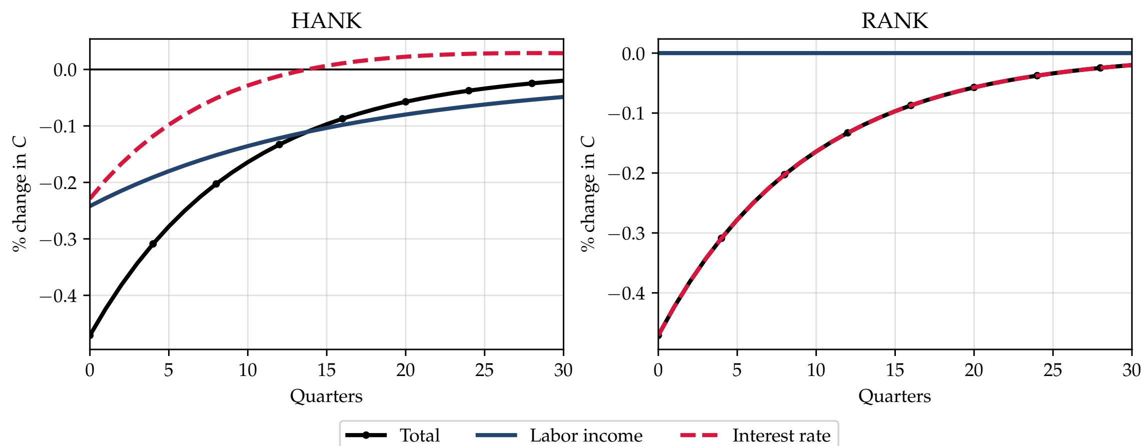
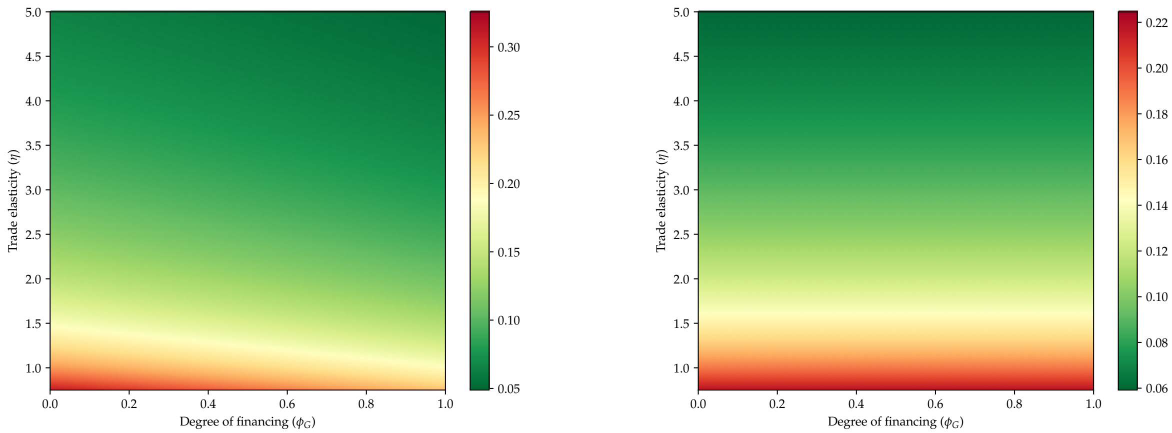


Figure A.4: Decomposition of consumption in HANK and RANK with $\eta = 1$

Note: The figure shows the decomposition of the response of consumption for the HANK and RANK models when $\eta = 1$ and the government maintains a balanced budget.



(a) HANK

(b) RANK

Figure A.5: Fiscal multipliers in the stylized model

Note: The figure shows the cumulative fiscal multipliers in the HANK and RANK models in the stylized setting, i.e. $\mathcal{M}^{\text{HA}}$ (left panel) and $\mathcal{M}^{\text{RA}}$ (right panel). The cumulative fiscal multipliers are computed over 5 years, i.e. $T = 20$ in (14).

A.6 Multipliers in the Stylized Model

Figure A.5 reports the cumulative fiscal multipliers underlying Figure 4 separately for the HANK and RANK models. The figure shows that the level of the fiscal multiplier in each of the two models is always moderate but non-negative.

B Numerical Appendix

This appendix provides some details behind the quantitative analysis in Section 4.

B.1 Model with Capital

We add capital to the model to see if this changes our results. Domestic firms produce using Cobb-Douglas technology with labor and lagged capital as inputs:

$$Y_t = N_t^{1-\alpha_K} K_{t-1}^{\alpha_K}.$$

Labor is rented from households at wage rate w_t, while capital is rented from capital firms at price r_t^K. Firms optimize subject to monopolistic competition. The first-order conditions are:

$$w_t = (1-\alpha_K)\frac{1}{\mu}\frac{P_{H,t}}{P_t}\frac{Y_t}{N_t},$$
$$r_{t+1}^K = \alpha_K \frac{1}{\mu}\frac{P_{H,t+1}}{P_{t+1}}\frac{Y_{t+1}}{K_t}.$$

Firms' (pre-tax) profits are then given by

$$D_t^f = \frac{P_{H,t}}{P_t} Y_t - w_t N_t - r_t^K K_{t-1} - F.$$

Capital firms invest and rent capital to final goods firms. Their profits are given by

$$D_t^k = r_t^K K_{t-1} - I_t - F_k,$$

where F_k is a fixed cost set to ensure that profits are zero for the capital firms in steady state. Capital firms maximize the discounted sum of profits facing a virtual adjustment cost of

$$f\left(\frac{I_t}{I_{t-1}}\right) = \frac{\phi^I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2,$$

subject to the law of motion for capital

$$K_t = \left(1-\delta^K\right) K_{t-1} + I_t.$$

Denoting by Q_t^I the Lagrange multiplier on the capital accumulation constraint, the

Lagrangian can be stated as:

$$\mathcal{L} = \sum_{t=0}^{\infty} d_t \left\{ \left[r_t^K K_{t-1} - I_t - f(I_t/I_{t-1}) I_t \right] - Q_t^I \left\{ K_t - \left(1 - \delta^K\right) K_{t-1} - I_t \right\} \right\},$$

where

$$\begin{aligned} d_t &\equiv (1 + r_0)^{-1} \ldots (1 + r_{t-1}), \quad \text{for } t = 1, 2, \ldots \\ d_0 &\equiv 1, \end{aligned}$$

is the discount factor. The derivative w.r.t. investment is

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial I_t} &= -d_t - d_t \frac{\partial f_t}{\partial I_t} I_t - d_t f_t + d_t Q_t^I - d_{t+1} \frac{\partial f_{t+1}}{\partial I_t} I_{t+1}, \\ &= -d_t - d_t \phi^I \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} - d_t \frac{\phi^I}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \\ &+ d_t Q_t^I + d_{t+1} \phi^I \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2, \end{aligned}$$

where we have defined the short-hand $f_t \equiv f(I_t/I_{t-1})$, and we have used that

$$\begin{aligned} \frac{\partial f_t}{\partial I_t} &= \phi^I \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{1}{I_{t-1}}, \\ \frac{\partial f_{t+1}}{\partial I_t} &= -\phi^I \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2. \end{aligned}$$

Setting this equal to zero and dividing by $-d_t$ yields

$$\begin{aligned} 0 = 1 + \phi^I \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} + \frac{\phi^I}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \\ - Q_t^I - \frac{1}{1 + r_t} \phi^I \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2. \end{aligned}$$

Rearrange to get:

$$\begin{aligned} 1 + \phi^I \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} + \frac{\phi^I}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \\ = Q_t^I + \frac{1}{1 + r_t} \phi^I \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2. \end{aligned}$$

The derivative w.r.t. capital is:

$$\frac{\partial \mathcal{L}}{\partial K_t} = d_{t+1} r^K_{t+1} - d_t Q^I_t + d_{t+1}(1+\delta^K) Q^I_{t+1}.$$

Set this to zero and divide by d_t:

$$Q^I_t = \frac{1}{1+r_t}\left[(1-\delta^K)Q^I_{t+1} + r^K_{t+1}\right].$$

In steady state, the first FOC implies that

$$Q^I_{ss} = 1.$$

The second FOC then implies that

$$r^K_{ss} = r_{ss} + \delta^K.$$

We assume that overall capital is a CES good assembled from domestic capital goods (owned by domestic firms) and imported capital goods using CES technology identical to the one used by households. With a dynamic elasticity of substitution this gives:

$$I_{F,t} = \omega_I \left(\hat{p}^I_{F,t}\right)^{-\eta} I_t, \quad \hat{p}^I_{F,t} = \left(\hat{p}^I_{F,t-1}\right)^{\rho_\eta} \left(\frac{P^I_{F,t}}{P^I_t}\right)^{1-\rho_\eta},$$
$$I_{H,t} = (1-\omega_I) \left(\hat{p}^I_{H,t}\right)^{-\eta} I_t, \quad \hat{p}^I_{H,t} = \left(\hat{p}^I_{H,t-1}\right)^{\rho_\eta} \left(\frac{P^I_{H,t}}{P^I_t}\right)^{1-\rho_\eta},$$

where ω_I is the steady-state share of investment goods imported from abroad. We assume that the price of imported final goods and investment goods are equal such that $P^I_{F,t} = P_{F,t}$. The price of domestic investment goods is simply $P^I_{H,t} = P_{H,t}$.
Goods market clearing is

$$Y_t = C_{H,t} + C^*_{H,t} + G_t + I_{H,t} + \frac{P_t}{P_{H,t}} F + \frac{P_t}{P_{H,t}} F_k.$$

The adjustment cost does not appear here, as it is virtual. For the calibration, we set a standard capital share of $\alpha_K = 0.33$, while we set $\delta^K = 0.05/4$ and $\phi^I = 9.6$ following Auclert et al. (2020). We assume that the import share of investment goods equals the import of share of final goods, $\omega_I = \alpha$, since investment goods display a high import content empirically, see e.g. Christiano et al. (2011), who report an import content in Sweden of roughly 43%. We calibrate the level of these import shares to match the same overall level of imports to GDP as in the baseline model. This gives $\omega_I = \alpha = 0.68$.

B.2 Additional Sensitivity Checks: Results

In the following, we report the results of the additional robustness checks conducted in Section 4.4.4. We first consider the IRFs under alternative assumptions about the monetary policy rule, trade elasticity, and source of nominal rigidities in Figure A.6. To shed further light on the case of a fixed exchange rate, we then display a larger set of IRFs from this case in Figure A.7.

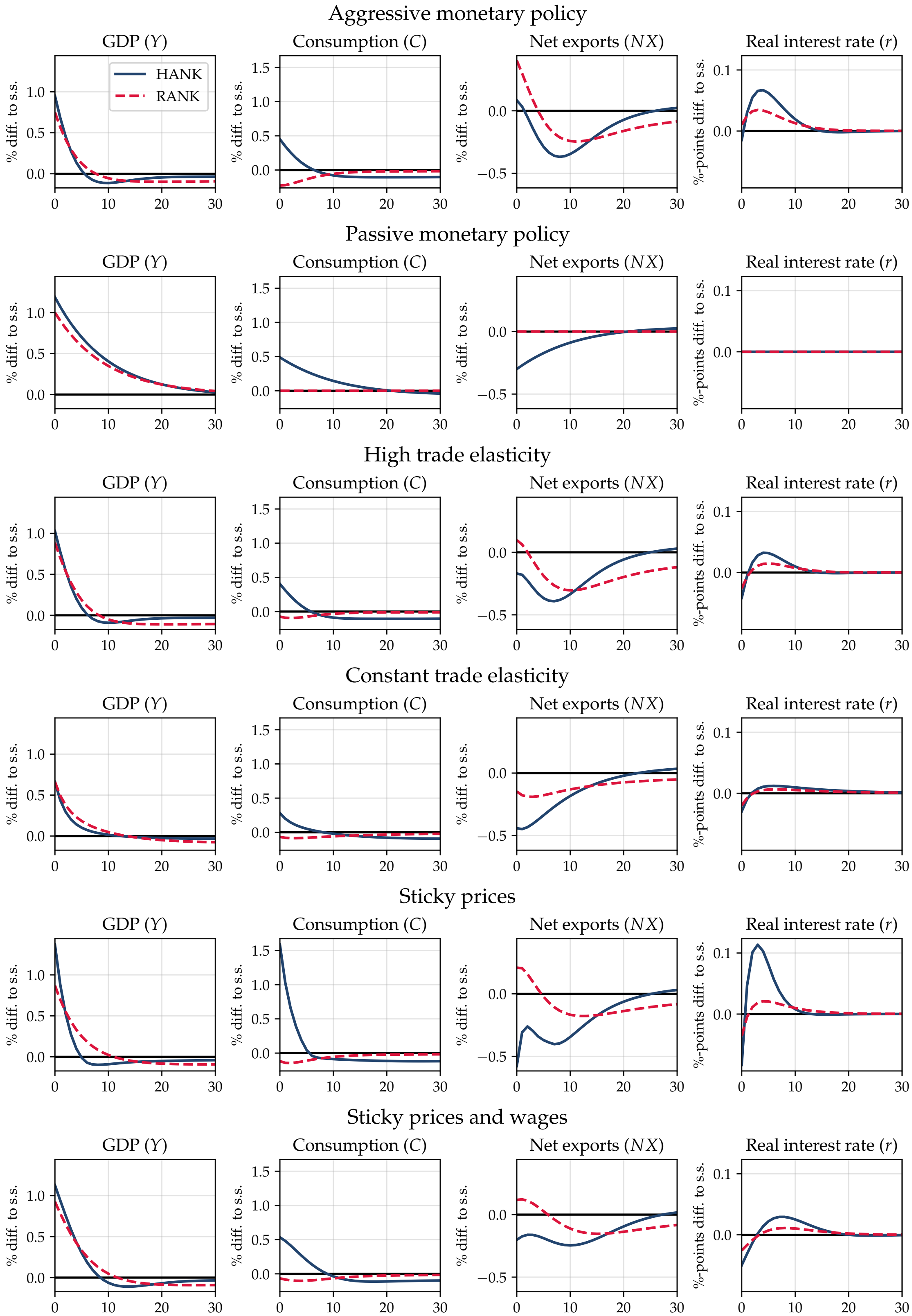


Figure A.6: IRFs to a government consumption shock: Sensitivity Checks

Note: The figure shows impulse response functions to a government consumption shock under various model assumptions regarding monetary policy (rows 1-2), the trade elasticity (rows 3-4), and price rigidities (rows 5-6).

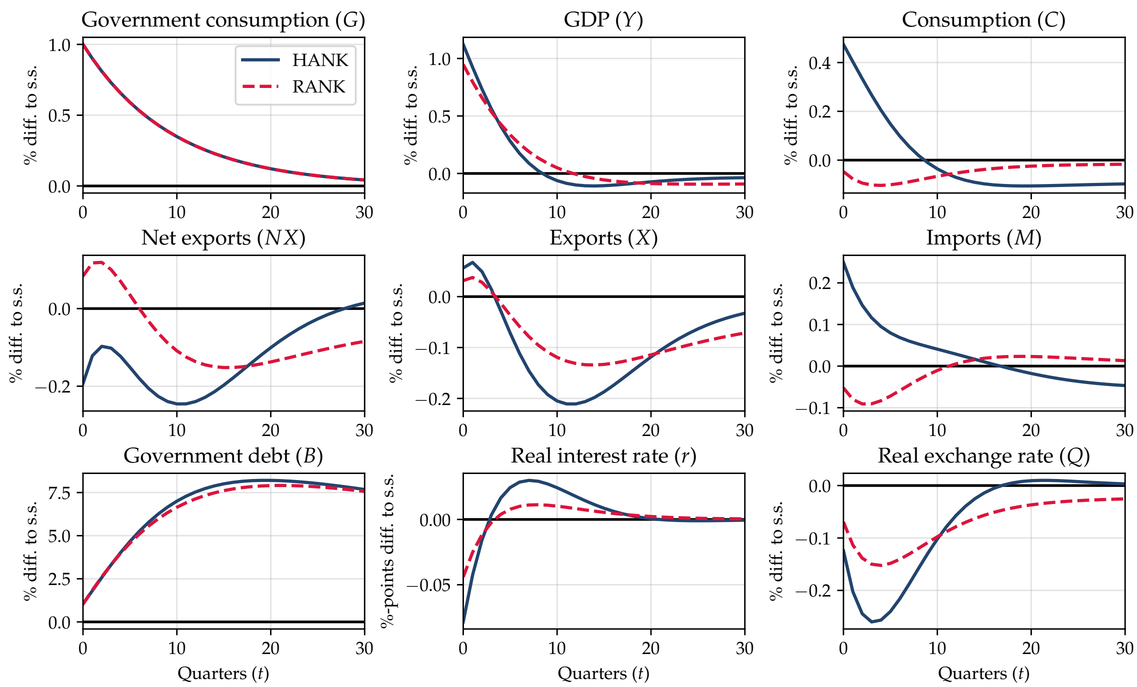


Figure A.7: IRFs to a government consumption shock with a fixed exchange rate

Note: The figure shows impulse response functions to a government consumption shock under a fixed exchange rate regime.