ISyE 4133 Advanced Optimization
Fall 2010
Course Project

THIS IS NOT AN EASY PROJECT. DO NOT WAIT TO START WORKING ON IT.

1. Overview

A telecommunications company is building networks that require the installation of expensive equipment (the major components cost at least $1 million). There are many places where the equipment might be installed, and by selectively choosing where to put the most-expensive pieces of equipment, the company might be able to reduce its expenditures by millions of dollars. The company would therefore like to know two things:

(1) What is the best (least-costly) configuration of equipment that you can find, and how much does it cost?

(2) If your best configuration isn't optimal, how much better could an optimal solution be? (I.e., is it worthwhile for them to spend more money to try to find a better solution, or can you prove that your solution is good enough that it's not worth spending money on an outside consultant?)

There are two different networks for which the company needs a solution: the Caldata network and the Barry network. Your task is to answer the two questions above for both networks.

1.1. Team Composition

Projects must be done by teams of 3-5 people. It is your responsibility to find teammates.

1.2. Deliverables and Due Dates

Dec 7 and 9 (in class): Each team will make an in-class presentation. Each team's presentation should be 5-8 minutes long, and focus on two things: (1) the answers to the two questions above, and (2) a description of the optimization techniques used to get those answers.

Dec 10 (by the end of the day): On t-square, submit (1) a writeup explaining your answers to the two questions above, and showing the details of the optimization techniques you used to get those answers; (2) the slides from your in-class presentation; and (3) Xpress code (and/or other code) that, when run, generates your team's solutions.

1.3. Grading

One main criterion for grading is the quality (in dollars) of your answers. Less-costly configurations and better bounds are worth more for your grade. A second important criterion is the correctness of your methods; if you've used optimization techniques incorrectly, or implemented them incorrectly, it'll obviously hurt your grade significantly. Finally, a third, less-important, criterion is how easily I can understand your write-up and presentation.

2. The ADM Selection and Routing Problem

The company is building two large SONET telecommunications networks in the western United States. Each network has a small number of hubs (you can think of these as cities), labeled by letters (because the actual locations are confidential). Hubs are connected by arcs, and in the network architecture the company is using, arcs are created in sets of parallel rings. For example, in Figure 1 (the Caldata network), there are 14 hubs (labeled A through L), and each hub has at least two rings going through it. The upper-left-hand rings, for example, connect hubs A, B, E, and C; as shown in the figure, there are four parallel rings that make up this cycle.

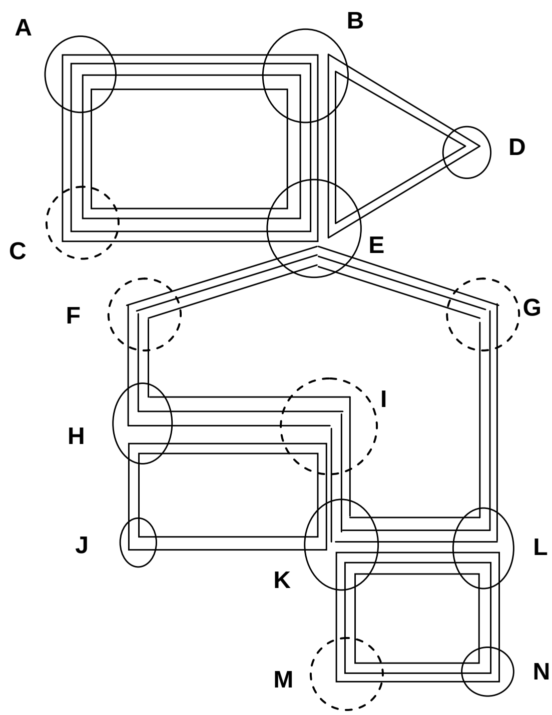

Figure 1. Plan for the company's Caldata network. Hubs with a solid circle (ABDEHJKLN) have a BBDX installed, and hubs with a dashed circle (CFGIM) do not.

As data flows from one hub to another in this network, it will often have to switch from one ring to another. For example, to go from A to D, the data will need to switch from one of the ABEC rings to one of the BED rings. Ring-switching is only possible at hubs that have a broadband digital cross-connect (BBDX) installed. In the figure above, all hubs with a solid circle (A,B,D,E,H,J,K,L,N) have a BBDX installed. Hubs with a dashed circle (C,F,G,I,M) do not have a BBDX installed. BBDX installation is based on equipment available from other networks (and each BBDX costs tens of millions of dollars), so the locations of BBDXes are fixed; they can't be changed.

Data can only enter or leave the network at hubs. Moreover, when data enters or leaves the network, it must do so on a specific ring. For example, data can enter the network at hub A on the first ring, or on the second ring, or the third ring, or the fourth ring. At hub B, there are 6 choices: data could enter on one of the four ABEC rings, or it could enter on one of the two BDE rings.

However, data can only enter or leave a ring at a hub if the ring has an add-drop multiplexer (ADM) installed at that hub. This restriction applies not only to data entering and leaving the network, but also to data switching from one ring to another (so in order for data to be able to switch rings at a hub, the hub needs to have a BBDX *and* the two rings both need to have an ADM; see Figure 2). The primary questions to be answered in this project are the locations of ADMs (at which hubs and on which rings should they be installed?).

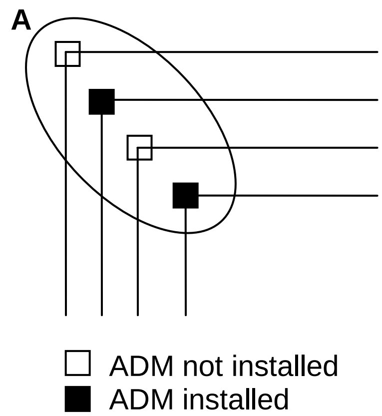

Figure 2. Hub A has four potential ADM locations, one on each ring. In this figure, ADMs are installed on the second and fourth rings (as shown by the dark squares). In this configuration, data can only enter and leave the network at Hub A at the second and fourth rings (because they have ADMs installed). Similarly, at Hub A, data can only transfer through the BBDX between the second and fourth rings. However, if another ADM was installed on the first or third ring, then data could enter, leave, and transfer on that ring as well.

In the Caldata network, each arc can carry a total of 24,000 units of data flow (sum of flow in the two directions). Because ADMs are attached to two arcs, one in each direction, they can handle a total of 48,000 units of data flow. BBDXes are large enough that they can handle as much flow as necessary (but note that because of the ring capacities, there would never be any reason to send more than 48,000 units of flow from one ADM to another at a BBDX). Figure 3 shows an example of these capacities.

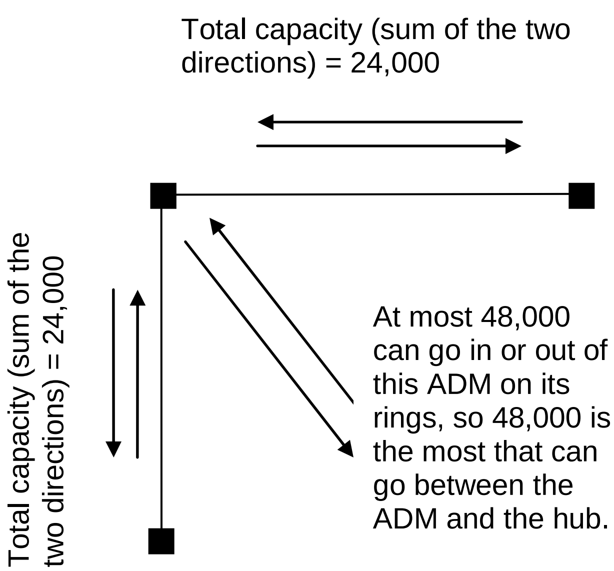

Figure 3. Example of the arc capacities and the *de facto* capacity of arcs between an ADM and its hub in the Caldata network. At most 24,000 units can flow in the left/right directions and at most 24,000 units can flow in the up/down direction, so the ADM will never need to handle more than a total of 48,000 units.

Each ADM costs $1,000,000 per year (this includes operating cost and prorated installation cost), so the company understandably does not want to install an ADM at every hub on every ring. However, they need to install enough ADMs that all of the network's demand can be met. Table 1 shows the hub-to-hub demand that the company expects its Caldata network.

Origin	Destination	Demand
A	B	38,000
A	D	7,000
B	K	13,000
B	D	19,000
B	C	1,000
B	A	42,000
B	N	31,000
B	E	7,000
B	F	6,000
B	J	9,000
B	H	2,000
K	I	1,000
K	M	2,000
K	A	11,000
K	N	32,000
K	J	14,000
K	H	6,000
K	L	6,000
D	C	1,000
D	A	12,000
D	N	1,000
D	J	2,000
M	J	1,000
C	A	1,000
A	N	27,000
A	E	5,000
A	J	1,000
N	J	20,000
N	L	11,000
E	G	3,000
F	J	2,000
J	H	4,000
J	L	2,000

Table 1. Expected Caldata network demand

In addition to considering the cost of ADMs, the company also needs to consider the cost of data flow. They estimate that because of maintenance, upkeep, etc., the annual cost per-unit of data using an ADM to enter or leave the network is $2, the annual cost per unit of data using a BBDX to change rings is a total of $8 ($2 per ADM used, plus $4 for the BBDX), and the annual cost per unit of data flow on each arc for the Caldata network is shown in Table 2. (Note that costs are symmetric, so, for example, the flow cost from K to M is the same as the flow cost from M to K.)

Hub 1	Hub 2	Cost per unit of flow
K	M	1
M	N	1
N	L	1
L	K	1
J	H	1
H	I	1
I	K	1
K	J	1
L	G	2
G	E	3
E	F	3
F	H	2
B	D	1
D	E	1
E	B	1
B	A	1
A	C	1
C	E	1

Table 2. Flow costs for the Caldata network

In addition to the Caldata network (Figure 1 and Tables 1 and 2), the company is planning to build a second network, called the Barry network (see Figure 4 and Tables 3 and 4). The fibers that will be used in the Barry network are twice as large, so they can carry 48,000 units of flow (double that of Caldata).

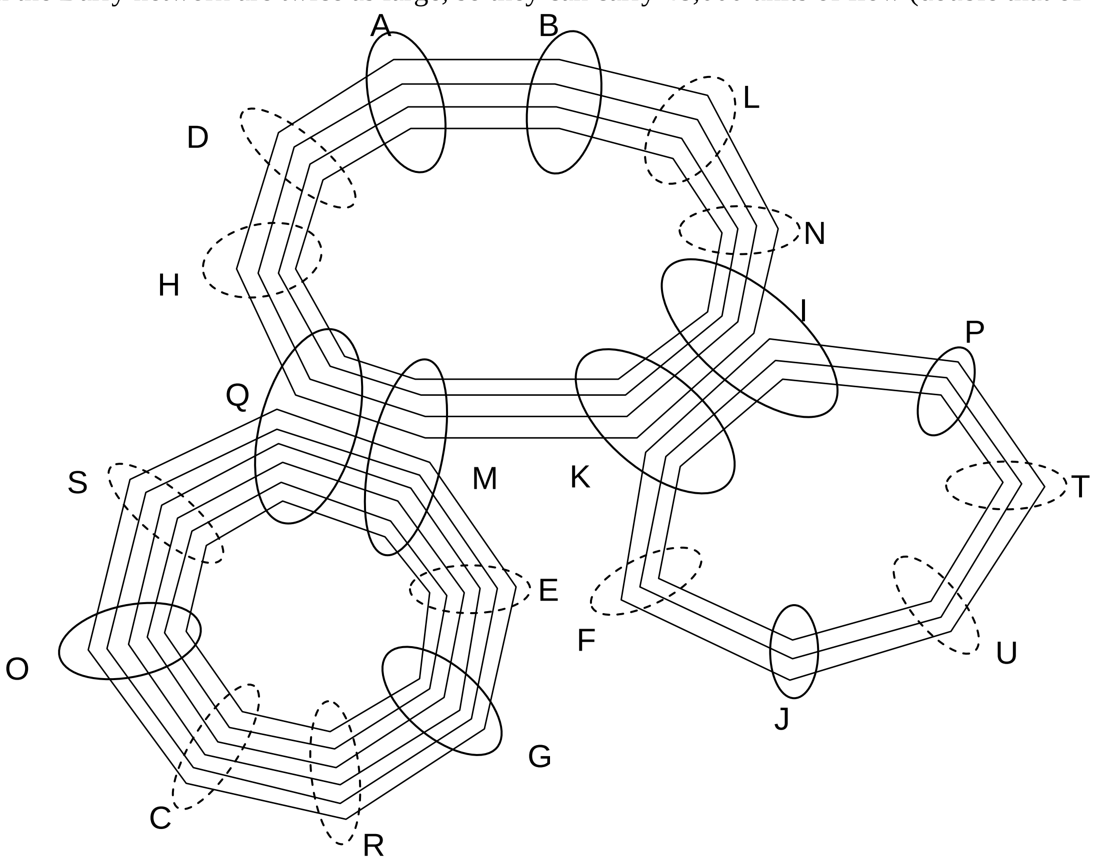

Figure 4. Plan for the company's Barry network. Hubs with solid oval have a BBDX installed, and hubs with a dashed oval do not.

Origin	Destination	Demand	Origin	Destination	Demand
A	B	66,000	F	G	6,000
A	C	1,000	F	I	19,000
A	D	34,000	F	J	61,000
A	E	2,000	F	K	10,000
A	F	2,000	F	P	2,000
A	G	23,000	F	S	1,000
A	H	10,000	G	I	13,000
A	I	16,000	G	J	23,000
A	J	13,000	G	K	4,000
A	K	6,000	G	M	17,000
A	L	9,000	G	O	118,000
A	M	23,000	G	P	6,000
A	N	17,000	G	R	177,000
A	O	57,000	G	S	15,000
A	P	9,000	H	M	1,000
A	Q	12,000	I	J	32,000
A	R	1,000	I	K	20,000
A	S	21,000	I	L	1,000
T	F	2,000	I	M	10,000
T	I	3,000	I	N	31,000
T	J	1,000	I	O	18,000
T	P	30,000	I	P	31,000
B	F	1,000	I	R	1,000
B	G	3,000	J	K	24,000
B	I	8,000	J	M	8,000
B	J	3,000	J	N	3,000
B	K	2,000	J	O	16,000
B	L	2,000	J	P	11,000
B	N	1,000	J	U	9,000
B	O	1,000	J	R	1,000
B	P	2,000	K	M	2,000
B	S	1,000	K	N	2,000
C	G	14,000	K	O	10,000
C	I	1,000	K	P	2,000
C	K	1,000	K	R	2,000
C	O	21,000	L	N	10,000
C	R	3,000	M	O	4,000
C	S	3,000	M	P	1,000
D	H	2,000	M	S	2,000
E	G	32,000	N	O	1,000
E	I	8,000	O	P	9,000
E	J	8,000	O	R	23,000
E	K	8,000	O	S	113,000
E	M	2,000	R	S	4,000
E	R	1,000			

Table 3. Expected Barry network demand

Hub 1	Hub 2	Cost per unit of flow
A	B	1.08
B	L	1.08
L	N	1.04
N	I	1.07
I	K	1.05
K	M	1.07
M	Q	1.06
Q	H	1.01
H	D	1.03
D	A	1.03
G	R	1.01
R	C	1.05
C	O	1.04
O	S	1.04
S	Q	1.10
M	E	1.06
E	G	1.05
J	F	1.01
F	K	1.03
I	P	1.04
P	T	1.03
T	U	1.02
U	J	1.02

Table 4. Flow costs for the Barry network

3. Data Files

The data for this problem is given in the file "ASR data.xls". In reality, you'd have to do a fair amount of coding to get the data into a format that's easy for Xpress to read and convenient to use for your formulation. To make this task easier, I provide an Excel macro for you to use. It'll create a new file in Xpress-friendly (and formulation-friendly) format for any data set (including those I gave you) that has the same format as those in "ASR data.xls". You might not need all the tables the macro creates, but it creates them just in case you might find them useful. If you want to run smaller problems for tests or as part of your algorithm, just create a new worksheet in the "ASR data.xls" workbook using the same format, copy the "Export" button to the new worksheet, and click it.

Part of the convenience of the Excel macro is that it names each table automatically (and makes it exactly the right size for its data). This makes it easy for you to read the data in a convenient way, without having to change your initializations code every time you solve a different-sized problem. The file that gets created contains tables called NodeTable, ArcTable, CommodityTable, HubTable, CycleTable, HubToHubTable, and TransmissionTable. Below, I've included an example of how you can read the table of hub data (which contains the hub's name, whether it has a BBDX, and which cycles run through it). I've attached the file "caldata.xls" (which the macro created from the Caldata worksheet in "ASR data.xls") so you can see how the file and the initializations fit together.

```
declarations
   NumHubs: integer
end-declarations

initializations from “mmdobc.odbc:caldata.xls”
   NumHubs as “[Z26:Z27]”
end-initializations

declarations         ! now that we know how many hubs there are,
                     ! declare the set of hubs and arrays
   Hubs = 1..NumHubs
   HubName: array(Hubs) of string
   HubBBDX: array(Hubs) of integer
   HubCycles: array(Hubs) of integer
end-declarations

initializations from “mmodbc.odbc:caldata.xls”
   [HubName,HubBBDX,HubCycles] as “HubTable” ! reads all three columns
end-initializations
```

[Note that just like reading a column of data from Xpress requires an extra row at the top, reading tables of data like this requires the table to have both an extra row at the top and an extra row on the left.]

In the Xpress-friendly data file, there will be two tables that might look a little confusing at first: NodeTable and ArcTable.

NodeTable lists all of the nodes in the network:

- Type 0 nodes: These are the hubs/cities themselves, where demand starts and ends.
- Type 1 nodes: These are potential ADM locations on rings, with one location per ring that goes through each hub.

ArcTable lists all of the arcs in the network.

- Type 0 arcs: These go from a ring to the hub/city. A commodity is allowed to flow on a Type 0 arc only if the hub is the commodity’s destination, and only if an ADM is installed on the ring at this hub.
- Type 1 arcs: These go from the hub/city to a ring. A commodity is allowed to flow on a Type 1 arc only if the hub is the commodity’s origin, and only if an ADM is installed on the ring at this hub.
- Type 2 arcs: These go from one hub to the next along a ring. Any commodity may flow on these arcs, whether or not an ADM is installed. An arc is created in each direction, so “Hub A to Hub B” and “Hub B to Hub A” are two different arcs.
- Type 3 arcs: These go from one ring to another at a hub that has a BBDX. Any commodity may flow on these arcs, but only if an ADM is installed at the hub on both rings. ArcTable automatically includes only arcs at hubs with a BBDX.

4. An Integer Programming Formulation for the Problem

The company’s problem can be formulated as an integer program. There are several ways to write a formulation for this problem, including the following:

Variables

- One binary variable per Type 1 node (each valid hub/ring combination), to show whether an ADM should be installed there or not
- One non-negative variable per commodity/arc pair to show how much of each commodity flows from the arc’s start node to the arc’s end node

Constraints

- The total bi-directional flow on an arc (the sum of the two directions) can be no more than the bi-directional capacity.
- No commodities may flow on a Type 0 arc unless their destination is at that hub.
- No commodities may flow on a Type 1 arc unless their origin is at that hub.
- No commodities may flow on a Type 0 arc unless an ADM is installed on that ring at that hub.
- No commodities may flow on a Type 1 arc unless an ADM is installed on that ring at that hub.
- No commodities may flow on a Type 3 arc unless an ADM is installed at that hub on both rings.
- The total flow of each commodity into its destination hub (i.e., into a Type 0 node on Type 0 arcs) must equal the demand for that commodity.
- The total flow of each commodity out of its origin hub (i.e., out of a Type 0 node on Type 1 arcs) must equal the demand for that commodity.
- The total flow of each commodity into each Type 1 node must equal the total flow of that commodity out of that Type 1 node

Costs

- ADM installation cost
- Flow cost on arcs and through ADMs and BBDXes

4.1. Formulation Check

In case you'd like to check your work on a smaller instance of the problem, the file "ASR Data.xls" includes a third network: the top two cycles of the Caldata network. The optimal objective value for this problem is 1.0921e7, with the following 10 ADMs selected:

Hub A: ring 1,2,4 on cycle ABEC
Hub B: ring 1,2,4 on cycle ABEC, and ring 1 on cycle BDE
Hub C: ring 4 on cycle ABEC
Hub D: ring 1 on cycle BDE
Hub E: ring 2 on cycle ABEC

Xpress should be able to solve this problem in less than a minute.

Of course, there are several symmetric solutions, so the answer you get might be slightly different (but the value should be the same).