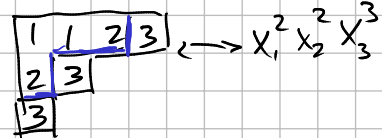


lozenge

SSYT

\longleftrightarrow

tilings



$$s_\lambda(x_1, \dots, x_n) = \sum_{T \in \text{SSYT}(\lambda | n)} \prod_{i=1}^n x_i^{\#i\text{'s in } T}$$

$\text{TESSYT}(\lambda | n) \longleftrightarrow$ Gelfand-Tsetlin pattern

$b_i^{(j)} = \# \text{ of boxes } \leq j$
in the row i of T

$$\tilde{b}_i^{(j)} = b_i^{(j)} + j - i$$

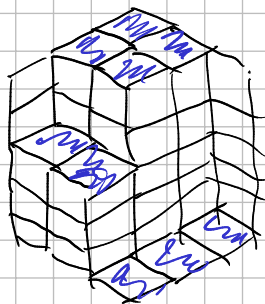
$$b_1^{(n)} \geq b_2^{(n)} \geq \dots \geq b_n^{(n)} \rightarrow (\lambda_1, \lambda_2, \dots, \lambda_n)$$

$$b_1^{(n-1)} \geq b_2^{(n-1)} \geq \dots \geq b_{n-1}^{(n-1)}$$

$$\vdots$$

$$b_1^{(1)} \geq b_2^{(1)} \geq \dots \geq b_n^{(1)}$$

$$\tilde{b}_1^{(n)} > \tilde{b}_2^{(n)} > \dots > \tilde{b}_n^{(n)} \rightarrow (\lambda_1, \lambda_2 - 1, \lambda_3 - 2, \dots)$$



$$\tilde{b}_1^{(n)} > \tilde{b}_2^{(n)} > \dots > \tilde{b}_n^{(n)} \rightarrow (\lambda_1, \lambda_2 - 1, \lambda_3 - 2, \dots)$$

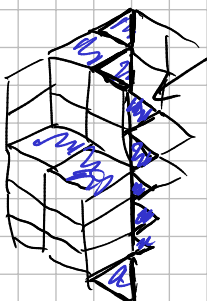
$$s_\lambda(x) s_\mu(y) = \left(\sum_{T \in \text{SSYT}(\lambda | n)} x^T \right) \left(\sum_{T' \in \text{SSYT}(\mu | k)} y^{T'} \right)$$

$$x^T y^{T'} \longleftrightarrow$$

tiling of a hexagon
("cubes in the corner")

skew Howe duality

$$s_\lambda(x) s_{\lambda'}(y) = \left(\sum_{T \in \text{SSYT}(\lambda | n)} x^T \right) \left(\sum_{T' \in \text{SSYT}(\lambda' | k)} y^{T'} \right)$$



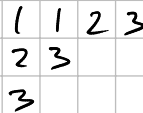
$-\lambda'_1 + i$ (or $\lambda'_1 - i$ from the top)

$\lambda_i - i$ from the bottom

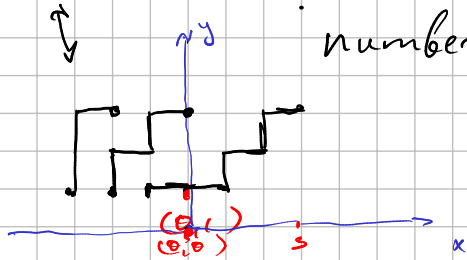
$$\mu_{nk}(d) = \frac{\dim V_{GL_n}^{(d)} \dim V_{GL_k}^{(d')}}{2^{nk}}$$

SSYT \longleftrightarrow Non-intersecting lattice paths (NILP)

Row-reading



row $i \longleftrightarrow$ path P_i — [(0,1) or (1,0)]
start at $(-i, 1)$, end at $(\lambda_i - i, n)$



number of horizontal steps on level $j = \#j^{\text{th}} \text{ row } i$

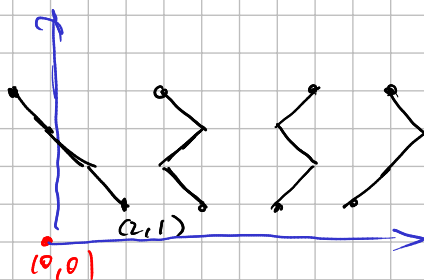


Column reading

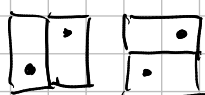
column $\leftrightarrow P_i' \setminus / (-1,1) \text{ or } (1,1)$
 start at $(2i, 1)$ end at $(2i - 2\lambda_i' + n, n)$

j -th step is $(-1,1)$ if j is in column i ,
 otherwise $(1,1)$

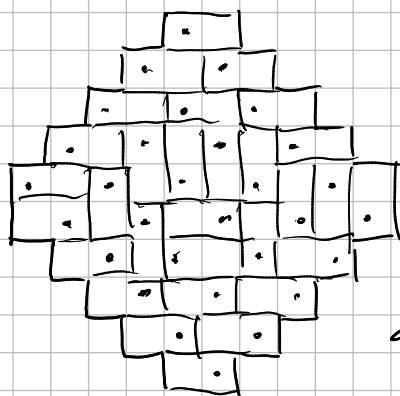
1 1 2 3
 2 3
 3



Aztec diamond, domino tilings

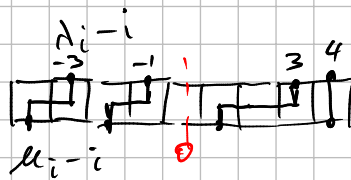
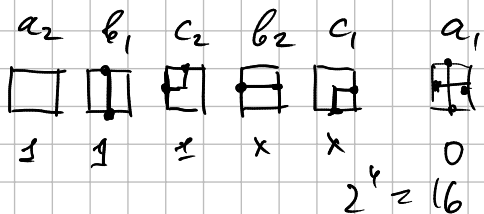


4 kinds of dominoes



$a_1, a_2 + b_1, b_2 - c_1, c_2 = 0$ free fermionic condition

Five vertex model



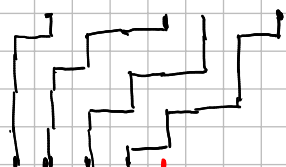
$\lambda_1 = 5 \quad \mu_1 = 5$
 $\lambda_2 = 5 \quad \mu_2 = 3$
 $\lambda_3 = 2 \quad \mu_3 = 1$
 $\lambda_4 = 1 \quad \mu_4 = 0$

$\lambda \geq \mu \quad \lambda_1 \geq \mu_1 \geq \lambda_2 \geq \mu_2 \dots$

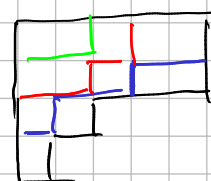
$wf = \prod_{\square} w(\square) = x^4$

SSYT

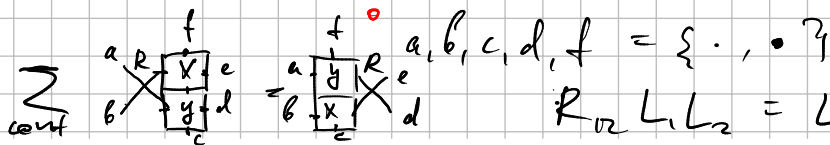
1 2 2 4
 2 3 3
 3 4 4 x_4
 4 x_3
 x_2
 x_1



$x_1 x_2^3 x_3^3 x_4^4$



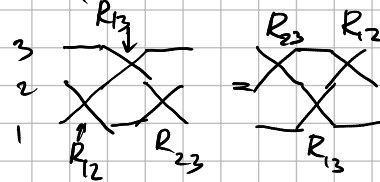
$S_{\lambda}(x_1, \dots, x_n) = \sum_{\text{conf}} w(\text{conf})$
 $= \sum_{\lambda}(x_1, \dots, x_n)$



$R_{12} L_1 L_2 = L_2 L_1 R_{12}$

$$R: \begin{array}{ccccc} \times & \times & \times & \times & \times \\ 1 & y/x & 1 & y/x & 1 - \frac{y}{x} \end{array}$$

Yang-Baxter equation



$$R_{23} R_{13} R_{12} = R_{12} R_{13} R_{23}$$

$$e^{\sum_{\ell=1}^{\infty} t_{\ell} \alpha_{\ell}}$$

$$t_{\ell} = \frac{1}{\ell} \sum_{i=1}^n x_i^{\ell}$$

$$e^{\sum_{i=1}^n \sum_{\ell=1}^{\infty} \frac{x_i^{\ell}}{\ell} \alpha_{\ell}} = \prod_{i=1}^n \underbrace{e^{\sum_{\ell=1}^{\infty} \frac{x_i^{\ell}}{\ell} \alpha_{\ell}}}_{T(x_i)}$$

$$T(x) = e^{\sum_{\ell=1}^{\infty} \frac{x^{\ell}}{\ell} \alpha_{\ell}}$$

$$S_{\lambda}(x, \dots) = \sum_{\phi = \lambda^{(0)}, \lambda^{(1)}, \dots, \lambda^{(n)} = \lambda} \prod_{i=1}^n \langle \lambda^{(i-1)} | T(x_i) | \lambda^{(i)} \rangle$$

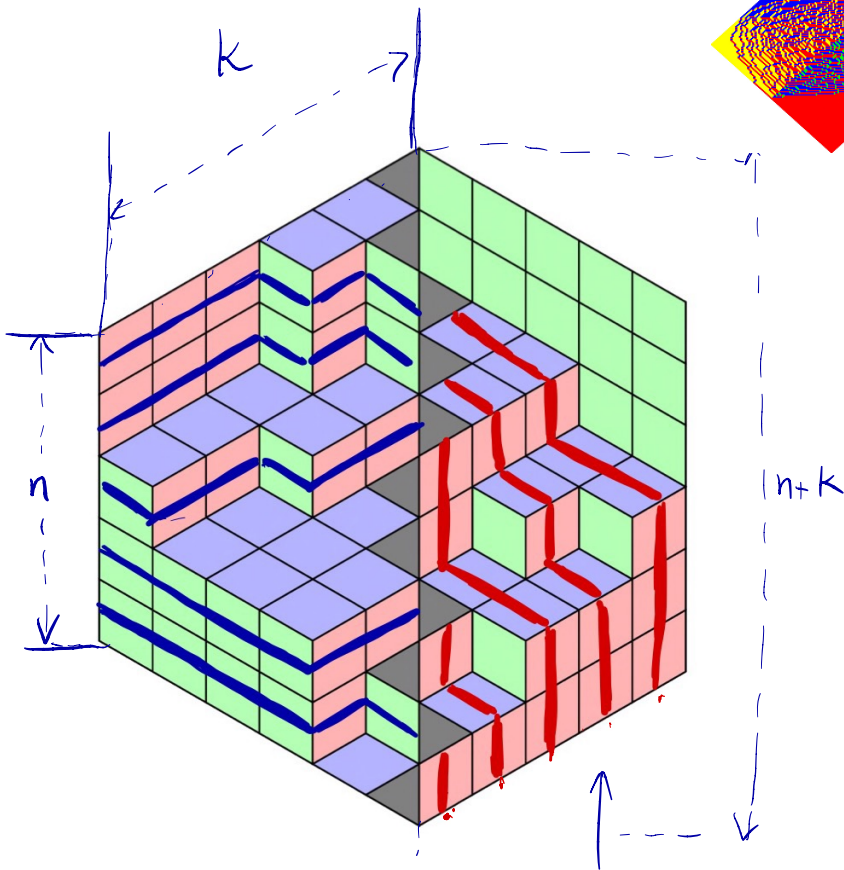
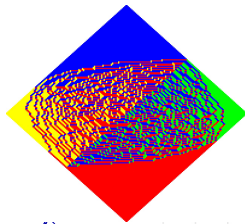
$$T(x) \rightarrow \begin{array}{c} \text{Diagram of a braid with 4 strands and red loops} \end{array}$$

$$L = \begin{array}{c} \begin{array}{|c|c|c|c|c|} \hline \square & \square & \square & \square & \\ \hline \square & 0 & 0 & 0 & 0 \\ \square & 0 & x & 1 & 0 \\ \square & 0 & x & 1 & 0 \\ \square & 0 & 0 & 0 & 1 \\ \hline \end{array} \end{array}$$

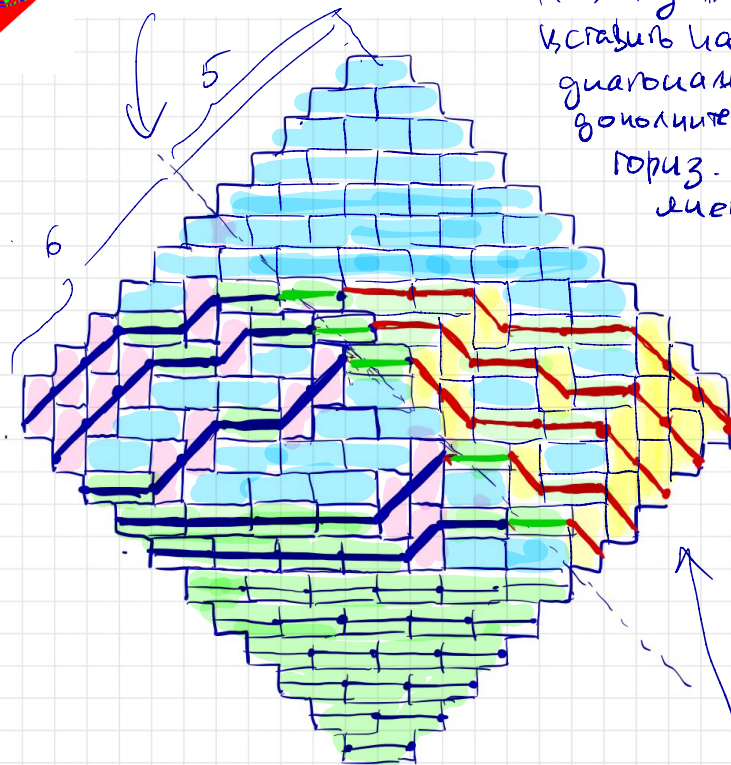
$$T(x) = \dots L_{a,-1}(x) L_{a,0}(x) L_{a,1}(x) \dots$$

$$R_{ab}(x, y) = L_{a,k}(x) L_{b,k}(y) = L_{b,k}(y) L_{a,k}(x) \cdot R_{a,b}(x, y)$$

$$h=5 \quad k=6$$



Я рисовала
эти пути



Поэтому пришлось
вставить на
диагональ
дополнительные
гориз.
линии

Но количество шаров у них на единицу
меньше чем нужно, чтобы они заканчивались
здесь

