CS 513

Probability Assignment

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1.1

Jerry and Susan have a joint bank account.
Jerry goes to the bank 20% of the days.
Susan goes there 30% of the days.
Together they are at the bank 8% of the days.

a.

P(J|S) = P(J &S) / P(S) = 0.08 / .3 = 0.267 = 26.7%

b.

(.20-0.08) / (1-.3) = 0.17143= 17.14%

c.

.08 / (1 - 0.58) = 0.19048 = 19.048

1.2

Harold and Sharon are studying for a test.
Harold's chances of getting a "B" are 80%.
Sharon's chances of getting a "B" are 90%.
The probability of at least one of them getting a "B" is 91%.
The probability of they all getting a B is as following:
Using $P(a \wedge b) = P(a) + P(b) - P(a \cup b)$
80% + 90% - 91% = 79%

a. What is the probability that only Harold gets a "B"?

80% - 79% = 1%

b. What is the probability that only Sharon gets a "B"?

90% - 79% = 11%

c. What is the probability that both won't get a "B"

1 – 91% = 9%

1.3

Jerry and Susan have a joint bank account. Jerry goes to the bank 20% of the days. Susan goes there 30% of the days. Together they are at the bank 8% of the days.
Are the events "Jerry is at the bank" and "Susan is at the bank" independent?

NO, because if they are independent events, the probability will be:
$P(A \wedge B) = P(A) * P(B)$
But
$P(A \wedge B) = 8\%$ which is not equal to {P(A) * P(B) = 60%}. Thus they are not independent.

1.4

You roll 2 dice.

Are the events "the sum is 6" and "the second die shows 5" independent?

Sample Space = {(2,4) ,(4,2), (1,5), (5,1), (3,3)}
P(sum=6) = 5/36

P(second_die=5) = 1/6
P(sum=6 and second_die=5) = 1/36
This is $\neq$ 5/ 216 { P(sum=6) * P(second_die=5)}
Thus, they are dependent events.

Are the events “the sum is 7” and “the first die shows 5” independent?

Sample Space = { (3,4) , (4,3), (5,2), (2,5), (6,1), (1,6)}
P(sum=7) = 6/36 = 1/6

P(first_die=5) = 1/6
P(sum=7 and first_die=5) = 1/36 = P(sum=7) * P(first_die=5)
Thus, they are independent events.

1.5

An oil company is considering drilling in either TX, AK and NJ. The company may operate in only one state. There is 60% chance the company will choose TX and 10% chance - NJ. There is 30% chance of finding oil in TX, 20% - in AK, and 10% - in NJ.

1. What's the probability of finding oil?

P(oil and TX) = 30% * 60% = 18%
P(oil and AK) = 20% * 30% = 6%
P(oil and NJ) = 10% * 10% = 1%
P(oil) = P(oil and TX) + P(oil and AK) + P(oil and NJ) = 25%

2.The company decided to drill and found oil. What is the probability that they drilled in TX?

P(TX|oil) = P(oil and TX) / P(oil) = 18% / 25% = 72%

1.6

Using the Titanic data

What is the probability that a passenger did not survive?

1490/2201 = 67.69%

What is the probability that a passenger was staying in the first class?

325/2201 = 24.696%

Given that a passenger survived, what is the probability that the passenger was a first class?

203 / 711 = 28.55%

Are survival and staying in the first class independent?

P(Survived) = 711/2201 = 32.303%
P(FirstClass) = 325/2201 = 14.766%
P(FirstClass & Survived) = 203/325 = 62.462%
If they are independent, P(F & S) should equals P(S) * P(F), but it isn't. So they are not independent event.

Given that a passenger survived, what is the probability that the passenger was staying in the first class and the passenger was a child?

6 / 711 = .8 %

Given that a passenger survived, what is the probability that the passenger was an adult?

654/711 = 91.9%

Given that a passenger survived, are age and staying in the first class independent?

P(Age|S) = 324/499 = 64.929%

P(FirstClass|S) = 203/499 = 40.681% P(Female & FirstClass|S) = 141/499 = 28.257%

Because P(Female|S)* P(FirstClass|S)≠ P(Female & FirstClass|S),

They are dependent .

7. Assuming between Age and Cabin

$$P(A \cap B) = P(A)\,P(B)$$

Total:

$P(\text{Adult} \cap 1^{st}) = 308.9$

$P(\text{Adult} \cap 2^{nd}) = 270.88$

$P(\text{Adult} \cap 3^{rd}) = 611.03$

$P(\text{Adult} \cap \text{Crew}) = 841.17$

$P(\text{Child} \cap 1^{st}) = 16.09$

$P(\text{Child} \cap 2^{nd}) = 14.11$

$P(\text{Child} \cap 3^{rd}) = 34.96$

$P(\text{Child} \cap \text{Crew}) = 43.83$

II. Assuming conditional on survival status

$$P(A \cap B \mid C) = P(A \mid C) \cdot P(B \mid C)$$

R.H.S $\Rightarrow 57/711 * 57/711 \rightarrow (1)$

LHS $\Rightarrow P(A \cap B \mid C) = 57/711 \rightarrow (2)$

Thus conditional Independent event

Survival:

P(Adult ∩ 1st) = 186.12
P(Adult ∩ 2nd) = 108.54
P(Adult ∩ 3rd) = 163.73
P(Adult ∩ Crew) = 195.00

P(Child ∩ 1st) = 16.27
P(Child ∩ 2nd) = 9.46
P(Child ∩ 3rd) = 14.27
P(Child ∩ Crew) = 16.99

Non-Survival:

P(Adult ∩ 1st) = 197.74
P(Adult ∩ 2nd) = 161.17
P(Adult ∩ 3rd) = 509.57
P(Adult ∩ Crew) = 649.51

P(Child ∩ 1st) = 4.258
P(Child ∩ 2nd) = 5.828
P(Child ∩ 3rd) = 18.427
P(Child ∩ Crew) = 23.487

Summed:

	1st	2nd	3rd	Grew	Total
Adult	186.72	108.54	163.73	195.00	653.99
Child	16.27	9.46	14.27	16.99	56.99
	203	118	178	212	711

Non-Summed:

	1st	2nd	3rd	Grew	Total
Adult	117.74	161.17	309.57	649.57	1437.99
Child	4.258	5.828	18.421	23.487	52
	122	167	528	673	1490