ELEN3007A Group 21 - Assignment 2024:

Application of Bayes' Theorem for Locating a Robot's Position in an Enclosed Area

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September 13, 2024

Introduction and Background

The assignment consideres that the position of β is known, and after recording N flashes at positions x_k, and inferes or answers the question: *where is the robot*?
The azimuth angles at which the flashes are emmited, at random intervals, are quantified by θ_k which is uniformly distributed distributed. Since θ_k is uniformly distributed, we expect more recordings of x_k near or around α which displays the vertical position underwhich the robot is expected to be. The mean value of x_k is distance away from the assumed value of α.

Assignment Criterion

- Geometry setup of the problem

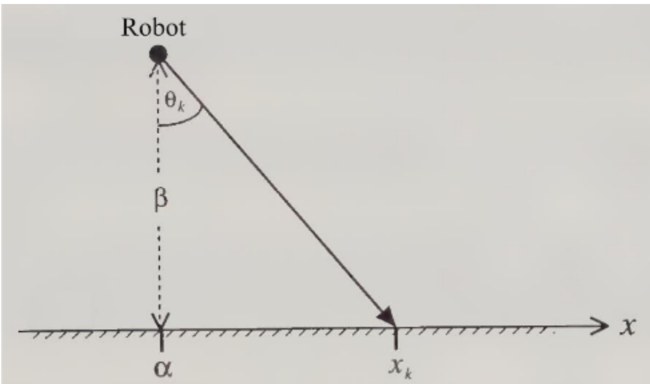

Figure 1: Geometry setup of the problem

- θ_k assumed to be uniform, azimuth angles lying between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, it has the PDF:

$$p(\theta_k|\alpha,\beta,B) = f_{\Theta|\Omega,\beta,B}(\theta|\alpha) = \frac{1}{\pi} \tag{1}$$

- In order to relate the readings of x_k to θ_k, using elementary trigonometry, the derived expression is

$$\beta tan\theta_k = x_k - \alpha \tag{2}$$

- x_k is independent and identically distributed from normal distribution.

Equations and Notations

$$p(\theta_k|\alpha, \beta, B) = f_{\Theta|\Omega,\beta,B}(\theta) \tag{3}$$

$$p(x_k|\alpha, \beta, B) = f_{X|\Omega,\beta,B}(x_k|\Omega = \alpha, \beta = b) \tag{4}$$

Assignment Answers

1. The given setup of the problem assumes that the photodetectors are placed on the x-axis above which the robot is located. Therefore, the signal comes from one side of the axis. This thus limits the range of the detectors to be within the range of π (that is $-\frac{\pi}{2}$ to $\frac{\pi}{2}$).

2.

3. Etc.

Conclusion

A ? Understanding the Assignment

- since we can't observe θ, we transform to x_k. so that means we transform and drop the random variable $\Theta = \theta_k \Rightarrow X = x_k$ and it's PDF(*prior*?) $p(\theta_k|\alpha, \beta, B)$ (given) to $p(x_k|\alpha, \beta, B)$ (proved)
- is α supposed to be assumed? if, then α is a constant and we want to find how far off we are from it as given by the measurements (observed data points)
- So, the α and β are supposed to have a joint distribution right? and so after we observe $\beta = b$, we condistion α that is $p_{\alpha=a|\beta=b}$. But we are also given data whhich is $\{x_k\}_{k=1}^N$ to infere the robot's position expressed by the posterior PDF $p_{\alpha|\{x_k\}_{k=1}^N}(\alpha = a)$

B ? Questions

1. is this the marginal PDF? $p(\theta_k|\alpha, \beta, B) = \frac{1}{\pi}$
2. is this the conditional distribution? $p(x_k|\alpha, \beta, B) = \frac{\beta}{\pi(\beta^2+(x_k-\alpha)^2)}$
3. in order to plot $p(\alpha|x_k, \beta, B)$, should we assume our own values for the parameters α and β?
4. is this notation correct? $p(x_k|\alpha, \beta, B) = f_{X|\Omega,\beta,B}(x_k|\alpha, \beta = b)$, where $b = constant$
5. if then is posterior notation? $p(\alpha|x_k, \beta, B) = f_{\Omega|X,\beta,B}(\alpha) = \frac{\beta}{\pi(\beta^2+(x_k-\alpha)^2)} \times \aleph$, where $\aleph$ is a *proportional constant* of bayes transformation to posterior PDF
6. and is? $p(\alpha|\{x_k\}_{k=1}^N, \beta, B) = f_{\Omega|X,\beta,B}(\alpha|x_1, ..., x_n, \beta = b) = \prod_{k=1}^N \frac{\beta}{\pi(\beta^2+(x_k-\alpha)^2)} \times \aleph$
7. is the x-position a matter of how far from the normal(that is $\alpha = 0 = x_0 = 0$ to $x_k = \mu_x$? is this the meaningful conclusion?
8. because accordring to question 7, it says estimate x-position using 30 measurements. what are the other extra data points for? demo?
9. is this professional style report clear? and what is meant by effective data representation(graphs of distro. with different no. of N)? *because* is (sub-&)heading numbering necessary?
10. what does the demonstration require? effect of *different number*(N) of observed data points?

C ? other Questions

1. do we need to apply maximum likelyhood solutions for the mean of data?
2. if (or not) then the goal is to estimate the posterior mean? which can be given by a compromise between the prior mean μ_0 and the maximum likelihood solution μ_{ML}?