ELEN3007A Group 27 - Assignment 2024:

Application of Bayes' Theorem for Locating a Robot's Position in an Enclosed Area

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QUESTION 1:

Why does θ_k not lie between $-\pi$ and π for which $p(\theta_k|\alpha, \beta, B)$ would then be $\frac{1}{2\pi}$?

Solution 1:

The given setup of the problem assumes that the photodetectors are placed on the x-axis above which the robot is located. Therefore, the signal comes from one side of the axis. This thus limits the range of the detectors to be within the range of π (that is $-\frac{\pi}{2}$ to $\frac{\pi}{2}$).

QUESTION 2:

Prove that

$$p(x_k|\alpha, \beta, B) = \frac{\beta}{\pi(\beta^2 + (x_k - \alpha)^2)} \tag{1}$$

Solution 2:

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QUESTION 3:

Plot $p(x_k|\alpha, \beta, B)$ and then relate its width at half maximum to the parameters of the PDF.

Solution 3:

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QUESTION 4:

Derive the expression for $p(\alpha|x_k, \beta, B)$.

Solution 4:

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QUESTION 5:

Finally derive an expression for $p(\alpha|\{x_k\}_{k=1}^N, \beta, B)$. (State all assumptions.)

Solution 5:

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QUESTION 6:

Explain how one obtains the x-position of the robot from the result in (5.).

Solution 6:

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QUESTION 7:

Implement the Bayesian position inference scheme in Matlab. Assume the robot is located inside a confined square region of size 10m $\times$ 10m. Demonstrate your Bayes estimator by inferring the robot's x-position from the data/measurements $\{x_k\}_{k=1}^{200}$ in *BayesData.mat*, with the robot's y-position 4.5 m. Plot the Bayes posterior distribution for the first $N = 1, 2, 5$ and 30 measurements. For this data, what is your best x-position estimate?

Solution 7:

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QUESTION 8:

The notation presented above, deliberately does *not* follow the conventions your Probs lecturer introduced. Throughout your assignment, strictly use the notation prescribed for use in Probs.

Solution 8:

Yes with the following Equations and Notations:

$$p(\theta_k|\alpha, \beta, B) = f_{\Theta|\Omega,\beta,B}(\theta) \tag{2}$$

$$p(x_k|\alpha, \beta, B) = f_{X|\Lambda,\mathfrak{B},B}(\,\cdot\,|\alpha, \beta) \tag{3}$$

$$\Lambda(\cdot) = \alpha;\ \mathfrak{B}(\cdot) = \beta;\ X(\cdot) = x_k;\ \Theta(\cdot) = \theta_k \tag{4}$$

QUESTION 9:

Professional report with clear and effective data representation. The report must not have a title page and is not allowed to exceed 5 pages in length.

Solution 9:

The report is presented in homework assignment form(statement and solution). The following reference materials have been used: [1]

(Bonus) QUESTION 10:

Infer both the x-position as well as the y-position of the robot. Estimate both α and β for the PDF in Eq. (1).

Solution 10:

References

[1] H Wang, TN Lamichhane, and MP Paranthaman. Review of additive manufacturing of permanent magnets for electrical machines: A prospective on wind turbine. *Materials Today Physics*, 24:100675, 2022.

Appendix A. - Matlab Inference Scheme Code

```
% Load the data
load('BayesData.mat');
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Appendix B. - Matlab Scheme Plots

Plots