# ELEN3007A Group 27 - Assignment 2024 (Consultation Questions):

*Application of Bayes' Theorem for Locating a Robot's Position in an Enclosed Area*

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## Introduction and Background

The assignment consideres that the position of $\beta$ is known, and after recording $N$ flashes at positions $x_k$, and inferes or answers the question: *where is the robot*?
The azimuth angles at which the flashes are emmited, at random intervals, are quantified by $\theta_k$ which is uniformly distributed distributed. Since $\theta_k$ is uniformly distributed, we expect more recordings of $x_k$ near or around $\alpha$ which displays the vertical position underwhich the robot is expected to be. The mean value of $x_k$ is distance away from the assumed value of $\alpha$.

### Assignment Criterion

- Geometry setup of the problem

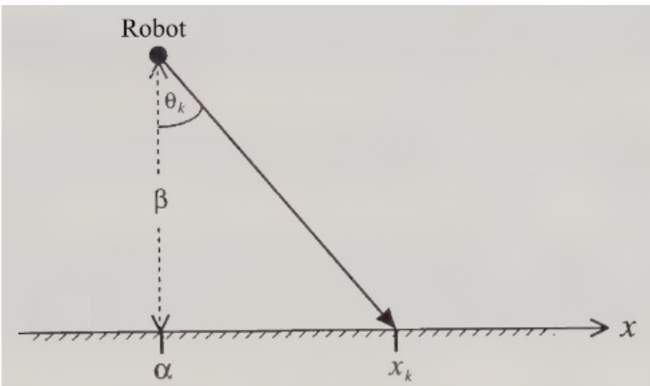


Figure 1: Geometry setup of the problem

- $\theta_k$ assumed to be uniform, azimuth angles lying between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, it has the PDF:

$$p(\theta_k|\alpha, \beta, B) = f_{\Theta|\Omega,\beta,B}(\theta|\alpha) = \frac{1}{\pi} \tag{1}$$

- In order to relate the readings of $x_k$ to $\theta_k$, using elementary trigonometry, the derived expression is

$$\beta tan\theta_k = x_k - \alpha \tag{2}$$

- $x_k$ is independent and identically distributed from normal distribution.

# 1 ? Understanding the Assignment

- since we can't observe $\theta$, we transform to $x_k$. so that means we transform and drop the random variable $\Theta = \theta_k \Rightarrow X = x_k$ and it's PDF(*prior*?) $p(\theta_k|\alpha, \beta, B)$ (given) to $p(x_k|\alpha, \beta, B)$ (proved)
- is $\alpha$ supposed to be assumed? if, then $\alpha$ is a constant and we want to find how far off we are from it as given by the measurements (observed data points)
- So, the $\alpha$ and $\beta$ are supposed to have a joint distribution right? and so after we observe $\beta = b$, we condistion $\alpha$ that is $p_{\alpha=a|\beta=b}$. But we are also given data whhich is $\{x_k\}_{k=1}^N$ to infere the robot's position expressed by the posterior PDF $p_{\alpha|\{x_k\}_{k=1}^N}(\alpha = a)$
- $\alpha$ and $\beta$, are they independent? but $\theta$ won't!

# 2 ? Questions

1. is this the marginal PDF? $p(\theta_k|\alpha, \beta, B) = \frac{1}{\pi}$
2. is this the conditional distribution? $p(x_k|\alpha, \beta, B) = \frac{\beta}{\pi(\beta^2+(x_k-\alpha)^2)}$
3. in order to plot $p(\alpha|x_k, \beta, B)$, should we assume our own values for the parameters $\alpha$ and $\beta$?
4. is this notation correct? $p(x_k|\alpha, \beta, B) = f_{X|\Omega,\beta,B}(x_k|\alpha, \beta = b)$, where $b = constant$
5. if then is posterior notation? $p(\alpha|x_k, \beta, B) = f_{\Omega|X,\beta,B}(\alpha) = \frac{\beta}{\pi(\beta^2+(x_k-\alpha)^2)} \times \aleph$, where $\aleph$ is a *proportional constant* of bayes transformation to posterior PDF
6. and is? $p(\alpha|\{x_k\}_{k=1}^N, \beta, B) = f_{\Omega|X,\beta,B}(\alpha|x_1, ..., x_n, \beta = b) = \prod_{k=1}^N \frac{\beta}{\pi(\beta^2+(x_k-\alpha)^2)} \times \aleph$
7. is the $x$-position a matter of how far from the normal(that is $\alpha = 0 = x_0 = 0$ to $x_k = \mu_x$? is this the meaningful conclusion?
8. because accordring to question 7, it says estimate $x$-position using 30 measurements. what are the other extra data points for? demo?
9. is this professional style report clear? and what is meant by effective data representation(graphs of distro. with different no. of N)? *because* is (sub-&)heading numbering necessary?
10. what does the demonstration require? effect of *different number*(N) of observed data points?

# 3 ? other Questions

1. do we need to apply maximum likelyhood solutions for the mean of data?
2. if (or not) then the goal is to estimate the posterior mean? which can be given by a compromise between the prior mean $\mu_0$ and the maximum likelihood solution $\mu_{ML}$?
3. are integrals or derivatives or both involved in finding the post-PDF? (marginal PDF)
4. do we use Maximum-A-Posterior (MAP) to estimate the range or point estimate of $\alpha$?

# A Consultation Answerers

### Understanding the Assignment ?

- Yes with the following Equations and Notations:

$$p(\theta_k|\alpha,\beta,B) = f_{\Theta|\Omega,\beta,B}(\theta) \tag{3}$$
$$p(x_k|\alpha,\beta,B) = f_{X|\Lambda,\mathfrak{B},B}(\,\cdot\,|\alpha,\beta) \tag{4}$$
$$\Lambda(\cdot) = \alpha;\ \mathfrak{B}(\cdot) = \beta;\ X(\cdot) = x_k;\ \Theta(\cdot) = \theta_k \tag{5}$$

- no, see answer below
- yes, sounds solid
- yes they have to be.. what do you think otherwise?

### Questions ?

1. yes, it is a prior (marginal) conditional PDF
2. yes, derivived from the above conditional PDF
3. yes, you can assume your values and compare as per question
4. yes, the notation is correct but $f_{X|\Omega,\beta,B}(\,\cdot\,|\alpha,\beta=b)$, where $b = \mathit{constant}$
5. yes, think some more about the proportionality constant
6. yes
7. no, the $\alpha \neq 0$.. phela this assignment point is to find or estimate the value of $\alpha$. but somewhat yes, you can I guess use $x_k \approx \mu_x$
8. use 30 measurements, but you can use random intervals of $x_{k=n}$ and see how this changes
9. yes but don't use the report style, use the homework style as per announcement. professional in this instance means typed
10. for demonstration, just the Matlab code in the appendix. BUT with meaningful comments. Still use the plots to estimate $\alpha = x$-position

### other Questions ?

1. yes, sounds like a good idea if you know whatever it means
2. sounds solid
3. the intergral is the proportionality constant, do you really need it? no? normalisation?
4. not really maybe?