Application of Bayes' Theorem for Locating a Robot's Position in an Enclosed Area

ELEN3007A Assignment 2024

Background

A robot is allowed to move freely on a horizontal flat rectangular area enclosed by walls all around. The robot is required to perform manoeuvres within this confined area in accordance with its operator's objectives. Identify the one side of this area to be the x-axis and the other to be the y-axis. Several ways exist for sensing the location of the robot. The approach opted for is based on probability theory (see below) and to test the concept, it is decided to first evaluate this scheme for inferring the robot's x-position and once successful, then also to extend it to infer the robot's y-position.

The Scheme

Mounted on top of the robot is a light module that continuously emits highly collimated[1] flashes at random intervals and hence at random azimuths. These pulses are intercepted along the side identified with the x-axis by photodetectors placed along the x-axis. The photodetectors only record the fact that a flash has occurred, but not the angle from which it came. Suppose N light flashes have so far been recorded at positions $\{x_k\}_{1=1}^{N}$. Where is the robot?

The geometry of the setup is shown below in Figure 1. To locate the robot's position from measurements would requires the estimation of both α and β. However, since we assume we are first only considering the case with photodetectors long the x-axis, we necessarily must assume β to be known. Based on Bayes' theorem, the inference of the robot's position is now expressed by the posterior PDF, $p(\alpha \mid \{x_k\}_{k=1}^{N}, \beta, B)$.

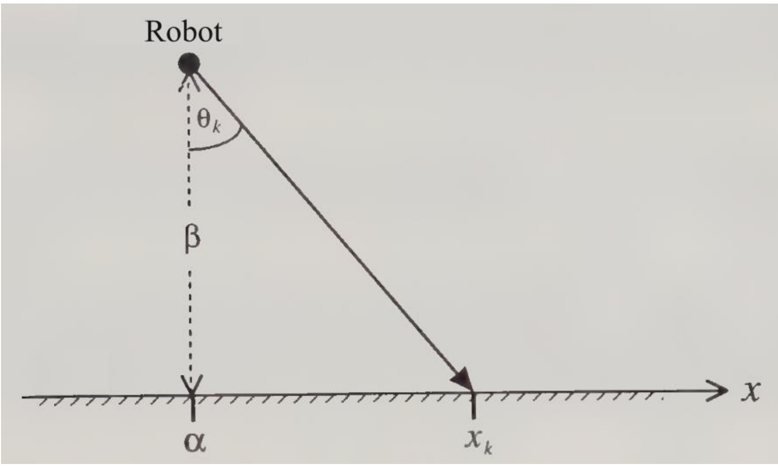

Figure 1: Geometry of the setup of the problem.

The background information, B, includes

- the assumed rectangular region and
- the particular sides identified as x-axis and y-axis.
- placement of the photodetectors.

[1] Brief, tightly focussed bursts of light.

Based on the assumed available background information, azimuth angles θ_k lies between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. Assume θ_k to be uniformly distributed, that is, has the PDF

$$p(\theta_k|\alpha,\beta,B) = \frac{1}{\pi} \tag{1}$$

Since the photodetectors are only sensitive to *position* along the x-axis rather than *direction*, we must relate θ_k to x_k. An inspection of Figure 1, using elementary trigonometry, we can derive the expression,

$$\beta \tan\theta_k = x_k - \alpha. \tag{2}$$

Questions

1. Why does θ_k not lie between $-\pi$ and π for which $p(\theta_k|\alpha,\beta,B)$ would then be $\frac{1}{2\pi}$? [2 marks]
2. Prove that [3 marks]
$$p(x_k \mid \alpha,\beta,B) = \frac{\beta}{\pi(\beta^2 + (x_k-\alpha)^2)}. \tag{3}$$
3. Plot $p(x_k \mid \alpha,\beta,B)$ and then relate its width at half maximum to the parameters of the PDF. [3 marks]
4. Derive the expression for $p(\alpha \mid x_k,\beta,B)$. [3 marks]
5. Finally derive an expression for $p(\alpha \mid \{x_k\}_{k=1}^{N},\beta,B)$. (State all assumptions.) [3 marks]
6. Explain how one obtains the x-position of the robot from the result in (5.). [3 marks]
7. Implement the Bayesian position inference scheme in Matlab.[2] Assume the robot is located inside a confined square region of size 10m × 10m. Demonstrate your Bayes estimator by inferring the robot's x-position from the data/measurements $\{x_k\}_{k=1}^{200}$ in BayesData.mat, with the robot's y-position 4.5 m. Plot the Bayes posterior distribution for the first $N = 1, 2, 5$ and 30 measurements. For this data, what is your best x-position estimate? [5 marks]
8. The notation presented above, deliberately does *not* follow the conventions your Probs lecturer introduced. Throughout your assignment, strictly use the notation prescribed for use in Probs. [3 marks]
9. Professional report with clear and effective data representation. The report must not have a title page and is not allowed to exceed 5 pages in length. [5 marks]

Bonus Question

10. Infer both the x-position as well as the y-position of the robot. [5 marks]
    Two options exist:
    a. Estimate both α and β for the PDF in Eq. (3).
    b. Extend your solution to also include photodetectors along the y-axis. (This would require additional data which is not available and hence this option is not viable.)

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[2] Each group will be interviewed to demonstrate their Matlab implementations.