We start out with the MAP decision rule

$$P(signal\ |\text{x}) > P(noise\ |\text{x})$$

Insert Bayes to introduce the prior

$$\frac{P(x\ |\text{signal})P(signal)}{P(x)} > \frac{P(x\ |\text{noise})P(noise)}{P(x)}$$

Use $P(signal) = 1 - P(noise)$ and rearrange

$$\frac{P(x\ |\text{signal})}{P(x\ |\text{noise})} > \frac{1 - P(signal)}{P(signal)}$$

Insert Gaussian probability density function, remember that $\sigma = 1$, take natural logarithm and rearrange

$$x^2 - (x - d')^2 > ln\left(\frac{1 - P(signal)}{P(signal)}\right)$$

Isolate x

$$x > \frac{ln\left(\frac{1 - P(signal)}{P(signal)}\right) + \frac{d'^2}{2}}{d'} = \frac{ln\left(\frac{1 - P(signal)}{P(signal)}\right)}{d'} + \frac{d'}{2}$$