The first question involves determining the probability of a fruit reflecting light with a wavelength in the spectrum 540-550 nm being ripe. Generally, it is assumed that ripe juju-fruits are orange and on average reflects light with a wavelength of about 600 nm, and unripe fruits are considered green and to reflect light with a wavelength of about 500 nm. A standard deviation of 50 nm is assumed for both fruit states. Finally, it is given that 15% of the fruits are ripe.

For this question, we have the following parameters:

- Posterior = the probability of a fruit being ripe, given the wavelength.
- Prior = the probability of a fruit being ripe.
- Likelihood = how likely we are to observe a wavelength in the spectrum from a ripe fruit.
- Terms discounted = the viewpoint of the fruit, the lighting of the environment, bias in the monkey's vision.

We have Bayes' rule:

$$P(ripe|wl) = \frac{P(wl|ripe) \cdot P(ripe)}{P(wl)}$$

However, due to the question involving two states of the fruit (namely being ripe or unripe), we must consider the probability of observing a given wavelength in the context of the fruit-states, and the equation must be expanded:

$$P(ripe|wl) = \frac{P(wl|ripe) \cdot P(ripe)}{P(wl|ripe) \cdot P(ripe) + P(wl|unripe) \cdot P(unripe)}$$

We determine each term in the denominator individually first, accounting for the wavelength-spectrum involving two wavelengths that determines the limits of the spectrum:

$$P(wl|ripe) = P\left(\frac{550-600}{50}\right) - P\left(\frac{540-600}{50}\right) = P(-1) - P(-1{,}2)$$

$$P(wl|unripe) = P\left(\frac{550-500}{50}\right) - P\left(\frac{540-500}{50}\right) = P(1) - P(0{,}8)$$

The question description states that 15% of the fruits are ripe, hence:

$$P(ripe) = 0{,}15$$

And, under the assumption that a fruit must assume one and only one of the two states (ripe or unripe), we have:

$$P(unripe) = P(\neg ripe) = 1 - P(ripe) = 1 - 0{,}15 = 0{,}85$$

Looking up the z-scores for a standard Gaussian distribution [1], we find:

$$P(wl|ripe) = P(-1) - P(-1{,}2) = 0{,}15866 - 0{,}11507 = 0{,}04359$$

$$P(wl|unripe) = P(1) - P(0{,}8) = 0{,}84134 - 0{,}78814 = 0{,}0532$$

We can now determine the probability of a juju-fruit reflecting light with a wavelength between 540-550 nm being ripe:

$$P(ripe|wl) = \frac{0{,}04359 \cdot 0{,}15}{0{,}04359 \cdot 0{,}15 + 0{,}0532 \cdot 0{,}85} = 0{,}126 = 12{,}6\%$$

Ergo, given the contextual values, the probability of observing a ripe juju-fruit in the jungle reflecting light with a wavelength between 540-550 nm is close to 13%.

- **Question 2**

The second question builds on the previous question but expands the number of fruits to be considered. We now must consider not only juju-fruits but also mongo berries and chakavas as well. Table 1 shows an overview of the information given about the fruits, including their relative distribution in the jungle, state distributions, wavelengths in nm and standard deviations in nm.

Fruit	% jungle	State	% state	Wl (nm)	Std (nm)
Juju	10	Ripe	15	600	50
		Unripe	85	500	50
Mongo	50	Ripe	80	580	20
		Unripe	20	520	20
Chakava	40	Ripe	10	400	100
		Unripe	90	550	100

Table 1. Overview of the given properties of the three fruits species as well as their relative distribution in the jungle.

For this question we have the following parameters:

- Posterior = the probability of a random fruit being ripe, given the wavelength.
- Prior = the probability of a random fruit being ripe.

[1] https://www.math.arizona.edu/~rsims/ma464/standardnormaltable.pdf

- Likelihood = how likely we are to observe a wavelength in the spectrum from a random ripe fruit.
- Terms discounted = the viewpoint of the fruits, the lighting of the environment, bias in the monkey's vision.

We have that

$$P(ripe, fruit|\, wl) = \sum_{fruit} P(ripe, fruit|wl)$$

We simply calculate the sum of the probability of each fruit reflecting a wavelength in the spectrum 540-550 nm being ripe. From the previous question we know that the probability of getting a ripe juju fruit reflecting light of a wavelength in the spectrum of 540-550 nm is 0,126 or 12,6%.

First, we calculate the probability for mongo fruits.

$$P(wl|ripe, mongo) = P\left(\frac{550-580}{20}\right) - P\left(\frac{540-580}{20}\right) = P(-1{,}5) - P(-2)$$

$$= 0{,}06681 - 0{,}02275 = 0{,}04406$$

$$P(wl|unripe, mongo) = P\left(\frac{550-520}{20}\right) - P\left(\frac{540-520}{20}\right) = P(1{,}5) - P(1)$$

$$= 0{,}93319 - 0{,}84132 = 0{,}09185$$

$$P(ripe, mongo) = 0{,}8 \leftrightarrow P(unripe, mongo) = 0{,}2$$

$$P(ripe, mongo|wl) = \frac{0{,}04406 \cdot 0{,}8}{0{,}04406 \cdot 0{,}8 + 0{,}09185 \cdot 0{,}2} = 0{,}6574 = 65{,}7\%$$

Thence, the probability of a mongo fruit reflecting light with a wavelength in the spectrum of 540-550 nm being ripe is 65,7%.

Finally, we calculate the probability for chakava fruits.

$$P(wl|ripe, chakava) = P\left(\frac{550-400}{100}\right) - P\left(\frac{540-400}{100}\right)$$

$$= P(1{,}5) - P(1{,}4) = 0{,}93319 - 0{,}91924 = 0{,}01395$$

$$P(wl|unripe, chakava) = P\left(\frac{550-550}{100}\right) - P\left(\frac{540-550}{100}\right)$$

$$= P(0) - P(-0{,}1) = 0{,}5 - 0{,}46017 = 0{,}03983$$

$$P(ripe, chakava|wl) = \frac{0{,}01395 \cdot 0{,}1}{0{,}01395 \cdot 0{,}1 + 0{,}9 \cdot 0{,}03983} = 0{,}0375 = 3{,}75\%$$

Thence, the probability a chakava fruit reflecting light with a wavelength in the spectrum of 540-550 nm being ripe is 3,75%.

We can now determine the probability of a random fruit being ripe.

$$P(ripe, fruit|wl) = 0{,}126 + 0{,}6574 + 0{,}0375 = 0{,}8209 = 82{,}1\%$$

Ergo, the probability of a random fruit in the jungle (among the three fruits of juju, mango and chakava) reflecting light with a wavelength in the spectrum of 540-550 nm is 82,1%.

As for how the result changes if another monkey is seen enjoying eating the fruit – it would make sense that the probability of that fruit being ripe increases seeing that the fruit has been "validated" by another monkey. However, in Bayes' logic, this observation depends on the probability of the other monkey eating the fruit, i.e. fx how hungry the other monkey is. A very hungry monkey might not care much for the ripeness of the fruit, which renders the observation of another monkey eating the fruit unreliable. Thence, observing another monkey eat the fruit shouldn't change the result.

- **Question 3**

The result of the implementation for a single monkey is evident from Fig. 4. The monkey seems fairly good at separating ripe from unripe fruit. In our tests, we found that the monkey on average picks around 90% of the ripe fruits and 10% of the unripe fruits, leaving 10% of the ripe fruits and 90% of the unripe fruits. There is some error, but assuming that the cost of picking an unripe fruit is the same as leaving a ripe fruit, then the monkey's performance seems good.

```
Command Window
There were 442 ripe fruits
The monkey picked 393 ripe fruits (88.9 percent of ripe fruits)
The monkey left 49 ripe fruits (11.1 percent of ripe fruits)
There were 558 unripe fruits
The monkey picked 64 unripe fruits (11.5 percent of unripe fruits)
The monkey left 494 unripe fruits (88.5 percent of unripe fruits)
fx >>
```

Fig. 4. Result of simulating one monkey picking fruits in MATLAB.

In our implementation, the maximum posterior decision rule has simply been implemented as an evaluation of the probability that a given fruit reflecting light in the seen (from the monkey's perspective) spectrum is ripe. If that probability is 50% or larger (we decided for inclusive), then that means that the posterior for the fruit being ripe is larger than that of the posterior for the fruit being unripe, and hence the monkey decides to pick the fruit.

Please refer to appendix for the MATLAB code.