Fluid Mechanics

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Fluid Mechanics - Why?

Applications and relevance in existing and upcoming technologies

Books

1. Transport Phenomena by Bird Stewart and Lightfoot
2. Fluid Mechanics by Fox and McDonald, Pitchard

Grading - Relative

Class Tests – 20
Mid-sem - 30
End-sem - 50

Topics

1. Conservation of mass – **Continuity Eqn**
   Control Volume, Control Surfaces
   Incompressible Fluid
   Body Forces/ Surface Force (gravity, viscous, pressure)

2. Conservation of MM – Newton's 2^{nd} law **Navier-Stoke's Eqn.**

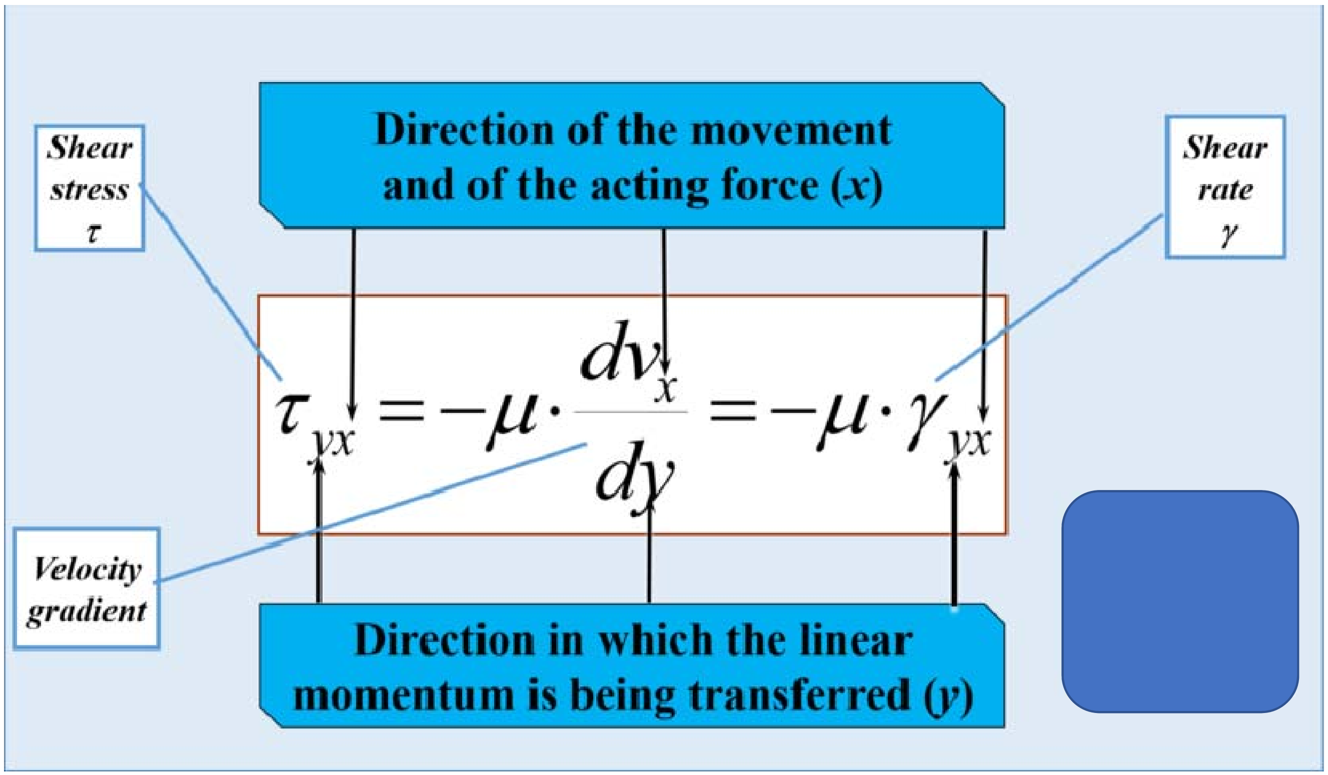
3. Ideal fluid- Eulers Eqn. ------- **Bernoulli Eqn**

4. Momentum Integral Equations

5. Pump Calculations

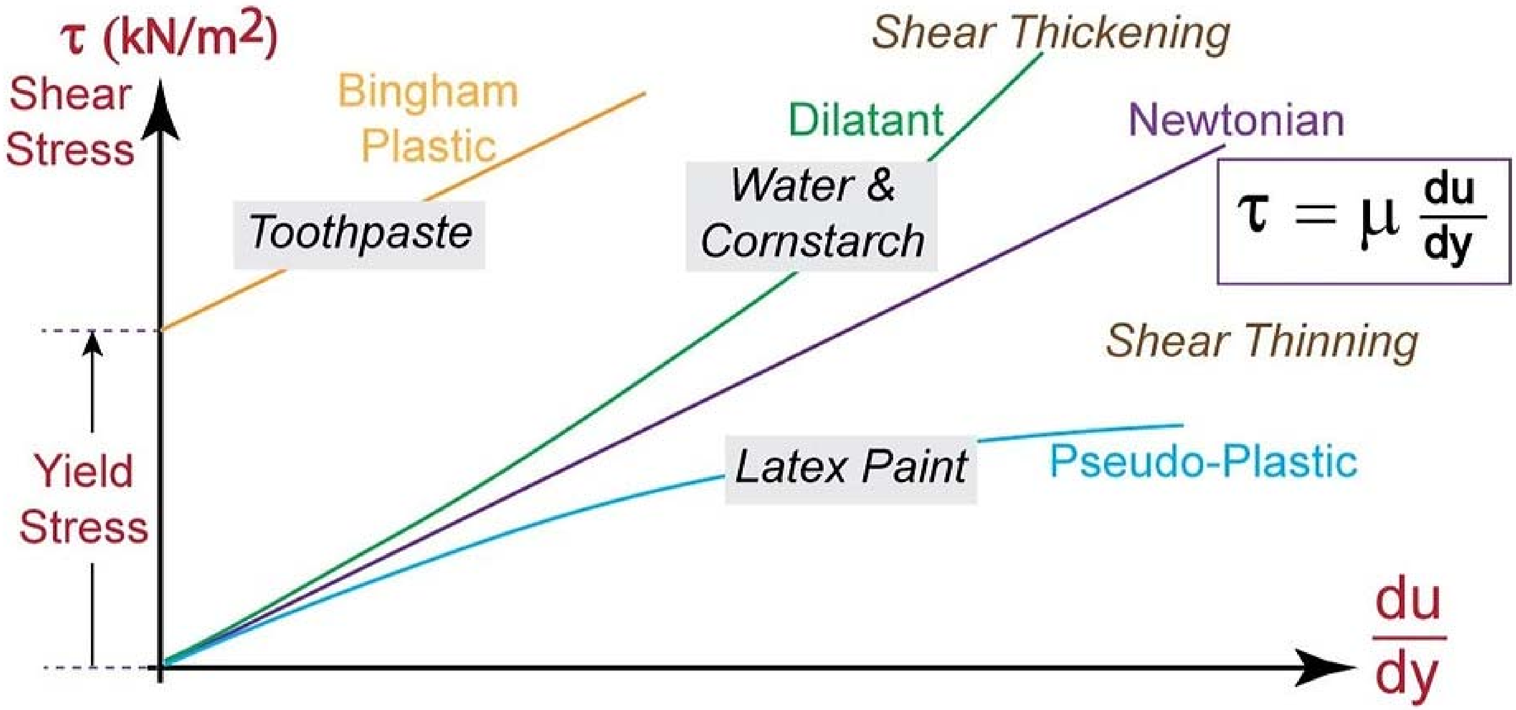
6. Flowmeters

Shear Stress – viscosity – Molecular Transport of MM



Components of shear stress Tensor – 9 components, 3 of which are termed as normal stresses

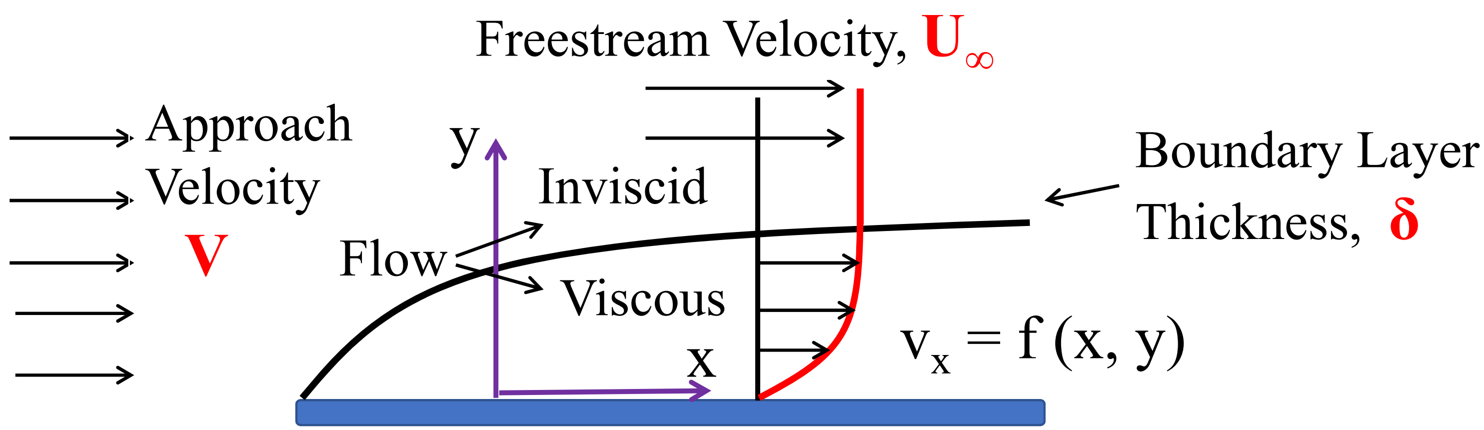
Newtonian and non-Newtonian Fluids



1 – Newtonian 2. Pseudoplastic 3. Dilatant 4. Bingham Plastic

K = consistency index, n = flow behaviour index

External Incompressible Viscous Flow – Boundary Layer



Flow over a flat plate

$v_x = f(x, y)$ Viscous 2D flow inside BL

$v_x = U_\infty$ Inviscid flow outside BL

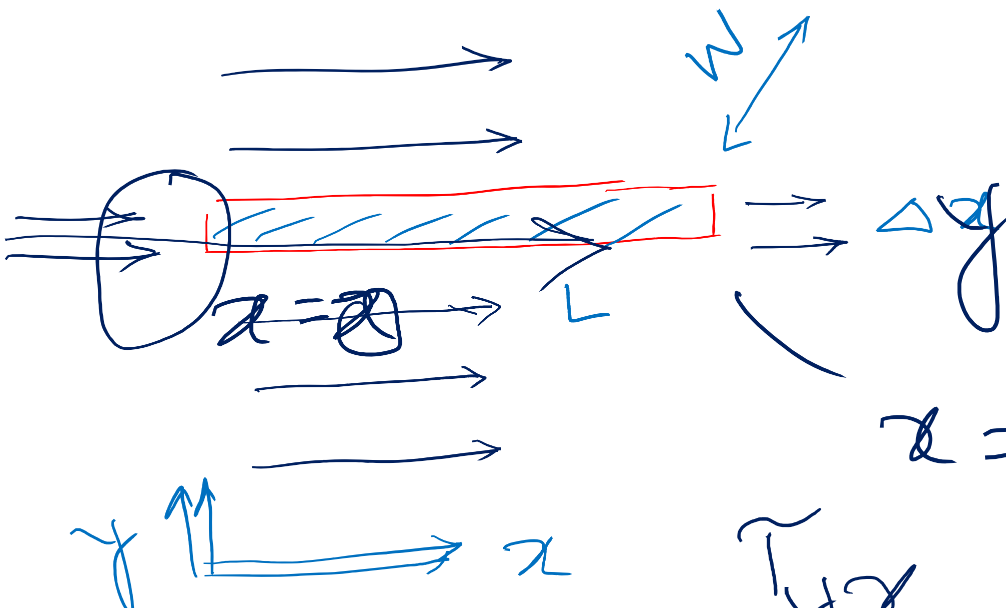
$v_x = 0.99U_\infty$
at $y = \delta$

δ - boundary layer thickness

Velocity Distributions in Laminar Flow

Shell MM Balance Rate of MM IN – Rate of MM OUT + Sum of all forces acting on it = 0

At Steady State/flow



IN

$$\rho \, \Delta y \, W \, v_x \Big|_{x=0} \; v_x \Big|$$

$x = 0$

OUT

$x = L$

$$\left(\quad \right) \Big|_{x=L}$$

τ_{yx}

Boundary Conditions

No Slip

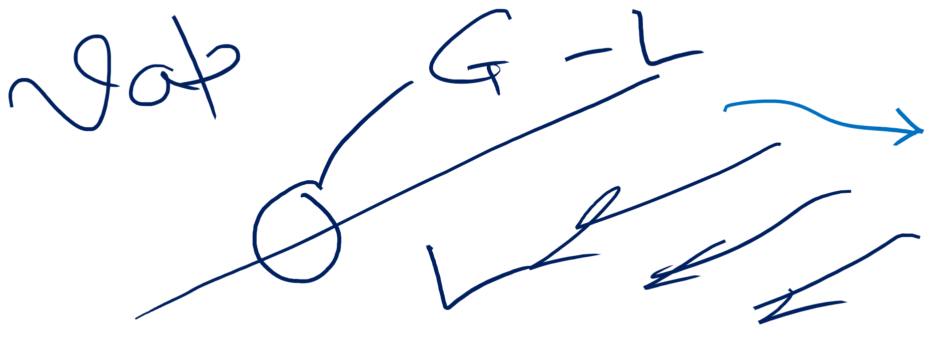
SL INTERFACE

$v = 0$

Relative vel. at SL interf.

$v_{rel} = 0$

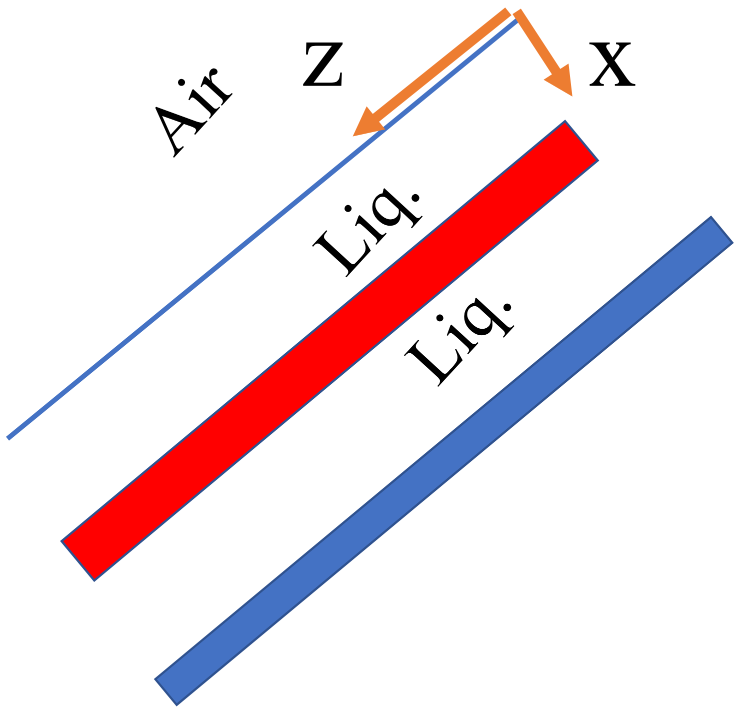
No Shear



Shear at L-V interface = 0

$\tau = 0$

Flow along an inclined plane



At Steady State/flow

Rate of MM IN – Rate of MM OUT + Σ F = 0

Assume Newtonian Fluid, v_z is a function of x only

$v_y, v_x = 0$

$v_z \neq 0$, $v_z \neq f(\text{time})$ at any given location

Flow along an inclined plane

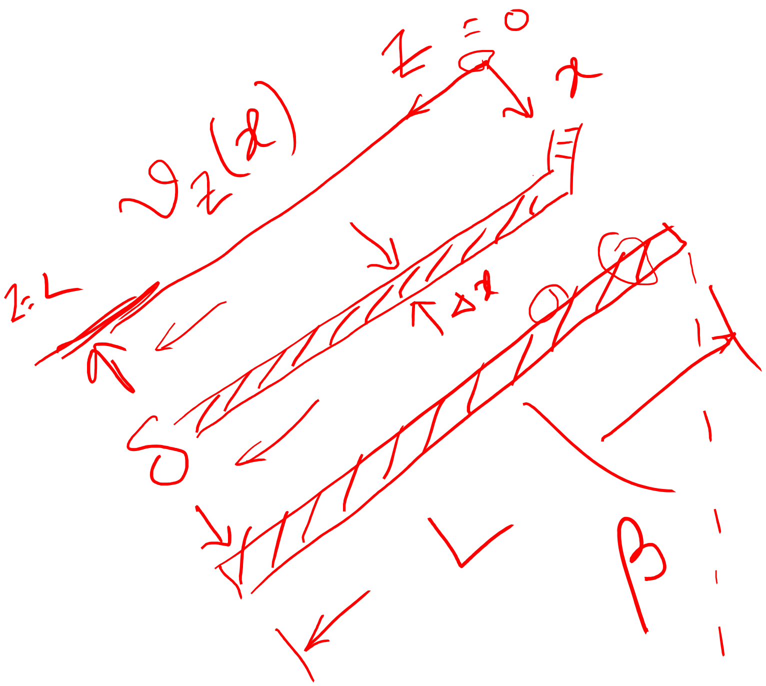
$v_z = f(x)$, $v_z \neq f(y)$

$z \rightarrow W$

WIDTH

SHELL

$LW\Delta x$ ← VOL. OF SHELL



Z M² EQ^N

RATE OF
Z M² IN BY CONV $= (\cancel{\rho} W \Delta x\, v_z\big|_{z=0} (\rho) v_z\big|_{z=0}$

" OUT " $\rho\, (W \Delta x\, v_z)\, v_z\big|_{z=L}$

M² IN BY COND. $= (LW)\, \tau_{xz}\big|_{x=x}$

M² OUT BY COND $= (LW)\, \tau_{xz}\big|_{x+\Delta x}$

BODY FORCE $(LW\Delta x)\rho g\cos\beta$

RATE

m^2

$$\text{IN} - \text{OUT} + \sum F = 0$$

$$LW\tau_{xz}\big|_{x} - LW\tau_{xz}\big|_{x+\Delta x} + W\Delta x\,\rho\,v_z^2\big|_{z=0}$$

$$- W\Delta x\,\rho\,v_z^2\big|_{z=L} + LW\Delta x\,\rho g\cos\beta = 0$$

$$v_z\big|_{z=0} \overset{?}{=} v_z\big|_{z=L}$$

$$\Rightarrow \quad \tau_{xz}\big|_{x+\Delta x} - \tau_{xz}\big|_{x} = \rho g\cos\beta\,\Delta x$$

$$\lim_{\Delta x \to 0} \frac{\tau_{xz}\big|_{x+\Delta x} - \tau_{xz}\big|_{x}}{\Delta x} = \rho g \cos\beta$$

$$\boxed{\frac{d}{dx}\left(\tau_{xz}\right) = \rho g \cos\beta}$$

GOV. EQN

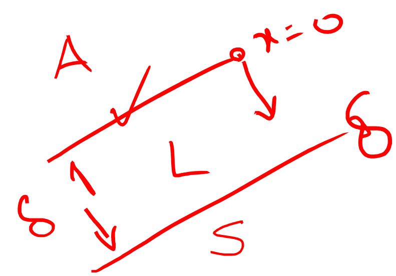
NEWTONIAN FLUID

$v_z = f^n_{(x)}$

$$\tau_{xz} = \rho g \cos\beta \, x + C_1$$

NO SHEAR AT L-V INTERFACE

$x = 0 \quad \tau = 0 \Rightarrow C_1 = 0$



$$\tau_{xz} = \rho g \cos\beta \; x$$

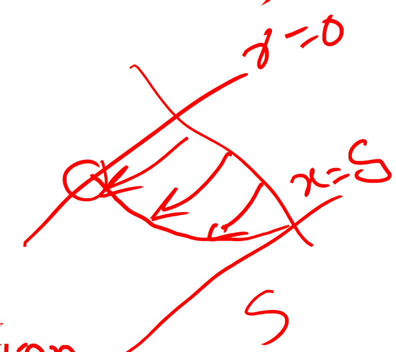
$$-\mu \frac{dv_z}{dx} = \rho g \cos\beta \; x$$

[NEWT. FLUID ONLY]

$$v_z = \frac{-\rho g \cos\beta}{\mu} \frac{x^2}{2} + C_2$$

BC. $x = \delta \quad v_z = 0 \Longrightarrow C_2$ (NO SLIP COND)

$$v_z = \frac{\rho g \delta^2 \cos\beta}{2\mu}\left[1 - \left(\frac{x}{\delta}\right)^2\right]$$



Parabolic distribution

$$v_z = \frac{\rho g \delta^2 \cos\beta}{2\mu}\left[1-\left(\frac{x}{\delta}\right)^2\right]$$

$$\Rightarrow v_{z,\,MAX}\Big|_{at\ x=0} \rightarrow \rho g \delta^2 \cos\beta / 2\mu .$$

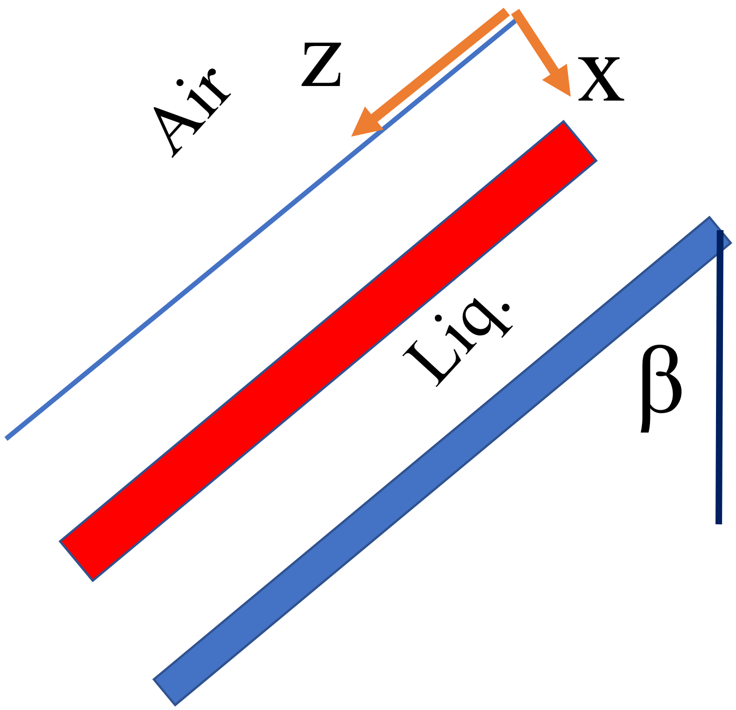
AV. VEL

$$\langle v_z \rangle = \frac{\int_0^W \int_0^\delta v_x \, dx \, dy}{W\delta}$$

VOL. FLOW RATE

$$\langle v_z \rangle = \frac{\rho g \delta^2 \cos\beta}{3\mu} .$$

$$Q = \langle v_z \rangle . W\delta$$

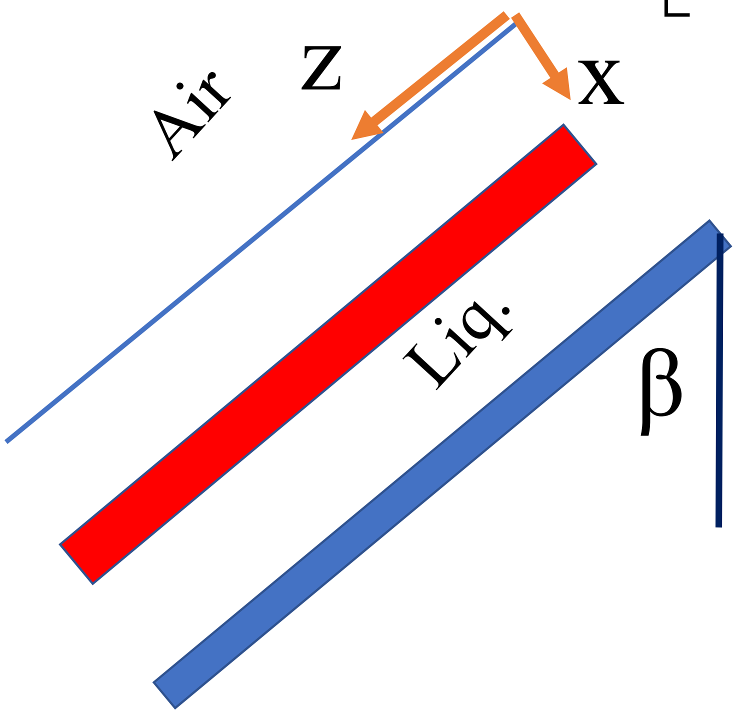


$$v_z = \frac{\rho g \cos\beta}{2\mu}\left[1-\left(\frac{x}{\delta}\right)^2\right]$$

$$v_{z,Max} = \frac{\rho g \cos\beta}{2\mu}$$

$$v_z = v_{z,Max}\left[1-\left(\frac{x}{\delta}\right)^2\right]$$

$$v_z = v_{z,Max}\left[1-\left(\frac{x}{\delta}\right)^2\right]$$



$$\langle v_z \rangle = \frac{\int_0^W \int_0^\delta v_z \, dx \, dy}{\int_0^W \int_0^\delta dx \, dy}$$

Flow Area Average Velocity

$$\langle v_z \rangle = \frac{\rho g \cos\beta}{3\mu}$$

$$Q = W\delta \langle v_z \rangle = \frac{W\delta\rho g \cos\beta}{3\mu}$$

Flow rate

$$F_z = \int_0^L \int_0^W \left(-\mu \frac{dv_z}{dx}\right)\Bigg|_{x=\delta} dy\, dz$$

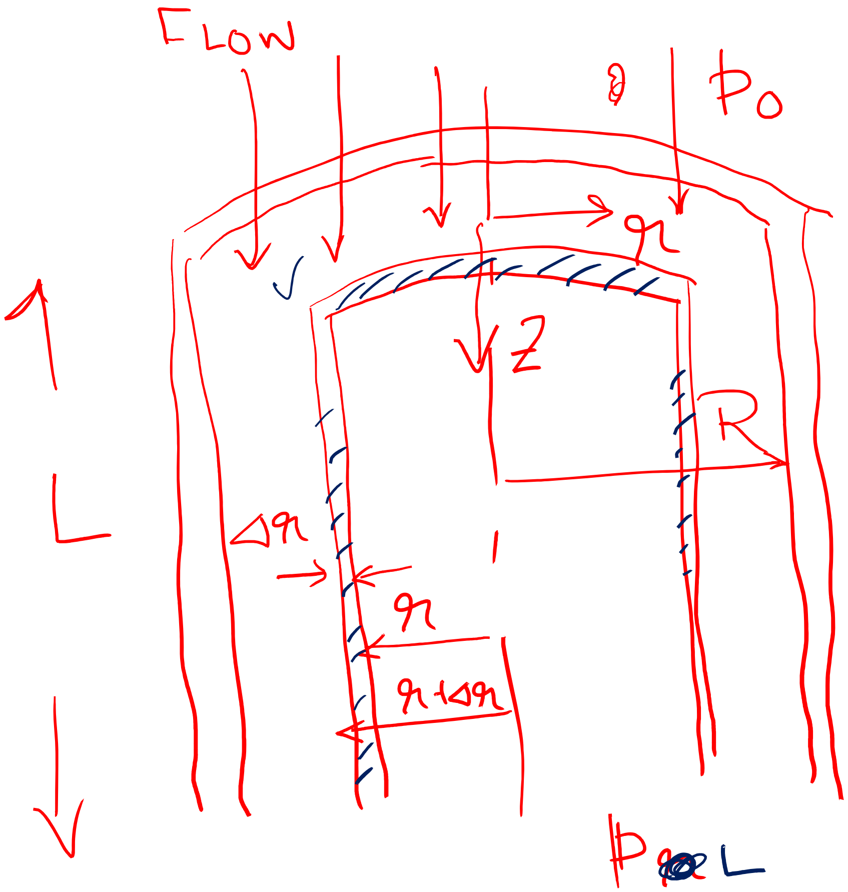
BSL

$$\boxed{F_z = \left(\rho g \delta \cos\beta\right) LW}$$

SHELL
m^2 BALANCE

BSL
Ch - 2

Flow Through a Circular Tube



$$v_z = f(r) \neq \theta, z$$

$$\tau_{rz}$$

M² IN

$$2\pi r \Delta r \cancel{v_z} \rho v_z \Big|_{z=0} - \cancel{(\quad)}\Big|_{z=L}$$

$$v_z \neq f(z)$$

M² IN COND.

$$L\left[\tau_{rz}\Big|_{r=r} 2\pi r - \tau_{rz}\Big|_{r+\Delta r} 2\pi(r+\Delta r)\right]$$

$$2\pi r \Delta r\, p_0 - 2\pi r \Delta r\, p_L$$

$$+ \underline{\underline{2\pi r \Delta r}} L \rho g = 0$$

$$\lim_{\Delta r \to 0} \left[\frac{(r\tau_{rz})\big|_{r+\Delta r} - r\tau_{rz}\big|_{r}}{\Delta r} \right] = \left(\frac{p_0 - p_L}{L} + \rho g \right) r$$

Define

$$\mathcal{P} \equiv p_z - \rho g z$$

$$\mathcal{P}_0 = p_0$$

$$\mathcal{P}_L = p_L - \rho g L$$

NEW FL

$$\frac{d}{dr}(r\tau_{rz}) = \left[\frac{p_0 - p_L}{L} + \rho g \right] r$$

BC

$\underline{1}$ = ? $\quad r = 0 \quad \boxed{\tau_{rz} = 0}$

2 $\Rightarrow$ $\quad r = R, \; v_z = 0$

$$v_z = \left[\qquad \right] \frac{R^2}{4\mu L} \left[1 - \left(\frac{r}{R} \right)^2 \right]$$

$$v_z = \frac{(P_0 - P_L) R^2}{4 \mu L} \left[1 - \left(\frac{r}{R} \right)^2 \right]$$

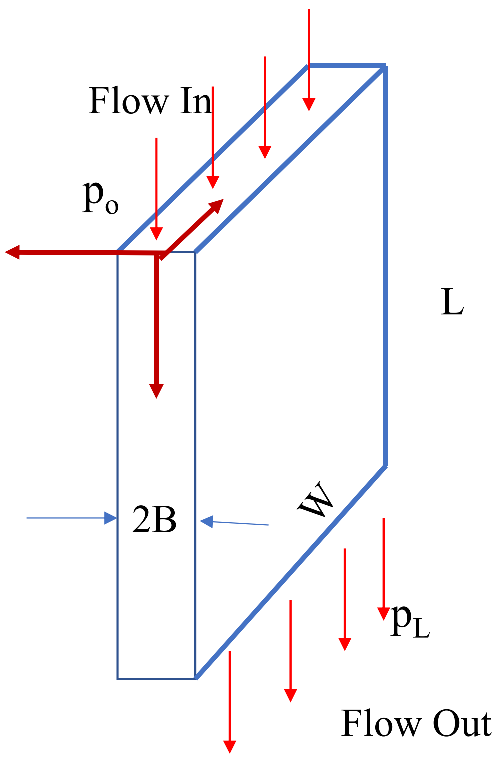
$$v_{z,max} = \frac{P_0 - P_L}{4 \mu L} R^2$$

$$\langle v_z \rangle = \frac{\int_0^{2\pi} \int_0^R v_z \, r \, dr \, d\theta}{\int_0^{2\pi} \int_0^R r \, dr \, d\theta} = \frac{(P_0 - P_L) R^2}{8 \mu L}$$

VOL. FLOW RATE, $Q = \langle \pi R^2 \rangle \langle v_z \rangle$

$$\boxed{Q = \frac{\pi (P_0 - P_L) R^4}{8 \mu L}}$$

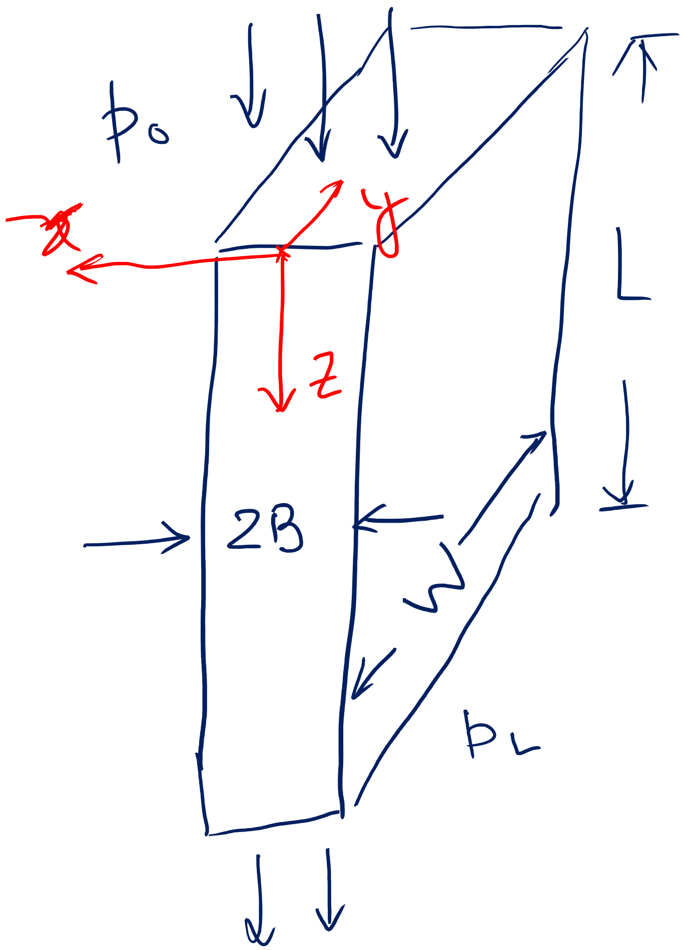
HAGEN POISEUILLE ~~EE~~ LAW



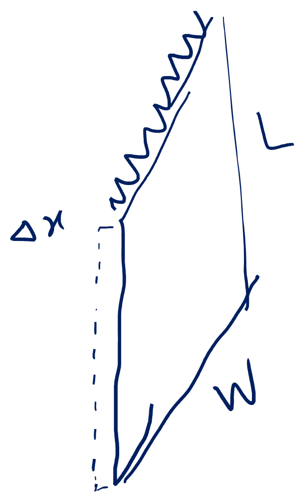
Consider flow between to large, vertical plates of length L and width W, separated by a small distance, 2B (separation << length, width, so that 1D flow assumption is justified). The flow is due to gravity, as well as an applied pressure gradient.

Evaluate expressions for the velocity distribution, the average velocity and the relation between the average and the maximum velocity.

LAMINAR FLOW IN A NARROW SLIT



FIND
1) VEL. PROFILE
2) AV. VEL.
3) RELATION BET^N $\langle v \rangle$ AND ~~$\langle v \rangle$~~ v_{max}



$M^2_{IN} - OUT + \Sigma F$ 0

~~CONV.~~ COND. GR + PR.

FULLY DEVELOPED

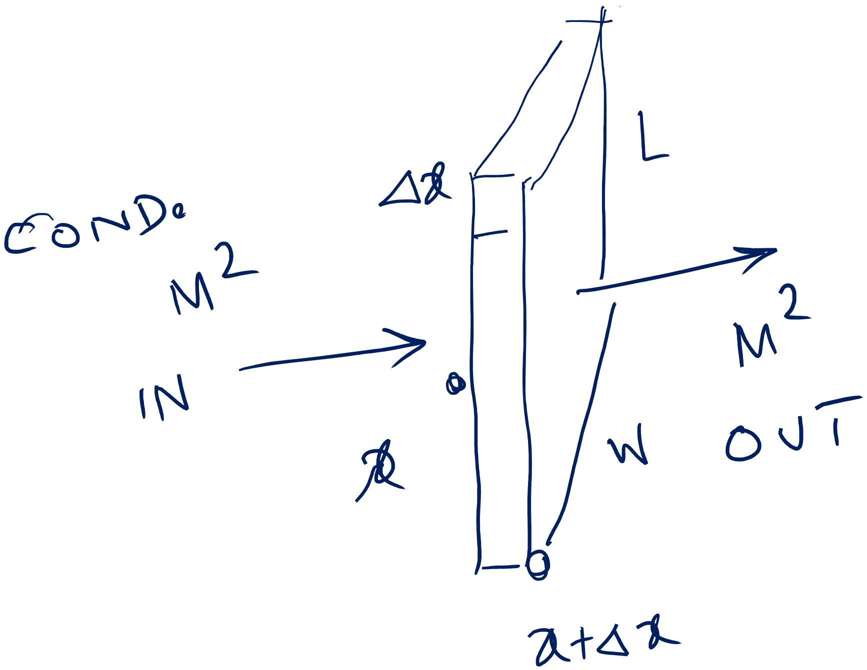
$v_z \neq f(z) \neq f(y)$

$W >> 2B$

τ_{xz}

$D \; v \; \rho \; \mu$

$\frac{D v \rho}{\mu}$



$$\tau_{xz}\Big|_{x} LW - \tau_{xz}\Big|_{x+\Delta x} LW$$

$$+ p_0 W \Delta x - p_L W \Delta x + (\Delta x W L \rho) g = 0$$

$$v_z = \frac{B^2}{2\mu}\left(\frac{P_0 - P_L}{L}\right)\left[1 - \left(\frac{x}{B}\right)^2\right]$$

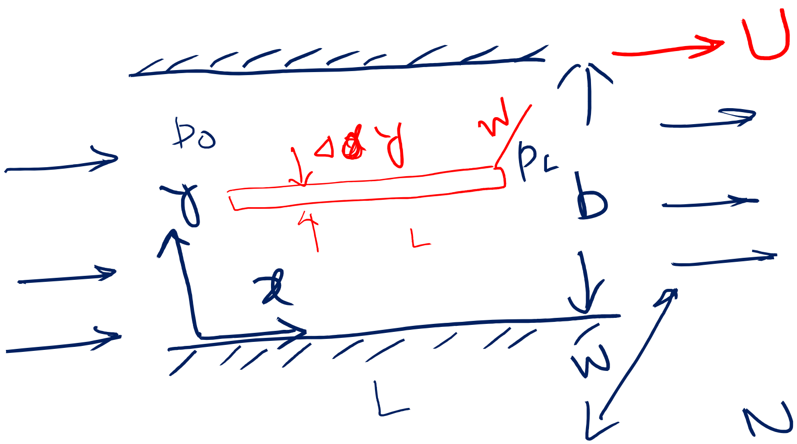
$$\boxed{\langle v_z \rangle = \frac{2}{3} v_{max}}$$

Flow between parallel plates, one moving the other stationary, with or without applied pressure gradient

Consider the case of a liquid between two parallel plates separated by a distance h. The origin of the coordinate system should be on the bottom plate. The top plate moves to the right with a constant velocity U. Obtain and sketch (qualitative) the velocity profile if an unfavorable pressure gradient is imposed on the flow such that the net flow rate is zero. Find the maximum favorable pressure gradient that can be applied so that the maximum velocity in the fluid will be the velocity of the top plate. Sketch the velocity profile for this case. Calculate the force necessary to move the plate in the second case.

FLOW BET^N || PLATES

$v_x = f(y)$ ONLY



$$\left(\tau_{yx}\Big|_{y}\right)LW - \left(\quad\right)\Big|_{y+\Delta y}LW$$

$$= \;\; + (p_0 - p_L)\Delta y\, W = 0$$

NEWT. LIQ.

$$\frac{d^2 v_x}{dy^2} = \frac{1}{\mu}\left(\frac{dp}{dx}\right)$$

$$\boxed{\frac{dp}{dx} = \frac{p_L - p_0}{L - 0}}$$

BCs

1) $v_x = U$ at $y = b$

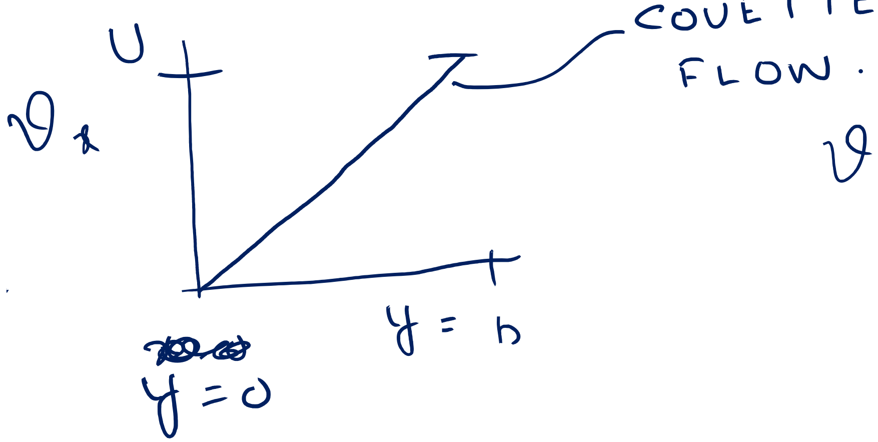
2) $v_x = 0$ at $y = 0$

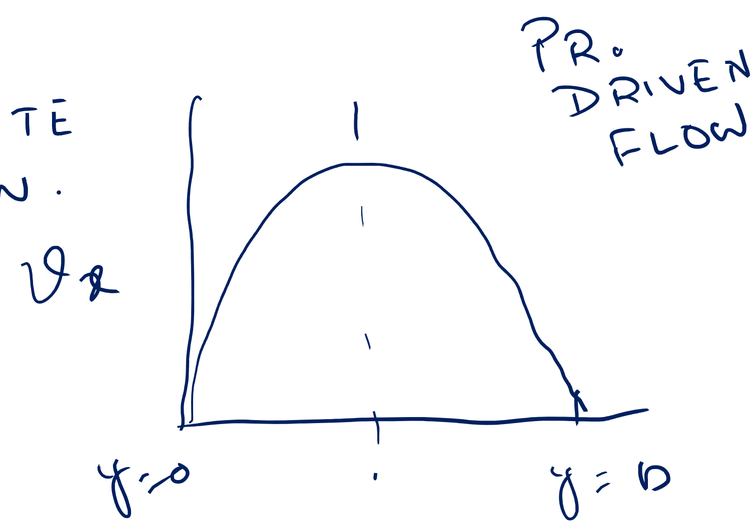
] NO SLIP

$$v_x = \frac{U}{b} y - \frac{1}{2\mu}\left(\frac{dp}{dx}\right) b^2 \left[\frac{y}{b} - \left(\frac{y}{b}\right)^2\right]$$

IF $\frac{dp}{dx} = 0$, $\boxed{v_x = \frac{U}{b} y}$ → COUETTE FLOW

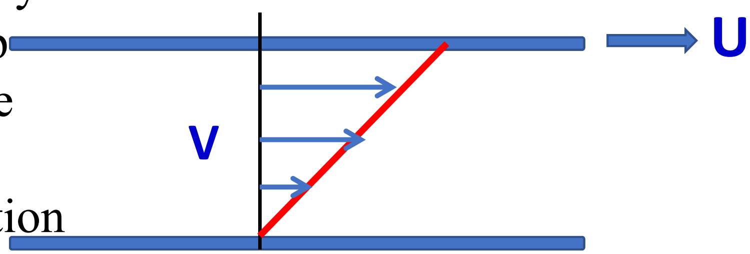
IF $U = 0$ v_x = PARABOLIC PROFILE

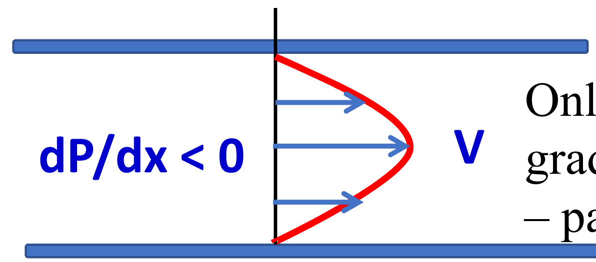




Flow between parallel plates

Couette flow (only motion of the top plate, no pressure gradient – linear velocity distribution



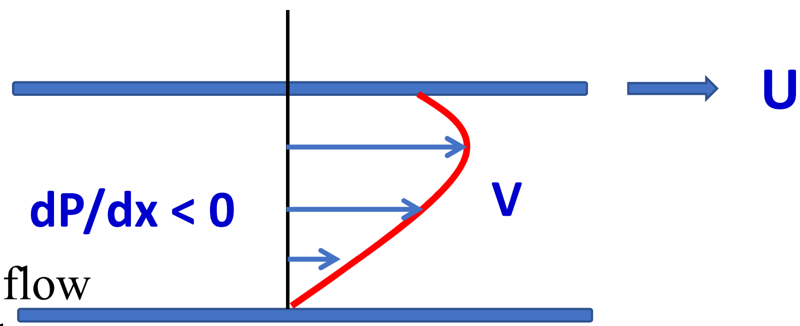


Only pressure gradient driven flow – parabolic profile

Couette Flow

Pressure gradient Driven Flow

Superposition of Couette flow and pressure gradient
Location of maximum velocity
Unfavorable pressure gradient and Couette flow – may lead to zero net flow
Force needed to move the moving plate

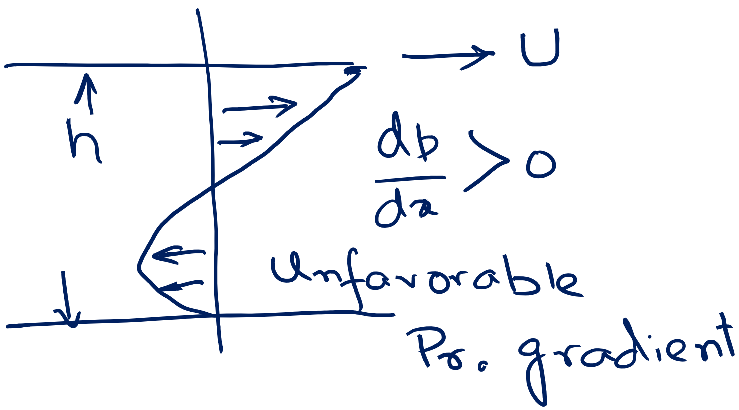


Couette and Pressure gradient Driven Flow

Obtain and sketch (qualitative) the velocity profile if an unfavorable pressure gradient is imposed on the flow such that the net flow rate is zero.

$$v_x = \frac{U}{h} y + \frac{1}{2\mu} A y^2 \left[1 - \frac{h}{y}\right], \quad A = \frac{dp}{dx}$$

For net flow to be zero, $\int_0^h v_x \, dy = 0$ (Av. vel = 0)



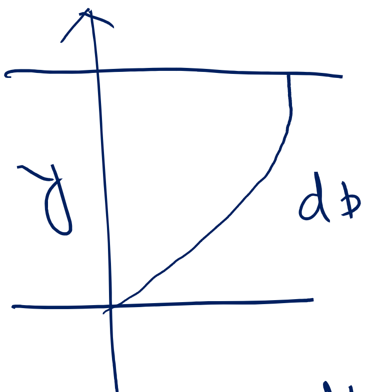
$$\Rightarrow A = \frac{6\mu U}{h^2} = \frac{dp}{dx}$$

And

$$v_x = \frac{U}{h} y + \frac{3U}{h^2} y^2 \left[1 - \frac{h}{y}\right]$$

Find the maximum favorable pressure gradient that can be applied so that the maximum velocity in the fluid will be the velocity of the top plate. Sketch the velocity profile for this case. Calculate the force necessary to move the plate in the second case.

Condition of the problem



$\rightarrow U$

$dp/dx > 0$

Any dp/dx larger than this would result in a velocity $> U$ in between $y = 0 \xrightarrow{\text{to}} h$.

Here $dv_x/dy\big|_{y=h} = 0$

$$\boxed{\Rightarrow A = -\frac{2\mu U}{h^2}}$$

Since Force $= \tau \times A$

$$= -\mu \frac{dv_x}{dy}\bigg|_{y=h} \times A$$

As $\frac{dv_x}{dy}\big|_{y=h} = 0$

$\therefore$ Force needed is zero for this case