Q1.

At maximum flow rate, the pressure at the throat will be minimum, and is equal to vapor pressure of water at 100°C, which is 10^5 Pa. ≈ 0 (gauge)

$$\frac{\left(\frac{Q}{0.01}\right)^2}{2} + \frac{100\times10^3 \text{ Pa}}{1000 \text{ kg/m}^3} = \frac{\left(\frac{Q}{0.001}\right)^2}{2} + \frac{0 \text{ Pa}}{1000 \text{ kg/m}^3}$$

Solve for Q

Q2 Using superposition

$$\psi = Uy + q\tan^{-1}\left(\frac{y}{x+a}\right) - q\tan^{-1}\left(\frac{y}{x-a}\right).$$

Since $\ln z = \ln(x+iy)$

$= \frac{1}{2}\ln(x^2+y^2) + i\tan^{-1}\left(\frac{y}{x}\right)$

and $\ln(z-a) = \ln((x-a)+iy)$

$= \frac{1}{2}\ln((x-a)^2+y^2) + i\tan^{-1}\left(\frac{y}{x-a}\right)$

similarly $\ln(z+a) = \ldots$

$$= Uy - q\tan^{-1}\left[\frac{2ay}{x^2+y^2-a^2}\right]$$

Since $\tan^{-1}(\alpha) - \tan^{-1}(\beta) = \tan^{-1}\left(\frac{\alpha-\beta}{1+\alpha\beta}\right).$

$$\Rightarrow u = \frac{\partial\psi}{\partial y} = U + q\frac{1}{\left[1+\frac{2ay}{x^2+y^2-a^2}\right]^2}$$

$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$

This should be equal to

$$= U + q\left[\frac{x+a}{(x+a)^2+y^2} - \frac{x-a}{(x-a)^2+y^2}\right]$$

$$\frac{d}{dy}\left[\frac{2ay}{x^2+y^2-a^2}\right] = \left(\frac{2a}{x^2+y^2-a^2}\right) - \frac{2ay(2y)}{(x^2+y^2-a^2)^2}$$

$$v = -\frac{\partial \psi}{\partial x} = q\,y\left[\frac{1}{(x+a)^2+y^2} - \frac{1}{(x-a)^2+y^2}\right]$$

For stagnation point, $u = v = 0$

$$= 0 \implies y = 0.$$

$u = 0 \implies$

$$U + q\left[\frac{x+a}{(x+a)^2} - \frac{x-a}{(x-a)^2}\right] = 0$$

$$\implies U + q\left[\frac{x-a-x-a}{x^2-a^2}\right] = 0$$

$$\implies U = q\,\frac{2a}{x^2-a^2} \implies x = \pm a\left[1+\frac{2q}{Ua}\right]^{1/2}$$

Two stagnation points as in Rankine Full Body.

The value of streamfunction passing through stagnation poin

$$\psi = Uy + q\tan^{-1}\left(\frac{y}{x+a}\right) - q\tan^{-1}\left(\frac{y}{x-a}\right)$$

Putting $y = 0$ and $x = \pm a\sqrt{1+\frac{2q}{Ua}}$

$$\psi_{\substack{\text{Body} \\ \text{Streamline}}} = 0$$