Basic Equations in Integral Form for a Control Volume

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Ch - 4

N = Arbitrary extensive property of a system

$$N\Big|_{SYSEM} = \int_{Mass(System)} \eta \, dm = \int_{\cancel{V}(System)} \eta \rho \, d\cancel{V}$$

η = Corresponding intensive property

Basic Equations in Integral Form for a CV

Conservation of Mass

Conservation of Momentum

Relation of System Derivatives to the Control Volume Formulation

$$\left.\frac{\partial N}{\partial t}\right|_{SYSEM} = \frac{\partial}{\partial t}\int_{CV} \eta\rho d\mathcal{V} + \int_{CS} \eta\rho\vec{V}.d\vec{A}$$

$\vec{V}$ is measured relative to the CV

$\left.\frac{\partial N}{\partial t}\right|_{SYSEM}$ = Total rate of change of any arbitrary extensive property of the system

$\frac{\partial}{\partial t}\int_{CV} \eta\rho d\mathcal{V}$ = Time rate of change of the arbitrary extensive property within the CV

$\int_{CS} \eta\rho\vec{V}.d\vec{A}$ = Net rate of efflux of the extensive property, N, through the control surface

Conservation of Mass N = Mass, η = 1

$$\left.\frac{\partial N}{\partial t}\right|_{SYSEM} = \frac{\partial}{\partial t}\int_{CV} \eta\rho d\forall + \int_{CS} \eta\rho\vec{V}.d\vec{A}$$

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho\vec{V}.d\vec{A}$$

Incompressible Fluid

$$0 = \int_{CS} \rho\vec{V}.d\vec{A}$$

The size of the CV is fixed

$$\int_{CS} \rho\vec{V}.d\vec{A} = \pm\left|\rho_n V_n A_n\right|$$

When uniform flow at section n is assumed

Momentum Equation for Inertial CV, N = Momentum, η = Velocity

$$\left.\frac{\partial N}{\partial t}\right|_{SYSEM} = \frac{\partial}{\partial t}\int_{CV} \eta\rho d\cancel{V} + \int_{CS} \eta\rho\vec{V}.d\vec{A}$$

$$\vec{F} = \vec{F}_S + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \vec{V}\rho d\cancel{V} + \int_{CS} \vec{V}\rho\vec{V}.d\vec{A}$$

$$\vec{F}_B = \int_{CV} \vec{B}\rho d\cancel{V} \qquad \vec{F}_S = \int_{A} -p\,d\vec{A}$$

Scalar Component

$$F_x = F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho\, d\cancel{V} + \int_{CS} u\,\rho\vec{V}.d\vec{A}$$

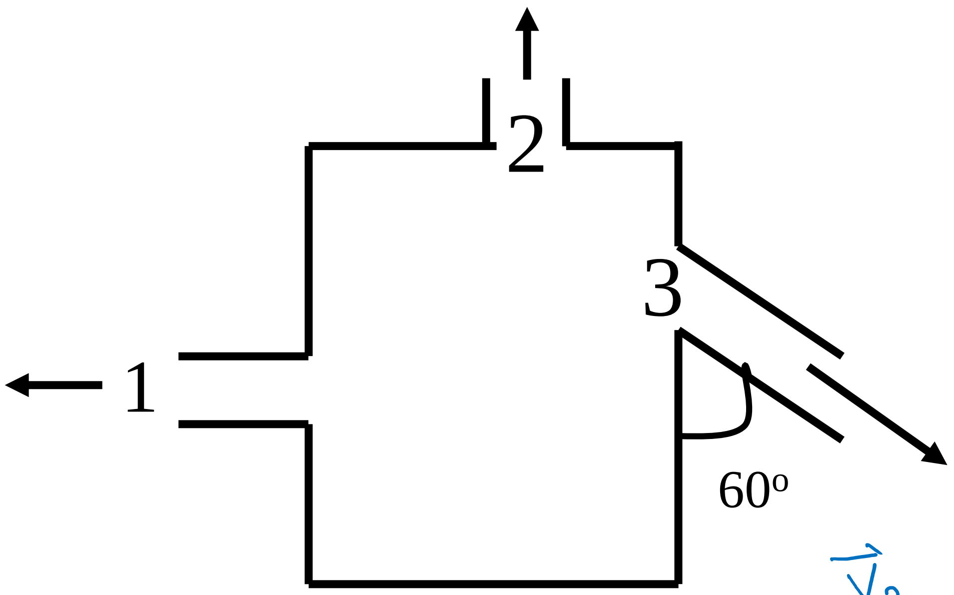
1. To determine the sign of

$$\rho\vec{V}.d\vec{A} = \pm\left|\rho V\, dA\cos\alpha\right|$$

2. To determine the sign of each velocity component

$$u\,\rho\vec{V}.d\vec{A} = u\left\{\pm\left|\rho V\, dA\cos\alpha\right|\right\}$$

Steady Incompressible Flow



Fluid with ρ = 1050 kg/m3 is flowing through the box,
$A_1 = 0.05\ m^2$, $A_2 = 0.01\ m^2$, $A_3 = 0.06 m^2$

$$\vec{V}_1 = 4\,\hat{\imath}\ m/s \qquad \vec{V}_2 = -8\,\hat{\jmath}\ m/s$$

Find V3

$$\vec{V}_1 \cdot \vec{A}_1 + \vec{V}_2 \cdot \vec{A}_2 + \vec{V}_3 \cdot \vec{A}_3 = 0$$

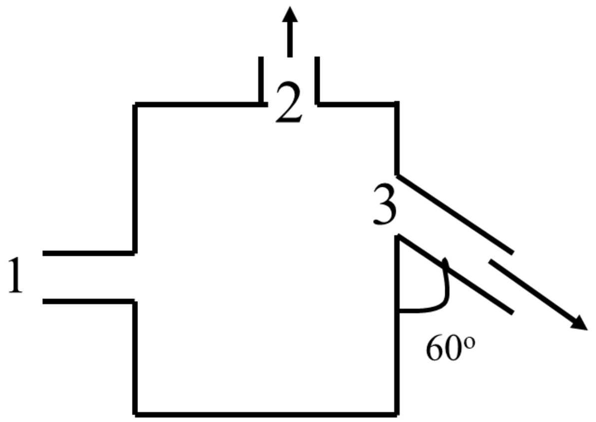
$$\vec{V}_3 \cdot \vec{A}_3 = -\vec{V}_1 \cdot \vec{A}_1 - \vec{V}_2 \cdot \vec{A}_2$$

$$= -4\hat{i} \cdot 0.05(-\hat{i}) - (-8\hat{j}) \cdot 0.01\hat{j}$$

$\vec{V}_3 \cdot \vec{A}_3 = 0.28\ m^3/s$. Since $\vec{V}_3 \cdot \vec{A}_3 > 0$, flow at section 3 is out of CV. $V_3 = \frac{1}{A_3} \times 0.28\ m^3/s = 4.67\ m/s$

From geometry $\vec{V}_3 = V_3 \sin\theta\,\hat{i} - V_3 \cos\theta\,\hat{j} = 4.04\hat{i} - 2.34\hat{j}$

Find the net rate of efflux of momentum through the CV



$\vec{V}_1 = 4\,\hat{i}\ m/s$ $\quad \vec{V}_2 = -8\,\hat{j}\ m/s$

The net rate of momentum efflux is given by $\int_{CS} \vec{V}\rho\vec{V}\cdot d\vec{A}$

$$= \vec{V}_1 \rho \vec{V}_1 \cdot \vec{A}_1 + \vec{V}_2 \rho \vec{V}_2 \cdot \vec{A}_2 + \vec{V}_3 \rho \vec{V}_3 \vec{A}_3$$

$$\overrightarrow{mf} = \left[u_1\{-|\rho V_1 A_1|\} + u_2\{-|\rho V_2 A_2|\} + u_3\{|\rho V_3 A_3|\}\right]\hat{i}$$

$u_1 = 4\ m/s$ $\quad u_2 = 0$ $\quad u_3 = 4.04\ m/s$

$$+\left[v_1\{-|\rho V_1 A_1|\} + v_2\{-|\rho V_2 A_2|\} + v_3\{|\rho V_3 A_3|\}\right]\hat{j}$$

$v_1 = 0$ $\quad v_2 = -8\ m/s$ $\quad v_3 = -2.33\ m/s$

$$\overline{mf} = 349\,\hat{i} - 13.5\,\hat{j}\ \ N$$