Heat Transfer

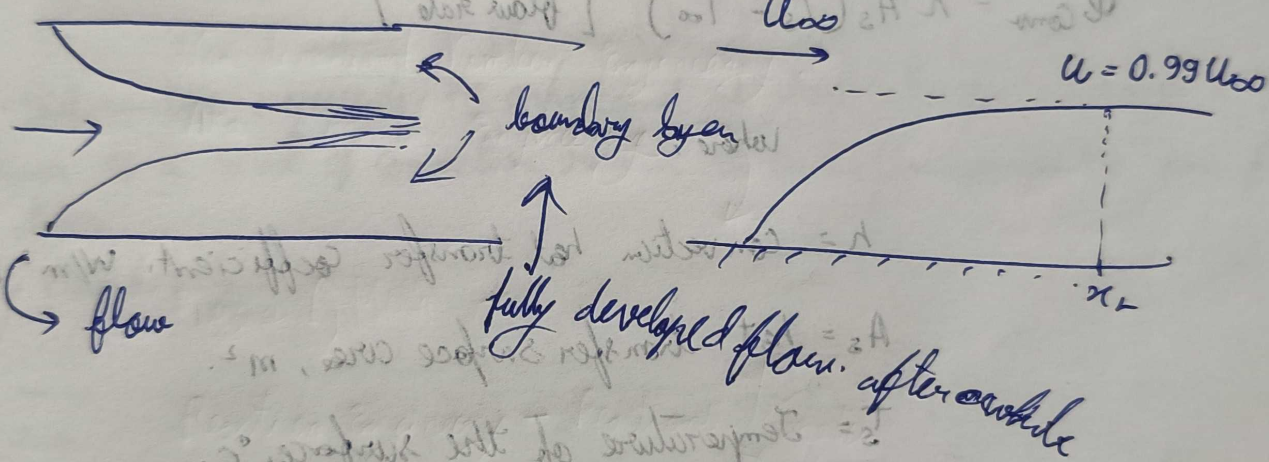
→ Thermodynamics with time dependence.

→ Mode and rate are discussed.

Advection: Bulk movement of fluid.

Conduction: The molecules transfer heat but don't move.

Conduction + Advection = Convection.



Newton's law of cooling:

area of heat transfer

$$q = h A \Delta T$$

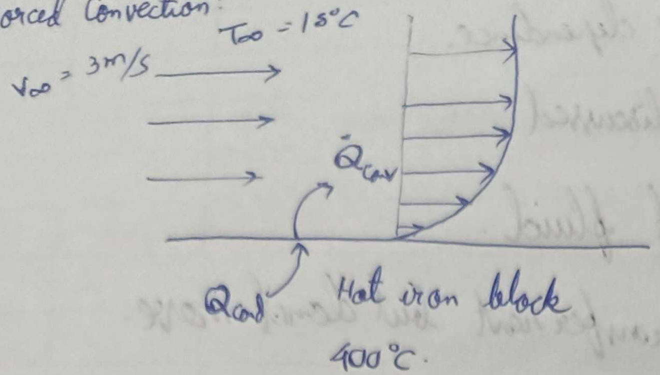
rate of Heat transfer

driving force (change in temp).

convection heat transfer coefficient.

h is dependent on, μ, ρ, C_p, C_v.

Forced Convection:



$$\dot{q}_{conv} = h(T_s - T_\infty) \quad [\text{flux}]$$

$$\dot{Q}_{conv} = h A_s (T_s - T_\infty) \quad [\text{flow rate}]$$

where,

h = Convection heat transfer coefficient, $W/m^2 \cdot °C$

A_s = heat transfer surface area, m^2.

T_s = Temperature of the surface, °C.

T_∞ = Temperature of the fluid sufficiently far away from the surface.

So.

$$\dot{q}_{conv} = \dot{q}_{cond} = -k_{fluid} \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

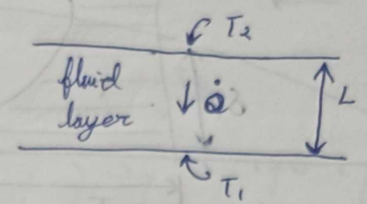
$$h(T_s - T_\infty) = -k_{fluid} \left.\frac{\partial T}{\partial y}\right|_{y=0}.$$

So.

$$\boxed{h = \frac{-k_{fluid} (\partial T/\partial y)_{y=0}}{T_s - T_\infty}}$$

Nusselt Number:

$$Nu = \frac{h L_c}{k}$$



$$\Delta T = T_2 - T_1$$

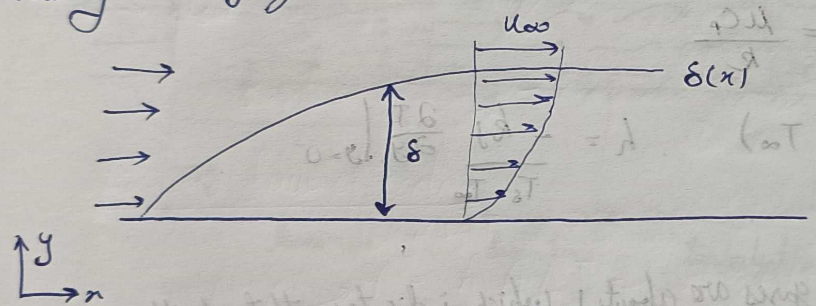
$$\dot{q}_{conv} = h \Delta T$$

$$\dot{q}_{cond} = k \frac{\Delta T}{L}$$

$$Nu = \frac{\dot{q}_{conv}}{\dot{q}_{cond}}$$, $Nu \geq 1$ as conduction + advection = convection.

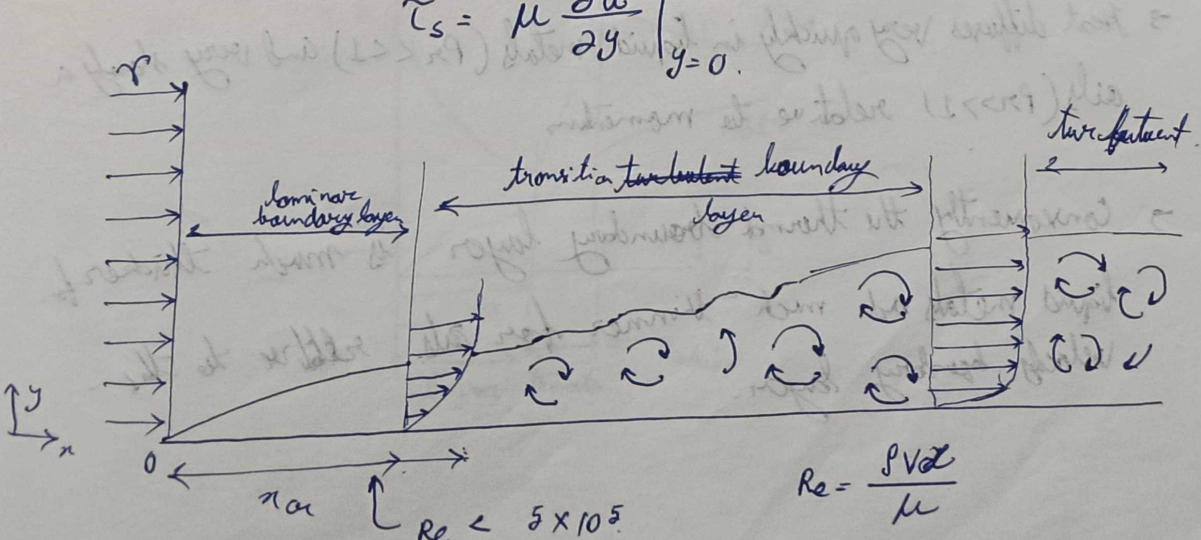
→ Nusselt number represents the enhancement of heat transfer through a fluid layer as a result of convection relative to conduction across the same fluid-layer.

velocity Boundary layer;

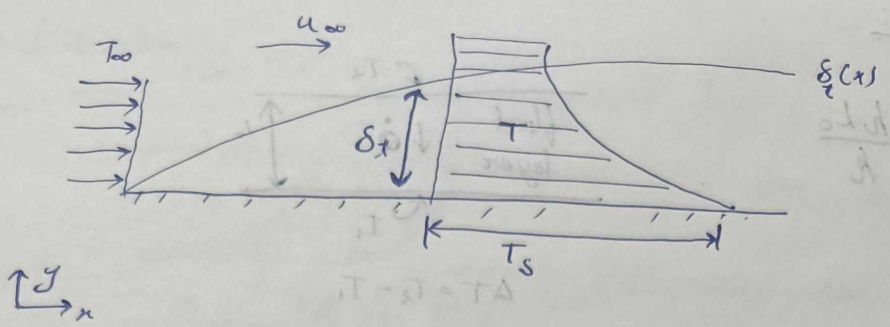


$$C_f \equiv \frac{\tau_s}{\rho u_\infty^2 / 2}$$

$$\tau_s = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0}.$$



$$Re = \frac{\rho V L}{\mu}$$



$$\frac{T_s - T}{T_s - T_\infty} = 0.99$$

Prandtl number (Pr).

$$Pr = \frac{\text{Molecular diffusivity of momentum}}{\text{molecular diffusivity of heat}}$$

$$= \frac{\nu}{\alpha}$$

$$= \frac{\mu C_p}{k}$$

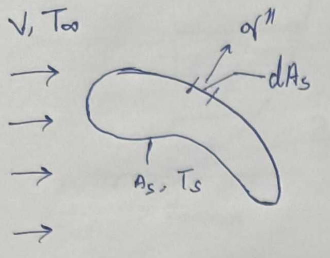
$$q''_s = h(T_s - T_\infty) \qquad h = \frac{-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{T_s - T_\infty}$$

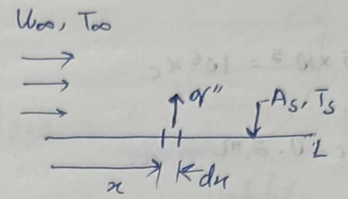
→ Prandtl numbers of gases are about 1, which indicates that both momentum and heat dissipate through the fluid at about the same rate.

→ Heat diffuses very quickly in liquid metals ($Pr << 1$) and very slowly in oils ($Pr >> 1$) relative to momentum

→ Consequently the thermal boundary layer is much thicker for liquid metals and much thinner for oils relative to the velocity boundary layer.

Local and Average Convection coefficients:





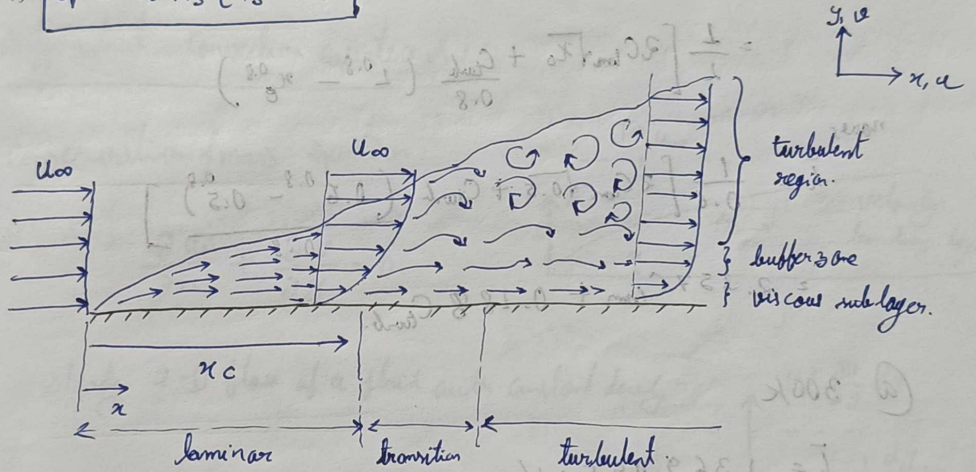
$$q = \int_{A_s} q'' \, dA_s$$

$$q = (T_s - T_\infty) \int_{A_s} h \, dA_s$$

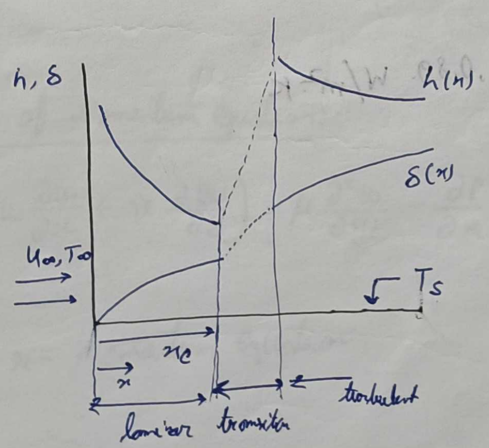
$$\boxed{q = \bar{h} A_s (T_s - T_\infty)}$$

$$\bar{h} = \frac{1}{A_s} \int_{A_s} h \, dA_s$$

$$\bar{h} = \frac{1}{L} \int_0^L h \, dx$$



$$Re_{x,c} \equiv \frac{\rho u_\infty x_c}{\mu} = 5 \times 10^5$$



Q.

$$Re_{x,c} = \frac{\rho u_\infty x_c}{\mu}$$

$$\Rightarrow 5 \times 10^5 = 10^6 x_c$$

$$\therefore x_c = 0.5 \, m$$

So.

$$\bar{h} = \frac{1}{L}\int h\,dx$$

$$= \frac{1}{L}\left[\int_0^{x_c} C_{lam}\, x^{-0.5}\,dx + \int_{x_c}^{L} C_{turb}\, x^{-0.2}\,dx\right]$$

$$= \frac{1}{L}\left[2C_{lam}\sqrt{x_c} + \frac{C_{turb}}{0.8}\left(L^{0.8} - x_c^{0.8}\right)\right]$$

nowes

$$\bar{h} = \frac{1}{0.6}\left[2C_{lam}\sqrt{0.5} + C_{turb}\frac{\left(0.6^{0.8} - 0.5^{0.8}\right)}{0.8}\right]$$

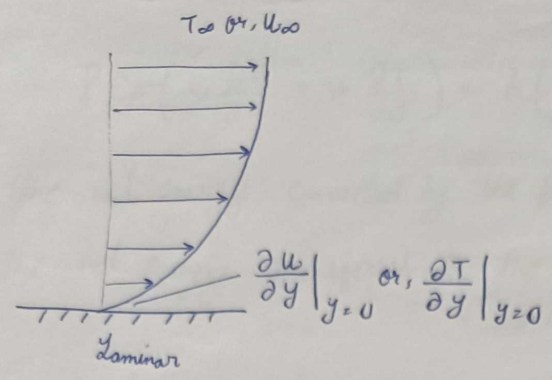
$$= 2.357\, C_{lam} + 0.188\, C_{turb}$$

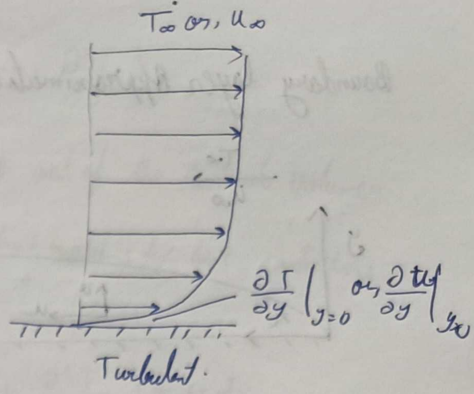
@ 300K

$$\bar{h} = 1369.055 \; W/m^2.K$$

@ 350K

$$\bar{h} = 1801.089 \; W/m^2.K$$

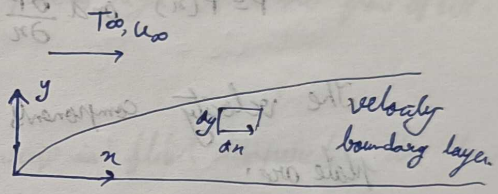




The velocity and temperature gradients at the wall and thus the wall shear stress and heat transfer rate, are much larger for turbulent flow than they are for laminar flow.

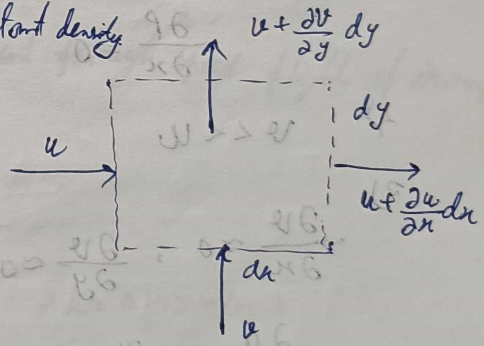
Differential Convection equations:

Conservation of mass equation:



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$$

steady 2-D flow of a fluid with constant density.

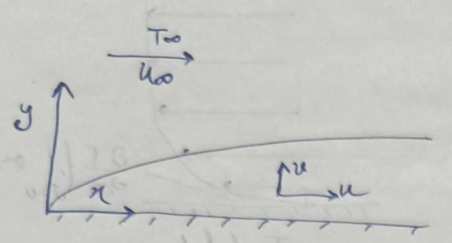


Conservation of Momentum equation:

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x}$$

x- momentum equation.

Boundary layer Approximation -



When body forces are negligible and the boundary layer approximations are valid, applying newton's 2nd law of motion on the volume element in the y-direction gives y-~~direction~~ momentum equation.

$$\frac{\partial P}{\partial y} = 0.$$

$$P = P(x) \text{ and } \frac{\partial P}{\partial x} = \frac{dP}{dx}.$$

The velocity components in the free stream region of a flat plate are:

$$u = u_\infty \quad , \quad v = 0$$

$$\frac{\partial P}{\partial x} \approx 0.$$

1) $v << u$

2) $\frac{\partial v}{\partial x} \approx 0 \quad , \quad \frac{\partial v}{\partial y} \approx 0$

$$\frac{\partial u}{\partial x} << \frac{\partial u}{\partial y}$$

3) $\frac{\partial T}{\partial x} << \frac{\partial T}{\partial y}$

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$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)$$

The net energy convected by the fluid out of the control volume = The net energy transferred into the control volume by heat conduction.

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\mu\Phi$$

viscous dissipation function.

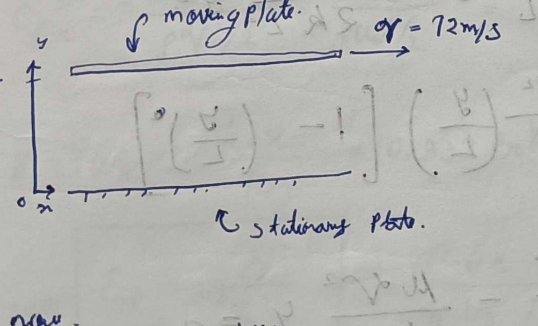
$$\Phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2$$

→ viscous dissipation may play a dominant role in high speed flows, especially when the viscosity of the fluid is high (like the flow of oil in Journal bearings)

→ This manifests itself as a significant rise in fluid temperature due to the conversion of the kinetic energy of the fluid to thermal energy.

→ viscous dissipation is also significant for high speed flights of aircraft.

Q.



$k = 0.145\ W/m.k$

$\mu = 0.800\ kg/m.s = 0.8\ N.s/m^2$

now, $u = u(y)$

so. $$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial P}{\partial x}$$

$$\therefore \frac{\partial^2 u}{\partial y^2}=0$$

$$\therefore u=\frac{y}{L}v$$

So

$$T = T(y)$$

So,

$$0 = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2$$

$$\Rightarrow 0 = k\frac{\partial^2 T}{\partial y^2} + \frac{\mu V^2}{L^2}$$

$$\Rightarrow \frac{\partial^2 T}{\partial y^2} = -\frac{\mu V^2}{kL^2}$$

$$\Rightarrow T = -\frac{\mu V^2}{2kL^2}y^2 + C_1 y + C_2.$$

So $C_2 = T_0$.

Al.

$$T(L) = T_0$$

So.

$$\frac{\mu V^2}{2kL} = C_1.$$

So,

$$\therefore T = T_0 + \frac{\mu V^2}{2kL}y - \frac{\mu V^2}{2kL^2}y^2.$$

$$\therefore T = T_0 + \frac{\mu V^2}{2k}\left(\frac{y}{L}\right)\left[1 - \left(\frac{y}{L}\right)\right]$$

S.

$$\frac{\partial T}{\partial y} = \frac{\mu V^2}{2kL} - \frac{\mu V^2}{kL^2}y = 0$$

$$\therefore \boxed{y = \frac{L}{2}}$$

So

$$T_{max} = T_0 + \frac{1}{8}\frac{\mu V^2}{k}$$

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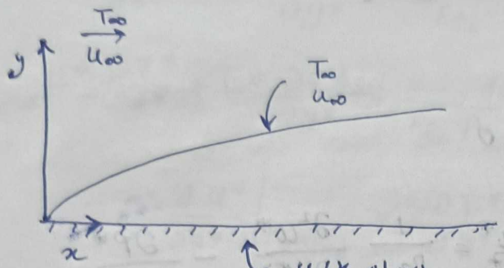
$$\dot{q}_0 = -k \frac{\partial T}{\partial y}\Big|_{y=0} = -\frac{\mu V^2}{2L}$$

$$\therefore \dot{q}_L = \frac{\mu V^2}{2L}$$

#



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

$u(x,0)=0$

$v(x,0)=0$

$T(x,0)=T_s$

At, $x=0$: $u(0,y)=u_\infty$, $T(0,y)=T_\infty$

At, $y=0$: $u(x,0)=0$, $v(x,0)=0$, $T(x,0)=T_s$.

As, $y\to\infty$: $u(x,\infty)=u_\infty$, $T(x,\infty)=T_\infty$

$$\delta = \frac{5}{\sqrt{\rho u_\infty/\mu x}} = \frac{5x}{\sqrt{Re_x}}$$

$$C_{f,x} = \frac{\tau_w}{\rho V^2/2} = \frac{\tau_w}{\rho u_\infty^2/2} = 0.664\, Re_x^{-1/2}$$

$$Nu_x = \frac{h_x x}{k} = 0.332\, Pr^{1/3}\, Re_x^{1/2}\,;\ Pr > 0.6$$

$$\boxed{\delta_t = \frac{\delta}{Pr^{1/3}}} = \frac{5x}{Pr^{1/3}\sqrt{Re_x}}$$

(laminar flow over an isothermal flat plate) now,

$$x^* = \frac{x}{L}, \quad y^* = \frac{y}{L}, \quad u^* = \frac{u}{V}, \quad v^* = \frac{v}{V}$$

$$p^* = \frac{p}{\rho V^2} \text{ and, } \quad T^* = \frac{T - T_s}{T_\infty - T_s}$$

So,

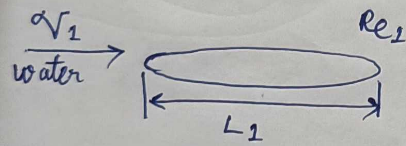
Continuity: $$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

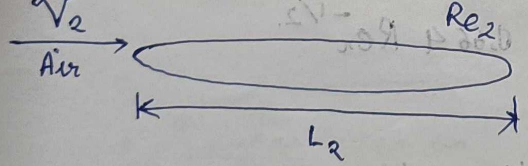
Momentum: $$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{1}{Re_L} \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\partial p^*}{\partial x^*}$$

Energy: $$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L Pr} \frac{\partial^2 T^*}{\partial y^{*2}}$$

$u^*(0, y^*) = 1, \quad u^*(x^*, 0) = 0, \quad u^*(x^*, \infty) = 1, \quad v^*(x^*, 0) = 0$

$T^*(0, y^*) = 1, \quad T^*(x^*, 0) = 0, \quad T^*(x^*, \infty) = 1$





Local Nusselt number,

$$Nu_x = f(x^*, Re_L, Pr)$$

Average Nusselt number,

$$Nu = f(Re_L, Pr)$$

If, $Re_1 = Re_2$ then, $C_{f_1} = C_{f_2}$

A common form of Nusselt number,

$$\boxed{Nu = C \, Re_L^m \, Pr^n}$$

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Momentum: Reynolds analogy.

Heat: Chilton-Colburn analogy.

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{1}{Re_L} \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\partial p^*}{\partial x^*}$$

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L Pr} \frac{\partial^2 T^*}{\partial y^{*2}}$$

$$\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} = \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}.$$

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \frac{\mu V}{L} \left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} = \frac{\mu V}{L} f_2(x^*, Re_L)$$

$$C_{f,x} = \frac{\tau_s}{\rho V^2/2} = \frac{\mu V/L}{\rho V^2/2} f_2(x^*, Re_L) = \frac{2}{Re_L} f_2(x^*, Re_L)$$

$$\therefore C_{f,x} = f_3(x^*, Re_L)$$

$$Nu_x = \frac{hL}{k} = \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0} = g_2(x^*, Re_L, Pr)$$

So,

$$C_{f,x} \frac{Re_L}{2} = Nu_x Pr^{-1/3}$$

Or

$$\frac{C_{f,x}}{2} = \frac{h_x}{\rho C_p V} Pr^{2/3} \equiv j_H \qquad 0.6 < Pr < 60$$

Colburn j factor.

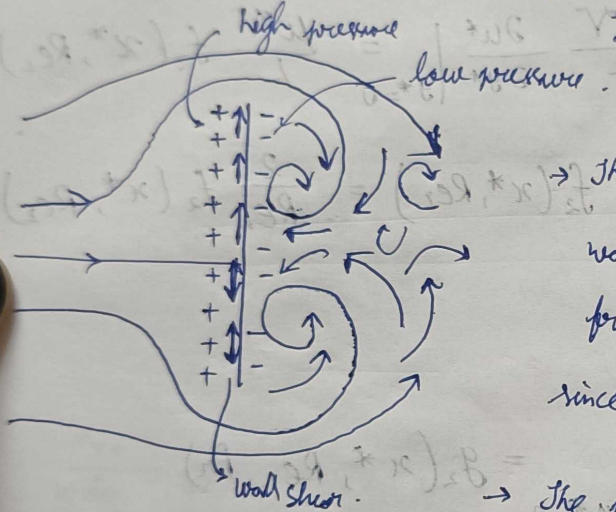
Friction and pressure Drag:

→ The force a flowing fluid exerts on a body in the flow direction is called drag.

→ The components of the pressure and wall shear forces in the normal direction to flow tend to move the body in that direction, and their sum is called lift.

Drag coefficient:

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$



→ The part of drag that is due directly to wall shear stress is called the skin-friction drag (or, just friction drag) since it is caused by frictional effects.

→ The part that is due directly to pressure P is called the pressure drag (also called the form drag because of its strong dependence on the form or shape of the body).

$$C_D = C_{D,friction} + C_{D,pressure}$$

→

Laminar: $Nu_x = \dfrac{h_x x}{k} = 0.332\, Re_x^{0.5} Pr^{1/3}$, $Pr > 0.6$

Turbulent: $Nu_x = \dfrac{h_x x}{k} = 0.0296\, Re_x^{0.8}\, Pr^{1/3}$, $0.6 \le Pr \le 60$

$5\times10^5 \le Re_x \le 10^7$.

$$h = \frac{1}{L}\left(\int_0^{x_{cr}} h_{x,\text{laminar}}\,dx + \int_{x_{cr}}^{L} h_{x,\text{turbulent}}\,dx\right)$$

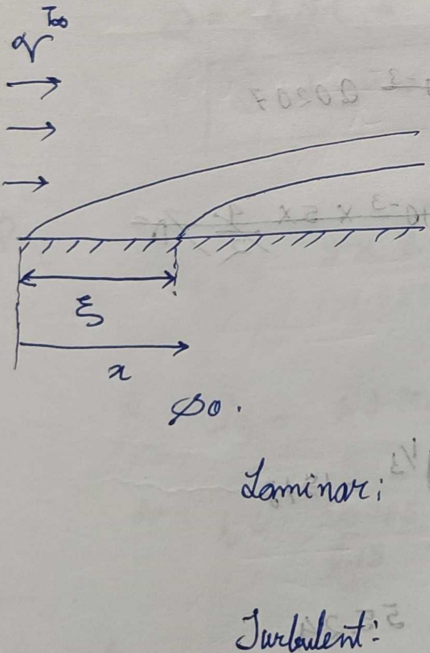
$$\frac{hL}{k} = Nu = \left(0.037\, Re_L^{0.8} - 871\right) Pr^{1/3} \qquad 5\times10^5 \le Re_L \le 10^7$$

$0.6 \le Pr < 60$

$$Nu = 0.565\,(Re_x Pr)^{1/2} \quad Pr < 0.05.$$

$$Nu_x = \frac{h_x x}{k} = \frac{0.3387\, Pr^{1/3}\, Re_x^{1/2}}{\left[1 + (0.0468/Pr)^{2/3}\right]^{1/4}}$$

Flat plate with unheated starting length:



Laminar: $$Nu_x = \frac{Nu_x|_{\xi=0}}{\left[1-\left(\frac{\xi}{x}\right)^{3/4}\right]^{1/3}} = \frac{0.332\, Re_x^{0.5} Pr^{1/3}}{\left[1-\left(\frac{\xi}{x}\right)^{3/4}\right]^{1/3}}$$

Turbulent: $$Nu_x = \frac{Nu_x|_{\xi=0}}{\left[1-\left(\frac{\xi}{x}\right)^{9/10}\right]^{1/9}} = \frac{0.0296\, Re_x^{0.8} Pr^{1/3}}{\left[1-\left(\xi/x\right)^{9/10}\right]^{1/9}}$$

So.

Laminar: $$h = \frac{2\left[1-\left(\xi/x\right)^{3/4}\right]}{\left(1-\xi/L\right)}\, h_{x=L}$$

Turbulent: $$h = \frac{5\left[1-\left(\xi/x\right)^{9/10}\right]}{\left(1-\xi/L\right)}\, h_{x=L}$$

Uniform Heat flux:

Laminar: $Nu_x = 0.453\, Re_x^{0.5}\, Pr^{1/3}$

Turbulent: $Nu_x = 0.0308\, Re_x^{0.8}\, Pr^{1/3}$

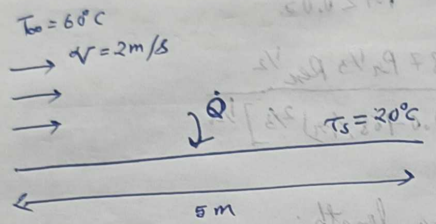
isothermal plate case.

Laminar: $Nu_x = 0.332\, Re_x^{0.5}\, Pr^{1/3}$

Turbulent: $Nu_x = 0.0296\, Re_x^{0.8}\, Pr^{1/3}$

$\dot{Q} = \dot{q}_s A_s \qquad \dot{q}_s = h_x [T_s(x) - T_\infty] \Rightarrow T_s = T_\infty + \dfrac{\dot{q}_s}{h_x}$

Q.



$\rho = 876\ kg/m^3 \ ; \ Pr = 2870$

$k = 0.144\ W/m \cdot ^\circ C \qquad \nu = 242 \times 10^{-6}\ m^2/s$

$Re_L = \dfrac{2 \times 5}{242} \times 10^6 = 41{,}322.$

$C_f = 1.328 \times (41322)^{-1/2} = \cancel{6.533 \times 10^{-3}}\ 0.0207$

So.

$F_D = \rho \cdot \overset{0.0207}{\cancel{6.533 \times 10^{-3}}}\, A\, \dfrac{v^2}{2} = \cancel{6.533 \times 10^{-3} \times 5 \times \frac{4}{2}\ N}$

$\therefore F_D = \cancel{0.06533\ N}\ 181\ N$

now

$Nu = 0.664 \times (41322)^{1/2} \times (2870)^{1/3} = 1918$

now $h = \dfrac{k}{L} Nu = \dfrac{0.144}{5} \times 1918 = 55.24$

So.

$Q = 55.24 \times 40 \times 5 = 11048\ W$

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Q.

$Re_x = \cancel{\frac{8 \times 6}{2.098 \times 10^{-5}}} = \cancel{22.88 \times 10^5}$

$v = \frac{v_{@1atm}}{P} = \frac{2.097 \times 10^{-5}}{0.823} = 2.548 \times 10^{-5}\ m^2/s$

$Re_x = \frac{48}{2548} \times 10^5 = 18.83 \times 10^5 = 1.883 \times 10^6$

$Pr = 0.7154$

So

$Nu = (0.037 Re_L^{0.8} - 871) Pr^{1/3} = \frac{hL}{k}$

$\Rightarrow h = \frac{k}{L}\left[(0.037 \times 104683.9873 - 871)\ 0.7154^{1/3}\right]$

$\therefore h = 13.2$

So.

$\dot{Q} = hA(T_s - T_\infty) = 9 \times 13.2 \times 120 = 14256\ W$

$\therefore \boxed{\dot{Q} = 1.4256 \times 10^4\ W}$

Q.

$Re_x = \frac{5 \times 0.15}{1.655} \times 10^5 = 4.53 \times 10^4$

So $Nu = 0.0296 \times (4.53 \times 10^4)^{0.8} \times (0.7268)^{1/3} = 141.25$

So $h = \frac{0.0265}{0.15} \times 141.25 = 24.95$

So.

$T_s = 20°C + \frac{15}{24.95}\ (0.15)^{-2}$

$\therefore \boxed{T_s = 46.72°C}$

Q.

$V_\infty = \frac{70\times10^3}{3600} = 19.44\ m/s$

So

$Re = \frac{19.44\times 8}{1.608}\times 10^5 = 9.67\times10^6$

$Pr = 0.7282$

So.

$h = \frac{k}{L}\left[0.037\times(9.67\times10^6)^{0.8} - 871\right]0.7282^{1/3}$

$h = 39.2$

So $\dot{q} = 39.2(T_s - T_\infty)$

So $\boxed{T_s = 35.1^\circ C}$

Q.

$Nu \propto Re^{0.5}$

and $h \propto Re^{0.5} \Rightarrow h \propto V_\infty^{0.5}$

So, $F_D \propto V_\infty^{1.5}$ and $Q \propto V_\infty^{0.5}$

So, $F_D \longrightarrow 2.83\, F_D|_1$

$Q \longrightarrow 1.414\, Q|_1$

Name: Nabarun Saha   Roll no: 22CH3[illegible]

midsem (2022-23)

2.

$$Nu = \cancel{0.664} Re_L^{0.8} (Pr)^{1/3} \times 0.037 - 871 (Pr)^{1/3}$$

and $$Nu_x = \cancel{0.664 R} \; 0.0296 Re_x^{0.8} Pr^{1/3}$$

now.

$$Re_L = \frac{15 \times 0.5}{0.0000195} = 3.85 \times 10^5$$

$$Nu = 0.037 (3.85 \times 10^5)^{0.8} \times (0.7)^{1/3} - 871 \times (0.7)^{1/3} \; \cancel{= 1545.5} = 192.545.$$

$$Nu_x = 772.74$$

now,

$$\frac{hL}{k} = Nu$$

$$\langle h \rangle = \frac{0.0292}{0.5} \times 192.545 = \cancel{90.25\ W/m^2} \; 11.24$$

So.

$$Q = \overset{11.24}{\cancel{90.25}} \times 0.25 \times 90 = \cancel{2.03\ kW} \; 252.9W$$

and; at the end:

$$h_L = 45.13\ W/m^2$$

now,

$$\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5 \times 0.5}{\sqrt{3.85 \times 10^5}} = 4.03\ mm$$

and $$\delta_t = \frac{\delta}{Pr^{1/3}} = 4.54\ mm$$

3. $$Nu_x = \frac{0.339\, Re_x^{1/2}\, Pr^{1/3}}{[1+(0.0468/Pr)^{2/3}]^{1/4}}. \quad Re_x \propto x^{1/2}$$

$$Nu_x = \cancel{\alpha Re_x}\ \alpha\, x^{1/2}$$

$$\Rightarrow \langle Nu \rangle = \alpha \frac{\int_0^L x^{1/2}\,dx}{\int_0^L dx} = \alpha \frac{\frac{2}{3} L^{3/2}}{L} = \alpha \frac{2}{3} L^{1/2}$$

&

$$\langle Nu \rangle = \frac{0.226\, Re_L^{1/2}\, Pr^{1/3}}{\left[1+\left(\frac{0.0468}{Pr}\right)^{2/3}\right]^{1/4}}$$

1.

$$\frac{C_f}{2} = \cancel{\quad} \; \frac{Nu}{Pr^{1/3}\, Re_L}$$

or.

$$\frac{C_f}{2} = \frac{Nu\ \nu}{Pr^{1/3}\, \cancel{u}\, L}$$

$$\nu \to 2\nu$$

$\Rightarrow$

$$C_f \to \frac{C_f}{2}$$

or,

h does not change.

Name: Nabarun Saha   Roll.no: 22CH30012

2017-18

Q.1.

$T = 150x^2 - 30x$

$W = 0.3m.$

$\frac{\partial T}{\partial x} = (300x - 30)$

at $\left.\frac{\partial T}{\partial x}\right|_{x=0} = -30 \ °C/m$

at $\left.\frac{\partial T}{\partial x}\right|_{x=0.3} = 90 - 30 = 60 \ °C/m$

and.

$\frac{d\dot{Q}}{dA} = -k \frac{\partial T}{\partial x}$

so,

$\frac{d\dot{Q}}{W dx} = -k (300x - 30)$

$\dot{Q} = \int_0^{0.3} (1.2 - 12x) 0.3 dx$

$= 0.3 [1.2 \times 0.3 - 6 \times 0.3^2]$

$= -0.054 \ W$

→ Cooling down.

Q.2.

$(T_s - T_\infty) = (300 - 50) = 250 °C$

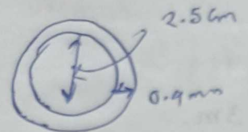
$h(T_s - T_\infty) = 75 \times 250 = 18.75 \ kW/m^2$

or

$h(T_s - T_\infty) = 50(300 - 30) = 13.5 \ kW/m^2$

max possible rate.

3.



$h = 100\ W/m^2 °C$ $\quad T_\infty = 40°C$

$T_s = 250°C$ and $k = 24\ W/m°C$

$$\frac{\partial^2 T}{\partial r^2} = -\frac{\dot{q}}{k}$$

$$\int d\left(\frac{\partial T}{\partial r}\right) = -\int \frac{\dot{q}}{k}\, dr \Rightarrow k\frac{\partial T}{\partial r} = \frac{\dot{q}}{\cancel{k}} \int_0^R dr$$

$$= \dot{q} R$$

now $\cancel{\dot{q} R = h(T_s - T_\infty)}$

$$\cancel{\dot{q} = \frac{h(T_s - T_\infty)}{R}}$$

$$\cancel{= \frac{100 \times 210}{0.4 \times 10^{-3}}}$$

$$= \cancel{52.5\ mW}$$

$D_{out} = 0.0258\ m$

$r_{out} = 0.0129\ m \quad , \quad r_{in} = 0.0125\ m$

no.

$$T_w = T_0 - \frac{\dot{q}}{4k} r_0^2$$

$$\therefore T_w = 523 - 1.733 \times 10^{-6} \dot{q}$$

and.

$$\dot{q} = hA(T_w - T_\infty)$$

w.

$$\dot{q} = \frac{hA}{V}(T_w - T_\infty)$$

Name: Nabasum Saha   Roll no: 22CH30010

$$\dot{q} = \frac{0.25 \quad (523 - 1.733\times10^{-6}\dot{q}) - 313)}{3.1918 \times 10^{-8}}$$

$\therefore \dot{q} = 1.622 \times 10^{6}\ MW/m^3$

Q. $Pr = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$

$Pr << 1$

then; $\nu << \alpha$.

so $v \approx 0$.

and, $\cancel{u\frac{\partial u}{\partial x}}$ $u\frac{\partial u}{\partial x} = 0$.

so $u = U_\infty \Rightarrow$ $v = 0$.

s.

$$\frac{\partial Tu}{\partial x} = T\frac{\partial u}{\partial x} + U_\infty \frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial y^2}$$

$$\Rightarrow U_\infty \frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial y^2}$$

(i)

$\cancel{\frac{U_\infty}{\alpha} x = \frac{\partial^2 T}{\partial y^2} \frac{\partial T}{\partial x}}$

$Nu = 0.53\ Pe^{1/2}$

$Pe = \sqrt{\frac{U_\infty x}{\alpha}}$

no. $Nu = \frac{hL}{k} = \frac{\dot{q}_{conv}}{\dot{q}_{cond}}$

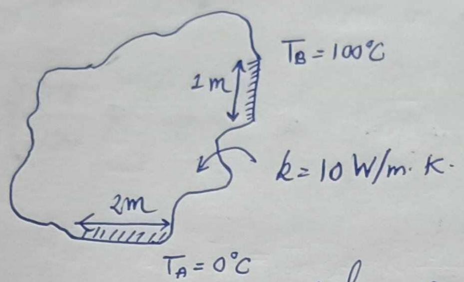
=

5.

$\dot{Q} = \dot{m}\, C_p\, (T_{out} - T_{in})$

$\dot{m} = 0.1\ kg/s.$

Q.



and, $\left.\frac{\partial T}{\partial y}\right|_A = 30\ K/m$

so

$\frac{Q}{A} = -k \left.\frac{\partial T}{\partial y}\right|_A = -900\ W/m^2$

So

$Q = -1800\ W/m$

So.

So. $1800 = 10 \left.\frac{\partial T}{\partial x}\right|_B$

$\therefore \left.\frac{\partial T}{\partial x}\right|_B = 180\ K/m.$

and. $\left.\frac{\partial T}{\partial y}\right|_B = 0.$

Q

Name: Nabayan Saha Roll no.: 22CH30018

Q. e^{-kx} Fourier int

Q.

$$\eta_f = \frac{\sqrt{hPkA_c}}{2A_f} \quad \frac{\tanh(mL) + \frac{h}{mk}}{1 + \frac{h}{mk}\tanh(mL)}$$

$$mL = \frac{\ln 199}{2} + \frac{1}{2}\ln\left(\frac{1-\frac{h}{mk}}{1+\frac{h}{mk}}\right)$$

$$\therefore e^{mL} = \sqrt{199}\cdot\sqrt{\frac{1-\frac{h}{mk}}{1+\frac{h}{mk}}} \quad , \quad e^{-mL} = \frac{1}{\sqrt{199}}\sqrt{\frac{1+\frac{h}{mk}}{1-\frac{h}{mk}}}$$

$$\tanh(mL) = \frac{e^{mL} - e^{-mL}}{e^{mL} + e^{-mL}} = \frac{199(1-\frac{h}{mk}) - (1+\frac{h}{mk})}{199(1-\frac{h}{mk}) + (1+\frac{h}{mk})}$$

$$= \left(\frac{198 - 200\frac{h}{mk}}{200 - 198\frac{h}{mk}}\right)$$

$$\frac{h}{mk} \approx \sqrt{\frac{ht}{2k}} \quad \text{for } \sqrt{\frac{ht}{2k}} \leq \frac{1}{2}$$

$$\tanh(mL) \approx \frac{198 - 100}{200 - 99} = \frac{98}{101}$$

$$\eta_f \approx \frac{0.970 + 0.5}{1 + 0.5 \times 0.970} = 0.99$$

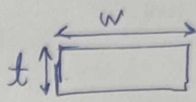
Q.

Name: Narayan Saha Roll no.: 22CH30018

$$L = \frac{\ln 199}{2m} + \frac{1}{2m}\ln\left(\frac{1-\frac{h}{mk}}{1+\frac{h}{mk}}\right)$$

$$L = \frac{\ln 199}{2m} - \frac{t}{2}$$

8. $$t = \frac{1}{m}\ln\left(\frac{1+\frac{h}{mk}}{1-\frac{h}{mk}}\right)$$



$$= \frac{1}{m}\ln\left(\frac{1-\frac{h}{mk}+\frac{2h}{mk}}{1-\frac{h}{mk}}\right)$$

$$\frac{P}{A} = \frac{2(w+t)}{wt} = 2\left(\frac{1}{w}+\frac{1}{t}\right)$$

$$= \frac{1}{m}\ln\left(1+\frac{\frac{2h}{mk}}{1-\frac{h}{mk}}\right)$$

$$m = \sqrt{\frac{hP}{kA}}$$

$$\longrightarrow \ln\left(1+\frac{2}{\frac{mk}{h}-1}\right)$$

$$\sqrt{\frac{ht}{2k}} \le \frac{1}{2}$$

$$\frac{2}{\frac{mk}{h}-1} << 1$$

$$1-\sqrt{\frac{ht}{2k}} \ge \frac{1}{2}$$

$$\left(\frac{mk}{h}-1\right) >> 2$$

assuming w to be large.

$$\frac{t}{1-\sqrt{\frac{ht}{2k}}} \ge 2t$$

$$\frac{mk}{h} >> 3$$

$$\frac{km}{h} = \sqrt{\frac{h}{k}\frac{2}{t}}\,\frac{k}{h} = \sqrt{\frac{2h}{kt}}\,\frac{k}{h} = \sqrt{\frac{2k}{ht}}$$

$$\sqrt{\frac{2k}{ht}} >> 3$$

$$\frac{1}{m}\,\frac{2}{\frac{mk}{h}-1}$$

$$\sqrt{\frac{ht}{2k}} << \frac{1}{3}$$

$$= \frac{2}{m^2\frac{k}{h}-m} = \frac{2}{\frac{2h}{kt}\frac{k}{h}-\sqrt{\frac{2h}{kt}}}$$

$$= \frac{2}{\frac{2}{t}-\sqrt{\frac{2h}{kt}}} = \frac{1}{\frac{1}{t}-\sqrt{\frac{h}{2kt}}}$$

$$\frac{t}{2} < \frac{t}{\left(1-\sqrt{\frac{ht}{2k}}\right)}$$

$$= \left(\frac{t}{1-\sqrt{\frac{ht}{2k}}}\right)$$

Name: Nabayan Saha Roll no.: 22CH30018

Q.

for, a finite fin with heat loss by convection from the end we can talk about it's L_∞.

as.

$$Q_f = M \frac{\tanh mL + \frac{h}{mk}}{1 + \frac{h}{mk}\tanh mL} \quad \text{where. } M = \sqrt{hPkA_c}\,\theta_0.$$

So.

$$Q_f \geq 0.99M$$

So

$$\tanh mL + \frac{h}{mk} \geq 0.99 + \frac{0.99h}{mk}\tanh mL.$$

$$\Rightarrow \tanh mL\left(1 - \frac{0.99h}{mk}\right) \geq \left(0.99 - \frac{h}{mk}\right)$$

$$\Rightarrow 2\sigma(2mL) - 1 \geq \frac{0.99 - \frac{h}{mk}}{1 - 0.99\frac{h}{mk}}$$

$$\left[\text{as. } \tanh(z) = 2\sigma(2z) - 1 \quad \sigma(z) = \frac{1}{1+e^{-z}} \ (\text{sigmoid})\right].$$

$$\Rightarrow \frac{2}{1+e^{-2mL}} - 1 \geq \frac{0.99 - \frac{h}{mk}}{1 - 0.99\frac{h}{mk}}$$

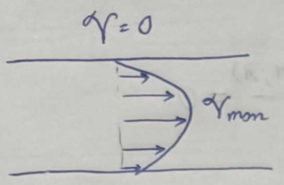
$$\Rightarrow \frac{2}{1+e^{-2mL}} \geq \frac{1 - 0.99\frac{h}{mk} + 0.99 - \frac{h}{mk}}{1 - 0.99\frac{h}{mk}}$$

$$\Rightarrow \frac{2\left(1 - 0.99\frac{h}{mk}\right)}{1.99\left(1 - \frac{h}{mk}\right)} \geq 1 + e^{-2mL}.$$

$$\Rightarrow e^{-2mL} \leq \frac{2 - 1.98\frac{h}{mk} - 1.99 + 1.99\frac{h}{mk}}{1.99\left(1 - \frac{h}{mk}\right)}$$

$$\Rightarrow e^{-2mL} \leq \frac{1 + \frac{h}{mk}}{199\left(1 - \frac{h}{mk}\right)}$$

$$\therefore -2mL \leq -\ln 199 - \ln\left(\frac{1 - \frac{h}{mk}}{1 + \frac{h}{mk}}\right)$$



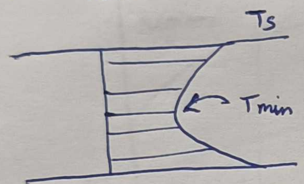
$$\dot{m} = \rho v_m A_c = \int_{A_c} \rho v(r,x)\, dA_c$$

and: $$v_m = \frac{\int_{A_c} \rho v(r,x)\, dA_c}{\rho A_c}$$

$$dA_c = 2\pi r\, dr \quad , \quad A_c = \pi R^2$$

$$\therefore v_m = \frac{2}{R^2} \int_0^R v(r,x)\, r\, dr$$

and now:



$$\dot{E}_{fluid} = \dot{m}\, c_p\, T_m$$

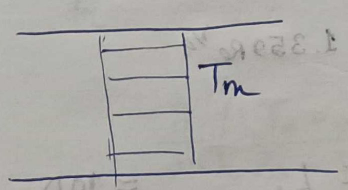
$$= \int_{\dot{m}} c_p T\, \delta\dot{m}$$

$$\Rightarrow T_m = \frac{\int_{\dot{m}} c_p T\, \delta\dot{m}}{c_p \dot{m}}$$

$$\Rightarrow T_m = \frac{\int_{A_c} T \rho v\, dA_c}{\rho v_m (\pi R^2)}$$

$$\therefore T_m = \frac{2}{v_m R^2} \int_0^R T(r,x)\, v(r,x)\, r\, dr$$

we can consider it like:



and $$T_b = \frac{T_{m,i} + T_{m,e}}{2}.$$

Hydrodynamically fully developed flow is when, $\dfrac{\partial v(r,x)}{\partial x} = 0$.

i.e, $v(r,x) = v(r)$

does not vary with x.

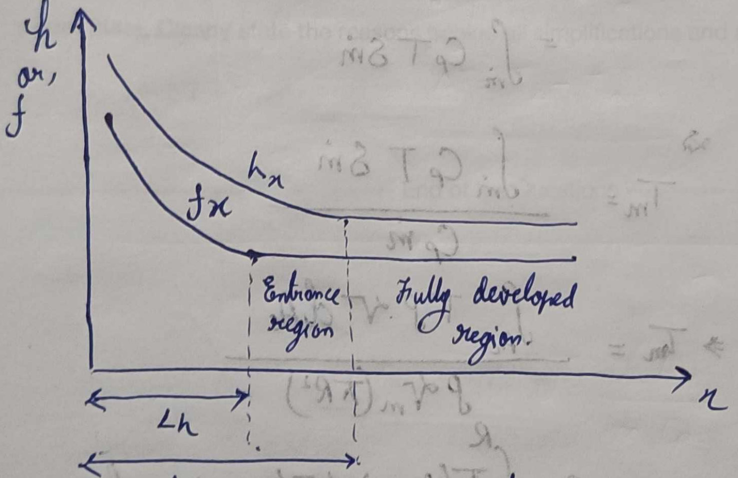
Thermally fully developed flow is:

$$\frac{\partial}{\partial x}\left(\frac{T_s(x) - T(r,x)}{T_s(x) - T_m(x)}\right) = 0$$

So.

$$\dot{q}_s = h_x(T_s - T_m) = k\frac{\partial T}{\partial r}\bigg|_{r=R}$$

$$\therefore h_x = \frac{k(\partial T/\partial r)|_{r=R}}{T_s - T_m}$$

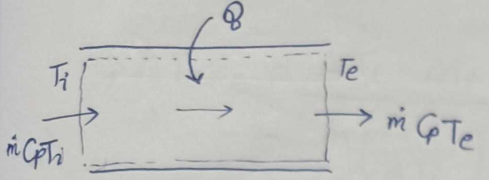


$$L_{h,\text{laminar}} \approx 0.05\, Re\, D$$

$$L_{t,\text{laminar}} \approx 0.05\, Re\, Pr\, D = Pr\, L_{h,\text{laminar}}$$

$$L_{h,\text{turbulent}} = 1.359\, Re^{1/4}$$

$$L_{h,\text{turbulent}} \approx L_{t,\text{turbulent}} \approx 10D$$



Energy balance,

$$\dot{Q} = \dot{m} C_p (T_e - T_i)$$

$$\cancel{\dot{Q} = \dot{m} C_p (T_s - T_i)}$$

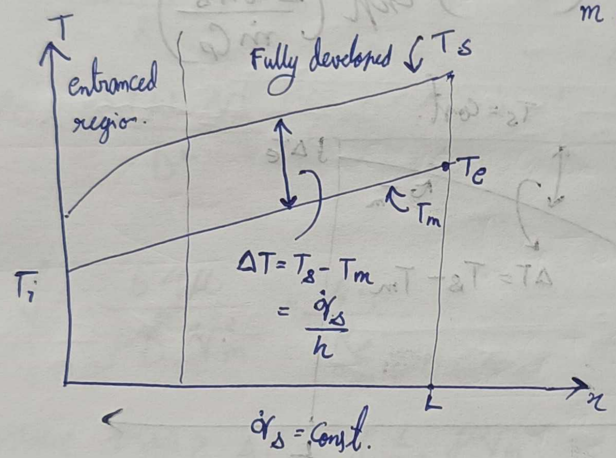
$$\dot{q}_s = h_x (T_s - T_m)$$

Const. surface heat flux:

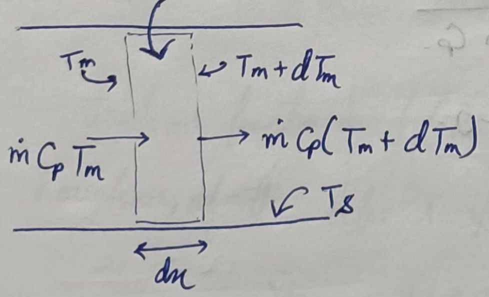
$$\dot{Q} = \dot{q}_s A_s = \dot{m} C_p (T_e - T_i)$$

$$\dot{q}_s = h (T_s - T_m) \quad \therefore T_s = T_m + \frac{\dot{q}_s}{h}$$

$$\therefore T_e = T_i + \frac{\dot{q}_s A_s}{\dot{m} C_p}$$



$$\delta \dot{Q} = h (T_s - T_m) dA$$



$$\dot{m} C_p dT_m = \dot{q}_s P dx$$

So.

$$\frac{dT_m}{dx} = \frac{\dot{q}_s P}{\dot{m} C_p}$$

$$\frac{\partial}{\partial x}\left(\frac{T_s - T}{T_s - T_m}\right) = 0.$$

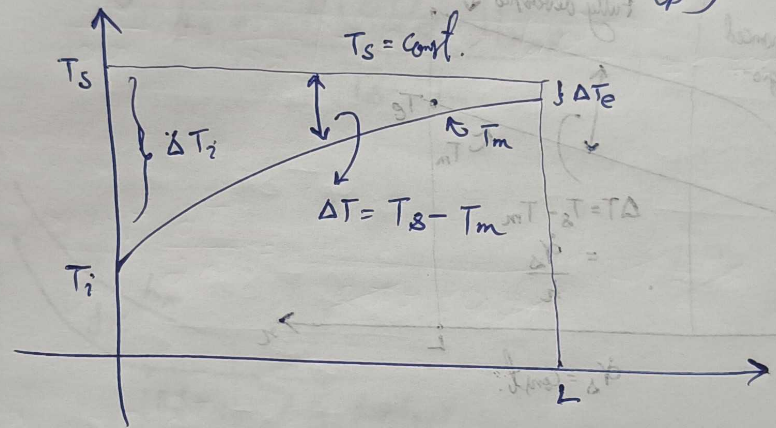
$$\text{S.} \quad \frac{\partial T_s}{\partial x} = \frac{\partial T}{\partial x} = \frac{\partial T_m}{\partial x} = \frac{\dot{q}_s P}{\dot{m} C_p}$$

$$\frac{d(T_s - T_m)}{T_s - T_m} = -\frac{hP}{\dot m C_p} dx.$$

$$\therefore \ln \frac{T_s - T_e}{T_s - T_i} = -\frac{hA_s}{\dot m C_p}$$

$$\therefore T_s - T_e = (T_s - T_i) \exp\left(-\frac{hA_s}{\dot m C_p}\right)$$

$$\therefore T_e = T_s - (T_s - T_i) \exp\left(\frac{-hA_s}{\dot m C_p}\right)$$



here, $NTU = \dfrac{hA_s}{\dot m C_p}$.

at nearly $5\,NTU$, $\Delta T_e \sim 0$.

$$\dot m C_p = \frac{-hA_s}{\ln\left[(T_s - T_e)/(T_s - T_i)\right]}$$

$$\therefore \dot Q = \dot q_s A_s = \dot m C_p (T_e - T_i)$$

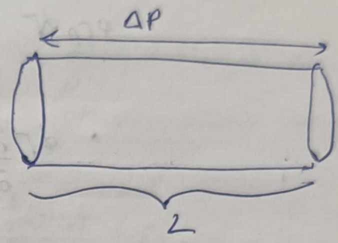
$$\therefore \dot Q = hA_s \Delta T_{ln}$$

$$\Delta T_{ln} = \frac{\Delta T_e - \Delta T_i}{\ln\left(\frac{\Delta T_e}{\Delta T_i}\right)}$$

Laminar flow in tubes:

$$v(r) = 2 v_m\left(1 - \frac{r^2}{R^2}\right)$$

$$v_{max} = 2 v_m.$$



$$\Delta P = \frac{8\mu L v_m}{R^2}$$

$$\Rightarrow \Delta P = \frac{32\mu L v_m}{D^2}$$

$$f = \frac{64\mu}{\rho D v_m} = \frac{64}{Re}$$

Pressure drop $\Rightarrow \Delta P = f \frac{L}{D} \frac{\rho v_m^2}{2}$

Drag friction factor $C_f = \tau_s\left(\rho v_m^2/2\right) = f/4$

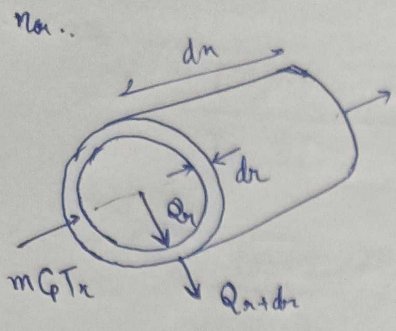
friction factor $= f(Re)$ only and does not depend upon tube Roughness of the tube surface.

$$\dot{W}_{pump} = \dot{V}\Delta P.$$

$$\boxed{\dot{V} = v_{ave} A_c = \frac{\Delta P R^2}{8\mu L}\pi R^2 = \frac{\pi R^4 \Delta P}{8\mu L} = \frac{\pi D^4 \Delta P}{128\mu L}}$$

Poiseuille flow.

Now..



$$\dot m C_p T_x - \dot m C_p T_{x+dx} + \dot Q_r - \dot Q_{r+dr} = 0.$$

$$\dot m = \rho v (2\pi r dr)$$

So..

$$\rho C_p v \frac{T_{x+dx} - T_x}{dx} = -\frac{1}{2\pi r dx} \cdot \frac{\dot Q_{r+dr} - \dot Q_r}{dr}.$$

$$\Rightarrow v \frac{\partial T}{\partial x} = -\frac{1}{2\rho C_p \pi r dx} \frac{\partial \dot Q}{\partial r}$$

as.

$$\frac{\partial \dot Q}{\partial r} = \frac{\partial}{\partial r}\left(-k\, 2\pi r dx \frac{\partial T}{\partial r}\right) = -2\pi k dx \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right)$$

So.

$$\Rightarrow v \frac{\partial T}{\partial x} = \frac{1}{\cancel{2\pi} r \rho C_p \cancel{dx}} \cancel{2\pi} k \cancel{dx} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right)$$

$$\Rightarrow v \frac{\partial T}{\partial x} = \frac{k}{\rho C_p r} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right)$$

$$\boxed{v \frac{\partial T}{\partial x} = \frac{\alpha}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right)}$$ where,

$$\alpha = \frac{k}{\rho C_p}.$$

For constant surface heat flux:

$$\frac{\partial T}{\partial x} = \frac{\partial T_s}{\partial x} = \frac{\partial T_m}{\partial x} = \frac{\dot{q}_s P}{\dot{m} C_p}$$

$$P = 2\pi R, \qquad \dot{m} = \rho v_m \pi R^2$$

So. $$v \frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

$$\Rightarrow \frac{4\dot{q}_s}{kR}\left(1 - \frac{r^2}{R^2}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

$$\Rightarrow \frac{4\dot{q}_s}{kR}\left(\frac{r^2}{2} - \frac{r^4}{4R^2}\right) + C_1 = r\frac{\partial T}{\partial r}$$

$$\Rightarrow \frac{\partial T}{\partial r} = \frac{4\dot{q}_s}{kR}\left(\frac{r}{2} - \frac{r^3}{4R^2}\right) + \frac{C_1}{r}$$

$\Rightarrow$ $\cancel{T = \frac{4\dot{q}_s}{kR}(\;)}, R$ $\qquad \partial T/\partial r = 0$ at $r = 0$; $T = T_s$ at $r = R$

So. $$T = T_s - \frac{\dot{q}_s R}{k}\left(\frac{3}{4} - \frac{r^2}{R^2} + \frac{r^4}{4R^4}\right)$$

$$T_m = \frac{\int_{\dot{m}} C_p T \, \delta\dot{m}}{\dot{m} C_p} = \frac{2}{v_m R^2}\int_0^R T(R,x)\, v(r,x)\, r\, dr.$$

$$T_m = T_s - \frac{11}{24}\frac{\dot{q}_s R}{k}$$

So. $$\frac{11}{24}\frac{\dot{q}_s R}{k} h = \dot{q}_s.$$

$$\therefore h = \frac{24}{11}\frac{k}{R} = \frac{48}{11}\frac{k}{D}$$

So. $$\boxed{\frac{hD}{k} = 4.36 = Nu}$$

So.

$Nu = 4.36$ (for laminar flow in circular tube).
($\dot{q}_s$ = const.)

and for, T_s = const.

$Nu = 3.66$ (laminar flow in circular tube)

→ when the difference b/w the surface and the fluid temperature is large it maybe necessary to account the variation of viscosity with temperature.

$$\boxed{Nu = 1.86\left(\frac{Re\,Pr\,D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}}$$

– Sieder and Tate

Laminar flow in non circular tubes.

$D_h = \frac{4A_c}{P}$, $Re = \frac{V_m D_h}{\nu}$ and $Nu = \frac{hD_h}{k}$

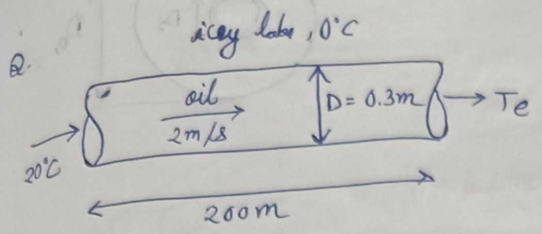
Q. Average Nusselt number for the thermal entrance region for circular tube:

$$Nu = 3.66 + \frac{0.065\left(\frac{D}{L}\right)Re\,Pr}{1+0.04\left[\left(\frac{D}{L}\right)Re\,Pr\right]^{2/3}}$$

Average Nusselt number for the thermal entrance region for flow b/w isothermal parallel plates:

$$Nu = 7.54 + \frac{0.03\left(\frac{D_h}{L}\right)Re\,Pr}{1+0.016\left[\left(\frac{D_h}{L}\right)Re\,Pr\right]^{2/3}}$$

Q.



$\rho = 888\ kg/m^3$

$k = 0.145\ W/m.°C$

$\nu = 901 \times 10^{-6}$

$C_p = 1880\ J/kg.°C$

$Pr = 10,400.$

$$Re = \frac{2 \times 0.3}{901} \times 10^6 = 665.92$$

$\Rightarrow$ Laminar.

$$Nu = 3.66 + \frac{0.065 \times \frac{0.3}{200} \times 665.92 \times 10,400}{1 + 0.04\left[\left(\frac{0.3}{200}\right) \times 665.92 \times 10400\right]^{2/3}}$$

$$= \frac{675.24}{20.04} = 33.69$$

$L_{h,laminar} = 0.05\, Re\, D = 0.05 \times 665.92 \times 0.3 = 9.98\ m$

$L_{t,laminar} = 103883\ m$

~~Hence~~ the velocity profile gets fully developed but the temperature profile does not.

$$\dot{m} = 2 \times \frac{\pi}{4} \times 0.09 \times 888 = 125.53\ kg/s.$$

$$NTU = \frac{hA_s}{\dot{m} C_p} = \left(\frac{(\pi \times 0.3 \times 200) \times (37.35 \times \frac{0.145}{0.3})}{125.53 \times 1880}\right).$$

$$\cancel{1.413 \times 10^{-3}} = 14 \times 10^{-3}$$

So

$$T_e = 20 \exp(-14 \times 10^{-3})$$

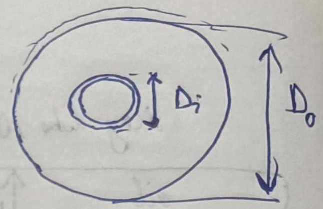
$$\therefore \boxed{T_e = 19.72°C}$$

$$\Delta T_{LMTD} = -\frac{20 - 19.72}{\ln\left(\frac{20}{19.72}\right)} = -19.86$$

$\Delta T_i = 20$

$\Delta T_e = 19.72$

$$Q = hA\,\Delta T_{LMTD} = -67.58\ kW$$



$$\Delta P = \frac{32 \mu L V_m}{D^2}$$

$$\Rightarrow \Delta P = \frac{32 \rho \nu L V_m}{D^2}$$

$$\Rightarrow \Delta P = \frac{32 \times 888 \times 901 \times 10^{-6} \times 200 \times 2}{0.09}$$

$$\Rightarrow \Delta P = 113790.\ Pa$$

$$\therefore \boxed{\Delta P = 113.8\ kPa} \approx 1.12\ atm$$

(b)

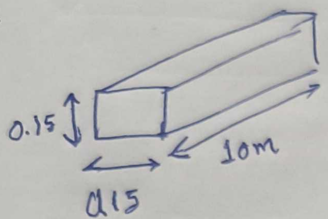
$$W = 113.8 \times 10^3 \times \frac{125.53}{888}$$

$$W = 16.08 \times 10^3\ W$$

$$\boxed{W = 16.08\ kW}$$

Ex210c.

Q.



$\dot{V} = 0.1\ m^3/s$, $T = 70°C$.

$T_\infty = 85°C$.

and. $V\ 0.15^2 = 0.1$

$\therefore V = \frac{0.1}{0.15 \times 0.15} = 4.44\ m/s$.

$Re = \frac{4.44 \times 0.15}{2.097 \times 10^{-5}} = 4 \times 0.079 \times 10^5 = 4 \times 7.9 \times 10^3$

$= 4 \times 7900 = 31,600$

(turbulent)

$Nu = 0.023\, Re^{0.8}\, Pr^{0.4}$ (heating)

$= 0.023\,(31600)^{0.8}\,(0.7154)^{0.4}$

$= 80.038$

$T_e = T_s - (\Delta T)\exp\left(-\frac{h A_s}{\dot{m} C_p}\right)$

Turbulent flow in tubes

$$f = (0.790 \ln Re - 1.64)^{-2} \qquad 10^4 < Re < 10^6$$

$$Nu = 0.125 f \, Re \, Pr^{1/3}$$

for fully developed turbulent flow in smooth tubes

$$f = 0.184 \, Re^{-0.2}$$

$$Nu = 0.023 \, Re^{0.8} Pr^{1/3}, \quad \begin{pmatrix} 0.7 \le Pr \le 160 \\ Re > 10{,}000 \end{pmatrix} \text{ colburn eq}^n.$$

$$\therefore \boxed{Nu = 0.023 \, Re^{0.8} \, Pr^n}$$

Dittus - Boelter equation.

$n = 0.4$ for heating and 0.3 for cooling.

for $0.004 < Pr < 0.01$

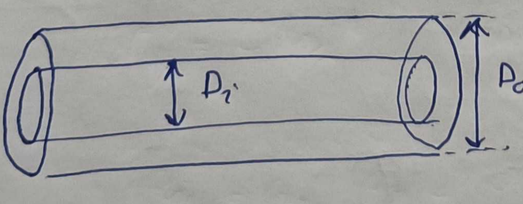
for liq metals.

$$T_s = \text{const.} : \quad Nu = 4.8 + 0.0156 \, Re^{0.85} \, Pr_s^{0.93}$$

$$\dot{q}_s = \text{const.} : \quad Nu = 6.3 + 0.0167 \, Re^{0.85} \, Pr_s^{0.93}$$

colebrook equation:

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\varepsilon / D}{3.7} + \frac{2.51}{Re \sqrt{f}} \right)$$



$$D_h = \frac{4 A_c}{P}$$

$$\therefore D_h = (D_o - D_i)$$

$$Nu_i = \frac{h_i D_h}{k}, \qquad Nu_o = \frac{h_o D_h}{k}.$$

Free convection:

$$F_{buoyancy} = \rho_{fluid}\, g\, V_{body}.$$

$$F_{net} = (\rho_{body} - \rho_{fluid})\, g\, V_{body}.$$

volume expansion coefficient.

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$$

$$\therefore \beta \approx -\frac{1}{\rho}\,\frac{\rho_\infty - \rho}{T_\infty - T} \quad (\text{at const. } P)$$

$$\therefore (\rho_\infty - \rho) = \beta\rho\,(T - T_\infty)$$

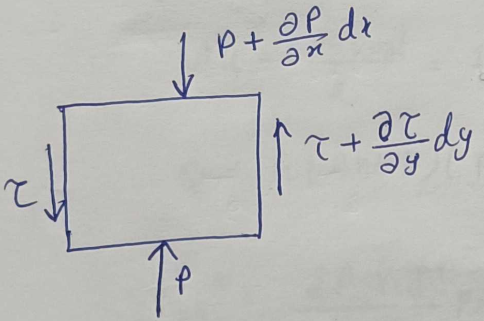
and $\beta_{ideal\ gas} = \dfrac{1}{T}$

$$PM = \rho R T$$

$$\Rightarrow PM\,\frac{1}{\rho} = R\cdot T$$

$$\Rightarrow PM\,\frac{d\rho}{dT}\cdot -\frac{1}{\rho^2} = R$$

$$-\frac{1}{\rho}\,\frac{d\rho}{dT} = \frac{1}{T}$$



$$\delta m \cdot a_x = F_x$$

$$\delta m = \rho\, dx\, dy.$$

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$$

$$= u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$F_x = \left(\frac{\partial \tau}{\partial y}\right)dy\,dx - \left(\frac{\partial p}{\partial x}dx\right)dy - \rho g\,(dx\,dy)$$

$$= \left(\mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial p}{\partial x} - \rho g\right)dx\,dy.$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial p}{\partial x} - \rho g.$$

$$\therefore \frac{\partial P_\infty}{\partial x} = -\rho_\infty g.$$

in the boundary layer,

$$v << u$$

$$\frac{\partial P}{\partial y} = 0, \quad \frac{\partial v}{\partial x} \simeq \frac{\partial v}{\partial y} \simeq 0$$

$$P = P(x)$$

$$\therefore \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} + (\rho_\infty - \rho)g.$$

$$\therefore u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)$$

$$u^* = \frac{u}{\nu_\infty}, \quad y^* = \frac{y}{L_c}, \quad x^* = \frac{x}{L_c}, \quad v^* = \frac{v}{\nu}$$

$$T^* = \frac{T - T_\infty}{T_s - T_\infty}$$

$$\therefore u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = \left[\frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\right]\frac{T^*}{Re_L^2} + \frac{1}{Re_L}\frac{\partial^2 u^*}{\partial y^{*2}}$$

Grashof number Gr_L

$$Gr_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$$

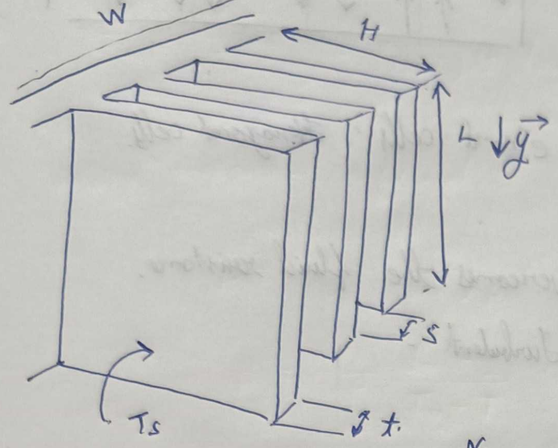
$$\hookrightarrow \frac{\text{buoyancy force}}{\text{viscous force}}.$$

$$Nu = C\,Ra_L^{\,n} \qquad n = \begin{cases} \frac{1}{4}, & \text{Laminar} \\ \frac{1}{3}, & \text{turbulent} \end{cases}$$

$$Ra_L = Gr_L\,Pr = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\,Pr$$

Constant surface heat flux.

$$Nu = \frac{hL}{k} = \frac{\dot{q}_s L}{k(T_{L/2} - T_\infty)}$$



$$Ra_s = \frac{g\beta(T_s - T_\infty)S^3}{\nu^2} Pr$$

$$Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} Pr = Ra_s\left(\frac{L}{S}\right)^3$$

$$Nu = \frac{hS}{k}$$

$$Nu = \left[\frac{576}{(Ra_s S/L)^2} + \frac{2.873}{(Ra_s S/L)^{0.5}}\right]^{-0.5}$$

$$\boxed{S_{opt} = 2.714 \frac{L}{Ra_L^{0.25}}}$$

$$Nu = \frac{h S_{opt}}{k} = 1.307 \quad , \quad T_{ave} = \frac{T_s + T_\infty}{2}$$

$$\dot{Q} = h(2nLH)(T_s - T_\infty) \quad , \quad n = \frac{W}{S+t} \approx \frac{W}{S}.$$

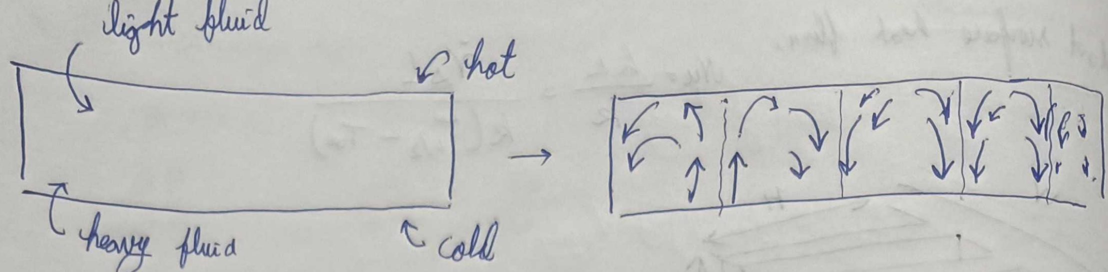
$$Ra_s^* = \frac{g\beta \dot{q}_s S^4}{k\nu^2} Pr$$

$$Nu_L = \frac{h_L S}{k} = \left[\frac{48}{Ra_s^* S/L} + \frac{2.51}{(Ra_L^* S/L)^{0.4}}\right]^{-0.5}.$$

$$S_{opt} = 2.12\left(\frac{S^4 L}{Ra_s^*}\right)^{0.2}$$

$$\dot{Q} = \dot{q}_s A_s = \dot{q}_s (2nLH)$$

$$\dot{q}_s = h_L (T_L - T_\infty)$$



Bénard-cells : Hexagonal cells.

$Nu = 1$: Pure conduction.

$Ra > 1708$: buoyant force overcomes the fluid resistance.

$Ra > 3 \times 10^5$: cells break down - Turbulent -

$$Ra_L = \frac{g\beta(T_1 - T_2) L_c^3}{\nu^2} Pr$$

$$\dot{Q} = h A_s (T_1 - T_2) = k\, Nu\, A_s \frac{T_1 - T_2}{L_c}$$

$$\dot{Q}_{cond} = k A_s \frac{T_1 - T_2}{L_c}$$

$$k_{eff} = k\, Nu$$

$$\dot{Q}_{rad} = \varepsilon \sigma A_s (T_s^4 - T_{surr}^4)$$

$\frac{Gr}{Re^2} < 0.1$: Natural convection is negligible.

$\frac{Gr}{Re^2} > 10$: forced convection is negligible.

$0.1 < Gr/Re^2 < 10$

$$Nu_{combined} = (Nu^n_{forced} \pm Nu^n_{natural})^{1/n}$$