H.T. Prof. Gargi Das

cont.

unsteady state (eg: working of coal by flue gas).

cooling of a hot metal ball.

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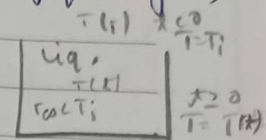
One-dimensionless

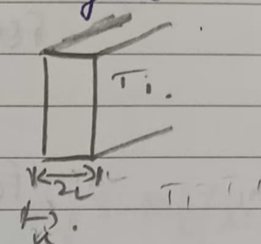


10 1) If temp gradient in one-dir is much less than temp gradient in other-dir.

11 $$\frac{\partial^2 T}{\partial x^2} + \cancel{\frac{\partial^2 T}{\partial y^2}} + \cancel{\frac{\partial^2 T}{\partial z^2}} + \cancel{\frac{q_{gen}}{k}} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

12 Assumption 1d, USS, no heat gen

1 $$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t},$$



B.C

2 1) at $t=0$, ~~x=0~~ $T=T_i$

at $t>0$ $x=0$ $T=T_1$

3 at $t>0$ $x=2L$ $T=T_1$

⇒ 1) $\theta=\theta_i$ at $t=0$

4 $\theta=0$ at $x=0$ $t>0$

$\theta=0$ at $x=2L$ $t>0$. $\theta_i = T_i - T_1$.

5

6 $$\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{\alpha}\frac{\partial \theta}{\partial t}.$$

$\theta = X(x)\tau(t)$

7 $$\cancel{\frac{\theta}{\theta_i} = \frac{T-T_1}{T_i-T_1}}$$

$$\frac{\partial^2\theta}{\partial x^2} = \tau\frac{d^2X}{dx^2}$$

$$\frac{1}{\alpha}\frac{\partial\theta}{\partial t} = \frac{X}{\alpha}\frac{d\tau}{dt}$$

$$\tau\frac{d^2X}{dx^2} = \frac{X}{\alpha}\frac{d\tau}{dt}$$

$$\Rightarrow \frac{1}{X}\frac{d^2X}{dx^2} = \frac{1}{\alpha\tau}\frac{d\tau}{dt} = -\lambda^2$$

λ^2 = square to ensure +ive value of separation factor

to keep the τ in temp bounded ($-\lambda^2$) is used

~~2020~~

For $\lambda^2 > 0$

$\Rightarrow \frac{d^2 x}{dx^2} + \lambda^2 x = 0. \Rightarrow X = c_1 \cos \lambda x + c_2 \sin \lambda x$

$\frac{dT}{dt} + \lambda^2 \alpha T = 0. \Rightarrow \cancel{T = e^{-c_3 \lambda^2 \alpha t}}$

$\underline{T = c_3 e^{-\lambda^2 \alpha t}}$

$\therefore \theta = (c_1 \cos \lambda x + c_2 \sin \lambda x) c_3 e^{-\lambda^2 \alpha t}$

$= (c_1' \cos \lambda x + c_2' \sin \lambda x) e^{-\lambda^2 \alpha t}$

For $\theta = \theta_i$ at $t = 0$, $0 < x < 2L$.

$\theta = \theta_i^\circ = c_1' \cancel{\cos \lambda x} + c_2' \sin \lambda x$.

$c_2' =$.

For $x = 0$, $t > 0$, $\theta = \theta_0$

$c_1' \underbrace{e^{-\lambda^2 \alpha t}}_{>0} = 0 \Rightarrow \underline{c_1' = 0}$.

For $x = 2L$, $\theta = 0$.

$c_2' \sin \lambda (2L) e^{-\lambda^2 \alpha t} = 0$.

c_2' can't be zero otherwise $\theta \neq f(x)$

$\Rightarrow \sin(\lambda 2L) = 0 \quad \because e^{-\lambda^2 \alpha t} \neq 0$.

$\Rightarrow (\sin(n\pi) = 0)$

$\therefore \boxed{\lambda = \frac{n\pi}{2L}}$.

$\theta = \sum_{n=1}^{\infty} c_n \sin(n\pi/2L \, x) \, e^{-(n\pi/2L)^2 \alpha t}$

where c_n ~~are to~~ obtain using at $t = 0$, $\theta = \theta_i$.
or $0 < x \le 2L$

$\therefore \frac{\theta}{\theta_i} = \frac{T - T_1}{T_i - T_1} = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} e^{-(n\pi/2L)^2 \alpha t} \sin \frac{n\pi x}{2L}$

$n = 1, 3, 5, \ldots$

$\theta = \theta_i = \sum_{n=1}^{\infty} c_n' \sin\left(\frac{n\pi x}{2L}\right)$.

$\therefore \boxed{c_n' = \frac{1}{L} \int_0^{2L} \theta_i \sin\left(\frac{n\pi x}{2L}\right) dx}$

$\therefore C_n' = \frac{4}{n\pi}\theta_i$ for $n = 1, 3, 5.$

$$\boxed{\therefore \frac{\theta}{\theta_i} = \frac{T - T_1}{T_i - T_1} = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n} e^{-[n\pi/2L]^2 \alpha\tau} \sin\frac{n\pi x}{2L} \quad n = 1, 3, 5 \ldots}$$

For infinite plate of thickness 2L

actual B.C. [at $x = 0$ (walls) cond = conv.].

for B.C. 1) $hA(T_\infty - T)\big|_{x=0} = -KA\frac{dT}{dx}\Big|_{x=0}$.

$X = x / 2\sqrt{\alpha\tau}$

T_i = initial temp. of solid

T_∞ = ambient temp.

$$\frac{T - T_i}{T_\infty - T_i} = 1 - \text{erf}\,X - \left[\exp\left(\frac{hx}{K} + \frac{h^2\alpha\tau}{K^2}\right)\right] * \left[1 - \text{erf}\left(X + \frac{h\sqrt{\alpha\tau}}{K}\right)\right]$$

except for Lumped ~~po~~ capacitance / Parameter model we use graphs. that gives us time. & extent of H.T.

for an infinite plate of thickness 2L.

$\theta_0 = T_0 - T_L$ → using this we can find temp. at midplane

$\theta_i = T_i - T_s$

$$\frac{\theta}{\theta_i} = \left(\frac{\theta}{\theta_0}\right) \cdot \left(\frac{\theta}{\theta_i}\right)$$

at any points ; mid plane

→ using the mid-plane temp. we can find temp at any other point.

$\therefore$ we can calculate (θ/θ_i).

$\frac{Q}{Q_0}$; $Q_0 = \rho C_p V(T_0 - T_i) = \rho C_p V \theta_0$

$Q = \rho C_p V(T - T_i) = \rho C_p V \theta$

$Q_0 = \rho C_p V(T_0 - T_i)$

$\frac{\rho C_p V \theta}{\rho C_p V \theta_0} = \frac{\theta}{\theta_0}$ → using this we can find amt. of H.T.

using this amt of H.T we can find time for H.T.

graphs - ~~H~~eissler's plot

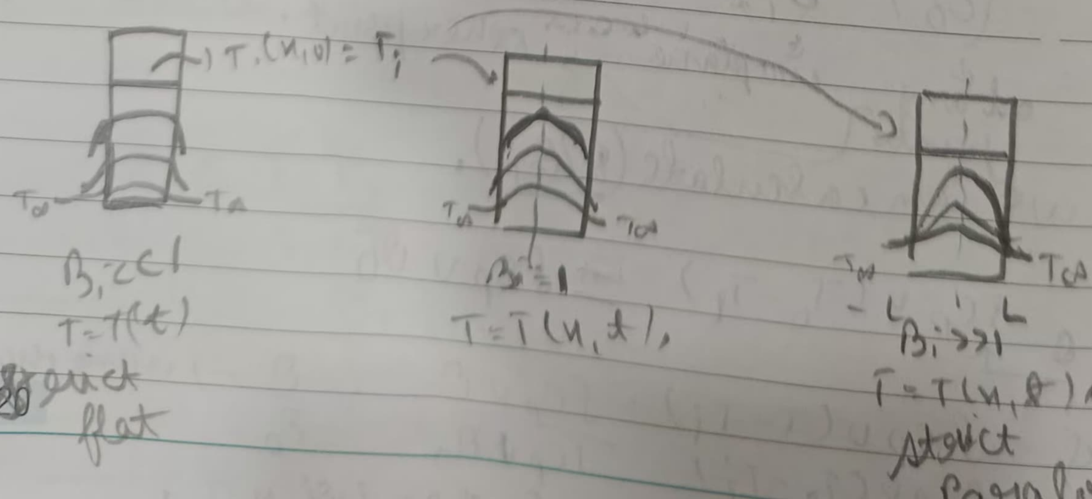
For Infinite cylinder
temp change only in axial dir.
graphs are used in same way as for infinite plate (here dimension = r_0).

only applicable for $F_0 > 0.2$
$\frac{\alpha t}{L^2} > 0.2$
for $Bi > 1$

For infinite sphere.
temp change only in axial dir, dimension = r_0.

Q1) A large plate of aluminium ($k = 215\ W/m°C$) 5cm thick is initially at uniform temp of 200°C. It is suddenly exposed to $T_\infty = 50°C$ with $h = 425\ W/m^2°C$. Calculate the temp. at a depth of 1.25cm from one of the 1 min after the plate is exposed to the environment. Also estimate the amt of heat removed from per unit area from the plate in this time. Given $\alpha = 8.4 \times 10^{-5}\ m^2/s$.

for any question find B_i first
if $Bi < 0.1$ → Lumped capacitance model.
$Bi \geq 0.1$ → find F_0
if $F_0 > 0.2$ → use Heissler's Plot.



Fins also help in ↑ the ~~con~~ conduction Resistance ~~that~~ along with ↑ in the surface area for heat transfer.

2-dim., S.S, no heat gen.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0. \qquad 4\ B.C\ \text{required.}$$

$$T = X(x)\,Y(y).$$

$$\frac{d^2T}{dx^2} = Y(y)\frac{d^2X}{dx^2}$$

$$\frac{d^2T}{dy^2} = X(x)\frac{d^2Y}{dy^2}.$$

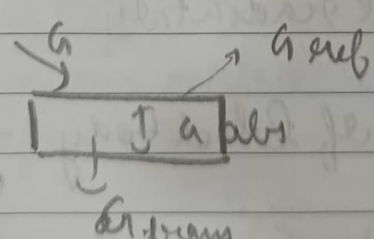
$$Y(y)\frac{d^2X}{dx^2} + X(x)\frac{d^2Y}{dy^2} = 0.$$

$$\cancel{Y(y)\,dy} \quad \frac{1}{X(x)}\frac{d^2X}{dx^2} + \frac{1}{Y(y)}\frac{d^2Y}{dy^2} = 0.$$

Radiation.

1) temp. gradient in req.
2) ~~depends on~~ not depends on material medium. only depends on temp diff.

energy in radiation - all in terms of energy flux.



Only Amt of thermal radiation that alters body temp. is → absorption.

$$\frac{G_{ref}}{G} = \rho$$

$$\frac{G_{abs}}{G} = \alpha$$

$$\frac{G_{trans}}{G} = \tau$$

$\rho + \alpha + \tau = 1$ (for semi transparent medium)

For opaque body $\rho + \alpha = 1$

For black body $\alpha = 1$ (all energy absorbed).

(black body absorbs all energy).

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Emissive power = energy emitted per unit area per unit ~~time~~ time.

Emissivity = ~~energy~~ emissive power of a body / emissive power of black body

Thermal Radiation - any radiation which is associated with ↑ in body temp.

Thermal Radiation is an electromagnetic radiation emitted by a body as a result of its temp.
(Infrared + visible) + part of U.V. ray spectrum.

$c = \lambda \nu$ $\quad c = 3\times10^{8}\ m/s$

$E = h\cdot\nu$ $\quad h = 6.626\times10^{-34}\ J\cdot s$

Black body = 1) To evaluate the ~~em~~ heat emitted / radiated black body acts as a reference.

For long wave thermal radiations → snow, ice behaves as black body.

Black body is an ideal concept.

Plane mirror undergoes diffusive reflection for thermal radiation

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Most common eg. of black body → pin hole camera (small inlet for incident light).

reflection
- Regular
- Diffusivity

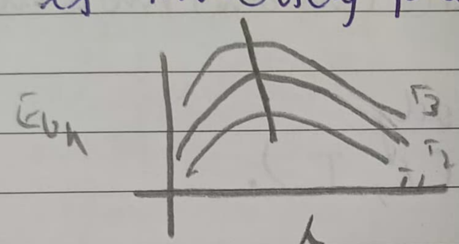
Emissive Power – Rate (W/m^2) at which Radiation is emitted from surface per unit surface area per unit time

Radiosity (W/m^2) of a surface accounts for all Radiant energy leaving surface.
Net radiative flux from surface (W/m^2).
Outgoing radiation – Incoming radiation.

Properties of Thermal Radiation

① If $E_{b\lambda}$ = Emissive power of black body as a function of λ is inversely proportional λ.

$T_3 > T_2 > T_1$



$\lambda_{max} T$ = constant = ~~Wynes~~ Weins. displacement Law

Assuming ~~surface~~ thermal radiation is a surface property

② Emissive power = $f(\lambda, \theta, T)$
i.e energy emitted from surface = f(nature of surface, λ, θ, Temp of surface)

$E = E(\lambda, \theta, T$, nature of surface).
For simplification
$E \neq f(\theta)$ → Diffuse emitter (no directional change of E).

∴ $E = E(\lambda, T$, nature of surface)
Directional & specular property

Grey Body Approximation
monochromatic emissivity
$\neq f(\lambda)$.

[Emissity $= \frac{E}{E_b}$
$\therefore \epsilon_b = 1$ = Blackbody]

For Grey Body $\epsilon_\lambda = \epsilon \neq f(\lambda)$.

monochromatic emissivity is the total emissivity.

→ at any λ & T, no surface can emit more energy than a black...

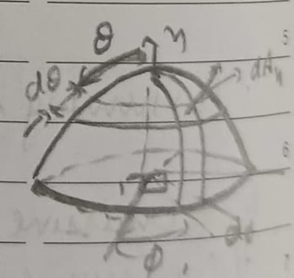
$\therefore$ for a black body
$E_b = E_b(\lambda, T)$.

ϵ = Emissity : ratio of emissive power of a body to Emissive power of black body at same T & λ.

Plack's Distribution Law / Plank's Law

$$E_{b,\lambda} = \frac{C_1}{\lambda^5 (e^{C_2/\lambda T} - 1)}$$

$C_1 = 3.743 \times 10^8 \ W.\mu m^4/m^2$
$C_2 = 1.4387 \times 10^4 \ \mu m.K$



Obtained from Energy density of radiation $u_\lambda = \frac{8\pi h c}{\lambda^5 (e^{hc/\lambda kT} - 1)}$

h = Planck's constant
k = Boltzmann constt
c = velocity of light.

$E_{b\lambda}$ at a λ $= \frac{u_\lambda c}{4} = \frac{C_1}{\lambda^5 (e^{(C_2/\lambda T)} - 1)}$.

9
10 1) 2)
11
12
1
2
3
4
5
6
7

$E_b = \int_0^\infty E_b(\lambda) d\lambda$ = Overall emmisive power of black body

$$\boxed{E_b = \int_0^\infty \frac{C_1}{\lambda^5 (e^{C_2/\lambda T} - 1)} d\lambda = \sigma T^4}$$ → Stefan Boltzmann eq^n.

From the curve we can explain

1) heating of a body making the body red hot.
2) greenhouse effect.

Radiation no material medium for transfer of heat.

let $C_2 / \lambda t = a$. $\qquad \lambda = C_2/at$

$-\frac{C_2}{\lambda^2 t} d\lambda = da$. $\qquad d\lambda = -\frac{\lambda^2 da}{t C_2} = \frac{-C_2 da}{a^2 t^2 t C_2}$.

$$E_b = \int_\infty^0 \frac{-C_1 a^5 t^5 \cancel{C_2} da}{C_2^5 (e^a - 1) a^2 t^3 \cancel{C_2}}$$

$$E_b = \int_0^\infty \frac{C_1 a^3 t^2}{C_2^5 (e^a - 1)} da.$$

$$E_b = \int_0^\infty \frac{C_1}{C_2^5} a^3 t^2 e^{-a} (1 - e^{-a})^{-1} da.$$

$$E_b = \int_0^\infty \frac{C_1}{C_2^5} a^3 t^2 \left[1 + \frac{e^{-a}}{1!} + \frac{e^{-2a}}{2!} + \frac{e^{-3a}}{3!} + \cdots \right]$$

$\int_0^\infty e^{-st} f(t) dt = \mathcal{L}\{f(t)\} =$

$$E_b = \int_0^\infty t^3 \sum_{n=1}^\infty e^{-(nC_2/T)t} dt. \qquad \because \sum_{n=1}^\infty \frac{1}{n^4} = \frac{\pi^4}{90}$$

$$= C_1 \sum_{n_1}^\infty \frac{6}{\left(\frac{nC_2}{T}\right)^4} \qquad \text{where } \sigma = \frac{6C_1}{C_2} \frac{\pi^4}{90}$$

$$= 6C_1 T^4 \sum_{n_1}^\infty \frac{1}{n^4 C_2^4} = \frac{6C_1}{C_2^4} \frac{\pi^4}{90} (T^4)$$

$$= \sigma T^4$$

$\boxed{\sigma = 5.67 \times 10^{-8} \, W/m^2 K^4}$

Reflection - a surface phenomenal.

amt / energy emitted depends on the nature of body

$E = E(\lambda, \theta, T)$

$E \neq E(\theta)$] $\rightarrow$ for diffuse emitter

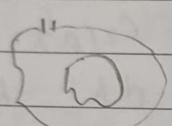
$E \neq (\lambda, \theta)$

all black body are diffuse emitters.

for thermal eqn absorbed heat = transmitted heat

$G_{abs} A = EA$

$G = G_{abs} = E_b$. [for black body].



For a pin hole camera => a black body.

$$\boxed{\begin{array}{l}\text{For a black body inside the pin hole camera} \\ G = G_{abs} = E_b\end{array}}$$

For a normal body inside the pin hole camera.

$$\boxed{\begin{array}{l}\text{For thermal eqn } G_{abs} = E = \alpha G = E = \epsilon E_b \\ \epsilon = \text{emissivity} = \text{avg. emissivity over entire range of } \lambda \ (0 \text{ to } \infty).\end{array}}$$

Emission - body releases due to its temp.

energy emitted by a black body to normal body

$\therefore G = E_b$

$E = G_{abs} = \alpha G = \alpha E_b$

$\alpha = \frac{G_{abs}}{G} = \frac{G_{abs}}{E_b} = \frac{E}{E_b} = \epsilon$

$\therefore$ For any body its absorbtivity = emissivity

$\alpha = \epsilon$

$\epsilon_b = 1$, $\alpha_b = \epsilon_b = 1$ $\rightarrow$ Kirchoff's law

$\lambda_{max} T$ = constant = Wien's ~~displacement~~ Displacement Law

$E_{b\lambda} = \sigma T^4$ = Stefan Boltzmann's Law.

with ↑ in λ E_b ↑, reaches maxima, then further ↓.
for maxima $\frac{dE_{b\lambda}}{d\lambda} = 0$ at const T.

The locus of this maxima points when joined by a line gives a straight line following

$$\boxed{\lambda_{max} T = 2898 \, \mu m \, K = C_3}$$ → Wein displacement Law

obtained by $\frac{dE_{b\lambda}}{d\lambda} = 0$.

Emitted radiation is a continuous function of λ.
at higher T for a fixed λ $E_{b\lambda}$ is high.

On heating a body the max. energy is emitted at a lower λ

$$\frac{dE_{b\lambda}}{d\lambda} = \frac{d \cdot \frac{C_1}{\lambda^5 (e^{C_2/\lambda T} - 1)}}{d\lambda}$$

$$0 = \frac{-C_1 \left[5\lambda^4 e^{C_2/\lambda T} + \lambda^5 \frac{C_2}{-\lambda^2 T} e^{C_2/\lambda T} - 5\lambda^4 \right]}{(\lambda^5 (e^{C_2/\lambda T} - 1))^2}$$

∴ On heating a body it becomes dull red → bright red → yellow → white.

$$0 = \frac{C_1 \left[5\lambda^4 e^{C_2/\lambda T} - \frac{\lambda^3 C_2}{T} e^{C_2/\lambda T} - 5\lambda^4 \right]}{\lambda^{10} (e^{C_2/\lambda T} - 1)^2}$$

at Room temp. the colour of every body depends on the energy it reradiates

on heating amt emitted > amt reflected.

$$5\lambda^4 T e^{C_2/\lambda T} - \lambda^3 C_2 e^{C_2/\lambda T} - 5\lambda^4 T = 0$$

$$5\lambda^4 T [-1 + e^{C_2/\lambda T}] = \lambda^3 C_2 e^{C_2/\lambda T}$$

$$5\lambda T [-1 + e^{C_2/\lambda T}] = C_2 e^{C_2/\lambda T}$$

$$5k [1 - e^{C_2/k}] = C_2 e^{C_2/k}$$

$k = 2898$

2020

at low temp → reflection
at high temp → temp. radiated.

M	T	W	T	F	S	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

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Sun visible white ∵ It is at 5800K and $E_{b,\lambda}$ max lies in visible region at this temp → Sun is a black body.

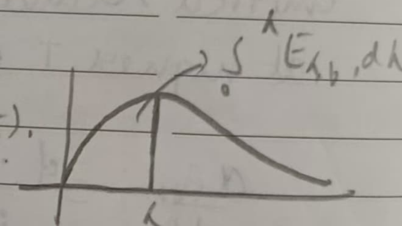
∴ any body on heating → appears white (eventhough if $E_{b\lambda}$ does not lie in visible range, but majority of radiations lie in the visible region → giving body its white colour)

→ Colour change of body on heating., colour of body on heating.

Solar radiation having EM spectrum is same as radiation emitted by black body at 5800K, ∴ spectral distribution by solar radiation is same as spectral distribution of B.B. at 5800K,

→. Band emission of B.B.

$$E_{b,\lambda} = \frac{C_1}{\lambda^5 (e^{C_2/\lambda T} - 1)}$$



$$f_{0-\lambda} = \frac{E_{b,0-\lambda}}{E_{b,0-\infty}} = \frac{\int_0^\lambda E_{b,\lambda}\, d\lambda}{\int_0^\infty E_{b,\lambda}\, d\lambda} \rightarrow \sigma T^4$$

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$$\cancel{\int_0^\lambda E_{b,\lambda}\, d\lambda} = \frac{E_b(\lambda)}{T^5} = \frac{C_1}{\lambda T^5 (e^{C_2/\lambda T} - 1)}.$$

$$\boxed{f_{0-\lambda} = \int_0^{\lambda T} \frac{E_{\lambda,b}}{\sigma T^5}\, d(\lambda T)}$$

· Glass - opaque to thermal radiation at R.T.

Opacity depends on temp. of thermal radiation.

For Glass $\tau = 1$ (transmissivity = 1).
for $\lambda \le 2.5\,\mu m$.

at T = 5800 K. For $\lambda = 2.5\,\mu m$.
$\lambda T = 2.5 \times 5800\,\mu mK$
$= 14500\,\mu mK$.
$f_{0\to\lambda} \approx 0.966 = 0.97$
∴ emitted radiation = 97% of incident radiation.

at T = 300 K $\lambda = 25\,\mu m$ $\lambda T = 7500\,\mu mK$.
$f_{0\to\lambda} = 0.000012$
∴ thermal radiation at room temp behaves as an opaque body & therefore capture these radiation
∴ forming greenhouses.

eg. Q Square glass plate = 30 cm side → used to view Radiation from a furnance $\tau = 0.5$ from 0.2 – 3.5 μm
$\tau = 0$ elsewhere.

$T_{furnace} = 4000°C = 4273 K$ $\epsilon = 0.3$, $\lambda < 3.5\,\mu m$.
$T_{furnance} = 2000°C = 2273 K$ $\epsilon = 0.9$ $\lambda > 3.5\,\mu m$.

Ans λT For $\lambda = 3.5\,\mu m$. $\lambda = 0.2\,\mu m$
$\lambda T = 14955.5\,\mu mK$ $\lambda T = 854.6\,\mu mK$
$f_{0\to\lambda} \approx 0.97$ $f_{0-0.2} \approx 0$.
~~E total~~ = incident on glass = E_b from furnance
$E_b = \sigma T^4 = 5.67 \times 10^{-8} \times (4273)^4 = G$ (irradiation on glass)
$= 18902299.07\ W/m^2 \approx 19000\ kW/m^2$
~~Ant transmitted~~ $\int_{0.2}^{3.5} \tau E_b$
for $0.2 < \lambda < 3.5$
$E_{b\,0.2\to3.5}$

$E_{b\,0\to\lambda_2} = \sigma T^4 f_{0\to\lambda_2}$
$= \sigma T^4 [f_{0\to\lambda_2} - f_{0\to\lambda_1}]$

$G_{incident} = 19002.3 \, kW/m^2$

$Q_{incident} = 19002.3 \times 30 \times 30 \times 10^{-4} = 1710.207$

$Q_{incident} = 19002.3 \times 30 - f_{0\to0.2}$

~~$Q_{tr} = G_{incident} f_{0\to3.5}$~~ $= 9501.15 \, kW/m^2$

$G_{tr} = 0.5 \times 19002.3$

$Q_{tr} = 0.5 \times 19002.3 \times 30 \times 30 \times 10^{-4} = 855.1035$

$E_{absorbed}$ for $\lambda > 3.5$.

$E_{abs} = G_{absorbed} A$

$\quad f_{(3.5\to\infty)} \epsilon G$

$E_{abs} = ((f_{0\to\infty} - f_{0\to3.5}) \epsilon G) A$

$E_{abs} = 461755.8$

$= 46.175$.

$0.2 < \lambda < 3.5$

$E_{abs} = 0.97 \times 0.3 \times 19002.3 \times$

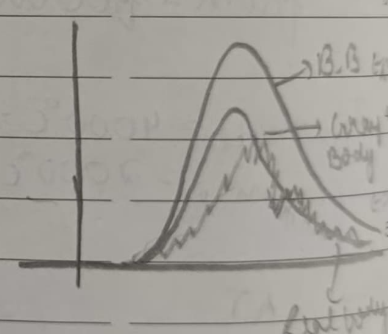
$= 4976702.3$

$= 497.67$

$E_{abs} = 1371.993 \, kW$ $E_{ref} = 287 \, kW$

Gray body – Monochromatic emissivity of body is independent of wavelength

$\epsilon_\lambda = \frac{E_\lambda}{E_{b\lambda}}$



$E = E(\lambda, \theta, T)$

for Diffuse emitter $E = E(\lambda, T)$

$E_b = \int_0^\infty E_{b\lambda} \, d\lambda$

$E = \int_0^\infty \epsilon_\lambda(T) E_{b\lambda} \, d\lambda$

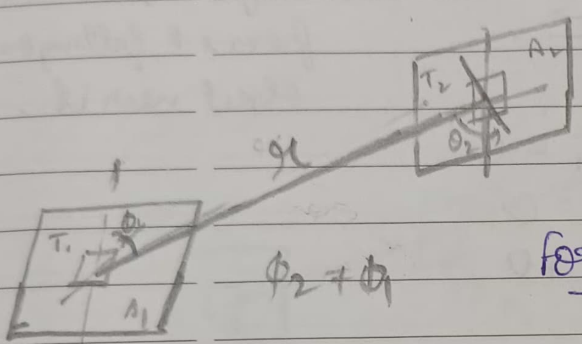
For Gray body – monochromatic emissity is consider i.e. no variation in emissivity with λ

Emissivity of highly polished metals is much lower when oxided its emissivity ↓.

with ↑ in temp, emissivity ↑ & generally holds for metals

Heat Interaction b/w two radiating bodies
Energy interaction b/w two bodies
depends on view factor F_{m-n}
which itself depends on

1) Shape of two bodies m & n
2) Distance b/w two bodies m & n.
3) Orientation of two bodies with respect of each other.



ϕ_2, ϕ_1 angles with r.

$E_{b_1} = f(\lambda, T_1)$
$E_{b_2} = f(\lambda, T_2)$.

Total energy radiated $= E_{b_1} A_1$

for normal body - all energy incident is not transmitted / absorbed.

for black body - all energy incident is absorbed & transmitted.

Imp.
view factor $= F_{mn}$ = is the energy emitted from m & incident on n.

Q_{12} = net Radiant energy exchange b/w 1 & 2 (from 1 to 2).
$= E_{b_1} A_1 F_{12} - E_{b_2} A_2 F_{21}$

if $T_2 > T_1$ then T_1 heats up
if $T_1 > T_2$, then T_2 heats up.

If both bodies are at same temp T_1 & T_2 → holds even for normal body
$Q_{12} = 0$
$E_{b_1} = E_{b_2} = \sigma T^4$ $\quad E_b = \int_0^\infty E(\lambda, T)\, d\lambda$.

Imp
$\boxed{A_2 F_{21} = A_1 F_{12}}$

$\boxed{F_{12} \neq F_{21}}$.

→ Reciprosity Relationship

at constt T $T_1 = T_2$

$\boxed{A_2 F_{21} = A_1 F_{12}}$ → Holds for all Temp.

~~for~~ → Summation Relationship

$$\sum_{j=1}^{1} F_{ij} = 1$$

within an enclosure the sum of energies radiation from i & falling on all object near it.

For plane surface $F_{ii} = 0$

For ~~concave~~ convex surface $F_{ii} = 0$

For irregular surface $F_{ii} \neq 0$

For N materials no. of viewfactors required N×N.

N×N − N (Sum rules) − N(N−1)/2 (Reciprosity) = N(N−1)/2 (actually to be found)

~~due to Reciprocity~~ due to energy transfer both bodies come at same temp.

f_{12} = Intensity radiated from 1 reaching 2.

Intensity = $E_b . dA_1 = I_b \times$ proj of dA_1 in dir. ϕ_1
$= I_b \, dA_1 \cos \phi_1$

If there is a dA_n body in b/w the two bodies then out of the total intensity radiated from 1 total $\times (dA_n / r^2)$ → i.e solid angle subtended by A_1 on A_n is the energy incident on A_n.

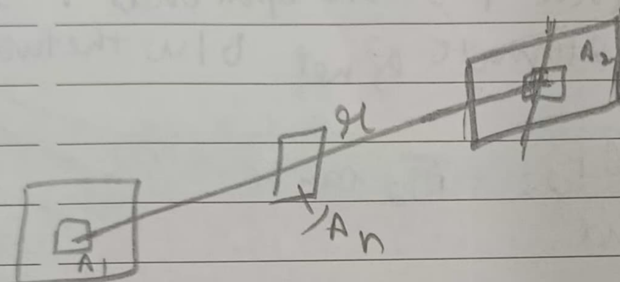
view factor ↔ shape factor ↔ Angle factor ↔ config factor

$$E_b\, dA_1 = I_b\, dA_1 \cos\phi \cdot \frac{dA_n}{r^2}$$

[Intensity = Power / Area × Solid Angle.]

where $dA_n = r^2 \sin\phi\, d\theta\, d\phi$

Emmisive power in terms of intensity of radiation from A_1



$$E_b\, dA_1 = I_b\, dA_1 \int_0^{2\pi}\int_0^{\pi/2} \sin\phi_1 \cos\phi_1\, d\phi_1\, d\theta.$$

$$= \pi I_b\, dA_1$$

$\underline{E_b = \pi I_b}$ .. $dA_n = dA_2 \cos\phi_2$

Energy leaving dA_1 & reaching dA_2

$$dQ_{12} = E_{b_1} \cos\phi_1 \cos\phi_2 \frac{dA_1\, dA_2}{\pi r^2}$$

$$dQ_{21} = E_{b_2} \cos\phi_2 \cos\phi_1 \frac{dA_2\, dA_1}{\pi r^2}.$$

$$\therefore Q_{net} = \int dQ_{12} - \int dQ_{21} = (E_{b_1} - E_{b_2}) \underbrace{\int_{A_1}\int_{A_2} \frac{\cos\phi_1 \cos\phi_2\, dA_1\, dA_2}{\pi r^2}}_{A_1 F_{12} \text{ or } A_2 F_{21}}$$

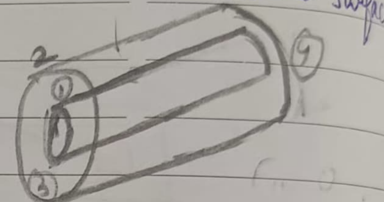
M	T	W	T	F	S
					1
3	4	5	6	7	8
10	11	12	13	14	15
17	18	19	20	21	22
24	25	26	27	28	29

non black body - absorbtion / reflection
emission / reflection ↑.

01

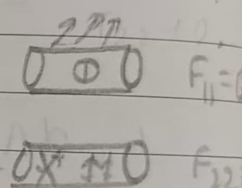
Q). 1m by 2 m Rectangles
$r = 0.5$ m.
$T_1 = 1600°c$. $T_2 = 800°c$.
find Qnet
$\because A_1 = A_2$
$Q = E_{b_1} A_1 F_{12} - E_{b_2} A_2 F_{21}$
$\therefore F_{12} = F_{21}$.
$= (E_{b_1} - E_{b_2}) A_1 F_{12}$.

-Q) . Two concentric cylinder $d_1 = 15$ cm 30 cm = d_2
$l = 30$ cm. Find F b/w open ends. If $T_0 = 2000°c$
$T_{in} = 800°c$; estimate Qnet b/w the two curved surface



$\cancel{F_{22}} = F_{34}$ F_{21} & F_{22} & F_{12} can be found

$A_1 / A_2 = \pi r_i l$.



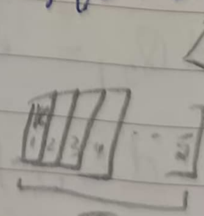
$\overset{0}{F_{11}} + F_{12} + F_{13} + F_{14} = 1$.
$F_{21} + F_{22} + F_{23} + F_{24} = 1$
$F_{13} = F_{14}$ {from sym.}
$F_{23} = F_{24}$

$F_{31} + F_{32} + \overset{0}{F_{33}} + F_{34} = 1$.

02 SUNDAY

Relation b/w view-factors
1) Reciprosity $A_1 F_{12} = A_2 F_{21}$
2) Summation, $\sum F_{ij} = 1$ (From conservation of Energy).
(Only for enclosure).

3). $F_{i-j} = \sum_{k=1}^{N} F_{i-k}$



k = Subpart of receiving body (only body receiving radiation).

'lly $\sum_{k=1}^{N} F_{k-i} = F_{j-i}$

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

Multiplying both sides by A_i

$$A_i F_{i-j} = A_i \sum_{k=1}^{N} F_{(i-k)}.$$

$$\boxed{A_j F_{j-i} = \sum_{k=1}^{N} A_k f_{k-i}}$$ → Reciprocity on each body.

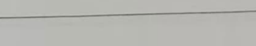
For a composite surface - we can break down it in parts.

$$\boxed{F_{j-i} = \frac{\sum_{k=1}^{N} A_k f_{k-i}}{A_j}}$$

(3) $F_{i-j} = \sum_{k=1}^{N} F_{i-k}.$

(4) $A_i F_{i-j} = A_j f_{j-i} = \sum_{k=1}^{N} A_k f_{k-i}$ { Assumed $f_{ii}=0$ plane, or convex surface }

$$F_{j-i} = \frac{\sum A_k f_{k-i}}{A_j}$$

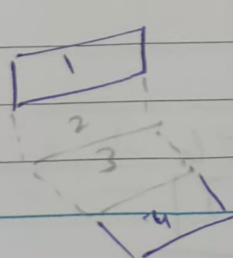
→ 

f_{12} f_{123}

$\sum f_{1i} = 1$ $f_{11} = 0$

$\underbrace{f_{12}}_{\text{Known}} + f_{13} = 1$

$\underline{f_{13} = 1 - f_{12}}.$

→ 

f_{12-34}, f_{12-3}, f_{2-34}, f_{2-3}

$A_1 f_{1-34} = \boxed{A_1 f_{13}} + A_1 f_{14}$

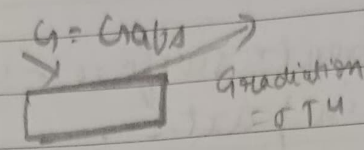
$A_{12} f_{12-34} = \boxed{A_1 f_{13}} + A_2 f_{23}$

$A_{12} f_{12-34} = A_1 f_{1-34} + A_2 f_{2-34}$

Irradiation → Total radiant energy received by surface per unit time per unit area.

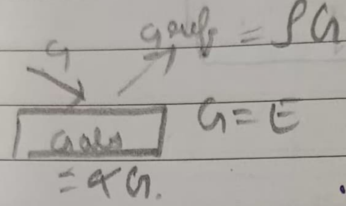
Assumptions
1. Opaque bod[y]
2. All surfac[es] are diffuse [...]
3. All surfaces [...] uniform in t[...]
4. ρ, ϵ const[ant over] the entire s[urface]
5. G, J are unif[orm] over each surf[ace]

For black body.



$\alpha = \epsilon = 1$

For opaque bodies.



$G = E$

$G = E$

$\alpha G = \epsilon E_b$.

∴ total energy leaving

$\epsilon E_b + \rho G = J$ — Radiosity

Radiosity → radiant energy leaving surface per unit time per unit area (W/m^2).

Net energy radiated from source $= J - G$

$= \epsilon E_b + \rho G - G$

$= \epsilon E_b + (\rho - 1) G$

$= \epsilon E_b - (1-\rho) G$.

$$\boxed{q_{net} = \epsilon E_b - \alpha G}$$

For thermal eqⁿ $G = E$

$\alpha G = \epsilon E_b$

$\therefore \alpha = \epsilon$.

$q_{net} = \epsilon (E_b) - \epsilon G$

$$\boxed{q_{net} = \epsilon [E_b - G]}$$

$J = \epsilon E_b + \rho G$

$= \epsilon E_b + (1-\alpha) G$

$J = \epsilon E_b + (1-\epsilon) G$

$J = G + \epsilon \underline{(E_b - G)}.$

$\cancel{G = J - \epsilon(E_b - G)}$

$$G = \frac{J - \epsilon E_b}{(1-\epsilon)}.$$

$J = G + q_{net}.$

$$J = \frac{J - \epsilon E_b}{(1-\epsilon)} + q_{net}$$

$\cancel{J} - J\epsilon = \cancel{J} - \epsilon E_b + (1-\epsilon) q_{net}.$

$$\boxed{\frac{\epsilon(E_b - J)}{1-\epsilon} = q_{net}.}$$

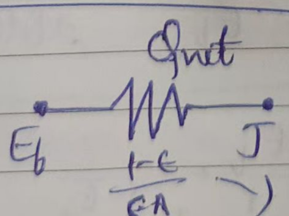
$$Q_{net} = \frac{\epsilon A (E_b - J)}{(1-\epsilon)}.$$

$$= \frac{E_b - J}{\left(\frac{1-\epsilon}{\epsilon A}\right)}.$$

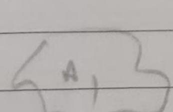
$\frac{1-\epsilon}{\epsilon A}$ = surface Resistance associated with the body [∵ it is not a black body]

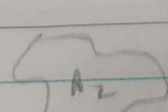
$E_b - J$ = Driving force. ~~for Q~~

$\frac{1-\epsilon}{\epsilon A}$ - Resistance for Q.



) surface resistance





$$Q_{net_1} = \frac{E_{b_1} - J_1}{\frac{1-\epsilon_1}{\epsilon_1 A_1}}, \quad Q_{net_2} = \frac{E_{b_2} - J_2}{\frac{1-\epsilon_2}{\epsilon \ldots}}$$

$Q_{net\,1-2}$ = Energy leaving from 1 & reaching 2 − energy leaving from 2 and reaching 1.

for only interaction b/w 1 & 2.
energy leaving from 1 and reaching 2 $= J_1 A_1 F_{12} = G_2 A_2$ = reaching 2.

energy leaving from 2 $= J_2 A_2$

~~energy~~ energy leaving from 2 and reaching 1 $= J_2 A_2 F_{21}$

$\therefore Q_{net\,1-2} = J_1 A_1 F_{12} - J_2 A_2 F_{21}$ = Net energy interchange b/w 1 & 2.

$A_1 F_{12} = A_2 F_{21}$

$$\boxed{Q_{net\,1-2} = (J_1 - J_2) A_1 F_{12}}.$$

due to Resistance D.F becomes $E_b - J$
for a black body DF only E_b, no resistance.

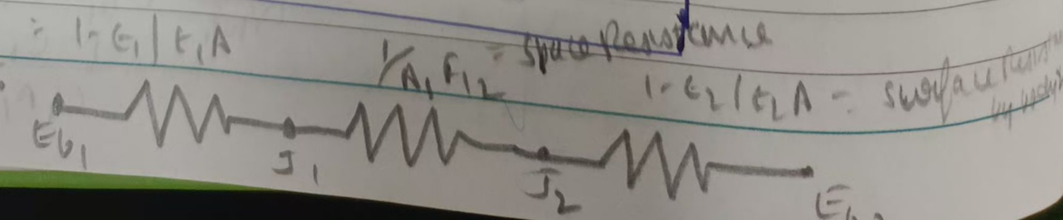
$Q_{net\,1-2} = (J_1 - J_2)$ → D.F.

$A_1 F_{12}$ – space Resistance

total D.F. $E_{b_1} - E_{b_2}$, total Resistance $= \frac{1-\epsilon_1}{\epsilon_1 A} + \frac{1}{A_1 F_{12}} + \frac{1-\epsilon_2}{\epsilon_2 A}$

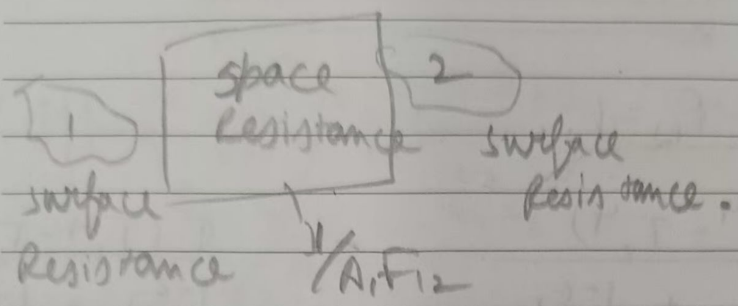
total HT b/w 1 & 2 $= Q_{1-2} = \dfrac{E_{b_1} - E_{b_2}}{\dfrac{1-\epsilon_1}{\epsilon_1 A} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\epsilon_2}{\epsilon_2 A}}$

surface Resistance by body 1: $1-\epsilon_1 / \epsilon_1 A$

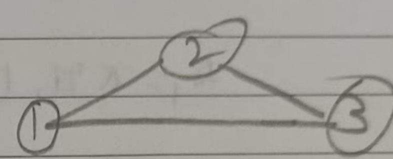


Q_{net} (~~Heat~~ Net HT b/w ① & ②)

$$= \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\epsilon_2}{\epsilon_2 A_2}}$$



→ For 3 bodies.



$$\frac{E_{b_1} - E_{b_2}}{\frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\epsilon_2}{\epsilon_2 A_2}} , \quad \frac{E_{b_1} - E_{b_3}}{\frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1 F_{13}} + \frac{1-\epsilon_3}{\epsilon_3 A_3}}$$

$$+ \frac{E_{b_2} - E_{b_3}}{\frac{1-\epsilon_2}{\epsilon_2 A_2} + \frac{1}{A_2 F_{23}} + \frac{1-\epsilon_3}{\epsilon_3 A_3}}$$

~~Heat~~ net heat Radiated by body 1 $= \dfrac{E_{b_1} - J_1}{\frac{1-\epsilon_1}{\epsilon_1 A_1}}$

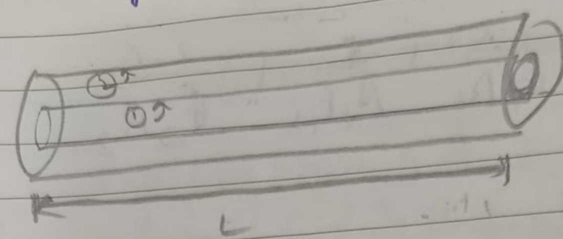
" " " " " 2 $= \dfrac{E_{b_2} - J_2}{\frac{1-\epsilon_2}{\epsilon_2 A_2}}$

net heat exchange b/w body 1 & 2 $\dfrac{E_{b_1} - E_{b_2}}{\frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\epsilon_2}{\epsilon_2 A_2}}$.

given the two bodies kept in a room

The room can be assumed to be a 3rd body.

Two long concentric cylinders.



$F_{11}=0$. $F_{12}=1$.

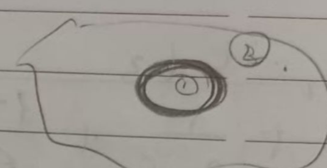
$$Q_{net} = \frac{\sigma A_1(T_1^4-T_2^4)}{\frac{1-\epsilon_1}{\epsilon_1 A_1}+\frac{1}{A_1}+\frac{1-\epsilon_2}{\epsilon_2 A_2}} = \frac{\sigma(T_1^4-T_2^4)}{\frac{1-\epsilon_1}{\epsilon_1}+1+\left(\frac{1-\epsilon_2}{\epsilon_2}\right)\left(\frac{A_1}{A_2}\right)}$$

$A_1=\pi r_1 L$, $A_2=\pi r_2 L$.

→ For small object in a large room

$A_2\to\infty$

$\therefore \frac{1-\epsilon_2}{\epsilon_2 A_2}\to 0$.



check.

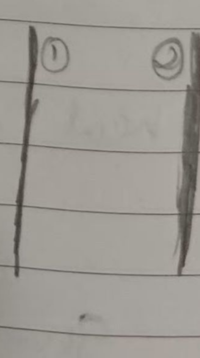
$\therefore Q_{net} = \sigma A_1\epsilon(T_1^4-T_2^4)$ $\because A_1\to$ small, $\epsilon < 1$.

→ Infinite Parallel plate

$F_{11}=0$

$F_{22}=0$.

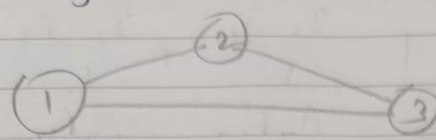
$A_1=A_2=A$.

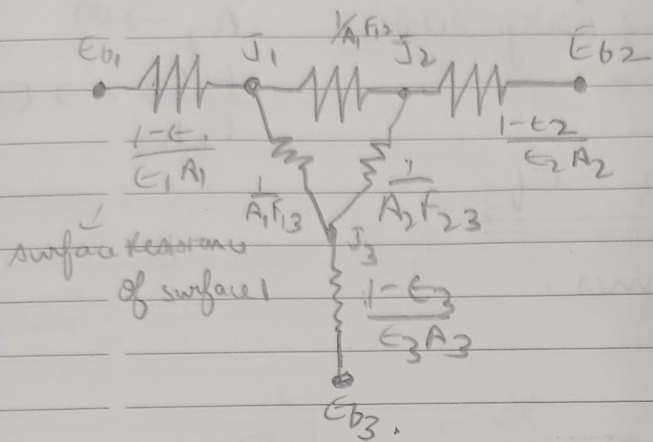


check

$$Q_{net}=\frac{\sigma A(T_1^4-T_2^4)}{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_2}-1}$$

3-body interaction.





overall circuits.

$$Q_{net-1} = \frac{E_{b_1} - J_1}{\frac{1-\epsilon_1}{\epsilon_1 A_1}}$$

J_i - are unknown.
To find J_1, J_2.
using Kirchoff's current Law (conservation of energy)

$$Q_{net\ 12} = \frac{J_1 - J_2}{1/A_1F_{12}}$$

- sum of the currents entering a node is zero

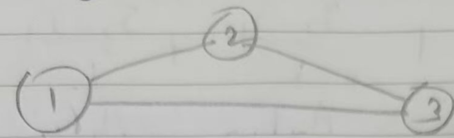
For node 1

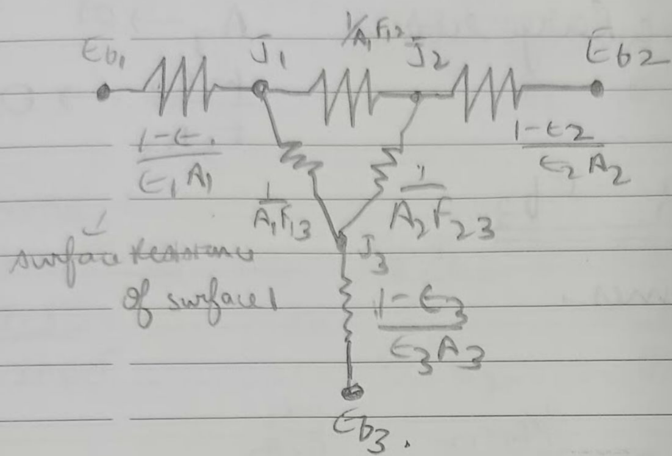
$$\frac{E_{b_1} - J_1}{\frac{1-\epsilon_1}{\epsilon_1 A_1}} + \frac{J_2 - J_1}{\frac{1}{A_1F_{12}}} + \frac{J_3 - J_1}{\frac{1}{A_3F_{13}}} = 0.$$

For node 2

$$\frac{E_{b_2} - J_2}{\frac{1-\epsilon_2}{\epsilon_2 A_2}} + \frac{J_2 - J_3}{\frac{1}{A_2F_{23}}} + \frac{J_2 - J_1}{\frac{1}{A_1F_{12}}} = 0;$$

3 - body interaction.





overall circuits.

$$Q_{net-1} = \frac{E_{b_1} - J_1}{\frac{1-\epsilon_1}{\epsilon_1 A_1}}$$

J_i = are unknown.
To find J_1, J.
using Kirchoff's current law (conservation of energy)

$$Q_{net\ 12} = \frac{J_1 - J_2}{1/A_1F_{12}}$$

- sum of the currents entering a node is zero

For node 1

$$\frac{E_{b_1} - J_1}{\frac{1-\epsilon_1}{\epsilon_1 A_1}} + \frac{J_2 - J_1}{1/A_1F_{12}} + \frac{J_3 - J_1}{\frac{1}{A_3F_{13}}} = 0.$$

For node 2

$$\frac{E_{b_2} - J_2}{\frac{1-\epsilon_2}{\epsilon_2 A_2}} + \frac{J_2 - J_3}{\frac{1}{A_2F_{23}}} + \frac{J_2 - J_1}{\frac{1}{A_1F_{12}}} = 0;$$

For node 3.

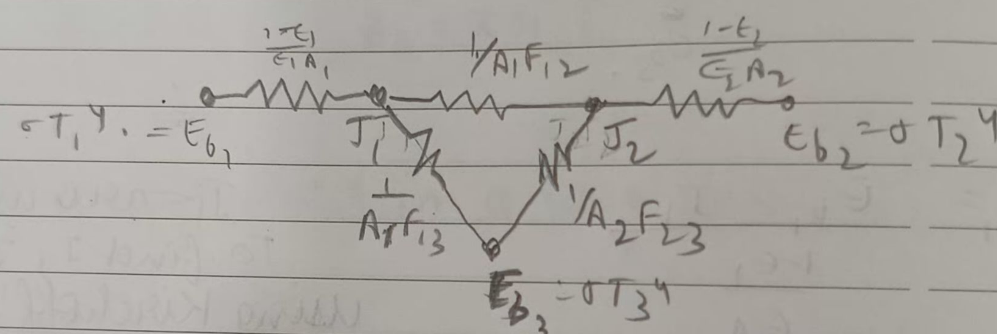
$$\frac{E_{b_3}-J_3}{\frac{1-\epsilon_3}{\epsilon_3 A_3}} + \frac{J_3-J_2}{\frac{1}{A_2F_{23}}} + \frac{J_3-J_1}{\frac{1}{A_1F_{13}}} = 0.$$

For 2 bodies in a large room $A_3 \to \infty$

$$\frac{1-\epsilon_3}{\epsilon_3 A_3} \to 0.$$

therefore $J_3 = E_{b_3}$

the circuit becomes.



Same circuit applicable for a third body is insulator (∵ It does not ~~hold any~~ absorb any heat it radiates it back) and for third body having very large area.

$J_3 = E_{b_3}$.

→ Q. Two parallel plates 0.5 m by 1 m placed 0.5 m apart $\epsilon = 0.5$ at $T = 1200°C$, other of emissivity $\epsilon = 0.2$ at $T = 2000°C$. Housed in large room is at 27°C. net heat transfer to each plate & the room.

Assuming - (a) Plates & room exchange radiant heat among themselves

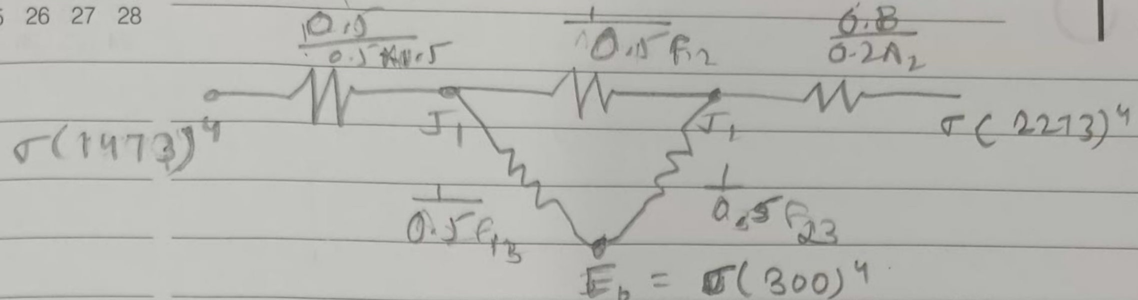
b) plate facing each other are consider for analysis.

Learn Fins formula.

$F_{33}=0$ $F_{13}=1$

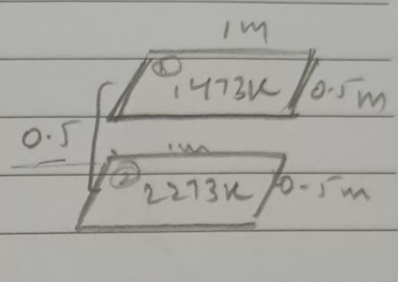
$F_{22}=0$ $F_{23}=1$

$F_{11}=0$ $F_{12}=1$



node 1

$$0=\frac{\sigma(1473)^4-J_1}{\frac{0.5}{0.5\times0.5}}+\frac{J_2-J_1}{\frac{1}{0.5F_{12}}}+\frac{\sigma(300)^4-J_1}{\frac{1}{0.5F_{13}}}$$



$A_1F_{12}=A_2F_{21}$

$A_1=A_2$

$F_{12}+F_{13}=1$

$F_{21}+F_{23}=1$

$A_1F_{13}\neq A_3F_{31}$

node 2

$$0=\frac{\sigma(2273)^4-J_2}{\frac{0.8}{0.2\times0.5}}+\frac{J_1-J_2}{\frac{1}{0.5F_{12}}}+\frac{\sigma(300)^4-J_2}{\frac{1}{0.5F_{23}}}$$

$$0=\frac{266927.941-J_1}{\frac{0.5}{0.5\times0.5}}+\frac{J_2-J_1}{\frac{1}{0.5F_{12}}}+\frac{459.27-J_1}{\frac{1}{0.5F_{13}}}$$

$$0=\frac{1246564.123-J_2}{\frac{0.8}{0.2\times0.5}}+\frac{J_1-J_2}{\frac{1}{0.5F_{12}}}+\frac{459.27-J_2}{\frac{1}{0.5F_{23}}}$$

Find F_{12}, F_{13}, F_{23} from the graphs to find J_1 & J_2.