Transport Phenomena by Bird, Stewart, Lightfoot

THE EQUATIONS OF CHANGE (ISOTHERMAL SYSTEM)

PARTIAL TIME DERIVATIVE $\frac{\partial c}{\partial t}$ (x, y, z const)

TOTAL TIME DERV, $\frac{dc}{dt}$

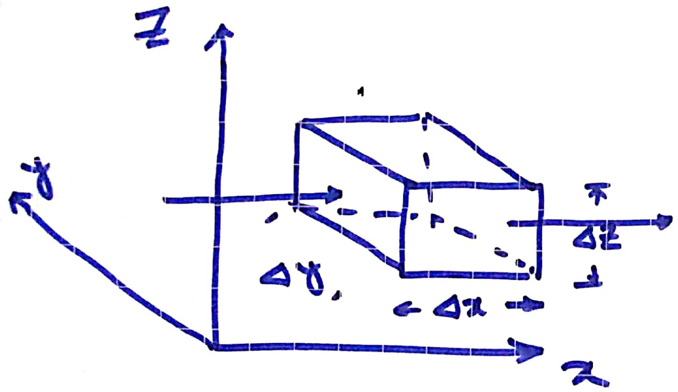
$$\frac{dc}{dt} = \frac{\partial c}{\partial t} + \frac{\partial c}{\partial x}\frac{dx}{dt} + \frac{\partial c}{\partial y}\frac{dy}{dt} + \frac{\partial c}{\partial z}\frac{dz}{dt}$$

SUBSTANTIAL TIME DERV., $\frac{Dc}{Dt}$

$$\frac{Dc}{Dt} = \frac{\partial c}{\partial t} + v_x\frac{\partial c}{\partial x} + v_y\frac{\partial c}{\partial y} + v_z\frac{\partial c}{\partial z}$$

NS-1

EQ^N OF CONTINUITY



RATE OF MASS IN THRU
x - FACE

$$\underbrace{\rho}_{kg/m^3}\ \underbrace{v_x}_{m/s}\Big|_x\ \underbrace{\Delta y \Delta z}_{m^2} \rightarrow kg/s$$

RATE OF MASS OUT x FACE

x 3 IN y z 3 OUT

$x+\Delta x$
$y+\Delta y$
$z+\Delta z$.

$$\rho v_x\Big|_{x+\Delta x} \Delta y \Delta z$$

$$\rho v_x \quad \frac{kg}{m^3}\cdot\frac{m}{s} = kg/m^2 s$$

$$\Delta x \Delta y \Delta z \frac{\partial \rho}{\partial t} \quad \frac{kg}{m^3 s} \quad = \frac{kg}{s}$$

RATE OF MASS ACCUM. = RATE OF MASS IN − RATE OF MASS OUT

CONT. EQ 1

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$$\Delta x \Delta y \Delta z \frac{\partial \rho}{\partial t} = \Delta y \Delta z \left[(\rho v_x)|_x - (\rho v_x)|_{x+\Delta x} \right] + \Delta x \Delta z \left[(\rho v_y)|_y - (\rho v_y)|_{y+\Delta y} \right] + \Delta x \Delta y \left[(\rho v_z)|_z - (\rho v_z)|_{z+\Delta z} \right]$$

$\Delta x \; \Delta y \; \Delta z$

$\Delta x, \Delta y, \Delta z \rightarrow 0$

$$\frac{\partial \rho}{\partial t} = -\left[\frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) \right]$$

$$\longrightarrow \left(\frac{\partial \rho}{\partial t} = -(\nabla . \underline{\underline{\rho v}}) \right)$$

$\frac{kg}{m^3} \frac{m}{s}$ $\quad \frac{kg}{m^2 s}$ — mass flux

CONT. EQN 2

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$$\frac{\partial \rho}{\partial t} = -\left[\frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z)\right]$$

$$= -\left[v_x \frac{\partial \rho}{\partial x} + \rho \frac{\partial v_x}{\partial x} + v_y \frac{\partial \rho}{\partial y} + \rho \frac{\partial v_y}{\partial y} + v_z \frac{\partial \rho}{\partial z} + \rho \frac{\partial v_z}{\partial z}\right]$$

$$\frac{\partial \rho}{\partial t} + v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} = -\rho\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)$$

$$\boxed{\frac{D\rho}{Dt} = -\rho(\nabla v)}$$

CONT EQ 3

$$\frac{\partial \rho}{\partial t} = -(\nabla \rho v)$$

$$\frac{D\rho}{Dt} = -\rho(\nabla v)$$

CONTINUITY EQUATION.

$$(\nabla v) = 0.$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

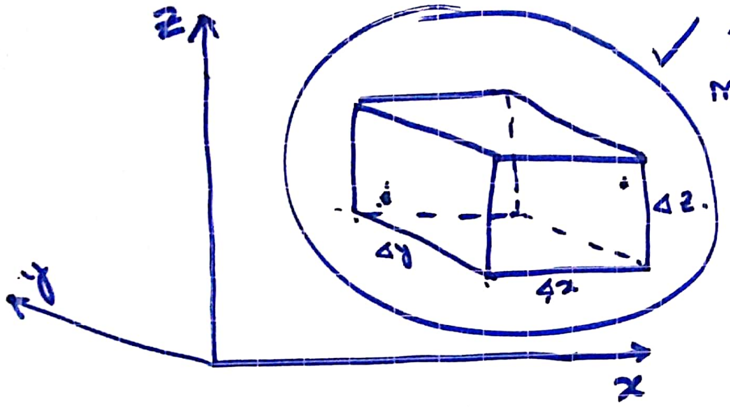
CY
SP.

INCOMPR. FLUID
ρ = CONST

BIRD STEWART LIGHTFOOT
TR. PHENOMENA

CONT EQ 4

THE EQUATION OF MOTION



RATE OF MOMENTUM ACCUMULATION = RATE OF M^2 IN − RATE OF M^2 OUT + $\sum$ FORCES ACTING ON THE SYSTEM.

x COMP.

IN OUT

$$\Delta y \Delta z \left(\rho v_x v_x \big|_x - \rho v_x v_x \big|_{x+\Delta x} \right) + \Delta x \Delta z \left(\rho v_y v_x \big|_y - \rho v_y v_x \big|_{y+\Delta y} \right) + \Delta x \Delta y \left(\rho v_z v_x \big|_z - \rho v_z v_x \big|_{z+\Delta z} \right)$$

MOL. M^2
CONDUCTIVE M^2
VISCOUS M^2

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$$\Delta y \Delta z \left(\overset{IN}{\tau_{xx}\big|_x} - \overset{OUT}{\tau_{xx}\big|_{x+\Delta x}} \right) + \Delta x \Delta z \left(\overset{IN}{\tau_{yx}\big|_y} - \overset{OUT}{\tau_{yx}\big|_{y+\Delta y}} \right) + \Delta x \Delta y \left(\overset{IN}{\tau_{zx}\big|_z} - \overset{OUT}{\tau_{zx}\big|_{z+\Delta z}} \right)$$

PR. FORCES

$$\boxed{\Delta y \Delta z \left(p\big|_x - p\big|_{x+\Delta x} \right)}$$

x COMP.

BODY FOR. $\rho g_x \Delta x \Delta y \Delta z$

RATE OF ACCUM. OF x MOMENTUM $\underbrace{\Delta x \Delta y \Delta z}_{m^3} \cdot \frac{\partial}{\partial t}(\rho v_x)$

$$\frac{kg}{m^3} \cdot \frac{m}{s} = \frac{kg}{m^2 s}.$$

EQ^N OF MOTION.

$$\boxed{\rho \frac{D\vec{v}}{Dt} = -\vec{\nabla} p - [\vec{\nabla} \cdot \vec{\tau}] + \rho \vec{g}}$$

+ CONST ρ, μ $\Rightarrow$

MASS PER UNIT VOL. X ACC.
(ρ) $\left(\frac{Dv}{Dt}\right)$

PR FORCE PER UNIT VOL

$\frac{\partial p}{\partial x}$ $\frac{N}{m^2}$

$\frac{N}{m^3}$

SHEAR FOR. VOL

$\frac{N}{m^3}$

gravitational force/vol.

EQ^N OF MOTION $\xrightarrow[\text{CONST. } \mu]{\text{CONST } \rho}$ + EQ^N OF CONT.

$$\boxed{\rho \frac{Dv}{Dt} = -\nabla p + \mu \nabla^2 \mu + \rho g}$$

NAVIER STOKE'S EQ^N.

VISCOUS EFFECTS ARE NOT PRESENT

$$\boxed{\rho \frac{Dv}{Dt} = -\nabla p + \rho g}$$

— EULER EQ^N.

The divergenve of a vector
field is a scalar

The gradient of a vector field
is a tensor

The gradient of a scalar field
is a vector