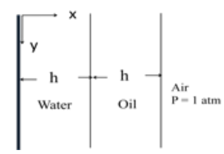
**Tutorial Problems on Navier Stokes Equation – Set 1**

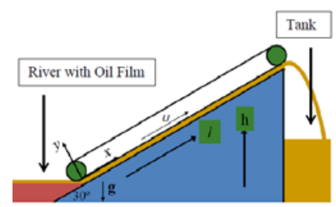
1. Consider incompressible, steady, fully developed laminar flow between infinite parallel plates. The upper plate moves to the right at U=3m/sec. There is no pressure variation in the x direction, but there is a constant body force due to an electric field, $\rho B_x$ = 800N/m$^3$. The clearance between the plates is h=0.1mm and the liquid viscosity is 0.02 Kg/m.s. Evaluate the velocity profile, u(y), if y=0 is located at the lower plate. Compute the volume flow rate past a vertical section.

2. The record-write head for a computer disk storage system floats above the spinning disk on a very thin film of air (the film thickness is 0.5micron). The head location is 150mm from the disk centreline, the disk spins at 3600rpm. The record-write head is 10mm x10mm square. Determine for standard air (density=1.23kg/m$^3$, viscosity=1.78x10$^{-5}$ kg/(m.s)) in the gap between the head and the disk -
(a) the Reynold's number of the flow, (b) the viscous shear stress and (c) the power required to overcome viscous shear stress. For such a small gap the flow can be considered as flow between parallel plates.

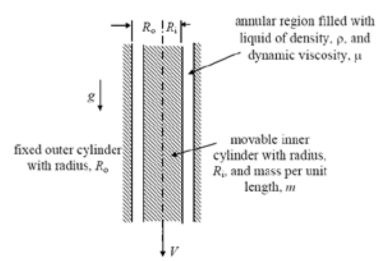
3. Water and oil flow down a vertical plane as shown. The flow is steady, laminar and fully developed. Simplify the Navier-Stokes equations separately for water and oil films with the relevant boundary conditions. Obtain and sketch (qualitatively) the two velocity profiles clearly emphasizing the region near the oil-water interface.



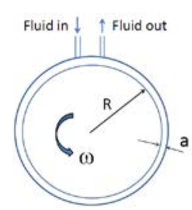

4. An oil skimmer uses a 5 m wide x 6 m long moving belt above a fixed platform (θ = 30º) to skim oil off of rivers (T = 10ºC). The belt travels at 3 m/s. The distance between the belt and the fixed platform is 2 mm. The belt discharges into an open tank on the ship. The fluid is actually a mixture of oil and water. To simplify the analysis, assume crude oil dominates. Find the discharge of oil into the tank on the ship, the force acting on the belt and the power required (kW) to move the belt. For oil: $\rho$ = 860 kg/m$^3$, viscosity, $\mu$ = 1x10$^{-2}$N.s/m$^2$



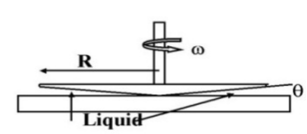

5. Consider two concentric cylinders with a Newtonian liquid of constant density, ρ, and constant dynamic viscosity, μ, contained between them. The outer pipe, with radius, *R*o, is fixed while the inner pipe, with radius, *R*i, and mass per unit length, *m*, <u>falls under the action of gravity at a constant speed</u>. There is no pressure gradient within the flow and no swirl velocity component. Determine the vertical speed, *V*, of the inner cylinder as a function of the following (subset of) parameters: *g*, $R_o$, $R_i$, *m*, ρ, and μ. The space between the two cylinders is **not** 'too small' compared to the radii of the cylinders.




6. A viscous-shear pump is made from a stationary housing with a close fitting rotating drum (angular velocity, ω) inside. The clearance, 'a' is small compared to the diameter of the drum. Fluid is dragged around the annulus by viscous forces. Evaluate the performance characteristics of the shear pump (pressure differential produced, torque needed to turn the drum and input power (torque times angular velocity)). Find an expression for the efficiency of the pump which is defined as the ratio of the output to the input power. The output power can be approximated as QΔP, where Q and ΔP denote the volumetric flow rate and the pressure differential. Assume the depth normal to the diagram is b.




7. The cone and plate viscometer consists of a flat plate and a rotating cone with a very obtuse angle (typically θ less than 0.5 degrees, making it reasonable to assume a linear velocity profile locally, i.e. at any r). The apex of the cone just touches the plate surface and the liquid to be tested fills the narrow gap formed by the cone and the plate. Derive an expression for the shear rate in the liquid that fills the gap in terms of the geometry (R, θ) and angular velocity (ω) of the system. Evaluate the torque in terms of the shear rate and geometry of the system. Assume that there is no mixing in the r and ϕ direction.



**Answers:**

1. Q/b = 1.53 x $10^{-7}$ ($m^3$/s).m
2. Re = 1.95, Stress 2.01 kN/$m^2$, Power = 11.4 W
3. $$v_{y_w} = -\frac{\rho_w g}{\mu_w}\frac{x^2}{2} + c_1\frac{x}{\mu_w} + c_2\ ;\ v_{y_0} = -\frac{\rho_0 g}{\mu_0}\frac{x^2}{2} + c_1\frac{x}{\mu_0} + c_2\ ;$$

   $$c_1 = \frac{g(\rho_0\mu_w - \rho_w\mu_0)}{2(\mu_0+\mu_w)}h\ ; c_2 = \frac{\rho_0 g h^2}{2\mu_0} - \frac{g(\rho_0\mu_w - \rho_w\mu_0)}{2\mu_0(\mu_0+\mu_w)}h^2$$

   It is to be noted that, while drawing the profiles, the continuity of stress and velocity at the liquid-liquid interface and no-slip at the solid-liquid interfaces must be depicted clearly.
4. Discharge = 0.0135 $m^3$/s, Stress = 19.21 N/$m^2$ power 1.73 kW
5. $$V = R_i \ln\frac{R_i}{R_0}\left(\frac{\rho g R_i}{2\mu} - \frac{mg}{2\pi R_i\mu}\right) - \frac{\rho g}{4\mu}\left(R_i^2 - R_0^2\right)$$
6. $$\eta = \frac{6Q}{abRW}\frac{\left(1-\frac{2Q}{abRW}\right)}{\left(4-\frac{6Q}{abRW}\right)} \qquad \frac{Q}{b} = \frac{U\,a}{2} - \frac{1}{12\,\mu}\left(\frac{dP}{dx}\right) a^3 \qquad \Delta P = \frac{6\,\mu\,L\,R\,\omega}{a^2}\left(1 - \frac{2\,Q}{abR\omega}\right)$$

   $$T = \tau\,R\,(bl),\quad Power = T\omega;\quad P = \frac{\mu\,Lb\,(r\omega^2)}{a}\,(\,4 - \frac{6\,Q}{abR\omega})$$
7. $$\tau = \frac{2\pi}{3}R^3\,\frac{\mu\omega}{\theta}$$