

→ Constant Pressure

↓  
flowrate with time  
decreases

DYE → impurities

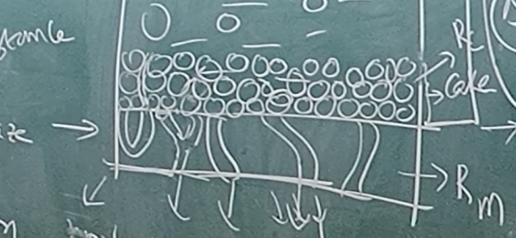
Benzillation ✓

Waste water

↓  
pressure

→ Filtration ✓

Separate solid from liquid  
(rystallization)



X Membrane filtration

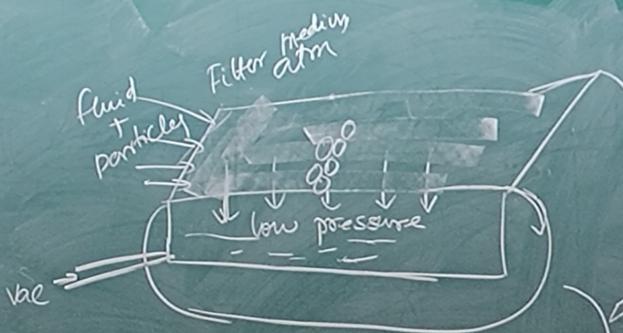
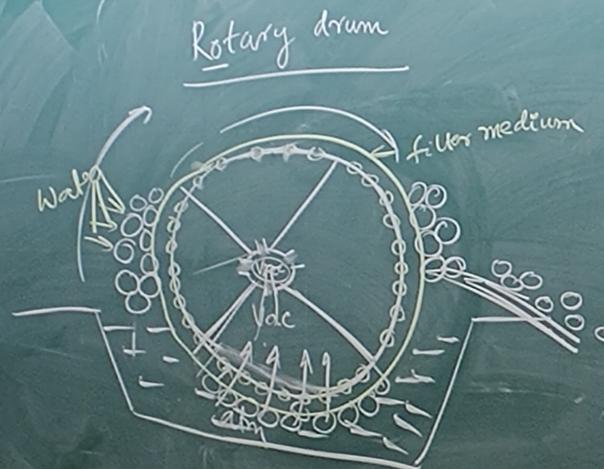
Pore size  
↓

→ Constant rate filtration  
P<sub>f</sub> ↑ flowrate constant

polymeric  
clot

filtrate ↓  
V<sub>f</sub>

↓



External  
P → Plate and frame filter press

Vac { → Rotary drum filter

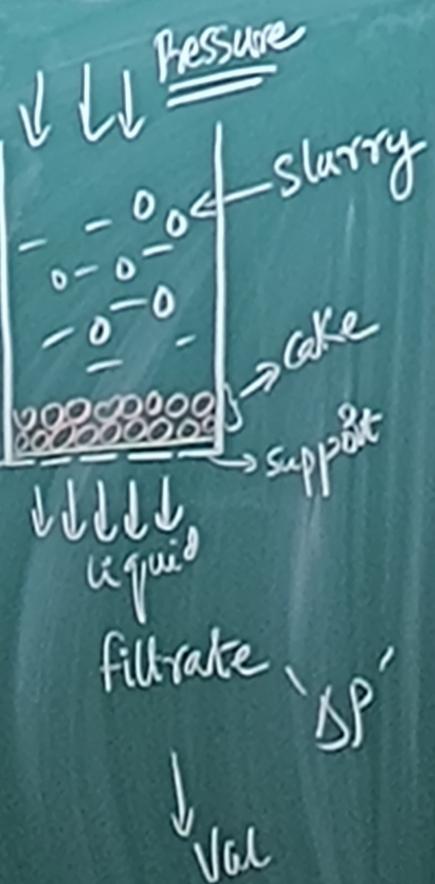
→ Belt filter -  
doctor's blade  
washing

ion equation

Cake filtration

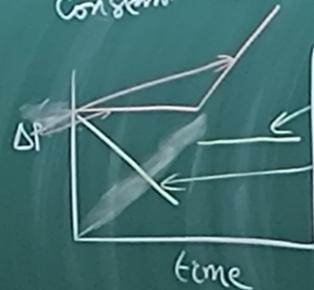
↓  
Concentrated  
Slurry

Porous filter medium  
ex - polymeric  
cloths  
filter paper



## Filtration Equation

Constant DP  
Constant Rate



Flow through cake  
resembles

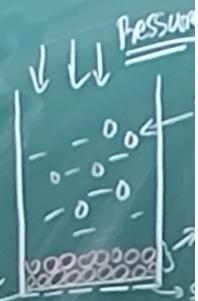
flow through packed

Usually Re in filtration is low

$$\frac{\Delta P}{L} = \frac{150 \mu u (1-\epsilon)}{d_p \epsilon^3}$$

## Cake filtration

Concentrated  
Glue



liquid  
filtrate

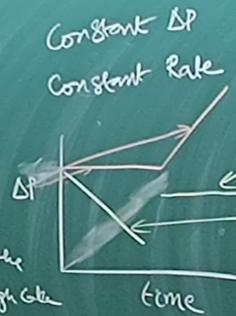
↓ Val

1

$$\frac{\Delta P}{L} = \left[ \frac{150}{36} \right] \mu L \frac{(1-\epsilon)^2}{\epsilon^3} \left( \frac{S_p}{V_p} \right)^2$$

$$d_p = \frac{6}{(S_p/V_p)}$$

$$\frac{\Delta P}{L} = \frac{150 \mu L (1-\epsilon)^2}{3c / (S_p/V_p)^2 \epsilon^3}$$



### Filtration Equation

Flow through cake resembles

flow through packed

usually Re in filtration is low

$$\frac{(\Delta P)}{L} = \frac{150 \mu (2) (1-\epsilon)^2}{d_p^2 \epsilon^3}$$

$\Delta P, L, u \sim$  Varies with time

### Cake filtration

↓ Concentrated slurry

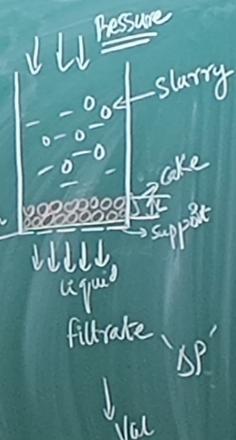
↓ Cake

↓ Porous filter medium  
ex - polymeric clothes  
filter paper

↓ Support liquid

↓ Filtrate  $\Delta P'$

↓ Val



$$\frac{\Delta P}{L} = \sqrt{\frac{150}{36}} \mu L \frac{(1-\epsilon)^2}{\epsilon^3} \left(\frac{s_p}{v_p}\right)^2$$

$$\textcircled{1} - \textcircled{2} \quad \frac{\Delta P}{L} = \frac{\text{constant}}{K} \mu L \frac{(1-\epsilon)^2}{\epsilon^3} \left(\frac{s_p}{v_p}\right)^2$$

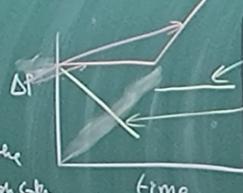
$\rightarrow \textcircled{2}$ ?

$$dp = \frac{6}{(s_p/v_p)}$$

$$\frac{\Delta P}{L} = \frac{150 \mu L (1-\epsilon)^2}{36 \left(\frac{s_p}{v_p}\right)^2 \epsilon^3}$$

$u \rightarrow$  Velocity of the fluid through cake

✓ Constant  $\Delta P$   
Constant Rate



### Filtration equation

Flow through cake  
resembles

flow through packed

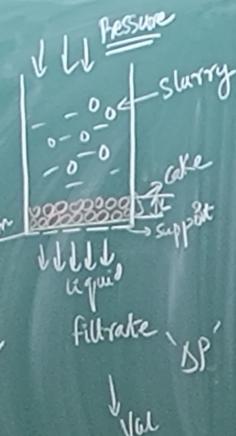
usually  $Re$  in filtration is low

$$\frac{\Delta P}{L} = \frac{150 \mu u (1-\epsilon)^2}{d_p^2 \epsilon^3}$$

$\Delta P, L, u \rightarrow$  varies with time

Cake filtration  
↓  
Concentrated  
Slurry

Boroukh filter  
medium  
Ex - polymeric  
clothes  
filter paper



$\downarrow$   
 $Vol$

$$C_f \rightarrow \frac{\text{mass}}{\text{volume of slurry}}$$

$$\frac{qV \times t}{\text{Volume of slurry}}$$

$$C_f \rightarrow \frac{\text{mass of solids}}{\text{Volume of liquid}}$$

Solid, Liquids       $\text{Volume of the cake} = A L_c$

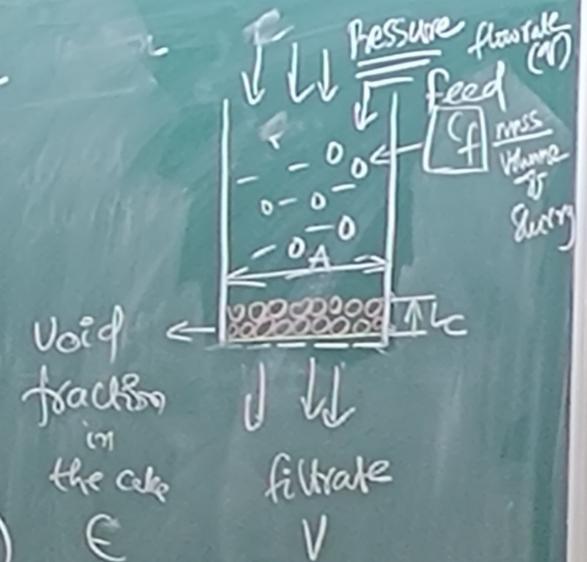
$C_f \rightarrow \frac{\text{mass}}{\text{volume of slurry}}$

$C_f \left( qV \times t \right) = A L_c (1 - \epsilon) P_s$        $\downarrow$  density of cake

$\frac{\text{mass}}{\text{volume of slurry}} \times t$

$C_f \times V_f = A L_c (1 - \epsilon) \epsilon$        $\downarrow$  length of the cake at any time  $t$  in the cake

$V_f$        $\downarrow$  volume of liquid interfaced at any instant



$$C_f \rightarrow \frac{\text{mass}}{\text{volume of slurry}}$$

$$\frac{qV \times t}{\text{Volume of slurry}}$$

$$C_f \rightarrow \frac{\text{Mass of Solids}}{\text{Volume of liquid}}$$

Solids  
in  
the feed  
slurry

Solids deposited  
on  
the filter medium

$$C_f [qV \times t]$$

$$C_f \times V_f$$

Solid, Liquids

Filtrate

$$\text{Volume of the cake} = A L_c$$

Solids + liquid

$$\text{Liquid volume} \text{ in } = A L_c \epsilon$$

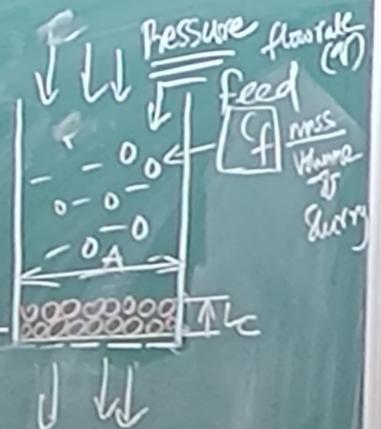
$$A L_c (1-\epsilon) p_s$$

$$\text{Solids Volume} \text{ in } = A L_c (1-\epsilon) \epsilon$$

at any time t

in the cake

$$\text{Void fraction in the cake} = \frac{V}{V_f}$$





Solid, Liquids Filter

$C_f \rightarrow \frac{\text{mass}}{\text{Volume of Slurry}}$

$\frac{C_f \times t}{V} = \frac{\text{Solids deposited in the feed slurry}}{\text{the filter medium}}$

$C_f \left( \frac{V \times t}{V} \right) = A L_c (1 - \epsilon) p_s$

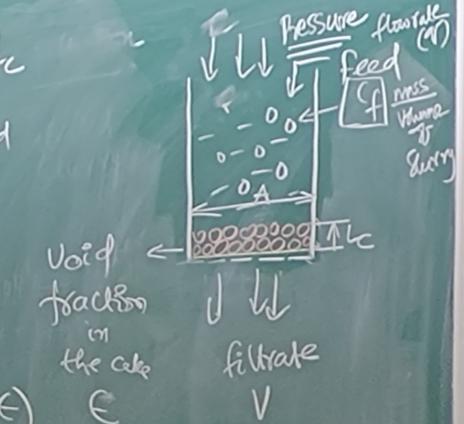
$\frac{\text{mass}}{\text{Vol. of slurry}} = \frac{\text{length of the cake}}{\text{density of cake}}$

$C_f \left( \frac{V}{V} \right) = A L_c (1 - \epsilon) p_s$

$V = \frac{\text{Volume of liquid in the feed at any time } t}{\text{mass of solids}}$

$V = A L_c (1 - \epsilon)$

$L_c \rightarrow \text{length of cake at any given time } t$



$$L_c = \frac{V C_f}{A(1-\epsilon) \rho_s - \epsilon A \rho}$$

$$L_c = \frac{V C_f}{A[(1-\epsilon) \rho_s - \epsilon \rho_f]}$$

$$V_f = A L_c \epsilon + V$$

$$C_f [A L_c \epsilon + V] = A L_c (1-\epsilon) \rho_s$$

$$[\epsilon A \rho] L_c + V C_f = [A (1-\epsilon) \rho_s] L_c$$

### Liquid Balance

liquid in the feed = liquid in the cake + filtrate  
 Slurry

volume of filtrate at any time  $t$

$$\frac{qV \times t}{\text{Volume of Slurry}}$$

$C_f \rightarrow$  mass  
Volume of Slurry

$$C_f \left( \frac{qV \times t}{V} \right) = A L_c (1-\epsilon) \rho_s$$

$\downarrow$   
mass/volume  
 $\frac{qV \times t}{V}$

length of the cake

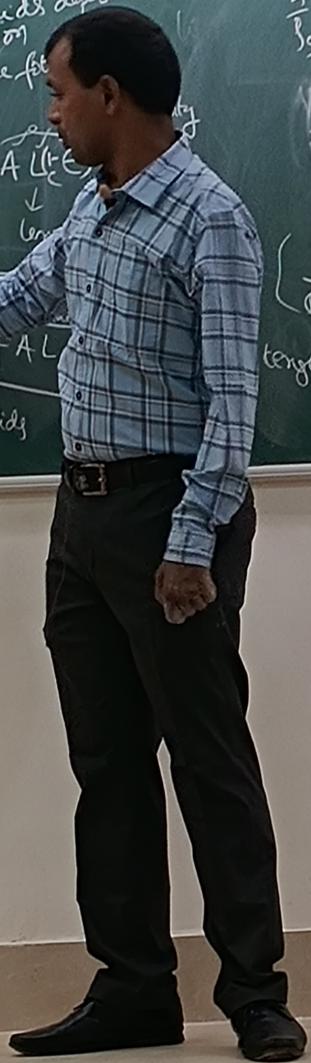
$$C_f \rightarrow$$
 mass of solids  

$$\frac{C_f \times V}{\text{Volume of liquid}} = A L_c (1-\epsilon) \rho_s$$

$\downarrow$   
mass of solids

Solid, liquid  
 Solids deposited  
 in the filter medium  
 Solid

Solids, Liquids  
 $c_f = \frac{\text{mass of Solids}}{\text{Volume of Slurry}}$   
 Solids in the feed = Solids deposited in the filter  
 $c_f = \frac{\text{Volume of Solids}}{\text{Volume of Slurry}}$   
 $\beta_s = \frac{V_f + V_s}{V_f}$   
 $\beta_s = \frac{V_s}{V_f} + 1$   
 $\left( \frac{V_s}{c_f} - 1 \right) = \frac{V_s}{V_f} \Rightarrow \frac{V_s}{V_f} = \frac{c_f}{\beta_s - c_f}$   
 tangent of slope at any given time  $t$



$$\text{C}_f = \frac{V_f}{A(1-\epsilon)P_s - C_f}$$

$$\text{C}_f = \frac{V_f}{A((1-\epsilon)P_s - C_f)}$$

$$L_c = \frac{V_f}{A}$$

$$\frac{dt}{dV} = \frac{\alpha \nu M_f}{A^2} \times \frac{V}{\Delta t}$$

$$\frac{\Delta P}{L_c} = K \alpha M_f \frac{(1-\epsilon)^{\gamma} \left(\frac{V_f}{V_p}\right)^{\gamma}}{\epsilon^3}$$

$$\Delta P = \left[ K \frac{(1-\epsilon)^{\gamma} \left(\frac{V_f}{V_p}\right)^{\gamma}}{\epsilon^3} \right] \mu \nu \frac{V_f}{A}$$

$$\alpha = \frac{\alpha \nu M_f V}{A^2} \left( \frac{dV}{dt} \right)$$

Volume of fluid  $\frac{dV}{dt}$   
duration time  $\frac{dt}{dV}$

$$C_f = \frac{\text{mass of solids}/\beta_s}{\text{Volume of slurry}}$$

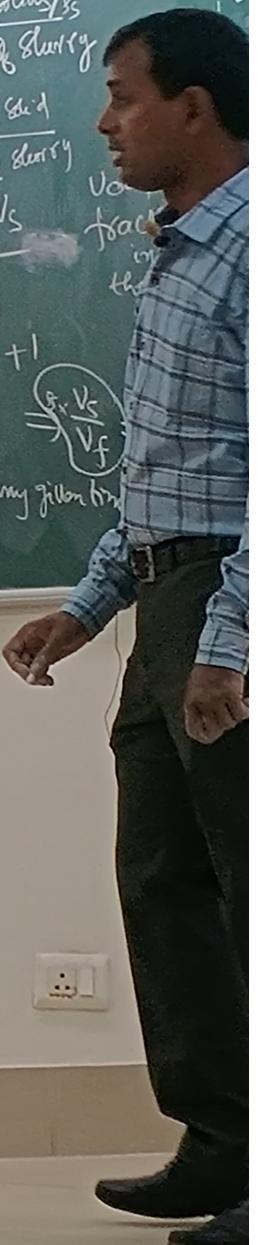
$$\frac{C_f}{\beta_s} = \frac{\text{Volume of solid}}{\text{Volume of slurry}}$$

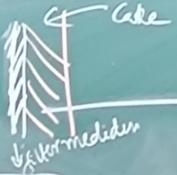
$$\frac{V_s}{C_f} = \frac{V_f + V_s}{V_s}$$

$$= \frac{V_f}{V_s} + 1$$

$$\left( \frac{P_s}{C_f} - 1 \right) = \frac{V_s}{V_f} = \frac{V_s}{V_f} = \frac{V_s}{V_f}$$

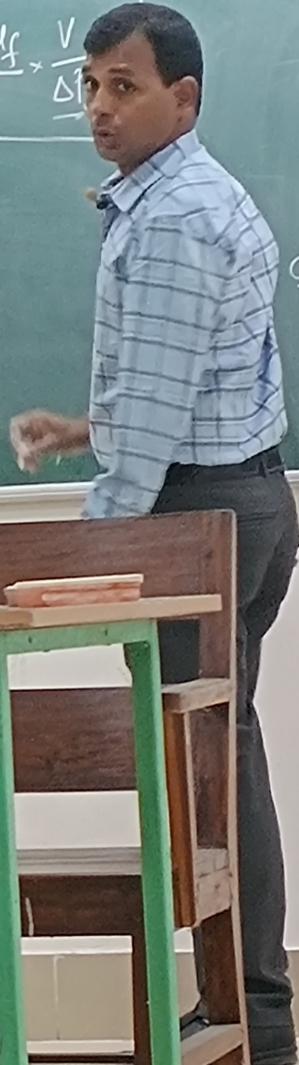
length of slake at any given time



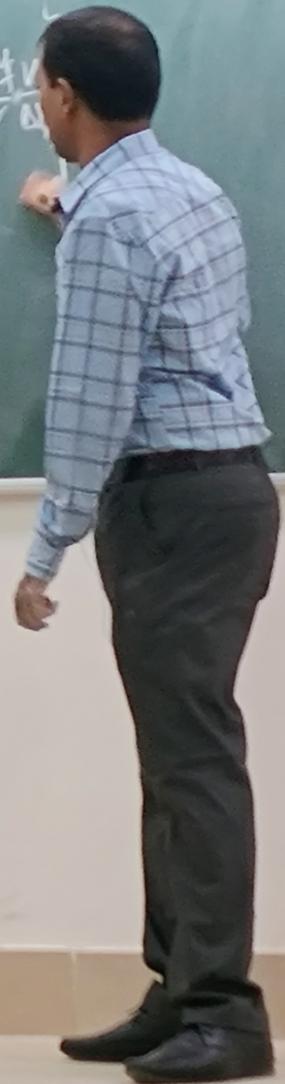


$$\textcircled{1} \quad L_c = \frac{V C_f}{A(1-\epsilon) \rho_s - g A \epsilon}$$
$$L_c = \frac{V C_f}{A \left( (1-\epsilon) \rho_s - \frac{g \epsilon}{\Delta t} \right)}$$
$$L_c = \frac{V (V)}{A}$$
$$\underline{\underline{V = \frac{L_c A}{V} \rightarrow \frac{\text{volume of cake}}{\text{Volume of filtrate}}}}$$

$$\frac{dt}{dV} = \frac{\alpha V M_f}{A^2} \times \frac{V}{\Delta t}$$



$$\frac{dV}{dt} / \frac{A}{V}$$



$$L_C = \frac{V C_f}{A(1-\epsilon) P_s - C_f A \epsilon}$$

$$L_C = \frac{V C_f}{A \left( \frac{(1-\epsilon) P_s - C_f \epsilon}{1 + \epsilon} \right)}$$

$$L_C = \frac{V C_f}{A}$$

$$V = \frac{L_A}{V} \xrightarrow{\text{Volume of cake}} \frac{\text{Volume of filtrate}}{\text{Volume of filtrate}}$$

$$\frac{dt}{dv} = \frac{\alpha \rho_f M_f}{A^2} \times \frac{V}{\Delta P_c} \Rightarrow \frac{dt}{dv} = \frac{\Delta P_m}{A^2} \times \frac{V}{\Delta P_f}$$

$$\Delta P = P_a - P_b$$

$$= \underbrace{P_a - P_f}_{\Delta P_c} + \underbrace{P_f - P_b}_{\Delta P_m}$$

$$\Delta P_c = \frac{\alpha \rho_f M_f}{A^2} V \frac{dv}{dt}$$

$$\Delta P_m = R_m$$

$C_f = \frac{\text{Volume of filtrate}}{\text{Volume of slurry}}$

$\beta_s = \frac{\text{Volume & solid}}{\text{Volume & slurry}}$

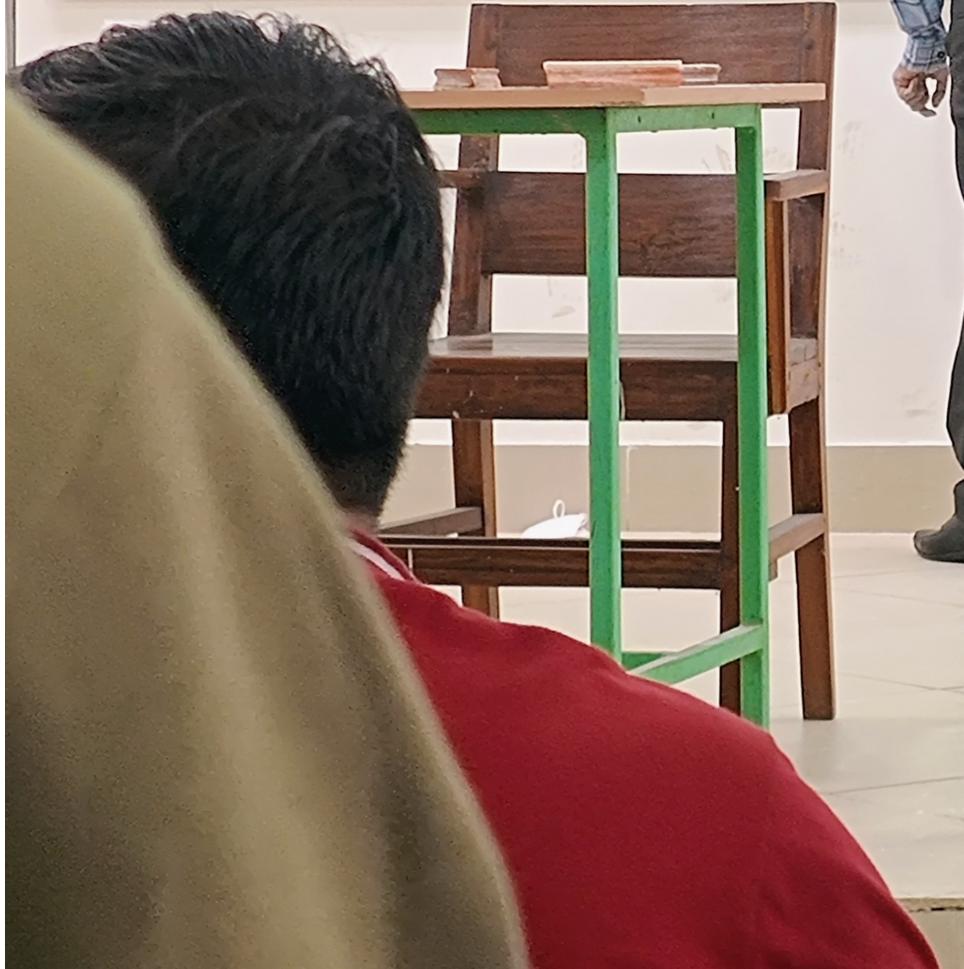
Void fraction in the cake  $\epsilon$

$$\frac{V \beta_s}{C_f} = \frac{V_f + V_s}{V_s}$$

$$= \frac{V_f}{V_s} + 1$$

$$\left[ \frac{V_s}{C_f} - 1 \right] = \frac{V_f}{V_s} \Rightarrow \frac{V_s}{V_f} = \frac{C_f}{\beta_s - C_f} \times \beta_s$$

length of cake at any given time  $t$



$\Delta P_c = \frac{\alpha \nu M_f}{\pi^2} V \frac{dV}{dt}$   
 $\Delta P_m = \frac{R_m}{A^2} M_f V \frac{dV}{dt}$   
 $\Delta P = \Delta P_c + \Delta P_m$   
 $= \alpha \nu + R_m$

$\frac{V_f}{V_s} = \frac{\text{Volume of Solid}}{\text{Volume of Slurry}}$   
 $\frac{V_f}{V_s} = \frac{V_f + V_s}{V_s}$   
 $\frac{V_f}{V_s} = \frac{V_f}{V_s} + 1$   
 $\left( \frac{V_f}{V_s} - 1 \right) = \frac{V_f}{V_s} \Rightarrow \frac{V_f}{V_s} = \frac{V_s - V_f}{V_s}$   
 Length of cake at any given time  $t$

Volume of Slurry  
 Volume & Solid  
 Volume & Slurry  
 Void fraction in the cake  
 C  
 filtrate V



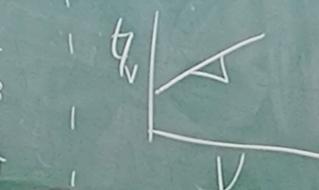
$$\int_0^x dt \frac{d(\Delta P)}{\Delta P} = \frac{M_f}{R}$$

$$\Delta P$$

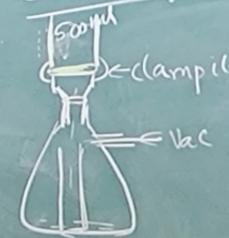
$$\Delta P \cdot t = \left[ \frac{\alpha \vartheta M_f V}{A^2} \frac{V}{2} + \frac{R_m M_f}{A} V \right]$$

$$\left( \frac{t}{V} \right) = \left( \frac{\alpha \vartheta M_f}{2 A^2 \Delta P} \right) V + \left( \frac{R_m M_f}{A \Delta P} \right)$$

$$t \quad V$$



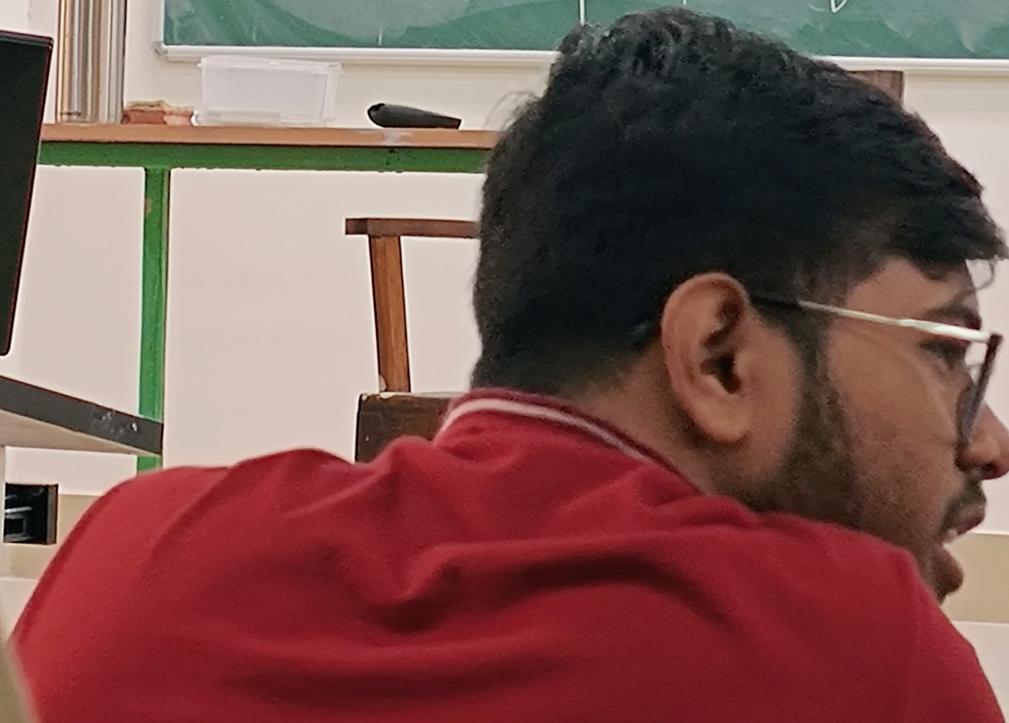
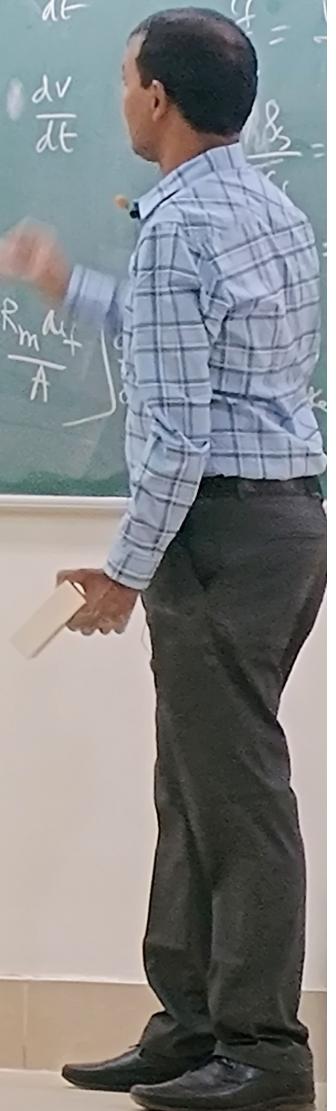
Laboratory filtration



$$\Delta P_c = \frac{\alpha \vartheta M_f}{A^2} V \frac{dV}{dt}$$

$$\Delta P_m = \frac{R_m M_f}{A} V \frac{dV}{dt}$$

$$\begin{aligned} \Delta P &= \Delta P_c + \Delta P_m \\ &= \left[ \frac{\alpha \vartheta M_f}{A^2} V + \frac{R_m M_f}{A} V \right] \end{aligned}$$



$$\frac{d\Delta P}{dt} = \frac{\alpha \vartheta M_f V}{A^2}$$

Constant Rate filtration

$$\frac{dV}{dt} = \frac{V}{t}$$

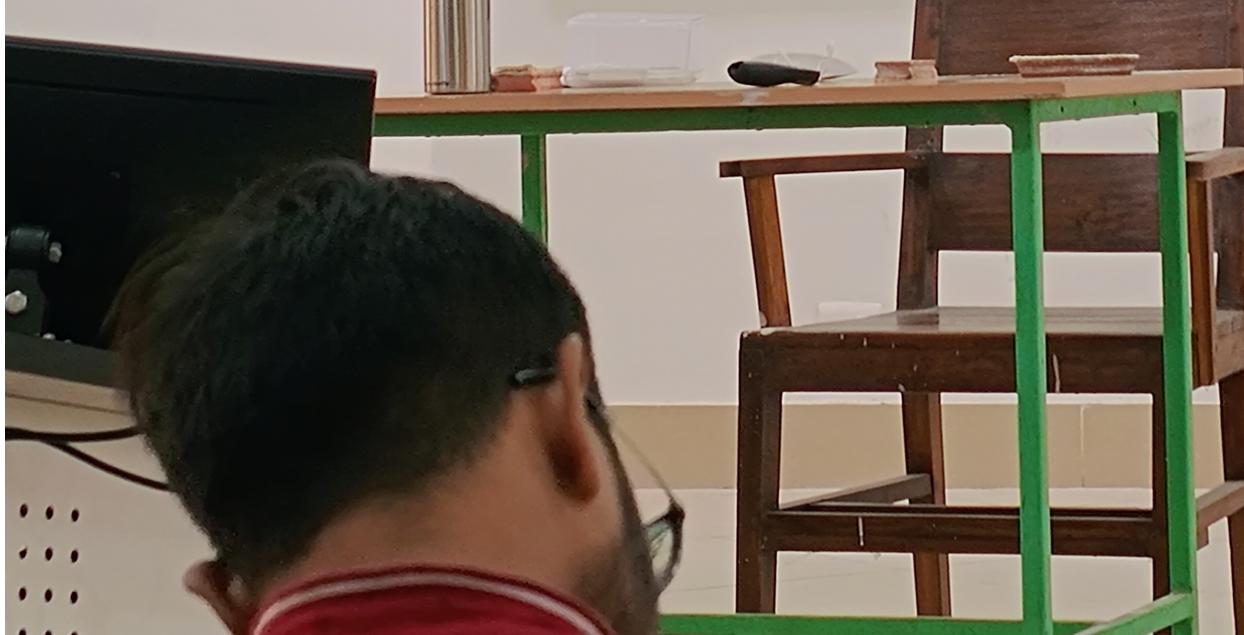
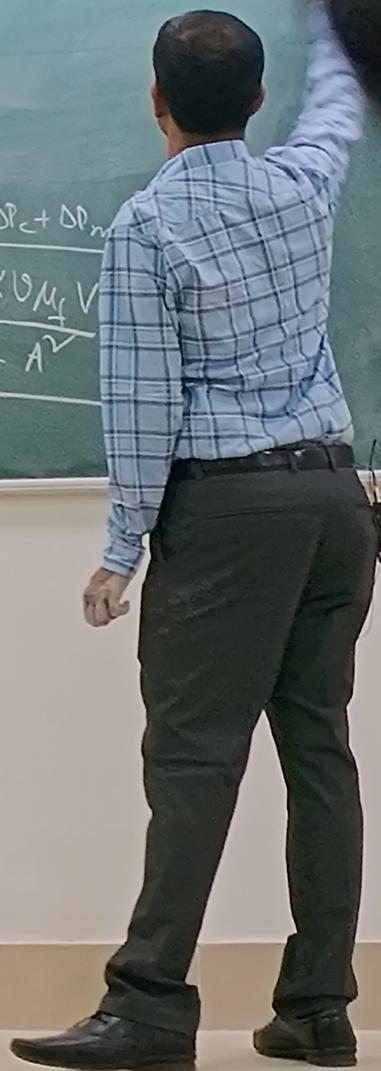
$$\Delta P \cdot t = \left[ \frac{\alpha \vartheta M_f}{A^2} \frac{V}{2} + R_m M_f V \right] t$$

$$\frac{t}{V} = \left( \frac{\alpha \vartheta M_f}{2 A^2 \Delta P} \right) V + \frac{R_m M_f}{A \Delta P}$$

$$\Delta P = \left( \frac{\alpha \vartheta M_f}{A^2} V + R_m M_f \right) \frac{V}{t}$$

$$\Delta P = \Delta P_c + \Delta P_n$$

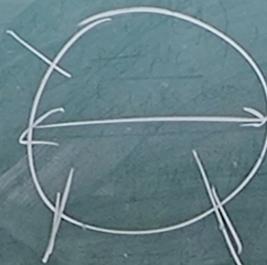
$$\Delta P = \left[ \frac{\alpha \vartheta M_f}{A^2} V \right]$$



Constant Rate filtration

$$\frac{dV}{dt} = \frac{V}{t}$$

✓  $(\Delta P) = \left( \frac{\alpha V \mu_f V}{A^2} + \frac{R_m \mu_f}{A} \right) t$



$$A = \pi \frac{D^2}{4}$$

$$f \times \frac{1}{t} = A$$

$$\frac{t}{1 \times f}$$

$$\Rightarrow -\frac{V}{t} + -\frac{V}{t} = 0$$

Constant  $P$  filtration

$$\frac{V}{t} = \frac{\alpha V \mu_f}{2(\Delta P)} V + \frac{R_m \mu_f}{A \Delta P}$$

filtration area

Design  $\frac{V}{t}$

$$\Delta P = \Delta P_c + \Delta P_m$$
$$\Delta P = \left[ \frac{\alpha V \mu_f V}{A^2} + \frac{R_m \mu_f}{A} \right] \frac{dV}{dt}$$