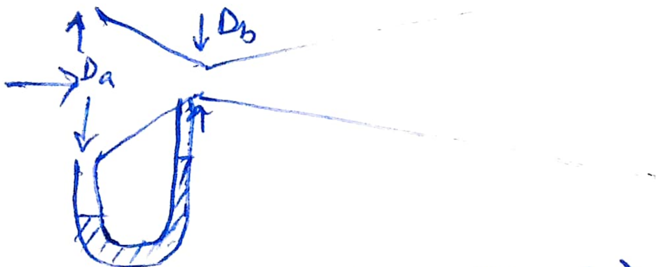
FLOW METERING

Venturi meter, orifice meter
Pitot Tube
Rotameter
Turbine meters, Thermal meter



$$\alpha_b \bar{V}_b^2 - \alpha_a \bar{V}_a^2 = \frac{2(P_a-P_b)}{\rho}$$

$$\bar{V}_a = \left(\frac{D_b}{D_a}\right)^2 \bar{V}_b$$

$$\Rightarrow \bar{V}_b = \frac{1}{\sqrt{\alpha_b - \left(\frac{D_b}{D_a}\right)^4 \alpha_a}} \sqrt{\frac{2(P_a-P_b)}{\rho}}$$

$$= \frac{C_v}{\sqrt{1-\beta^4}} \sqrt{\frac{2(P_a-P_b)}{\rho}}$$

Kinetic energy correction factor

For an element of cross-sectional area ds, the mass flow rate

$= \rho u \, ds$.

where each mass unit carries energy $\frac{u^2}{2}$

$\Rightarrow$ Total energy through ds per unit time

$$d\dot{E}_K = (\rho u \, ds) \frac{u^2}{2}$$

For entire cross-section

$$\dot{E}_K = \frac{\rho}{2} \int_s u^3 \, ds$$

$\Rightarrow$ Kinetic energy per unit mass ~~for con~~ to include in Bernoulli's eqn.

$$\frac{\dot{E}_K}{\dot{m}} = \frac{\frac{1}{2} \int_s u^3 \, ds}{\int_s u \, ds} = \frac{\frac{1}{2} \int_s u^3 \, ds}{\bar{V} S}$$

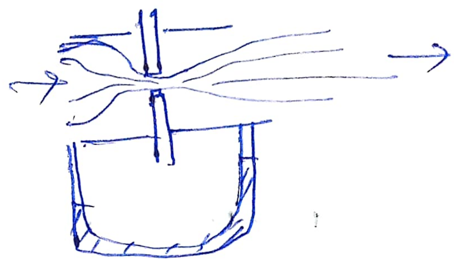
$\alpha \frac{\bar{V}^2}{2}$

Flow Metering ..... contd.

For an orificemeter (measures flowrate)

'o' refers to parameters at orifice

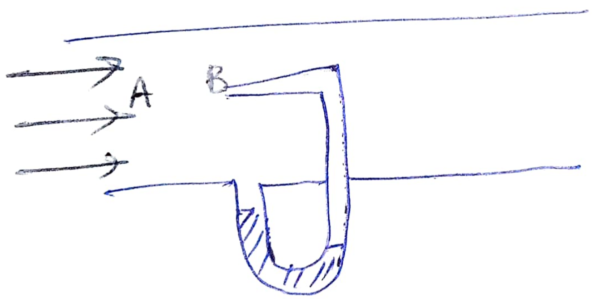
$$u_0 = \frac{C_0}{\sqrt{1-\beta^4}} \sqrt{\frac{2(P_a - P_b)}{\rho}}$$



$C_0 = 0.61$ for $Re_0 = \frac{D_0 u_0 \rho}{\mu} > 30{,}000$

- Vena Contracta
- Significant energy loss at orifice
- Availability of straight pipe required at upstream and downstream.

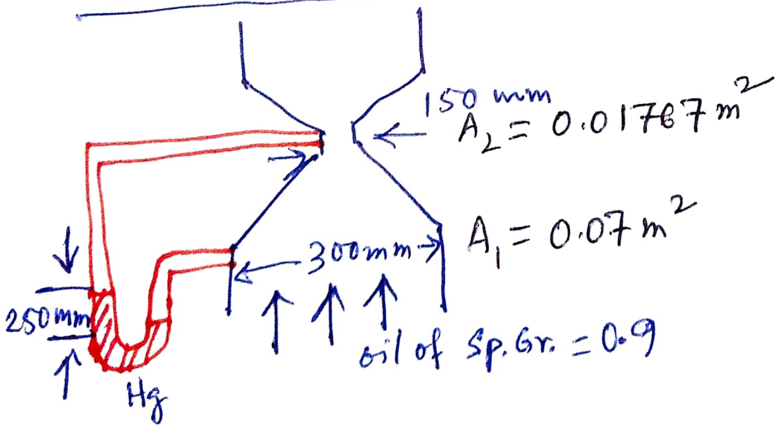
Pitot tube (measures local velocity)



Static tube measures P_0 (static pressure)

Streamline AB terminates at B (the stagnation point); $\frac{u_0^2}{2}$ gets converted to $\frac{P_s - P_0}{\rho}$ $\Rightarrow u_0 = \sqrt{\frac{2(P_s - P_0)}{\rho}}$

Venturimeter Problem



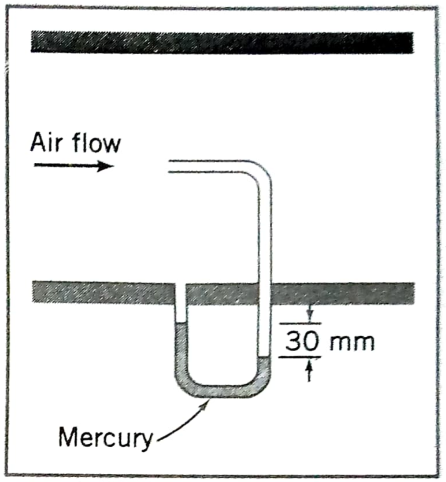
- Calculate oil flowrate ($\approx 0.1489\ m^3/s$)
- Plot the change in pressure, as fluid travels through venturimete
- Calculate $P_1 - P_2$ (≈ 33.8 kPa)

EXAMPLE PROBLEM 6.2

GIVEN: A pitot tube inserted in a flow as shown. The flowing fluid is air and the manometer liquid is mercury.

FIND: The flow speed.

SOLUTION:



Basic equation: $$\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

Assumptions:
(1) Steady flow
(2) Incompressible flow
(3) Flow along a streamline
(4) Frictionless deceleration along stagnation streamline

Writing Bernoulli's equation along the stagnation streamline (with $\Delta z = 0$) yields

$$\frac{p_0}{\rho} = \frac{p}{\rho} + \frac{V^2}{2}$$

p_0 is the stagnation pressure at the tube opening where the speed has been reduced, without friction, to zero. Solving for V gives

$$V = \sqrt{\frac{2(p_0 - p)}{\rho_{\text{air}}}}$$

From the diagram,

$$p_0 - p = \rho_{\text{Hg}} g h = \rho_{\text{H}_2\text{O}} g h(\text{SG}_{\text{Hg}})$$

and

$$V = \sqrt{\frac{2\rho_{\text{H}_2\text{O}} g h(\text{SG}_{\text{Hg}})}{\rho_{\text{air}}}}$$

$$= \sqrt{2 \times \frac{1000 \text{ kg}}{\text{m}^3} \times \frac{9.81 \text{ m}}{\text{s}^2} \times 30 \text{ mm} \times \frac{\text{m}}{1000 \text{ mm}} \times 13.6 \times \frac{\text{m}^3}{1.23 \text{ kg}}}$$

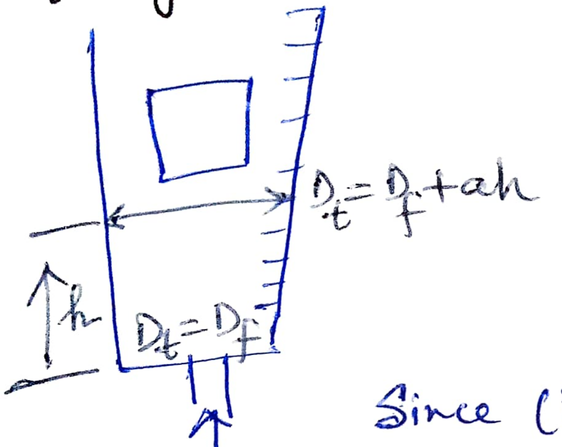
$V = 80.8$ m/s $\qquad V$

At $T = 20°\text{C}$, the speed of sound in air is 343 m/s. Hence, $M = 0.236$ and the assumption of incompressible flow is valid.

{This problem illustrates the use of a pitot tube to determine the flow speed at a point.}

Definition of DRAG

Theory of Rotameter



Balance of three forces

(i) the weight of the float $= (\text{vol. of float})(\text{density})g$

(ii) the buoyancy force $= (\text{vol. of float})(\text{fluid density})g$

(iii) the drag force on the float $= A_f C_D \rho \frac{u^2}{2}$

$$(i) - (ii) = (iii)$$

Since (i) and (ii) do not depend on flow rate or velocity, the drag force for equilibration of three forces is constant, even when the flowrate changes. i.e., u must remain constant

$$\text{Flow rate} = u\frac{\pi}{4}(D_t^2 - D_f^2) = u\frac{\pi}{4}\left[2D_f ah + a^2h^2\right] \simeq u\frac{\pi}{2}D_f ah$$

where D_f is the diameter of the float

and D_t is the inner diameter of linearly tapered tube $= D_f + ah$

$\Rightarrow$ Rotameter has linear relationship between the flowrate and reading. In case of venturimeter/orificemeter, flow rate $\propto \sqrt{\text{reading}}$.

Straight pipe section at inlet and outlet is not required.