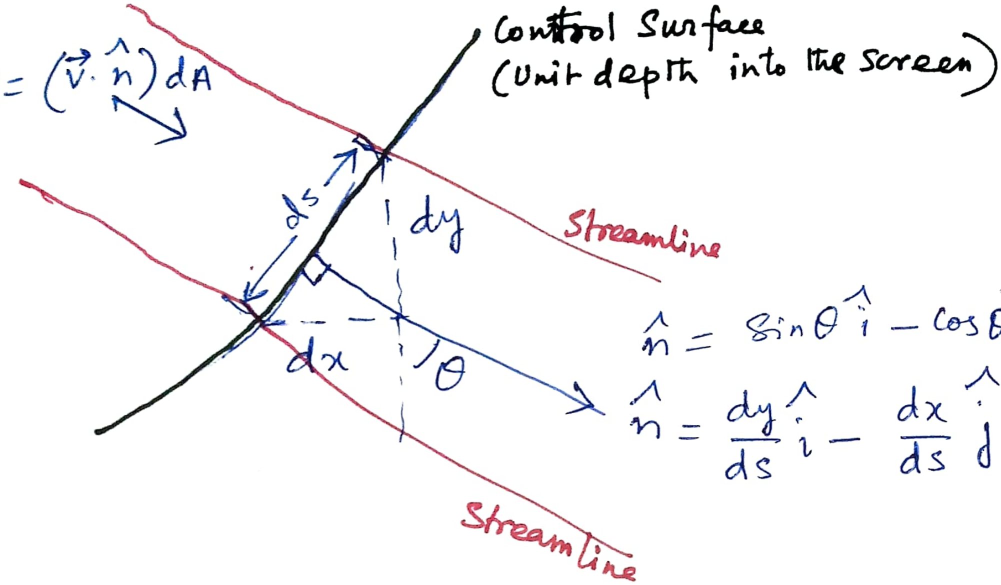
$dQ = (\vec{V}\cdot\hat{n})\,dA$



$\hat{n} = \sin\theta\,\hat{i} - \cos\theta\,\hat{j}$

$\hat{n} = \frac{dy}{ds}\hat{i} - \frac{dx}{ds}\hat{j}$

Volume flow dQ through an element ds of control surface of unit depth:

$$dQ = (\vec{V}\cdot\hat{n})\,dA = \left[\left(\hat{i}\frac{\partial\psi}{\partial y} - \hat{j}\frac{\partial\psi}{\partial x}\right)\cdot\left(\hat{i}\frac{dy}{ds} - \hat{j}\frac{dx}{ds}\right)\right]\left[ds\,(\text{Unit depth})\right]$$

$$= \frac{\partial\psi}{\partial x}dx + \frac{\partial\psi}{\partial y}dy = d\psi$$

$\Rightarrow$ The volume flow between any two streamlines in the flow field is equal to change in stream function between those streamlines.

$$Q_{1\to2} = \int_1^2 (\vec{V}\cdot\hat{n})\,dA = \int_1^2 d\psi = \psi_2 - \psi_1$$

For compressible flow, at steady state ~~at an instant of time~~ $\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial x}(\rho v) = 0$

Stream function is defined as

$\rho u = \frac{\partial\psi}{\partial y}$; $\rho v = -\frac{\partial\psi}{\partial x}$ $\Rightarrow$ $d\dot{m} = \rho(\vec{V}\cdot\hat{n})\,dA = d\psi$

$$\dot{m}_{1\to2} = \int_1^2 \rho\,(\vec{V}\cdot\hat{n})\,dA = \psi_2 - \psi_1$$