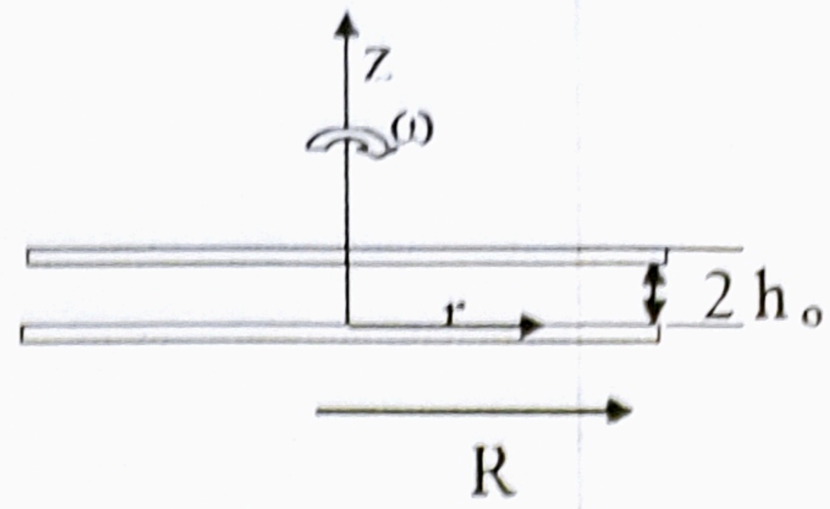
A parallel plate viscometer consists of a stationary, circular plate of radius R, over which another identical plate rotates as shown in the figure. A liquid is placed in the gap and the torque on the lower plate is measured. The gap between the two plates is equal to $2h_o$. It is safe to assume that no liquid is lost through the small gap.



(a) Simplify the Navier Stokes equation, clearly stating all assumptions with proper justifications. Show that

$$V_\theta = \frac{\omega r Z}{2 h_o}$$

can be a solution for this situation.

(b) Using this expression for V_θ obtain an expression for viscosity of the liquid in terms of the measured torque on the lower plate.

3+7=10

Here $v_r = v_z = 0$, $\frac{\partial}{\partial t} = 0$ (SS), $\frac{\partial}{\partial \theta} = 0$ (symmetry)

NS eq^n $\quad r$ comp. $\Rightarrow \frac{\partial p}{\partial r} = \frac{\rho v_\theta^2}{r}$, $\quad z$ comp.: $\Rightarrow \frac{\partial p}{\partial z} = 0$.

θ comp.

$$\rho\left(\cancel{\frac{\partial v_\theta}{\partial t}} + \cancel{v_r \frac{\partial v_\theta}{\partial r}} + \cancel{\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}} + \cancel{v_z \frac{\partial v_\theta}{\partial z}} + \cancel{\frac{v_r v_\theta}{r}}\right) = -\cancel{\frac{1}{r}\frac{\partial p}{\partial \theta}}$$

$$+ \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \cancel{\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}} + \frac{\partial^2 v_\theta}{\partial z^2} + \cancel{\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}}\right] + \cancel{\rho g_\theta}$$

2

$$\Rightarrow \frac{\partial^2 v_\theta}{\partial z^2} = -\frac{\partial}{\partial r}\left[r \left\{\frac{\partial}{\partial r}(r v_\theta)\right\}\right].$$

1

if $v_\theta = \frac{\omega r z}{2h_o}$, $\quad$ RHS = 0 = LHS

Thus it satisfies NS eq^n and is a sol^n for this situation.

b) $\tau = \mu \frac{\partial v_\theta}{\partial z} = \mu \frac{\omega r}{2h_o}$.

3

Shear force $= \frac{\mu \omega r}{2h_o} 2\pi r\, dr$

Torque $= \int_0^R \frac{\mu \omega r}{2h_o} 2\pi r\, dr \cdot r$.

3

$$T = \frac{\mu \omega \pi R^4}{4h_o}$$

$$\mu = \frac{4h_o T}{\omega \pi R^4}$$

1