10

Storage and Flow of Powders – Hopper Design

10.1 INTRODUCTION

The short-term storage of raw materials, intermediates and products in the form of particulate solids in process plants presents problems which are often underestimated and which, as was pointed out in the introduction of this text, may frequently be responsible for production stoppages.

One common problem in such plants is the interruption of flow from the discharge orifice in the hopper, or converging section beneath a storage vessel for powders. However, a technology is available which will allow us to design such storage vessels to ensure flow of the powders when desired. Within the bounds of a single chapter it is not possible to cover all aspects of the gravity flow of unaerated powders, and so here we will confine ourselves to a study of the design philosophy to ensure flow from conical hoppers when required. The approach used is that first proposed by Jenike (1964).

10.2 MASS FLOW AND CORE FLOW

Mass flow. In perfect mass flow, all the powder in a silo is in motion whenever any of it is drawn from the outlet as shown in Figure 10.1(b). The flowing channel coincides with the walls of the silo. Mass flow hoppers are smooth and steep. Figure 10.2(a–d) shows sketches taken from a sequence of photographs of a hopper operating in mass flow. The use of alternate layers of coloured powder in this sequence clearly shows the key features of the flow pattern. Note how the powder surface remains level until it reaches the sloping section.

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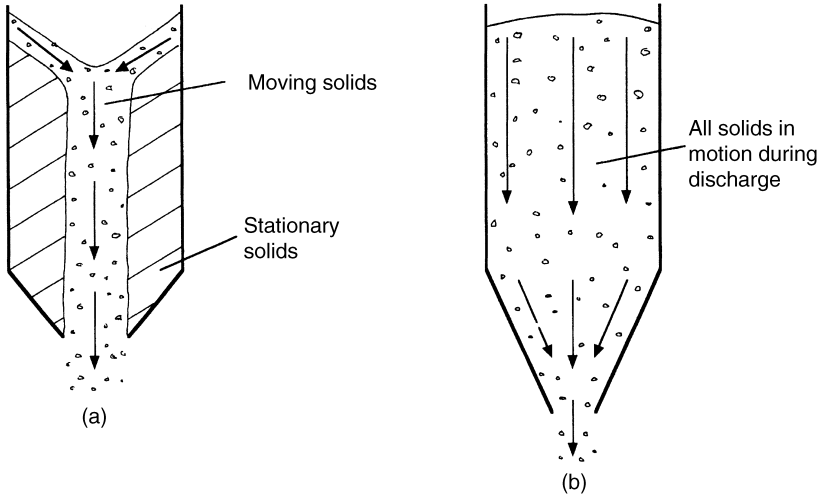


Figure 10.1 Mass flow and core flow in hoppers: (a) core flow; (b) mass flow

Core flow. This occurs when the powder flows towards the outlet of a silo in a channel formed within the powder itself [Figure 10.1(a)]. We will not concern ourselves with core flow silo design. Figure 10.3 (a–d) shows sketches taken from a sequence of photographs of a hopper operating in core flow. Note the regions of

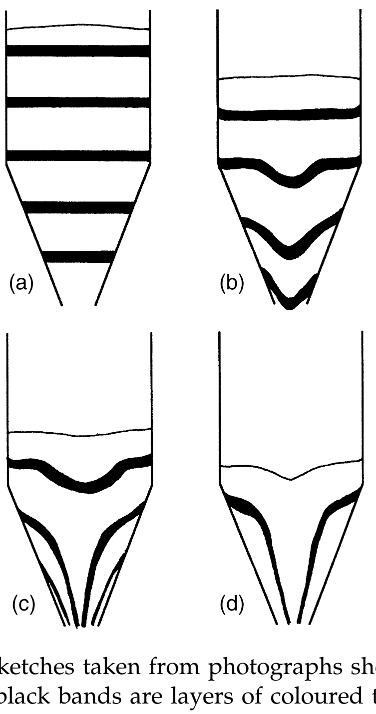


Figure 10.2 Sequence of sketches taken from photographs showing a mass flow pattern as a hopper empties. (The black bands are layers of coloured tracer particles)

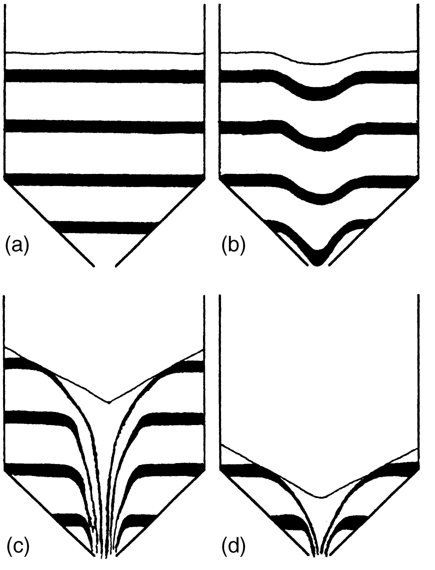


Figure 10.3 Sequence of sketches taken from photographs showing a core flow pattern as a hopper empties. (The black bands are layers of coloured tracer particles)

powder lower down in the hopper are stagnant until the hopper is almost empty. The inclined surface of the powder gives rise to size segregation (see Chapter 11).

Mass flow has many advantages over core flow. In mass flow, the motion of the powder is uniform and steady state can be closely approximated. The bulk density of the discharged powder is constant and practically independent of the height in the silo. In mass flow stresses are generally low throughout the mass of solids, giving low compaction of the powder. There are no stagnant regions in the mass flow hopper. Thus the risk of product degradation is small compared with the case of the core flow hopper. The first-in–first-out flow pattern of the mass flow hopper ensures a narrow range of residence times for solids in the silo. Also, segregation of particles according to size is far less of a problem in mass flow than in core flow. Mass flow has one disadvantage which may be overriding in certain cases. Friction between the moving solids and the silo and hopper walls result in erosion of the wall, which gives rise to contamination of the solids by the material of the hopper wall. If either contamination of the solids or serious erosion of the wall material are unacceptable, then a core flow hopper should be considered.

For conical hoppers the slope angle required to ensure mass flow depends on the powder/powder friction and the powder/wall friction. Later we will see how these are quantified and how it is possible to determine the conditions which give rise to mass flow. Note that there is no such thing as a mass flow hopper; a hopper which gives mass flow with one powder may give core flow with another.

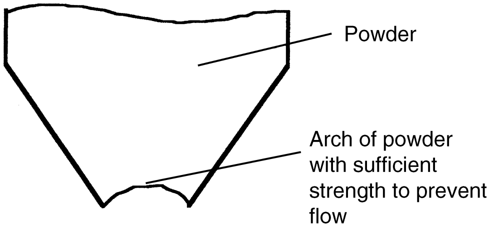


Figure 10.4 Arching in the flow of powder from a hopper

10.3 THE DESIGN PHILOSOPHY

We will consider the blockage or obstruction to flow called arching and assume that if this does not occur then flow will take place (Figure 10.4). Now, in general, powders develop strength under the action of compacting stresses. The greater the compacting stress, the greater the strength developed (Figure 10.5). (Free-flowing solids such as dry coarse sand do not develop strength as the result of compacting stresses and will always flow.)

10.3.1 Flow–No Flow Criterion

Gravity flow of a solid in a channel will take place provided the strength developed by the solids under the action of consolidating pressures is insufficient to support an obstruction to flow. An arch occurs when the strength developed by the solids is greater than the stresses acting within the surface of the arch.

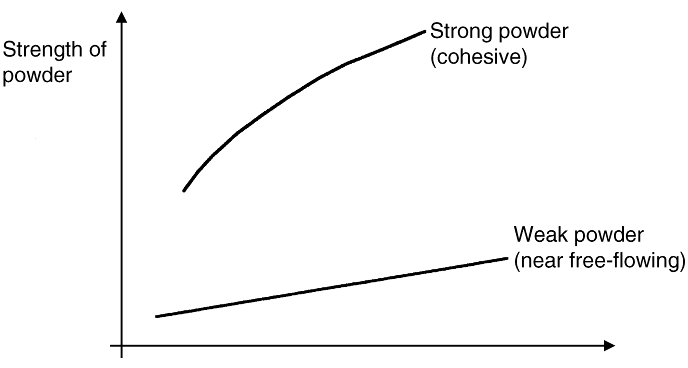


Figure 10.5 Variation of strength of powder with compacting stress for cohesive and free-flowing powders

10.3.2 The Hopper Flow Factor, *ff*

The hopper flow factor, *ff*, relates the stress developed in a particulate solid with the compacting stress acting in a particular hopper. The hopper flow factor is defined as:

$$ff = \frac{\sigma_C}{\sigma_D} = \frac{\text{compacting stress in the hopper}}{\text{stress developed in the powder}} \tag{10.1}$$

A high value of *ff* means low flowability since high σ_C means greater compaction, and a low value of σ_D means more chance of an arch forming.

The hopper flow factor depends on:

- the nature of the solid;
- the nature of the wall material;
- the slope of the hopper wall.

These relationships will be quantified later.

10.3.3 Unconfined Yield Stress, σ_y

We are interested in the strength developed by the powder in the arch surface. Suppose that the yield stress (i.e. the stress which causes flow) of the powder in the exposed surface of the arch is σ_y. The stress σ_y is known as the unconfined yield stress of the powder. Then if the stresses developed in the powder forming the arch are greater than the unconfined yield stress of the powder in the arch, flow will occur. That is, for flow:

$$\sigma_D > \sigma_y \tag{10.2}$$

Incorporating Equation (10.1), this criterion may be rewritten as:

$$\frac{\sigma_C}{ff} > \sigma_y \tag{10.3}$$

10.3.4 Powder Flow Function

Obviously, the unconfined yield stress, σ_y, of the solids varies with compacting stress, σ_C:

$$\sigma_y = \text{fn}(\sigma_C)$$

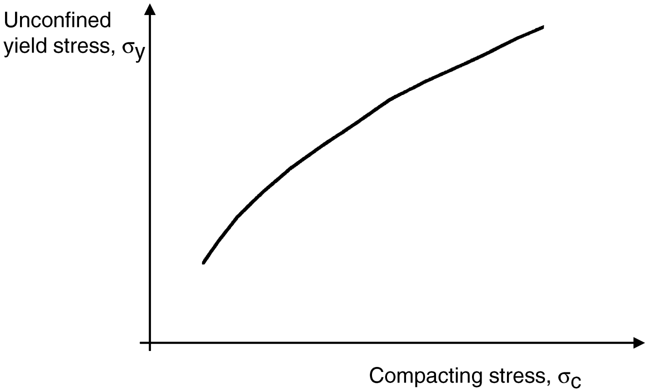


Figure 10.6 Powder flow function (a property of the solids only)

This relationship is determined experimentally and is usually presented graphically (Figure 10.6). This relationship has several different names, some of which are misleading. Here we will call it the *powder flow function*. Note that it is a function *only* of the powder properties.

10.3.5 Critical Conditions for Flow

From Equation (10.3), the limiting condition for flow is:

$$\frac{\sigma_C}{ff} = \sigma_y$$

This may be plotted on the same axes as the powder flow function (unconfined yield stress, σ_y and compacting stress, σ_C) in order to reveal the conditions under which flow will occur for this powder in the hopper. The limiting condition gives a straight line of slope $1/ff$. Figure 10.7 shows such a plot.

Where the powder has a yield stress greater than σ_C/ff, no flow occurs [powder flow function (a)]. Where the powder has a yield stress less than σ_C/ff flow occurs [powder flow function (c)]. For powder flow function (b) there is a critical condition, where unconfined yield stress, σ_y, is equal to stress developed in the powder, σ_C/ff. This gives rise to a critical value of stress, σ_{crit}, which is the critical stress developed in the surface of the arch:

$$\text{If actual stress developed} < \sigma_{crit} \Rightarrow \text{no flow}$$
$$\text{If actual stress developed} > \sigma_{crit} \Rightarrow \text{flow}$$

10.3.6 Critical Outlet Dimension

Intuitively, for a given hopper geometry, one would expect the stress developed in the arch to increase with the span of the arch and the weight of solids in the arch. In practice this is the case and the stress developed in the arch is related to

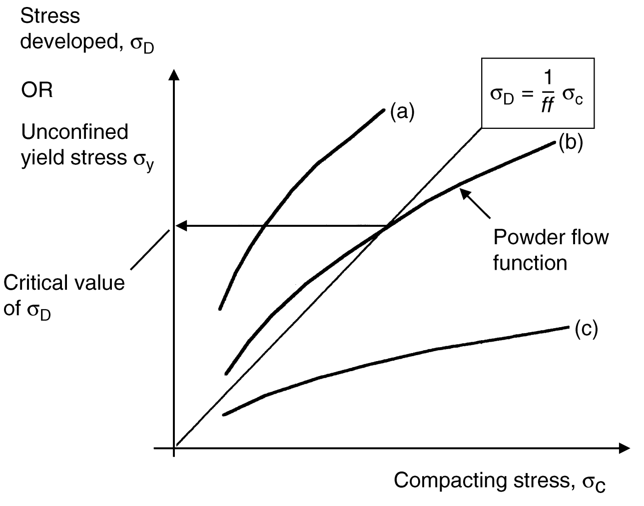


Figure 10.7 Determination of critical conditions for flow

the size of the hopper outlet, B, and the bulk density, ρ_B, of the material by the relationship:

$$\text{minimum outlet dimension}, B = \frac{H(\theta)\sigma_{\text{crit}}}{\rho_B g} \tag{10.4}$$

where $H(\theta)$ is a factor determined by the slope of the hopper wall and g is the acceleration due to gravity. An approximate expression for $H(\theta)$ for conical hoppers is

$$H(\theta) = 2.0 + \frac{\theta}{60} \tag{10.5}$$

10.3.7 Summary

From the above discussion of the design philosophy for ensuring mass flow from a conical hopper, we see that the following are required:

(1) the relationship between the strength of the powder in the arch, σ_y (unconfined yield stress) with the compacting stress acting on the powder, σ_C;

(2) the variation of hopper flow factor, *ff*, with:

(a) the nature of the powder (characterized by the effective angle of internal friction, δ);

(b) the nature of the hopper wall (characterized by the angle of wall friction, Φ_W);

(c) the slope of the hopper wall (characterized by θ, the semi-included angle of the conical section, i.e. the angle between the sloping hopper wall and the vertical).

Knowing δ, Φ_w, and θ, the hopper flow factor, *ff*, can be fixed. The hopper flow factor is therefore a function both of powder properties and of the hopper properties (geometry and the material of construction of the hopper walls).

Knowing the hopper flow factor and the powder flow function (σ_y versus σ_C) the critical stress in the arch can be determined and the minimum size of outlet found corresponding to this stress.

10.4 SHEAR CELL TEST

The data listed above can be found by performing shear cell tests on the powder.

The Jenike shear cell (Figure 10.8) allows powders to be compacted to any degree and sheared under controlled load conditions. At the same time the shear force (and hence stress) can be measured.

Generally powders change bulk density under shear. Under the action of shear, for a specific normal load:

- a loosely packed powder would contract (increase bulk density);
- a very tightly packed powder would expand (decrease bulk density);
- a critically packed powder would not change in volume.

For a particular bulk density there is a critical normal load which gives failure (yield) without volume change. A powder flowing in a hopper is in this critical condition. Yield without volume change is therefore of particular interest to us in design.

Using a standardized test procedure five or six samples of powder are prepared all having the same bulk density. Referring to the diagram of the Jenike shear cell shown in Figure 10.8 a normal load is applied to the lid of the cell and the horizontal force applied to the sample via the bracket and loading pin

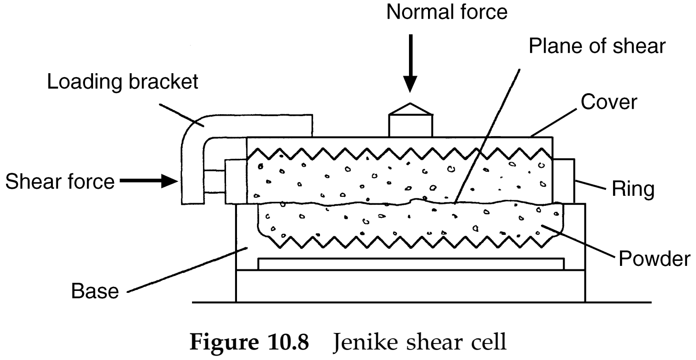


Figure 10.8 Jenike shear cell

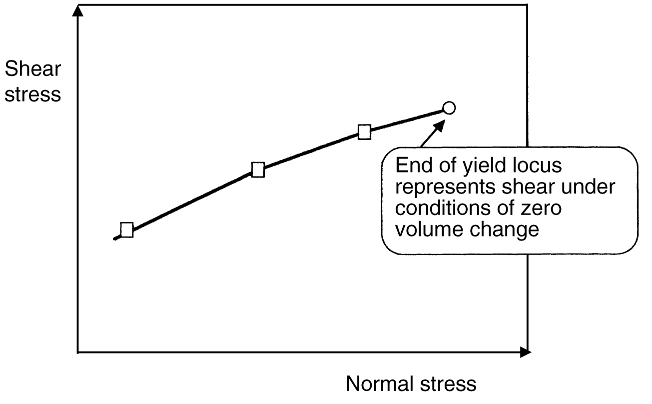


Figure 10.9 A single yield locus

is recorded. That horizontal force necessary to initiate shear of flow of the powder sample is noted. This procedure is repeated for each identical powder sample but with a greater normal load applied to the lid each time. This test thus generates a set of five or six pairs of values for normal load and shear force and hence pairs of values of compacting stress and shear stress for a powder of a particular bulk density. The pairs of values are plotted to give a yield locus (Figure 10.9). The end point of the yield locus corresponds to critical flow conditions where initiation of flow is not accompanied by a change in bulk density. Experience with the procedure permits the operator to select combinations of normal and shear force which achieve the critical conditions. This entire test procedure is repeated two or three times with samples prepared to different bulk densities. In this way a family of yield loci is generated (Figure 10.10).

These yield loci characterize the flow properties of the unaerated powder. The following section deals with the generation of the powder flow function from this family of yield loci.

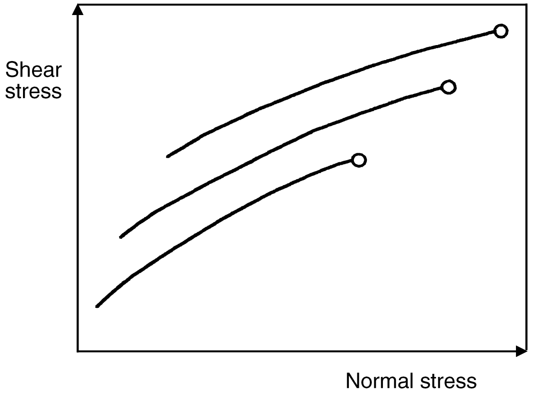


Figure 10.10 A family of yield loci

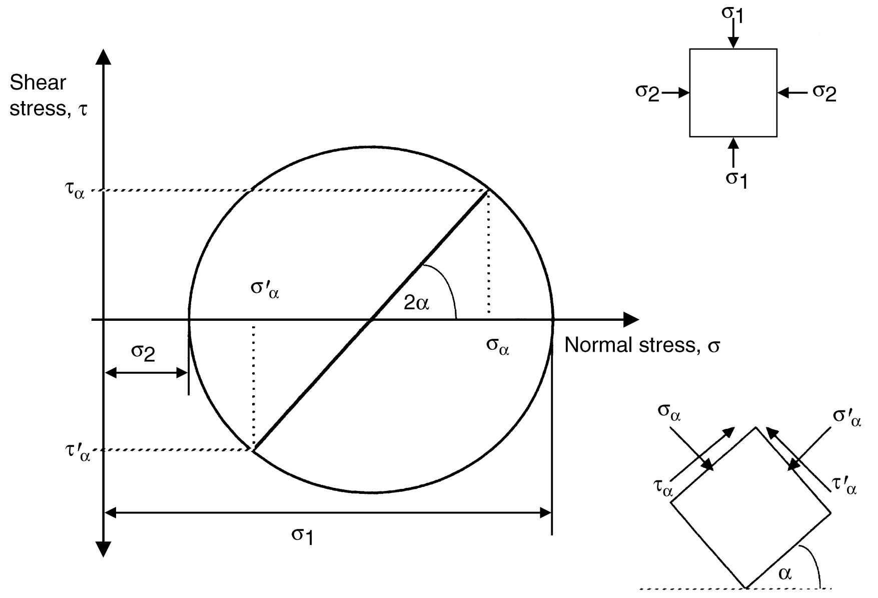


Figure 10.11 Mohr's circle construction

10.5 ANALYSIS OF SHEAR CELL TEST RESULTS

The mathematical stress analysis of the flow of unaerated powders in a hopper requires the use of principal stresses. We therefore need to use the Mohr's stress circle in order to determine principal stresses from the results of the shear tests.

10.5.1 Mohr's Circle – in Brief

Principal stresses – in any stress system there are two planes at right angles to each other in which the shear stresses are zero. The normal stresses acting on these planes are called the principal stresses.

The Mohr's circle represents the possible combinations of normal and shear stresses acting on any plane in a body (or powder) under stress. Figure 10.11 shows how the Mohr's circle relates to the stress system. Further information on the background to the use of Mohr's circles may be found in most texts dealing with the strength of materials and the analysis of stress and strain in solids.

10.5.2 Application of Mohr's Circle to Analysis of the Yield Locus

Each point on a yield locus represents that point on a particular Mohr's circle for which failure or yield of the powder occurs. A yield locus is then tangent to all the Mohr's circles representing stress systems under which the powder will fail (flow).

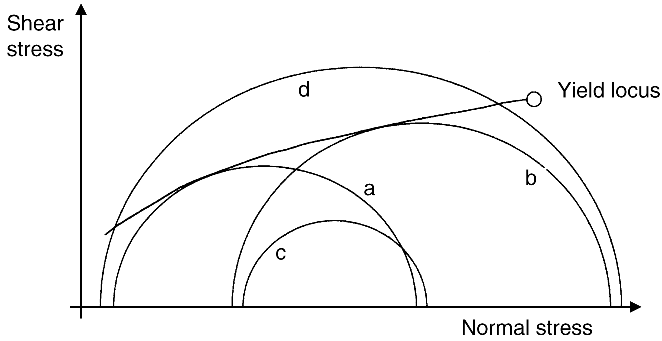


Figure 10.12 Identification of the applicable Mohr's circle

For example, in Figure 10.12 Mohr's circles (a) and (b) represent stress systems under which the powder would fail. In circle (c) the stresses are insufficient to cause flow. Circle (d) is not relevant since the system under consideration cannot support stress combinations above the yield locus. It is therefore Mohr's circles which are tangential to yield loci that are important to our analysis.

10.5.3 Determination of σ_y and σ_c

Two tangential Mohr's circles are of particular interest. Referring to Figure 10.13, the smaller Mohr's circle represents conditions at the free surface of the arch: this free surface is a plane in which there is zero shear and zero normal stress and so the Mohr's circle which represents flow (failure) under these conditions must pass through the origin of the shear stress versus normal stress plot. This Mohr's circle gives the (major principal) unconfined yield stress, and this is the value we use for σ_y. The larger Mohr's circle is tangent to the yield locus at its end point and therefore represents conditions for critical failure. The major principal stress from this Mohr's circle is taken as our value of compacting stress, σ_C.

Pairs of values of σ_y and σ_C are found from each yield locus and plotted against each other to give the powder flow function (Figure 10.6).

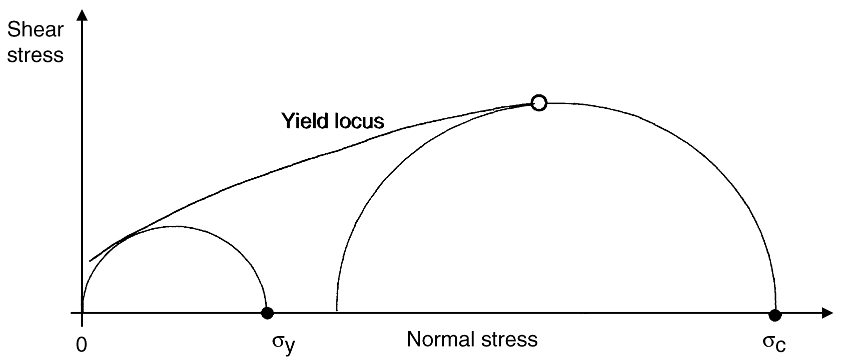


Figure 10.13 Determination of unconfined yield stress, σ_y and compacting stress, σ_C

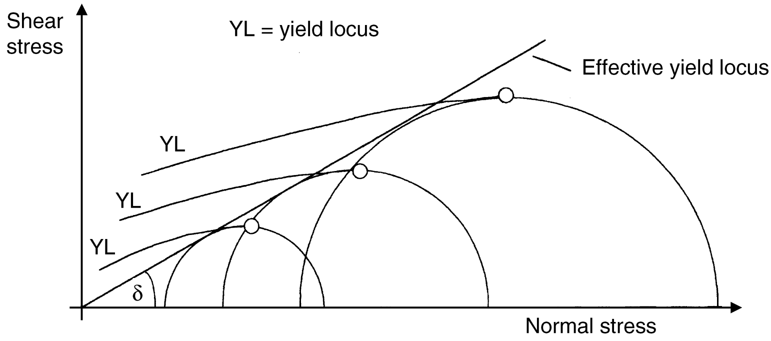


Figure 10.14 Definition of effective yield locus and effective angle of internal friction, δ

10.5.4 Determination of δ from Shear Cell Tests

Experiments carried out on hundreds of bulk solids (Jenike, 1964) have demonstrated that for an element of powder flowing in a hopper:

$$\frac{\sigma_1}{\sigma_2} = \frac{\text{major principal stress on the element}}{\text{minor principal stress on the element}} = \text{a constant}$$

This property of bulk solids is expressed by the relationship:

$$\frac{\sigma_1}{\sigma_2} = \frac{1 + \sin\delta}{1 - \sin\delta} \tag{10.6}$$

where δ is the effective angle of internal friction of the solid. In terms of the Mohr's stress circle this means that Mohr's circles for the critical failure are all tangent to a straight line through the origin, the slope of the line being tan δ (Figure 10.14).

This straight line is called the effective yield locus of the powder. By drawing in this line, δ can be determined. Note that δ is not a real physical angle within the powder; it is the tangent of the ratio of shear stress to normal stress. Note also that for a free-flowing solid, which does not gain strength under compaction, there is only one yield locus and this locus coincides with the effective yield locus (Figure 10.15). (This type of relationship between normal stress and shear stress is known as Coulomb friction.)

10.5.5 The Kinematic Angle of Friction between Powder and Hopper Wall, Φ_w

The kinematic angle of friction between powder and hopper wall is otherwise known as the angle of wall friction, Φ_w This gives us the relationship between normal stress acting between powder and wall and the shear stress under flow conditions. To determine Φ_w it is necessary to first construct the wall yield locus

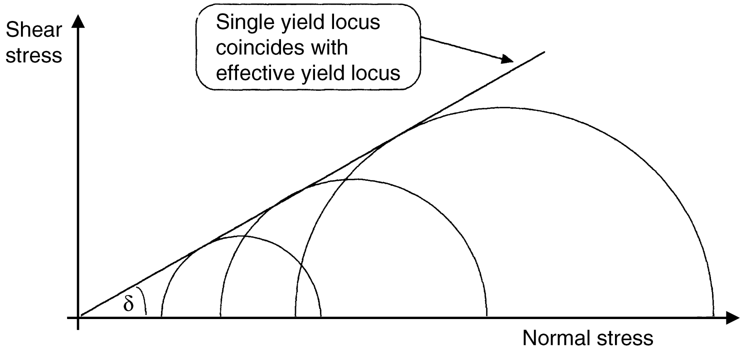


Figure 10.15 Yield locus for a free-flowing powder

from shear cell tests. The wall yield locus is determined by shearing the powder against a sample of the wall material under various normal loads. The apparatus used is shown in Figure 10.16, and a typical wall yield locus is shown in Figure 10.17.

The kinematic angle of wall friction is given by the gradient of the wall yield locus (Figure 10.17), i.e.

$$\tan \Phi_{\mathrm{w}} = \frac{\text{shear stress at the wall}}{\text{normal stress at the wall}}$$

10.5.6 Determination of the Hopper Flow Factor, *ff*

The hopper flow factor, *ff*, is a function of δ, Φ_{w}, and θ and can be calculated from first principles. However, Jenike (1964) obtained values for a conical hopper and for a wedge-shaped hopper with a slot outlet for values of δ of 30°, 40°, 50°, 60° and 70°. Examples of the 'flow factor charts' for conical hoppers are shown in

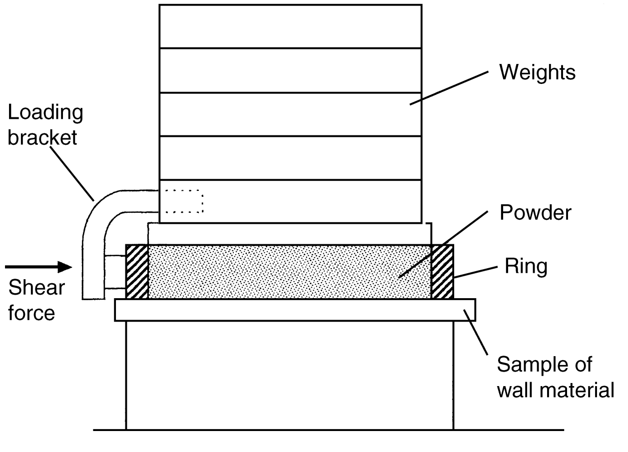


Figure 10.16 Apparatus for the measurement of kinematic angle of wall friction, Φ_{w}

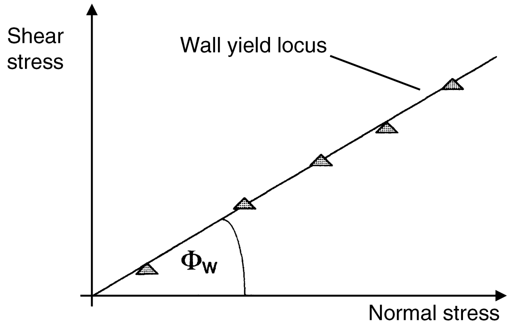


Figure 10.17 Kinematic angle of wall friction, Φ_w

Figure 10.18. It will be noticed that values of flow factor exist only in a triangular region; this defines the conditions under which mass flow is possible.

The following is an example of the use of these flow factor charts. Suppose that shear cell tests have given us δ and Φ_w equal to 30° and 19°, respectively, then entering the chart for conical hoppers with effective angle of friction $\delta = 30°$, we find that the limiting value of wall slope, θ, to ensure mass flow is 30.5° (point X in Figure 10.19). In practice it is usual to allow a safety margin of 3°, and so, in this case the semi-included angle of the conical hopper θ would be chosen as 27.5°, giving a hopper flow factor, $ff = 1.8$ (point Y, Figure 10.19).

10.6 SUMMARY OF DESIGN PROCEDURE

The following is a summary of the procedure for the design of conical hoppers for mass flows:

(i) Shear cell tests on powder give a family of yield loci.

(ii) Mohr's circle stress analysis gives pairs of values of unconfined yield stress, σ_y, and compacting stress, σ_C, and the value of the effective angle of internal friction, δ.

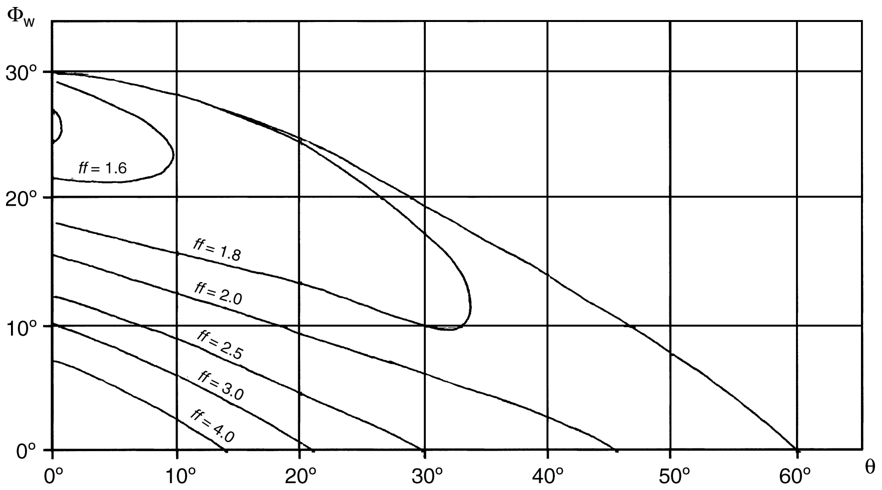
(iii) Pairs of values of σ_y and σ_C give the powder flow function.

(iv) Shear cell tests on the powder and the material of the hopper wall give the kinematic angle of wall friction, Φ_w.

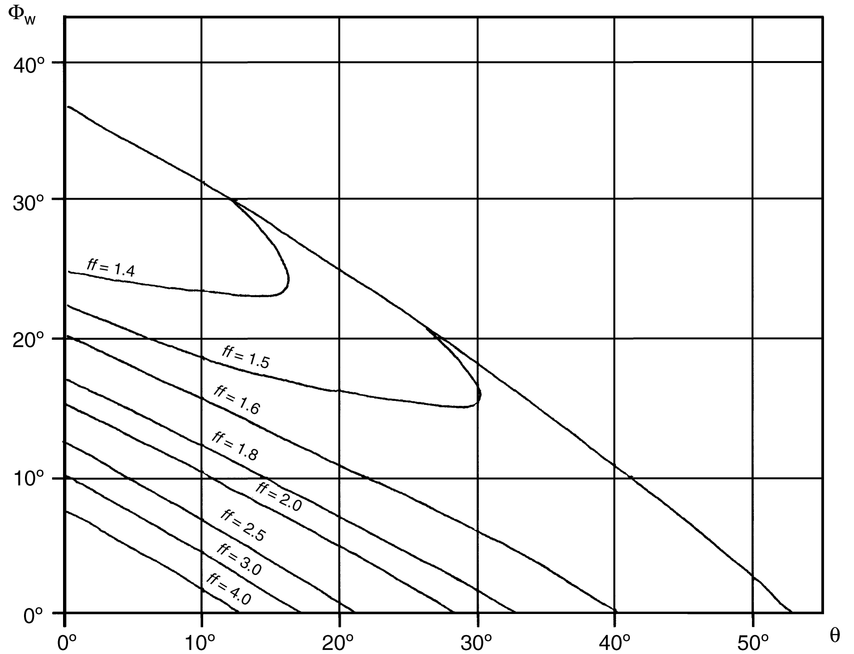
(v) Φ_w and δ are used to obtain hopper flow factor, ff, and semi-included angle of conical hopper wall slope, θ.

(vi) Powder flow function and hopper flow factor are combined to give the stress corresponding to the critical flow – no flow condition, σ_{crit}.

(vii) σ_{crit}, $H(\theta)$ and bulk density, ρ_B, are used to calculate the minimum diameter of the conical hopper outlet B.



(a)



(b)

Figure 10.18 (a) Hopper flow factor values for conical channels, $\delta = 30°$. (b) Hopper flow factor values for conical channels, $\delta = 40°$. (c) Hopper flow factor values for conical channels, $\delta = 50°$. (d) Hopper flow factor values for conical channels, $\delta = 60°$

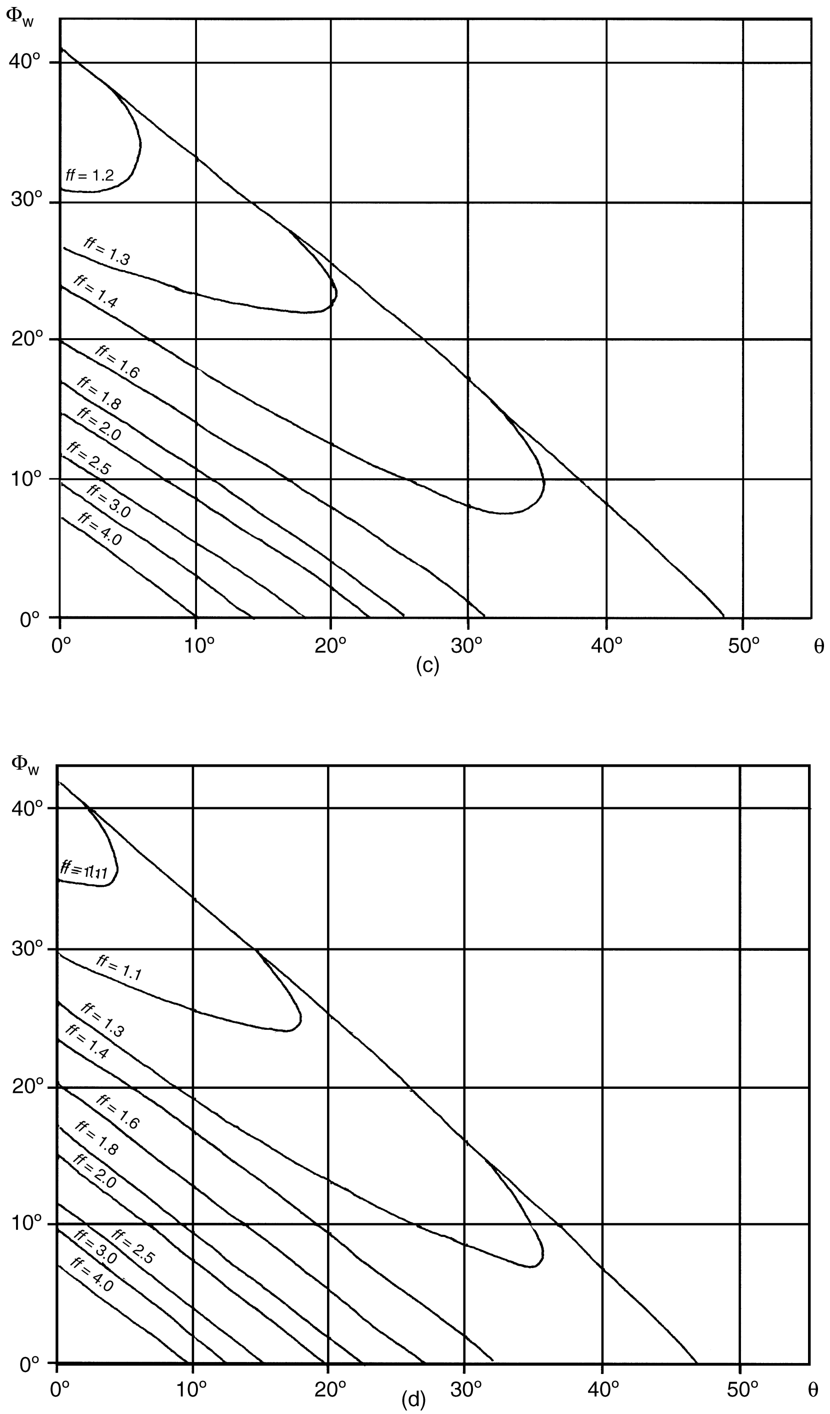


Figure 10.18 (*Continued*)

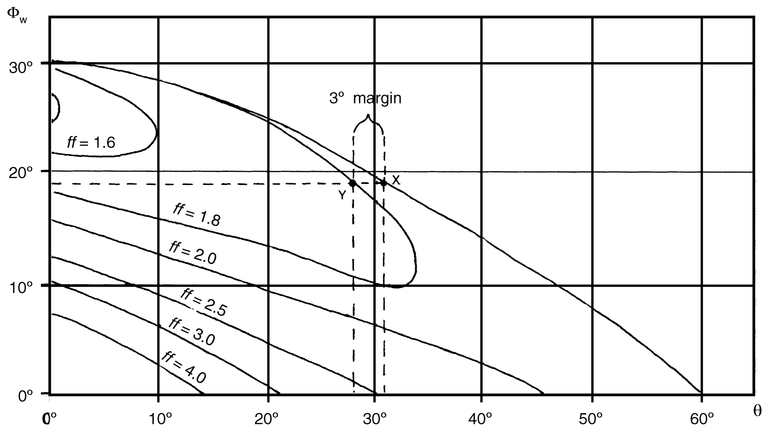


Figure 10.19 Worked example of the use of hopper flow factor charts. Hopper flow factor values for conical channels, $\delta = 30^\circ$

10.7 DISCHARGE AIDS

A range of devices designed to facilitate flow of powders from silos and hoppers are commercially available. These are known as discharge aids or silo activators. These should not, however, be employed as an alternative to good hopper design.

Discharge aids may be used where proper design recommends an unacceptably large hopper outlet incompatible with the device immediately downstream. In this case the hopper should be designed to deliver uninterrupted mass flow to the inlet of the discharge aid, i.e. the slope of the hopper wall and inlet dimensions of the discharge aid are those calculated according to the procedure outlined in this chapter.

10.8 PRESSURE ON THE BASE OF A TALL CYLINDRICAL BIN

It is interesting to examine the variation of stress exerted on the base of a bin with increasing depth of powder. For simplicity we will assume that the powder is non-cohesive (i.e. does not gain strength on compaction). Referring to Figure 10.20, consider a slice of thickness ΔH at a depth H below the surface of the powder. The downward force is

$$\frac{\pi D^2}{4}\sigma_v \tag{10.7}$$

where D is the bin diameter and σ_v is the stress acting on the top surface of the slice. Assuming stress increases with depth, the reaction of the powder below the slice acts upwards and is

$$\frac{\pi D^2}{4}(\sigma_v + \Delta\sigma_v) \tag{10.8}$$

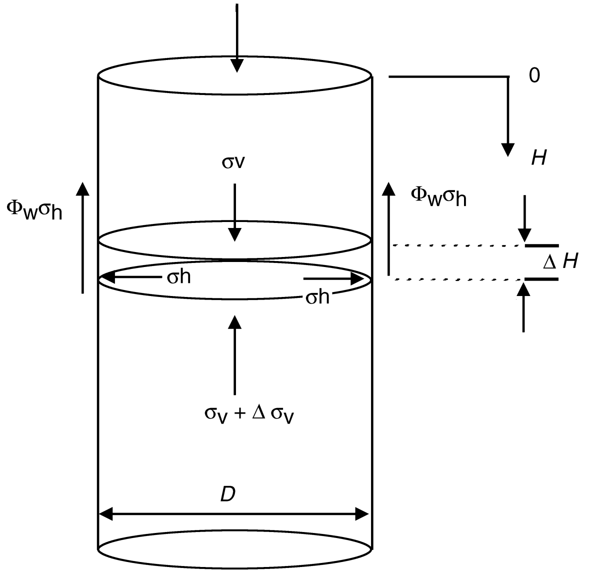


Figure 10.20 Forces acting on a horizontal slice of powder in a tall cylinder

The net upward force on the slice is then

$$\frac{\pi D^2}{4}\Delta\sigma_{\mathrm{v}} \tag{10.9}$$

If the stress exerted on the wall by the powder in the slice is σ_{h} and the wall friction is tan Φ_{w}, then the friction force (upwards) on the slice is

$$\pi D\Delta H \tan\Phi_{\mathrm{w}}\sigma_{\mathrm{h}} \tag{10.10}$$

The gravitational force on the slice is

$$\frac{\pi D^2}{4}\rho_{\mathrm{B}} g\Delta H \text{ acting downwards} \tag{10.11}$$

where ρ_{B} is the bulk density of the powder, assumed to be constant throughout the powder (independent of depth).

If the slice is in equilibrium the upward and downward forces are equated, giving

$$D\Delta\sigma_{\mathrm{v}} + 4\tan\Phi_{\mathrm{w}}\sigma_{\mathrm{h}}\Delta H = D\rho_{\mathrm{B}} g\Delta H \tag{10.12}$$

If we assume that the horizontal stress is proportional to the vertical stress and that the relationship does not vary with depth,

$$\sigma_{\mathrm{h}} = k\sigma_{\mathrm{v}} \tag{10.13}$$

and so as ΔH tends to zero,

$$\frac{\mathrm{d}\sigma_{\mathrm{v}}}{\mathrm{d}H} + \left(\frac{4\tan\Phi_{\mathrm{w}}k}{D}\right)\sigma_{\mathrm{v}} = \rho_{\mathrm{B}}g \tag{10.14}$$

Noting that this is the same as

$$\frac{\mathrm{d}\sigma_{\mathrm{v}}}{\mathrm{d}H}(\mathrm{e}^{(4\tan\Phi_{\mathrm{w}}k/D)H}\sigma_{\mathrm{v}}) = \rho_{\mathrm{B}}g\mathrm{e}^{(4\tan\Phi_{\mathrm{w}}k/D)H} \tag{10.15}$$

and integrating, we have

$$\sigma_{\mathrm{v}}\mathrm{e}^{(4\tan\Phi_{\mathrm{w}}k/D)H} = \frac{D\rho_{\mathrm{B}}g}{4\tan\Phi_{\mathrm{w}}k}\mathrm{e}^{(4\tan\Phi_{\mathrm{w}}k/D)H} + \text{constant} \tag{10.16}$$

If, in general the stress acting on the surface of the powder is $\sigma_{\mathrm{v}0}$(at $H = 0$) the result is

$$\sigma_{\mathrm{v}} = \frac{D\rho_{\mathrm{B}}g}{4\tan\Phi_{\mathrm{w}}k}[1 - \mathrm{e}^{-(4\tan\Phi_{\mathrm{w}}k/D)H}] + \sigma_{\mathrm{v}0}\mathrm{e}^{-(4\tan\Phi_{\mathrm{w}}k/D)H} \tag{10.17}$$

This result was first demonstrated by Janssen (1895).

If there is no force acting on the free surface of the powder, $\sigma_{\mathrm{v}0} = 0$ and so

$$\sigma_{\mathrm{v}} = \frac{D\rho_{\mathrm{B}}g}{4\tan\Phi_{\mathrm{w}}k}(1 - \mathrm{e}^{-(4\tan\Phi_{\mathrm{w}}k/D)H}) \tag{10.18}$$

When H is very small

$$\begin{gathered}\sigma_{\mathrm{v}} \cong \rho_{\mathrm{B}}Hg \\ \text{(since for very small } z, \mathrm{e}^{-z} \cong 1 - z\text{)}\end{gathered} \tag{10.19}$$

equivalent to the static pressure at a depth H in fluid of density ρ_{B}.

When H is large, inspection of Equation (10.18) gives

$$\sigma_{\mathrm{v}} \cong \frac{D\rho_{\mathrm{B}}g}{4\tan\Phi_{\mathrm{w}}k} \tag{10.20}$$

and so the vertical stress developed becomes independent of depth of powder above. The variation in stress with depth of powder for the case of no force acting on the free surface of the powder ($\sigma_{\mathrm{v}0} = 0$) is shown in Figure 10.21. Thus, contrary to intuition (which is usually based on our experience with fluids), the force exerted by a bed of powder becomes independent of depth if the bed is deep

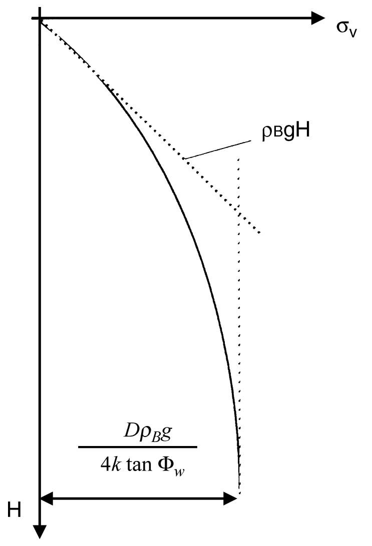


Figure 10.21 Variation in vertical pressure with depth of powder (for $\sigma_{v0} = 0$)

enough. Hence most of the weight of the powder is supported by the walls of the bin. In practice, the stress becomes independent of depth (and also independent of any load applied to the powder surface) beyond a depth of about $4D$.

10.9 MASS FLOW RATES

The rate of discharge of powder from an orifice at the base of a bin is found to be independent of the depth of powder unless the bin is nearly empty. This means that the observation for a static powder that the pressure exerted by the powder is independent of depth for large depths is also true for a dynamic system. It confirms that fluid flow theory cannot be applied to the flow of a powder. For flow through an orifice in the flat-based cylinder, experiment shows that:

$$\text{mass flow rate}, M_p \propto (B - a)^{2.5} \text{for a circular orifice of diameter } B$$

where a is a correction factor dependent on particle size. [For example, for solids discharge from conical apertures in flat-based cylinders, Beverloo *et al*. (1961) give $M_p = 0.58\rho_B g^{0.5}(B - kx)^{2.5}$.]

For cohesionless coarse particles free falling over a distance h their velocity, neglecting drag and interaction, will be $u = \sqrt{2gh}$.

If these particles are flowing at a bulk density ρ_B through a circular orifice of diameter B, then the theortical mass flow rate will be:

$$M_p = \frac{\pi}{4}\sqrt{2}\rho_B g^{0.5} h^{0.5} B^2$$

The practical observation that flow rate is proportional to $B^{2.5}$ suggests that, in practice, particles only approach the free fall model when h is the same order as the orifice diameter.

10.10 CONCLUSIONS

Within the confines of a single chapter it has been possible only to outline the principles involved in the analysis of the flow of unaerated powders. This has been done by reference to the specific example of the design of conical hoppers for mass flow. Other important considerations in the design of hoppers such as time consolidation effects and determination of the stress acting in the hopper and bin wall have been omitted. These aspects together with the details of shear cell testing procedure are covered in texts specific to the subject. Readers wishing to pursue the analysis of failure (flow) in particulate solids in greater detail may refer to texts on soil mechanics.

10.11 WORKED EXAMPLES

WORKED EXAMPLE 10.1

The results of shear cell tests on a powder are shown in Figure 10W1.1. In addition, it is known that the angle of friction on stainless steel is 19° for this powder, and under flow conditions the bulk density of the powder is 1300 kg/m^3. A conical stainless steel hopper is to be designed to hold this powder.

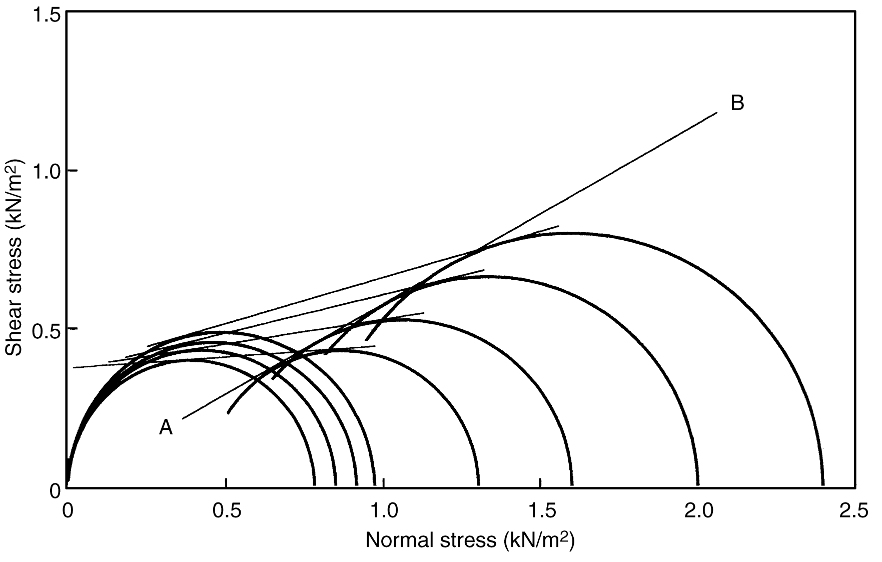


Figure 10W1.1 Shear cell test data

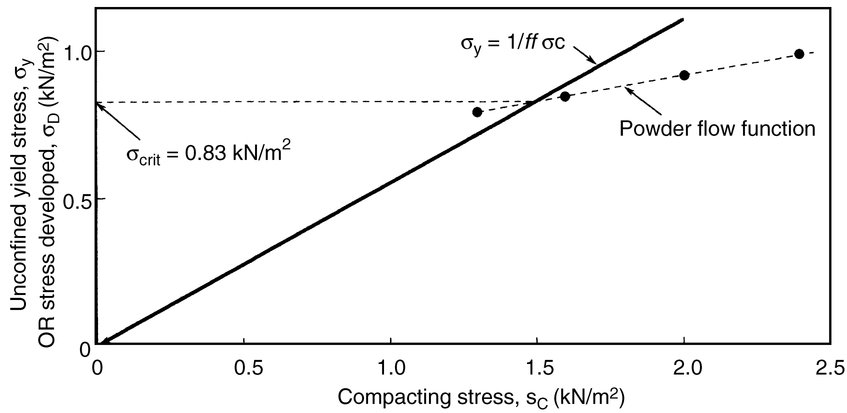


Figure 10W1.2 Determination of critical stress

Determine:

(a) the effective angle of internal friction;

(b) the maximum semi-included angle of the conical hopper which will confidently give mass flow;

(c) the minimum diameter of the circular hopper outlet necessary to ensure flow when the outlet slide valve is opened.

Solution

(a) From Figure 10W1.1, determine the slope of the effective yield locus (line AB). Slope = 0.578.

Hence, the effective angle of internal friction, $\delta = \tan^{-1}(0.578) = 30°$

(b) From Figure 10W1.1, determine the pairs of values of σ_C and σ_y necessary to plot the powder flow function (Figure 10W1.2).

σ_C	2.4	2.0	1.6	1.3
σ_y	0.97	0.91	0.85	0.78

Using the flow factor chart for $\delta = 30°$ [Figure 10.18(a)] with $\Phi_v = 19°$ and a 3° margin of safety gives a hopper flow factor, $ff = 1.8$, and the semi-included angle of hopper wall, $\theta = 27.5°$ (see Figure 10W1.3).

(c) The relationship $\sigma_y = \sigma_C/ff$ is plotted on the same axes as the powder flow function (Figure 10W1.2) and where this line intersepts the powder flow function we find a value of critical unconfined yield stress, $\sigma_{crit} = 0.83\,\mathrm{kN/m^2}$. From Equation (10.5),

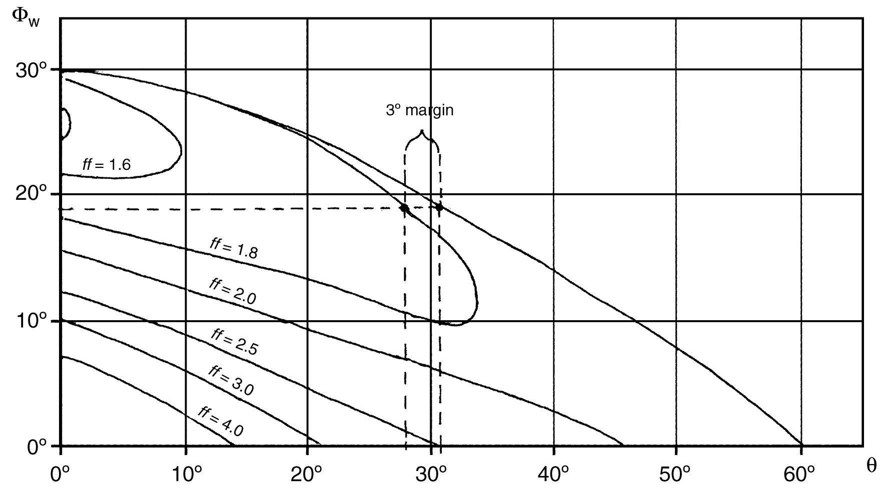


Figure 10W1.3 Determination of θ and ff

$$H(\theta) = 2.46 \text{ when } \theta = 27.5^\circ$$

and from Equation (10.4), the minimum outlet diameter for mass flow, B, is

$$B = \frac{2.46 \times 0.83 \times 10^3}{1300 \times 9.81} = 0.160\,\text{m}$$

Summarizing, then, to achieve mass flow without risk of blockage using the powder in question we require a stainless steel conical hopper with a maximum semi-included angle of cone, 27.5° and a circular outlet with a diameter of at least 16.0 cm.

WORKED EXAMPLE 10.2

Shear cell tests on a powder give the following information:
Effective angle of internal friction, $\delta = 40^\circ$
Kinematic angle of wall friction on mild steel, $\Phi_w = 16^\circ$
Bulk density under flow condition, $\rho_B = 2000\,\text{kg/m}^3$
The powder flow function which can be represented by the relationship, $\sigma_y = \sigma_C^{0.6}$, where σ_y is unconfined yield stress (kN/m^2) and σ_C is consolidating stress (kN/m^2)
Determine (a) the maximum semi-included angle of a conical mild steel hopper that will confidently ensure mass flow, and (b) the minimum diameter of circular outlet to ensure flow when the outlet is opened.

Solution

(a) With an effective angle of internal friction $\delta = 40^\circ$ we refer to the flow factor chart in Figure 10.18(b), from which at $\Phi_w = 16^\circ$ and with a safety margin of 3° we obtain the hopper flow factor, $ff = 1.5$ and hopper semi-included angle for mass flow, $\theta = 30^\circ$ (Figure 10W2.1).

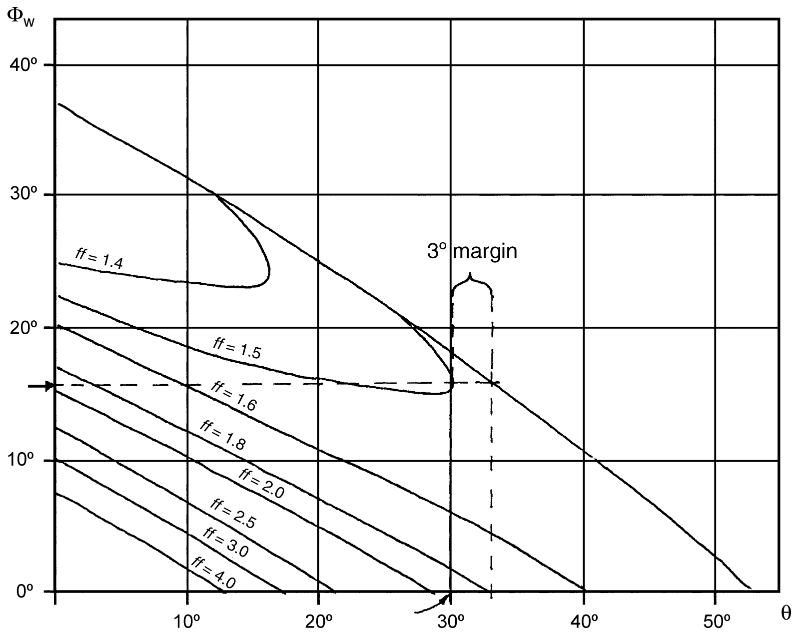


Figure 10W2.1 Determination of θ and ff

(b) For flow: $\dfrac{\sigma_C}{ff} > \sigma_y$ [Equation (10.3)]

but for the powder in question σ_y and σ_C are related by the material flow function: $\sigma_y = \sigma_C^{0.6}$.

Thus, the criterion for flow becomes

$$\left(\frac{\sigma_y^{1/0.6}}{ff}\right) > \sigma_y$$

and so the critical value of unconfined yield stress σ_{crit} is found when $\left(\dfrac{\sigma_y^{1/0.6}}{ff}\right) = \sigma_y$ hence, $\sigma_{crit} = 1.837\,\mathrm{kN/m^2}$.

From Equation (10.5), $H(\theta) = 2.5$ when $\theta = 30^\circ$ and hence, from Equation (10.4), minimum diameter of circular outlet,

$$B = \frac{2.5 \times 1.837 \times 10^3}{2000 \times 9.81} = 0.234\,\mathrm{m}$$

Summarizing, mass flow without blockages is ensured by using a mild steel hopper with maximum semi-included cone angle 30° and a circular outlet diameter of at least 23.4 cm.

TEST YOURSELF

10.1 Explain with the aid of sketches what is meant by the terms *mass flow* and *core flow* with respect to solids flow in storage hoppers.

10.2 The starting point for the design philosophy presented in this chapter is the *flow-no flow criterion*. What is the flow-no flow criterion?

10.3 Which quantity describes the strength developed by a powder in an arch preventing flow from the base of a hopper? How is this quantity related to the hopper flow factor?

10.4 What is the *powder flow function?* Is the powder flow function dependent on (a) the powder properties, (b) the hopper geometry, (c) both the powder properties and the hopper geometry?

10.5 Show how the critical value of stress is determined from a knowledge of the *hopper flow factor* and the *powder flow function*.

10.6 What is meant by critical failure (yield) of a powder? What is its significance?

10.7 With the aid of a sketch plot of shear stress versus normal stress, show how the effective angle of internal of a powder is determined from a family of yield loci.

10.8 What is the kinematic angle of wall friction and how is it determined?

10.9 A powder is poured gradually into a measuring cylinder of diameter 3 cm. At the base of the cylinder is a load cell which measures the normal force exerted by the powder on the base. Produce a sketch plot showing how the normal force on the cylinder base would be expected to vary with powder depth, up to a depth of 18 cm.

10.10 How would you expect the mass flow rate of particulate solids from a hole in the base of a flat-bottomed container to vary with (a) the hole diameter and (b) the depth of solids?

EXERCISES

10.1 Shear cell tests on a powder show that its effective angle of internal friction is 40° and its powder flow function can be represented by the equation: $\sigma_y = \sigma_C^{0.45}$, where σ_y is the unconfined yield stress and σ_C is the compacting stress, both in kN/m^2. The bulk density of the powder is $1000\,kg/m^3$ and angle of friction on a mild steel plate is 16°. It is proposed to store the powder in a mild steel conical hopper of semi-included angle 30° and having a circular discharge opening of 0.30 m diameter. What is the critical outlet diameter to give mass flow? Will mass flow occur?

(Answer: 0.355 m; no flow.)

10.2 Describe how you would use shear cell tests to determine the effective angle of internal friction of a powder.

A powder has an effective angle of internal friction of 60° and has a powder flow function represented in the graph shown in Figure 10E2.1. If the bulk density of the powder is

1500 kg/m^3 and its angle of friction on mild steel plate is 24.5°, determine, for a mild steel hopper, the maximum semi-included angle of cone required to safely ensure mass flow, and the minimum size of circular outlet to ensure flow when the outlet is opened.

(Answer: 17.5°; 18.92 cm.)

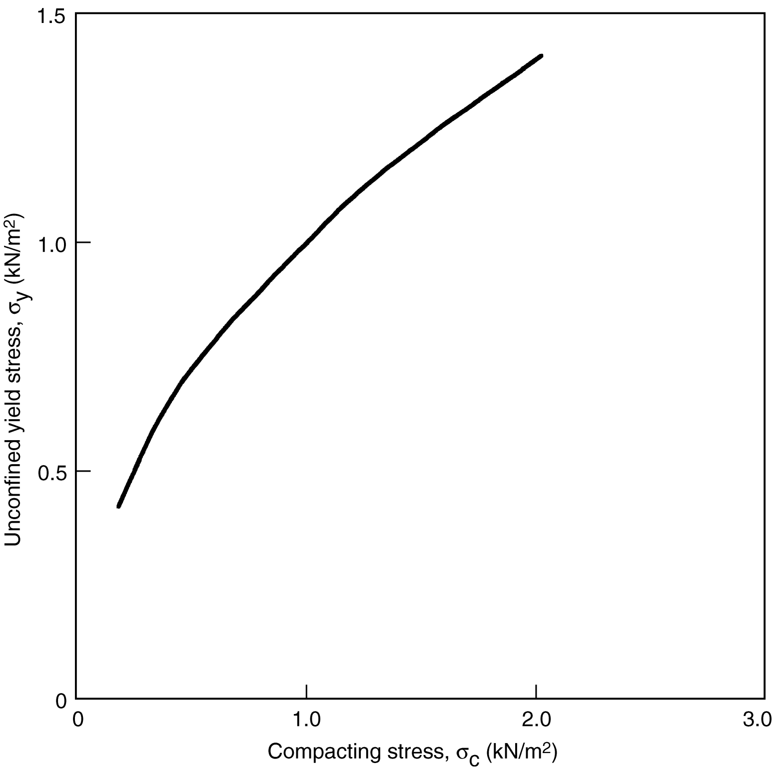


Figure 10E2.1 Powder flow function

10.3

(a) Summarize the philosophy used in the design of conical hoppers to ensure flow from the outlet when the outlet valve is opened.

(b) Explain how the powder flow function and the effective angle of internal friction are extracted from the results of shear cell tests on a powder.

(c) A firm having serious hopper problems takes on a chemical engineering graduate.

The hopper in question feeds a conveyor belt and periodically blocks at the outlet and needs to be 'encouraged' to restart. The graduate makes an investigation on the hopper, commissions shear cell tests on the powder and recommends a minor modification to the hopper. After the modification the hopper gives no further trouble and the graduate's reputation is established. Given the information below, what was the graduate's recommendation?

Existing design: Material of wall – mild steel

Semi-included angle of conical hopper -33°

Outlet – circular, fitted with 25 cm diameter slide valve

Shear cell test data: Effective angle of internal friction, $\delta = 60^\circ$

Angle of wall friction on mild steel, $\Phi_w = 8^\circ$

Bulk density, $\rho_B = 1250\,\mathrm{kg/m^3}$

Powder flow function: $\sigma_y = \sigma_C^{0.55}$ (σ_y and σ_C in $\mathrm{kN/m^2}$)

10.4 Shear cell tests are carried out on a powder for which a stainless steel conical hopper is to be designed. The results of the tests are shown graphically in Figure 10E4.1. In addition it is found that the friction between the powder on stainless steel can be described by an angle of wall friction of 11°, and that the relevant bulk density of the powder is $900\,\mathrm{kg/m^3}$.

(a) From the shear cell results of Figure 10E4.1, deduce the effective angle of internal friction δ of the powder.

(b) Determine:

(i) the semi-included hopper angle safely ensuring mass flow;

(ii) the hopper flow factor, *ff*.

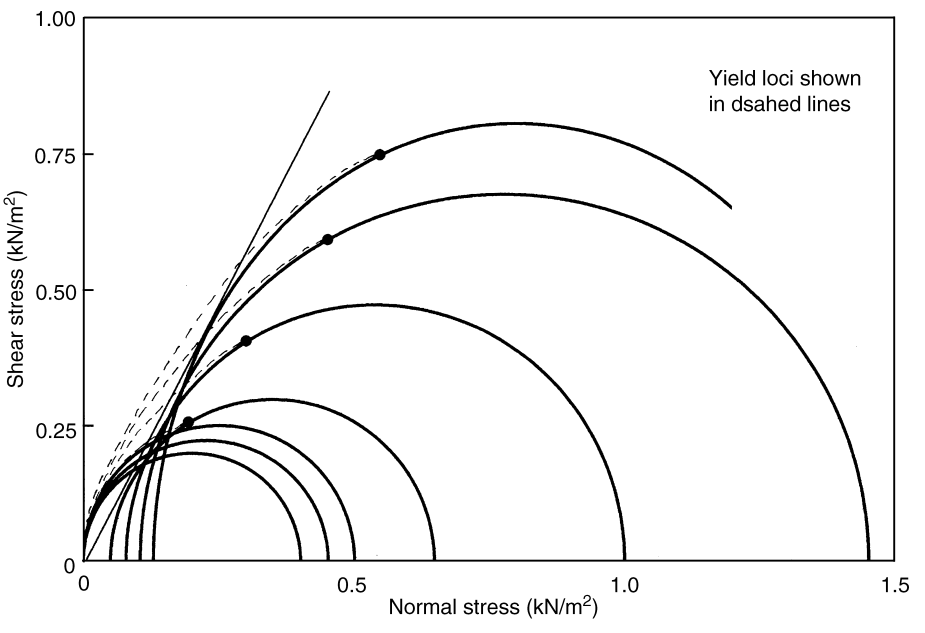


Figure 10E4.1 Shear cell test data

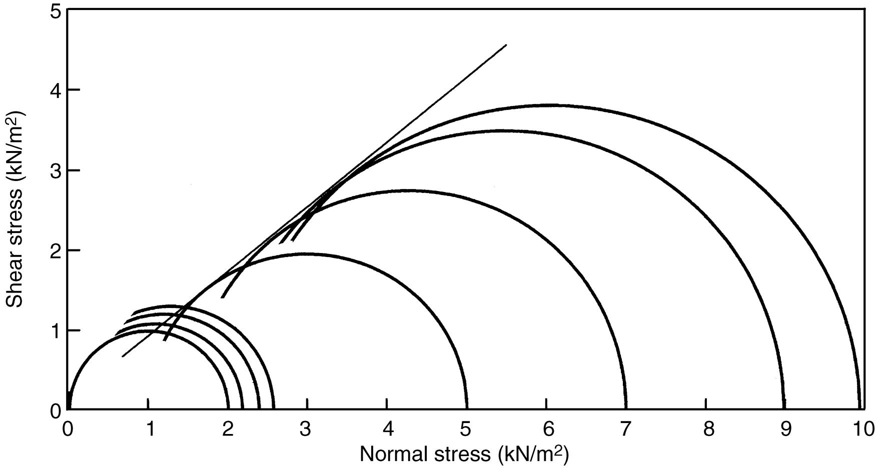


Figure 10E5.1 Shear cell test data

(c) Combine this information with further information gathered from Figure 10E4.1 in order to determine the minimum diameter of outlet to ensure flow when required. (*Note*: Extrapolation is necessary here.)

(d) What do you understand by 'angle of wall friction' and 'effective angle of internal friction'?

[Answer: (a) 60°; (b) (i) 32.5°, (ii) 1.29; (c) 0.110 m.]

10.5 The results of shear cell tests on a powder are given in Figure 10E5.1. An aluminium conical hopper is to be designed to suit this powder. It is known that the angle of wall friction between the powder and aluminium is 16° and that the relevant bulk density is 900 kg/m^3.

(a) From Figure 10E5.1 determine the effective angle of internal friction of the powder.

(b) Determine:

(i) the semi-included hopper angle safely ensuring mass flow;

(ii) the hopper flow factor, *ff*.

(c) Combine the information with further information gathered from Figure 10E5.1 in order to determine the minimum diameter of circular outlet to ensure flow when required. (*Note*: Extrapolation of these experimental results may be necessary.)

[Answer: (a) 40°; (b)(i) 29.5°; (ii) 1.5; (c) 0.5 m ± approximately 7% depending on the extrapolation.]