Q1. A two dimensional flow field is described by the following velocity components

$u = 3x(t+1) \qquad v = 3y(t-1)$

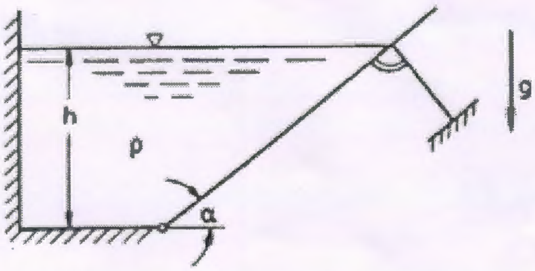
Determine the trajectory of the fluid particle that passes through the point (x_p, y_p) at $t = 0$.

(5 Marks)

Q2. Find the stream function associated with the two dimensional incompressible flow

$v_r = U\left\{1 - \frac{a^2}{r^2}\right\}\cos\theta \qquad v_\theta = -U\left\{1 + \frac{a^2}{r^2}\right\}\sin\theta$ **(5 Marks)**

Q3. A pivoted wall of a water container with width B is supported with a rod. Here, h = 3m; B = 1m; $\alpha = 30°$; $\rho = 10^3$ kg m^{-3}; g = 10 m s^{-2}. Determine the force in the rod. **(5 Marks)**



Helpful Equation

$$\frac{\partial(rV_r)}{\partial r} + \frac{\partial V_\theta}{\partial \theta} = 0$$

$$V_r \equiv \frac{1}{r}\frac{\partial\psi}{\partial\theta} \quad \text{and} \quad V_\theta \equiv -\frac{\partial\psi}{\partial r}$$

Q.2

$$V_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} \Rightarrow \psi = \int U\left(1 - \frac{a^2}{r^2}\right)\cos\theta\, dr$$

$$= U\left(r - \frac{a^2}{r}\right)\sin\theta + f(r)$$

$$\Rightarrow \frac{\partial\psi}{\partial r} = -V_\theta = U\left(1 + \frac{a^2}{r^2}\right)\sin\theta + \frac{df(r)}{dr}$$

$$\Rightarrow U\left[1 + \frac{a^2}{r^2}\right]\sin\theta + \frac{df(r)}{dr} = U\left(1 + \frac{a^2}{r^2}\right)\sin\theta$$

$\Rightarrow \frac{df(r)}{dr} = 0 \Rightarrow f(r) =$ Constant, and can be set at any value of convenience (say, 0)

$$\Rightarrow \psi = U\left(r - \frac{a^2}{r}\right)\sin\theta$$

Q.1.

$u = \frac{dx}{dt} = 3x(t+1)$ $\qquad v = \frac{dy}{dt} = 3y(t-1)$

$\Downarrow$

$y = C_2 \exp\left[\frac{3}{2}(t-1)^2\right]$

Upon integration,

$$\ln x = \frac{3}{2}(t+1)^2 + \ln C_1$$

$$\Rightarrow x = C_1 \exp\left\{\frac{3}{2}(t+1)^2\right\}$$

At $t=0$, $x = x_p$ and $y = y_p$

$$\Rightarrow C_1 = \frac{x_p}{e^{3/2}} \qquad C_2 = \frac{y_p}{e^{3/2}}$$

$$\Rightarrow x = x_p e^{(t+1)^2} \quad \ldots(1)$$

$$y = y_p e^{(t-1)^2} \quad \ldots(2)$$

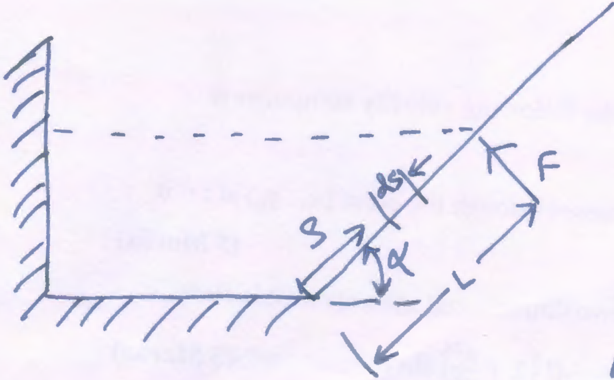
Eliminating t from Equation (1)

$$t = \sqrt{\ln\left(\frac{x}{x_p}\right)} + 1$$

and substituting in Equation (2)

$$y = y_p \exp\left[\left(\sqrt{\ln\left(\frac{x}{x_p}\right)} + 1\right) - 1\right]^2$$

Q.3



$$F \cdot L = \int_{s=0}^{s=L} (P - P_{atm})\, s B\, ds$$

where $P = P_{atm} + \rho g s [L - s] \sin\alpha$

$$F = \frac{1}{L} \int_{s=0}^{s=L} \rho g [L-s] s \sin\alpha \; B\, ds$$

$$= \frac{1}{L} \rho g B \sin\alpha \left[\frac{L^3}{2} - \frac{L^3}{3} \right]$$

$$= \frac{\rho g B (\sin\alpha) L^2}{6} = 3 \times 10^4 \, N$$