Bernoulli Equation

$$\frac{p}{\rho} + \frac{V^2}{2} + g\,Z = Const.$$

Relates pressure changes to velocity and elevation changes along a streamline

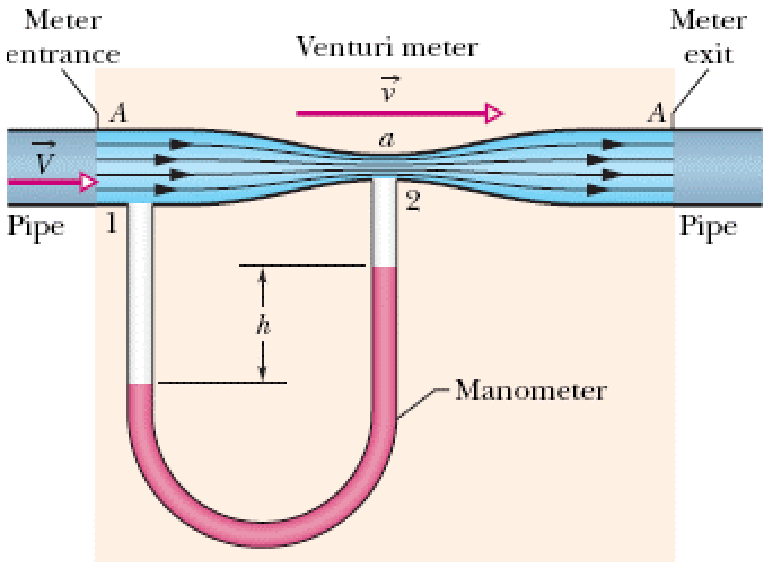
Restriction to the use of Bernoulli's equation

i) Steady flow
ii) No friction
iii) Flow along a streamline
iv) Incompressible flow

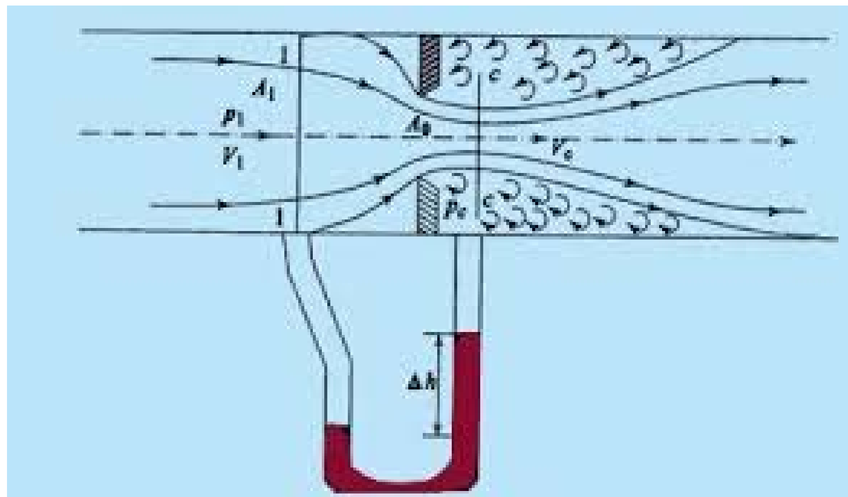
Cautions on the use of Bernoulli's equation

1. Friction should have a negligible effect
2. No flow separation and B. L. on the walls
3. Diverging passage and sudden expansion cannot be modelled.
4. Reasonable model for well rounded entrance, gentle bends, short overall lengths
5. Cannot be applied through a machine, e.g. a propeller, pump etc.
6. Compressibility (for gases) has to be considered. If Ma is about 0.3 and above, property variation may not be neglected.
7. However, temperature change (effect on density for gases) will cause non-applicability of B. Eqn. e.g. for flow through a heating element

Venturi meter



Orifice meter



Modified form of Bernoulli's Eqn – to account for head losses

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\overline{V_1^2}}{2} + g\,Z_1\right) = \left(\frac{p_2}{\rho} + \alpha_2 \frac{\overline{V_2^2}}{2} + g\,Z_2\right) + h_{LT}$$

Features

1. The velocities are average velocities
2. Significance of α, the kinetic energy coefficient
3. h_{LT} – total head loss – major and minor losses, what are they?
4. Dimension of h_{LT} – energy per unit mass
5. If the flow is frictionless, $\alpha_1 = \alpha_2$ and no head losses

This equation can be used to calculate the pressure difference between any two points in a piping system, provided the head loss, h_{LT} is known.

Kinetic Energy Coefficient

$$\int_A \frac{V^2}{2}\rho V\,dA = \alpha \int_A \frac{\overline{V}^2}{2}\rho V\,dA = \alpha\,\frac{\dot{m}\,\overline{V^2}}{2}$$

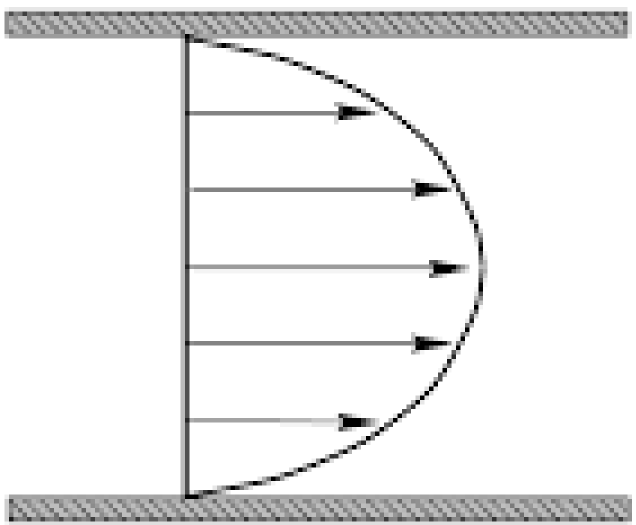
$$\alpha = \frac{\int_A \rho V^3\,dA}{\dot{m}\,\overline{V^2}}$$

For laminar flow in a pipe $\alpha = 2.0$

For turbulent flow, large Reynold's number, $\alpha \approx 1.0$

Velocity Profiles

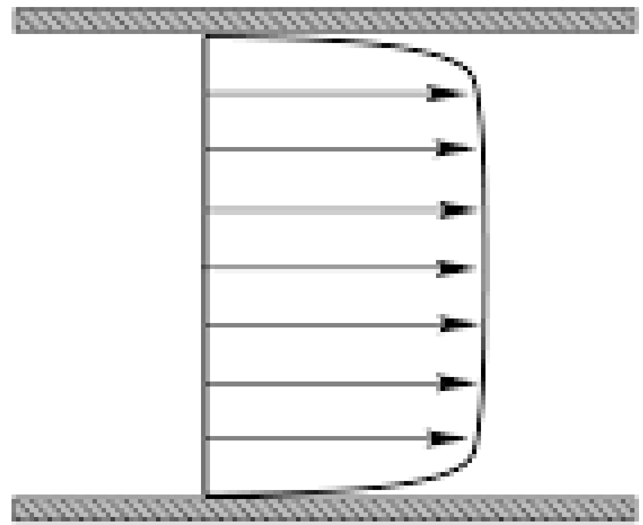
Laminar Flow



(a)

$\alpha = 2.0$

Turbulent Flow



(b)

$\alpha = 1.0$

Calculation of head loss

$$h_{LT} = h_L + h_{LM}$$

h_L = Major loss due to frictional effects in fully developed flow

h_{LM} = Minor losses due to fittings, entrance, area changes

h_L Major loss

For FD flow in a horizontal pipe (from B eqn)

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\overline{V_1^2}}{2} + g\, Z_1 \right) = \left(\frac{p_2}{\rho} + \alpha_2 \frac{\overline{V_2^2}}{2} + g\, Z_2 \right) + h_{LT}$$

$$\frac{p_1 - p_2}{\rho} = \frac{\Delta P}{\rho} = h_L$$

$$h_{LT} = h_L + h_{LM}$$

Laminar Flow

$$\Delta P = \frac{128\mu L Q}{\pi D^4} = \frac{128\mu L V\left(\pi D^2 / 4\right)}{\pi D^4} = 32\frac{L}{D}\frac{\mu V}{D}$$

$$h_L = \frac{\Delta P}{\rho} = \frac{64}{\mathrm{Re}}\frac{L}{D}\frac{\overline{V}^2}{2} = f\frac{L}{D}\frac{\overline{V}^2}{2}$$

f ≡ Friction factor

Turbulent Flow

$$\Delta P = \Delta P\left(D, L, e, \overline{V}, \rho, \mu\right)$$

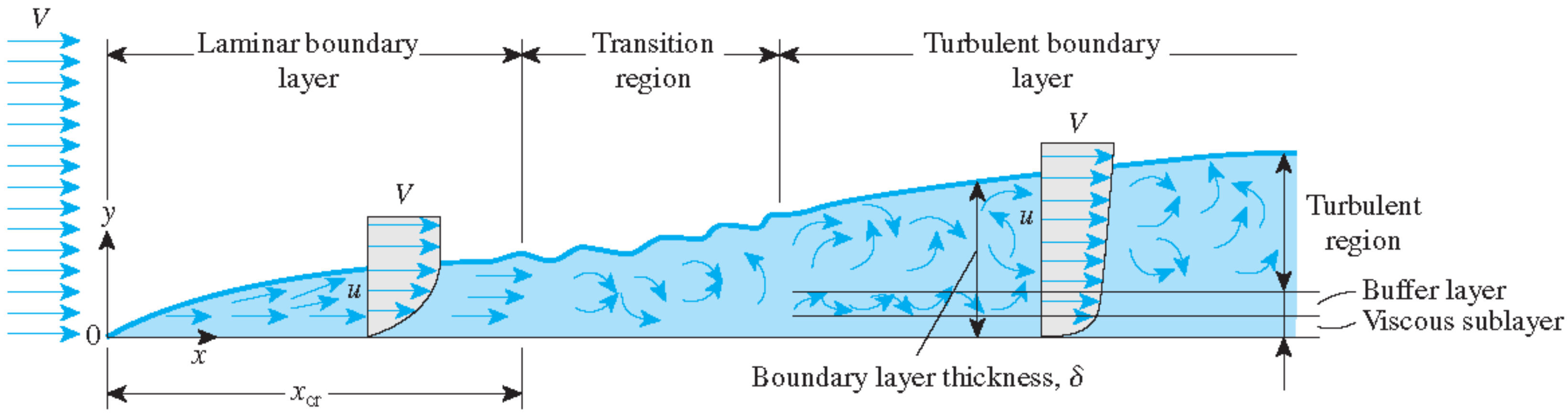
$$h_L = f\frac{L}{D}\frac{\overline{V}^2}{2}$$

f ≡ Friction factor,
determined experimentally

Value of f is needed to calculate the pressure drop

Velocity Profile in Turbulent Flow

Viscous sublayer, transition and turbulent core, 1/7 th Power Law

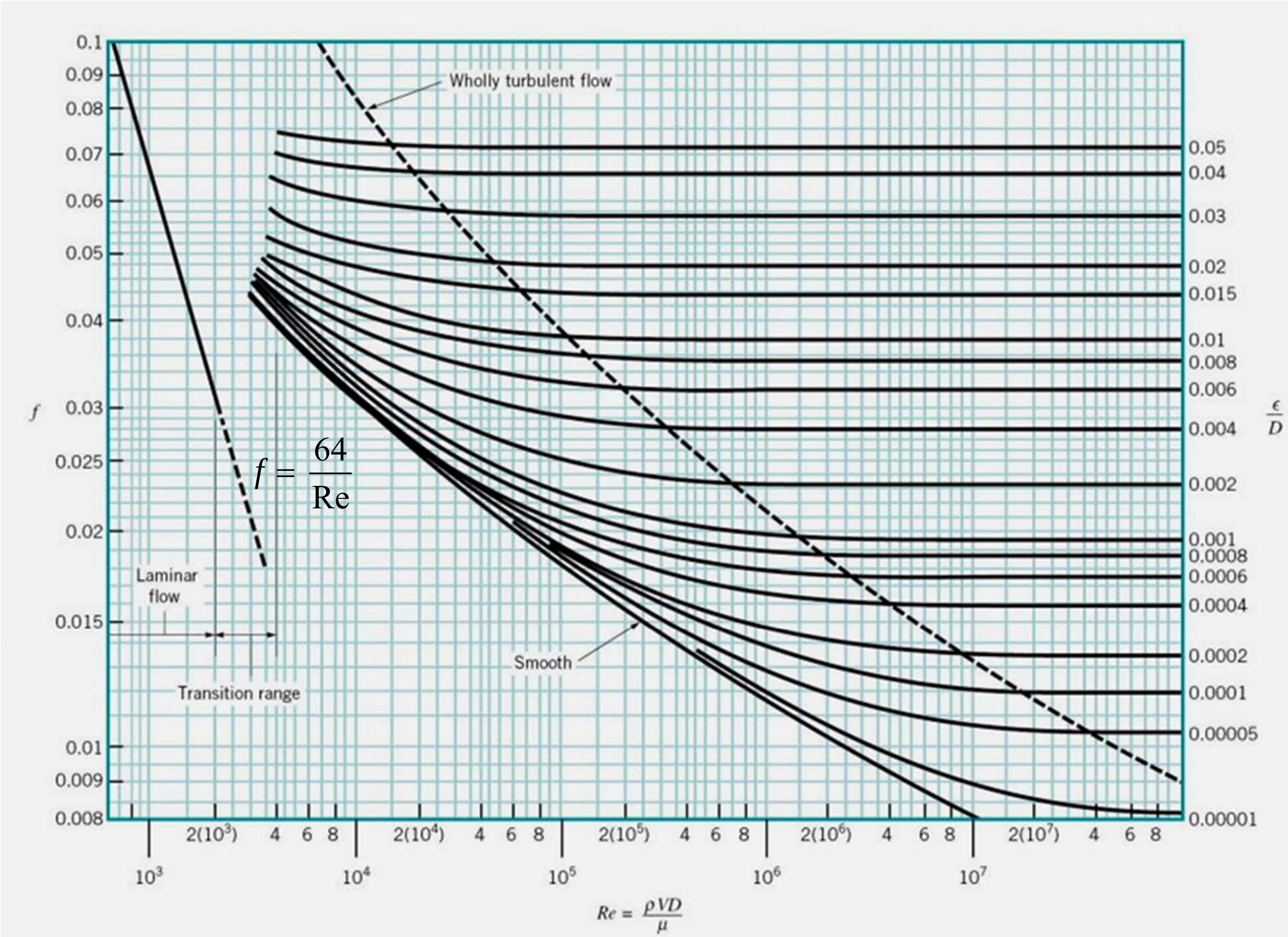


Turbulent Boundary Layer

- All BL variables [*U(y), δ, δ*, θ*] are determined empirically.
- One common empirical approximation for the time-averaged velocity profile is the **one-seventh-power law**

$$\frac{U}{U_e} = \left(\frac{y}{\delta}\right)^{1/7} \qquad y \leq \delta$$

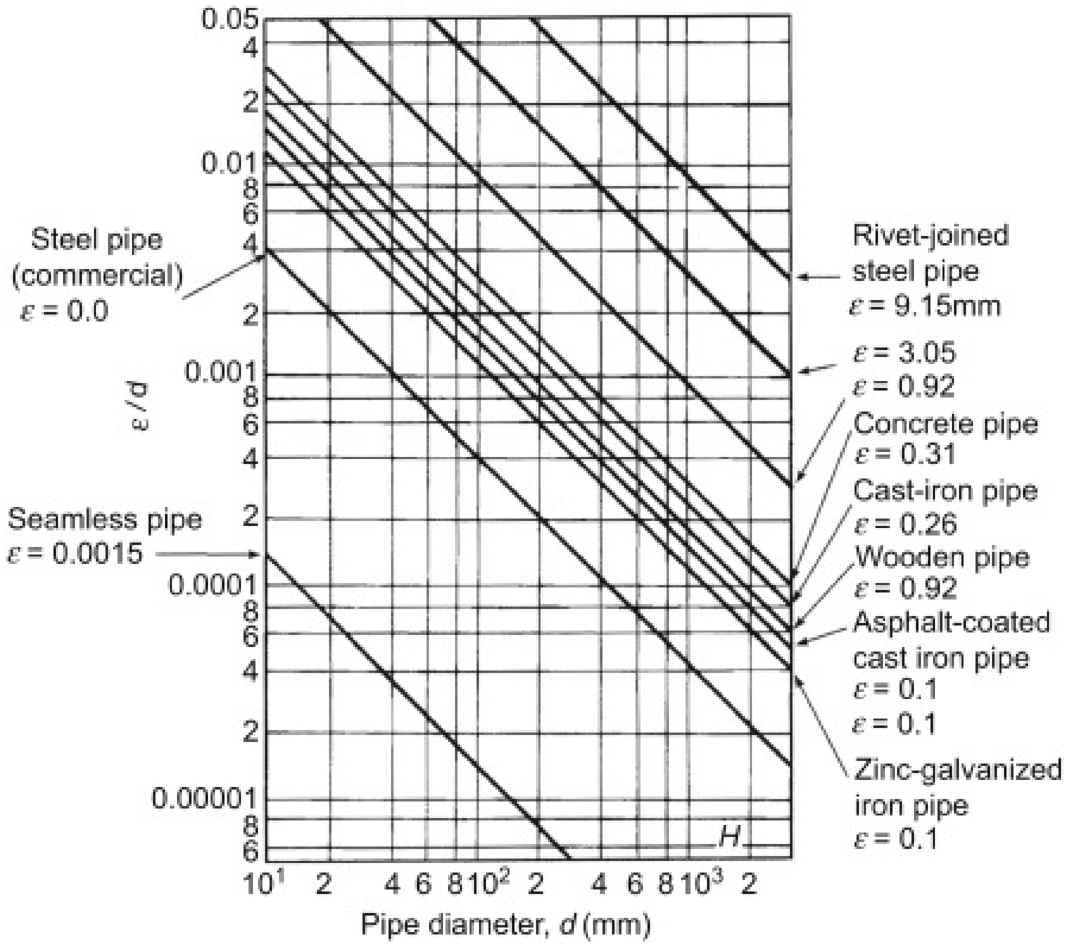
$$\frac{U}{U_e} \cong 1 \qquad y > \delta$$



Moody Diagram (Friction Factor - to calculate major losses)

$$h_{LT} = h_L + h_{LM}$$

To evaluate ε/D



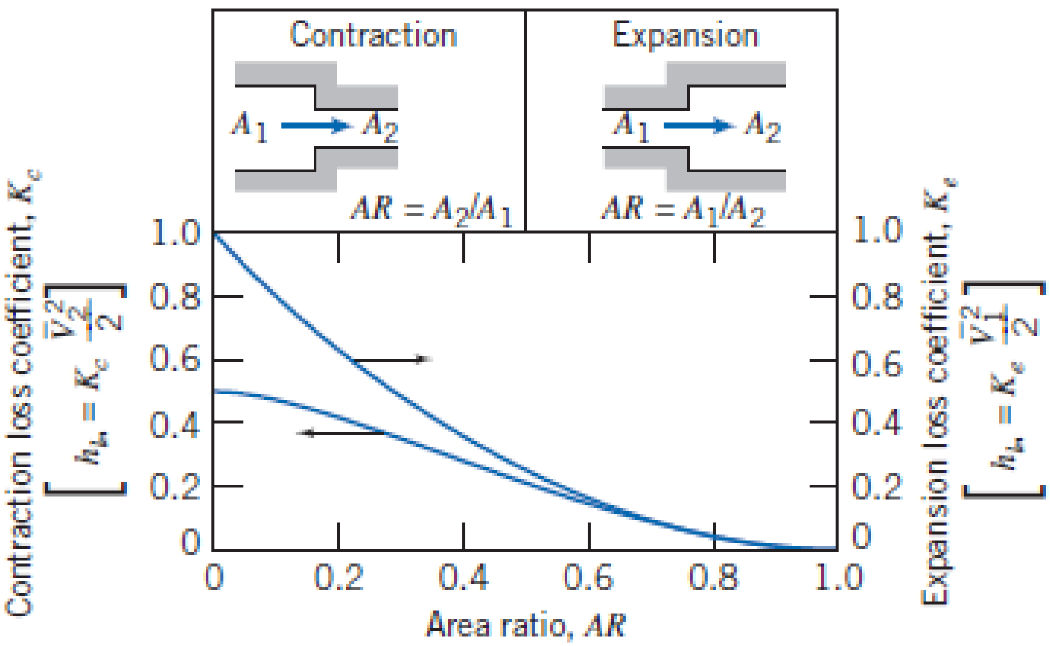
Steps to calculate head loss

1. Find Re
2. Find ε/D from figure
3. Find f from Moody's diagram (for turbulent flow)
4. For laminar flow f = 64/Re, independent of roughness, viscous layer is quite thick, wall roughness has no effect
5. Find h_L

$$h_L = 64/\mathrm{Re} \quad (Lam) \qquad\qquad h_L = f\frac{L}{D}\frac{\overline{V}^2}{2} \quad (Turb)$$

$$h_{LT} = h_L + h_{LM}$$

Minor Losses – Sudden Contraction/Expansion



$$h_{LM} = K \frac{\overline{V}^2}{2}$$

$$h_{LM} = f \frac{L_e}{D} \frac{\overline{V}^2}{2}$$

K is the loss coefficient, to be determined experimentally

L_e is the equivalent length of straight pipe

Minor Losses – Equivalent Lengths

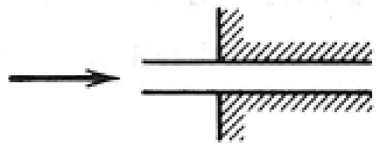
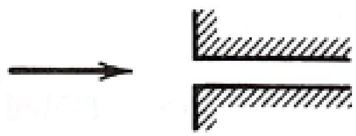
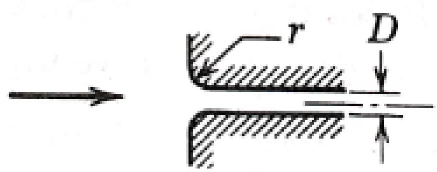
$$h_{LM} = f\frac{L_e}{D}\frac{\overline{V}^2}{2}$$

Representative Dimensionless Equivalent Lengths (L_e/D) for Valves and Fittings

Fitting Type	Equivalent Length,[a] L_e/D
Valves (fully open)	
Gate valve	8
Globe valve	340
Angle valve	150
Ball valve	3
Lift check valve: globe lift	600
angle lift	55
Foot valve with strainer: poppet disk	420
hinged disk	75
Standard elbow: 90°	30
45°	16
Return bend, close pattern	50
Standard tee: flow through run	20
flow through branch	60

[a]Based on $h_{l_m} = f(L_e/D)(\overline{V}^2/2)$.

Minor loss coefficients for pipe entrances

Entrance Type		Minor Loss Coefficient, K^a
Reentrant		0.78
Square-edged		0.5
Rounded		r/D: 0.02 | 0.06 | ≥ 0.15 K: 0.28 | 0.15 | 0.04

Based on $h_{LM} = K\dfrac{\overline{V}^2}{2}$ where $\overline{V}$ is the mean velocity in the pipe

Solution of Pipe Flow Problems

Relevant Equations

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{\overline{V_1^2}}{2}+gZ_1\right)=\left(\frac{p_2}{\rho}+\alpha_2\frac{\overline{V_2^2}}{2}+gZ_2\right)+h_{LT} \qquad (A)$$

All terms are energy per unit mass

$$h_L = f\frac{L}{D}\frac{\overline{V^2}}{2}, \quad \textit{major head loss}, \qquad (B)$$

f = 64/Re, for laminar flow

f from Moody diagram or $f=\dfrac{0.3164}{\mathrm{Re}^{0.25}}$ for smooth pipes for turbulent flow

$$h_{LM} = K\frac{\overline{V^2}}{2} \quad (\text{minor loss, fittings, bends, abrupt area change etc}) \qquad (C1)$$

$K = \textit{Loss coefficient}$ (experimentally determined)

$$h_{LM} = f\frac{L_e}{D}\frac{\overline{V^2}}{2} \quad \textit{mostly for valves fittings and bends} \qquad (C2)$$

L_e *Equivalent length of straight pipe*

Solution of Pipe Flow Problems – contd.

Head at 1 + Pump Head = Head at 2 + Losses

$$\dot{W}_{in} = \dot{m}\left[\left(\frac{p_2}{\rho} + \alpha_2 \frac{\overline{V_2^2}}{2} + g\,Z_2\right) + h_{LT} - \left(\frac{p_1}{\rho} + \alpha_1 \frac{\overline{V_1^2}}{2} + g\,Z_1\right)\right]$$

All terms are energy per unit mass

$$Pump\ Head = \frac{\dot{W}_{in}}{\dot{m}} \left(in\ \ \frac{m^2}{s^2}\right), \quad Power = \rho Q \times Pump\ Head, (W)$$

$$\Delta P = \phi\left(L, Q, D, e, \Delta Z, \ system\ config., \rho, \mu\right)$$

Solution of Pipe Flow Problems – contd.

$$\Delta P = \phi\left(L, Q, D, e, \Delta Z, \ system\ config., \rho, \mu\right)$$

Once the pipeline layout and the fluid properties are fixed

$$\Delta P = \phi\left(L, Q, D\right)$$

Possible cases

$$Case\ i) \quad L, Q, D \quad known \quad \Delta P \ unknown$$

$$Case\ ii) \quad \Delta P, Q, D \quad known \quad L \quad unknown$$

$$Case\ iii) \quad \Delta P, L, D \quad known \quad Q \quad unknown$$

$$Case\ iv) \quad \Delta P, L, Q \quad known \quad D \quad unknown$$

$Case \;\; i) \quad L, Q, D \qquad known \quad \Delta P \;\; unknown$

- Calculate Re
- Obtain f
- Calculate h_L Eq. (B)
- Calculate h_{LM} Eq. (C1/C2)
- Calculate ΔP from Eq. A

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\overline{V_1^2}}{2} + g Z_1\right) = \left(\frac{p_2}{\rho} + \alpha_2 \frac{\overline{V_2^2}}{2} + g Z_2\right) + h_{LT} \qquad (A)$$

$$h_L = f \frac{L}{D} \frac{\overline{V^2}}{2}, \quad major\ head\ loss, \qquad (B)$$

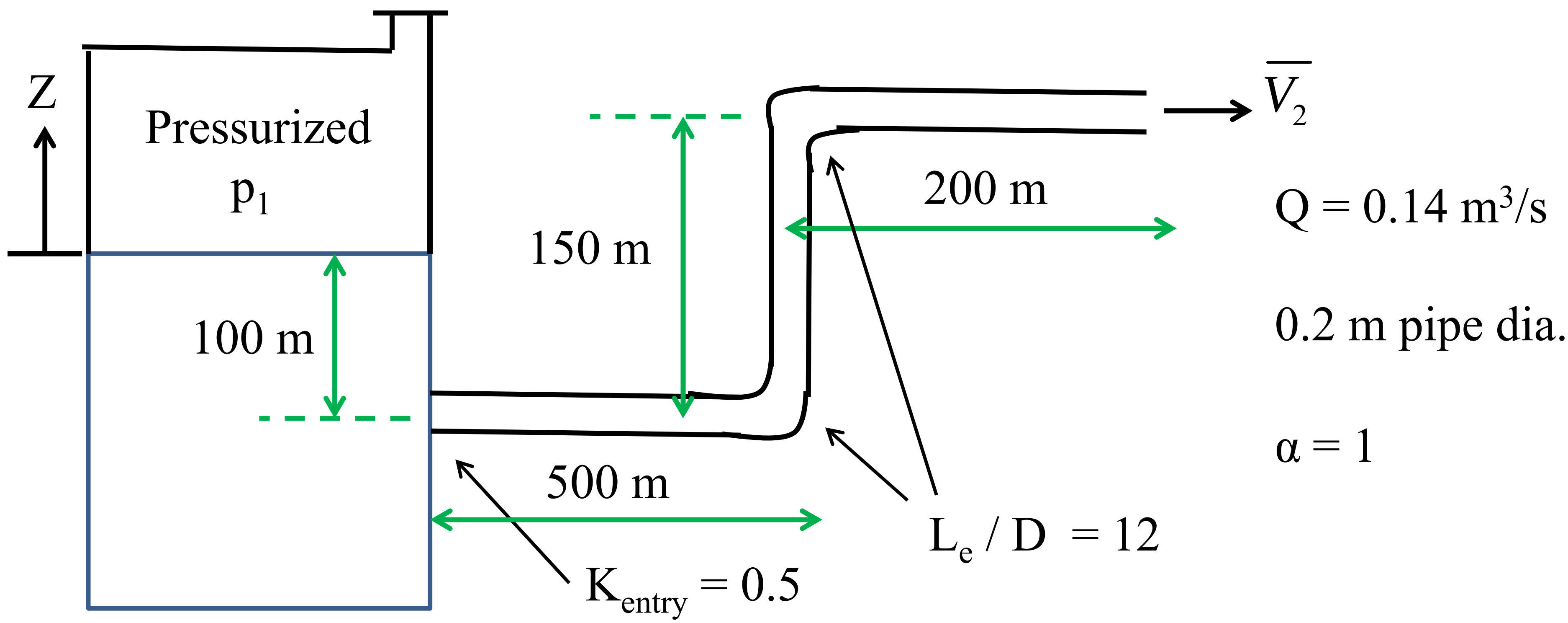
$$f = {}^{64}\!/_{\mathrm{Re}} - Lamninar \quad OR\ Moody\ diagram - Turbulent$$

$$h_{LM} = K \frac{\overline{V^2}}{2} \qquad (C1) \qquad\qquad h_{LM} = f \frac{L_e}{D} \frac{\overline{V^2}}{2} \qquad (C2)$$

$K = Loss\ coefficient$ $\qquad\qquad$ L_e $Equiv.\ length\ of\ straight\ pipe$

Example of Case (i)

L, Q, D known, ΔP unknown



Water flows from a reservoir at $0.14\ m^3/s$ through a 0.2m id pipe.
Properties: $\mu = 1.3 \times 10^{-3} Ns/m^2$, $\varepsilon/D = 0.0013$. Find the gage pressure p_1

$$\left(\overset{=0}{\cancel{\frac{p_1}{\rho}}} + \alpha_1 \overset{=0}{\cancel{\frac{\overline{V_1^2}}{2}}} + g \overset{=0}{\cancel{Z_1}}\right) = \left(\cancel{\frac{p_2}{\rho}} + \alpha_2 \frac{\overline{V_2^2}}{2} + g Z_2\right) + h_{LT} \qquad (A)$$

$$h_L = f \frac{L}{D} \frac{\overline{V}^2}{2}, \quad \textit{major head loss}, \qquad (B)$$

$$h_{LM} = K \frac{\overline{V}^2}{2} \qquad (C1) \qquad\qquad h_{LM} = f \frac{L_e}{D} \frac{\overline{V}^2}{2} \qquad (C2)$$

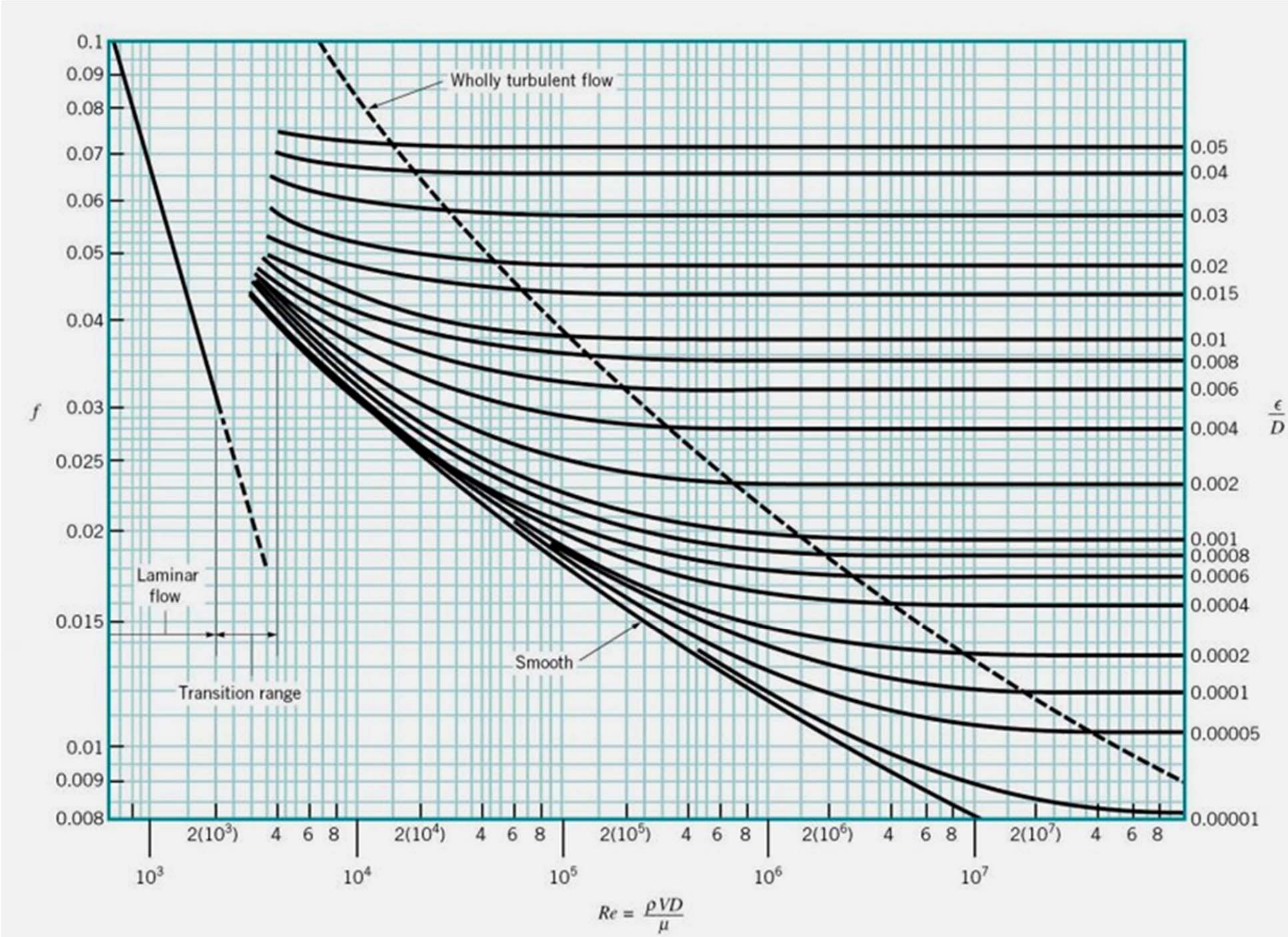
$K = \textit{Loss coefficient}$ $\qquad$ L_e *Equiv. length of straight pipe*

$$V_2 = \frac{Q}{A_2} = 0.14 \times \frac{4}{\pi} \cdot \frac{1}{(0.2)^2} = 4.46 \text{ m/s}$$

$$Re = \frac{\rho V D}{\mu} = 6.83 \times 10^5, \quad \frac{\epsilon}{D} = 0.0013$$

$\Rightarrow$ from Moody diagram $\rightarrow$ $f = 0.021$

$L = 850\text{m}$, $\alpha_2 = 1$



Moody Diagram

(to calculate major losses)

$$h_{Lm} = h_{L\,entry} + 2\,h_{L,bend}$$

$$= \left(K_{ent} + 2f\frac{L_e}{D}\right)\frac{V_2^2}{2}$$

$$K_{ent} = 0.5, \quad L_e/D = 12$$

$$\therefore\ h_{LT} = f\frac{L}{D}\frac{V_2^2}{2} + K_{ent}\frac{V_2^2}{2} + 2f\frac{L_e}{D}\frac{V_2^2}{2}$$

$$= \frac{V_2^2}{2}\left[f\left(\frac{L}{D} + 2\frac{L_e}{D}\right) + K_{ent}\right]$$

$$= \frac{1}{2}(4.04)^2\left[0.021\left(\frac{850}{0.2} + 2\times 12\right) + 0.5\right] = 898\ \frac{m^2}{s^2}$$

$$p_1 = \rho\left(g z_2 + \frac{V_2^2}{2} + h_{LT}\right)$$

$$= 999\left(9.81\times 50 + \frac{1}{2}(4.46)^2 + 898\right)$$

$$\Rightarrow\ p_1 = 1.40\ MPa\ (gauge)$$

$Case \;\; ii) \quad \Delta P, Q, D \quad known \quad L \;\; unknown$

- Calculate h_{LT} from (A)
- Calculate Re, Obtain f
- Solve for L using Eq.

  (B) and/or C1,, C2

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\overline{V_1^2}}{2} + g Z_1\right) = \left(\frac{p_2}{\rho} + \alpha_2 \frac{\overline{V_2^2}}{2} + g Z_2\right) + h_{LT} \qquad (A)$$

$$h_L = f \frac{L}{D} \frac{\overline{V^2}}{2}, \quad major\ head\ loss, \qquad (B)$$

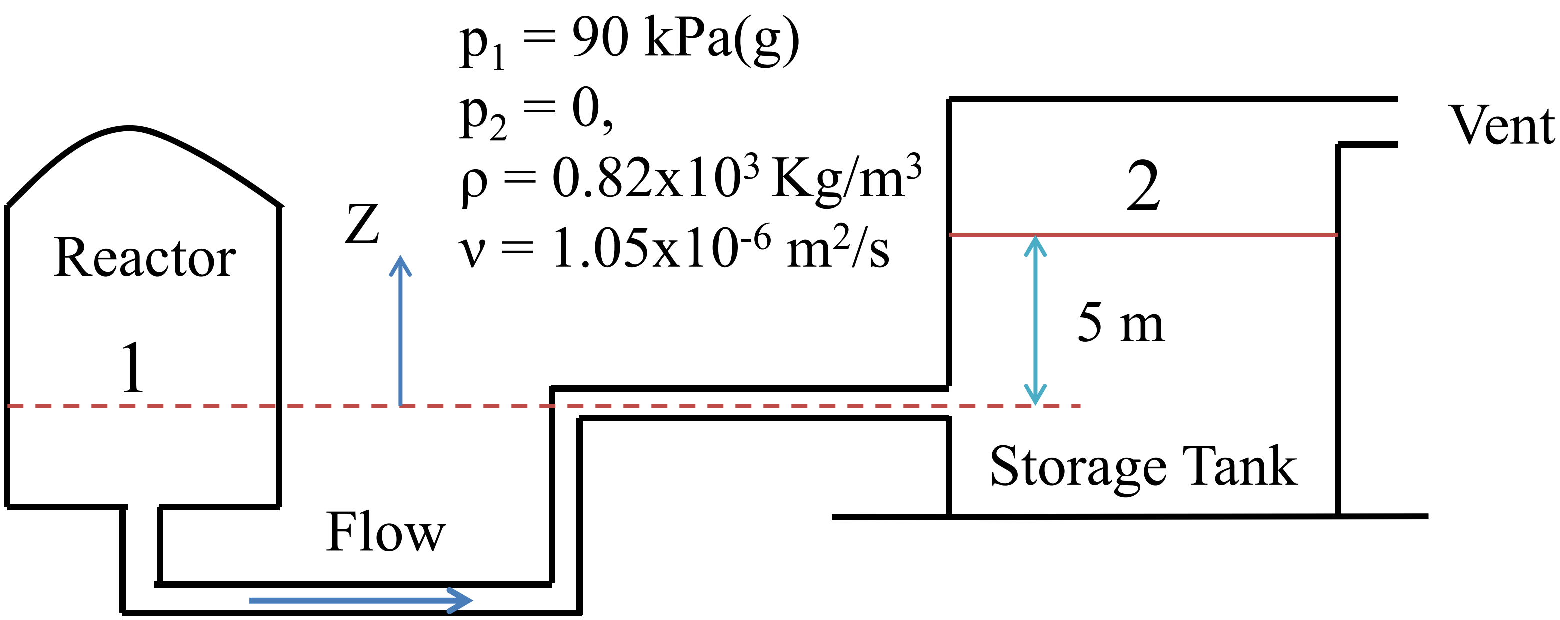
$$f = {}^{64}\!/_{\mathrm{Re}} - Lamninar \quad OR\ Moody\ diagram - Turbulent$$

$$h_{LM} = K \frac{\overline{V^2}}{2} \qquad (C1) \qquad\qquad h_{LM} = f \frac{L_e}{D} \frac{\overline{V^2}}{2} \qquad (C2)$$

$K = Loss\ coefficient$ $\qquad\qquad$ L_e $Equiv.\ length\ of\ straight\ pipe$

Example of Case (ii)

$$\Delta P, Q, D \ known, L \ \ unknown$$



$\varepsilon/D = 0.0003$, $K_{entry} = 0.5$, $K_{exit} = 1.0$, $L_e/D = 12$, pipe dia. = 0.15m

Find the total length of the straight pipe in the system

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{\overline{V_1^2}}{2}+gZ_1\right)=\left(\frac{p_2}{\rho}+\alpha_2\frac{\overline{V_2^2}}{2}+gZ_2\right)+h_{LT} \qquad (A)$$

$$h_L = f\frac{L}{D}\frac{\overline{V^2}}{2}, \quad major\ head\ loss, \qquad (B)$$

$$f = {}^{64}\!/_{\mathrm{Re}} - Lamninar \quad OR\ Moody\ diagram - Turbulent$$

$$h_{LM} = K\frac{\overline{V^2}}{2} \qquad (C1) \qquad\qquad h_{LM} = f\frac{L_e}{D}\frac{\overline{V^2}}{2} \qquad (C2)$$

$$K = Loss\ coefficient \qquad\qquad L_e\ Equiv.\ length\ of\ straight\ pipe$$

$$\frac{p_1}{\rho} = g z_2 + f \frac{L}{D} \frac{V^2}{2} + h_{INLET} + h_{EXIT} + 3 h_{L\,ELBOWS}$$

$$\epsilon / D = 0.0003, \quad \nu = 1.03 \times 10^{-6}\ m^2/s$$

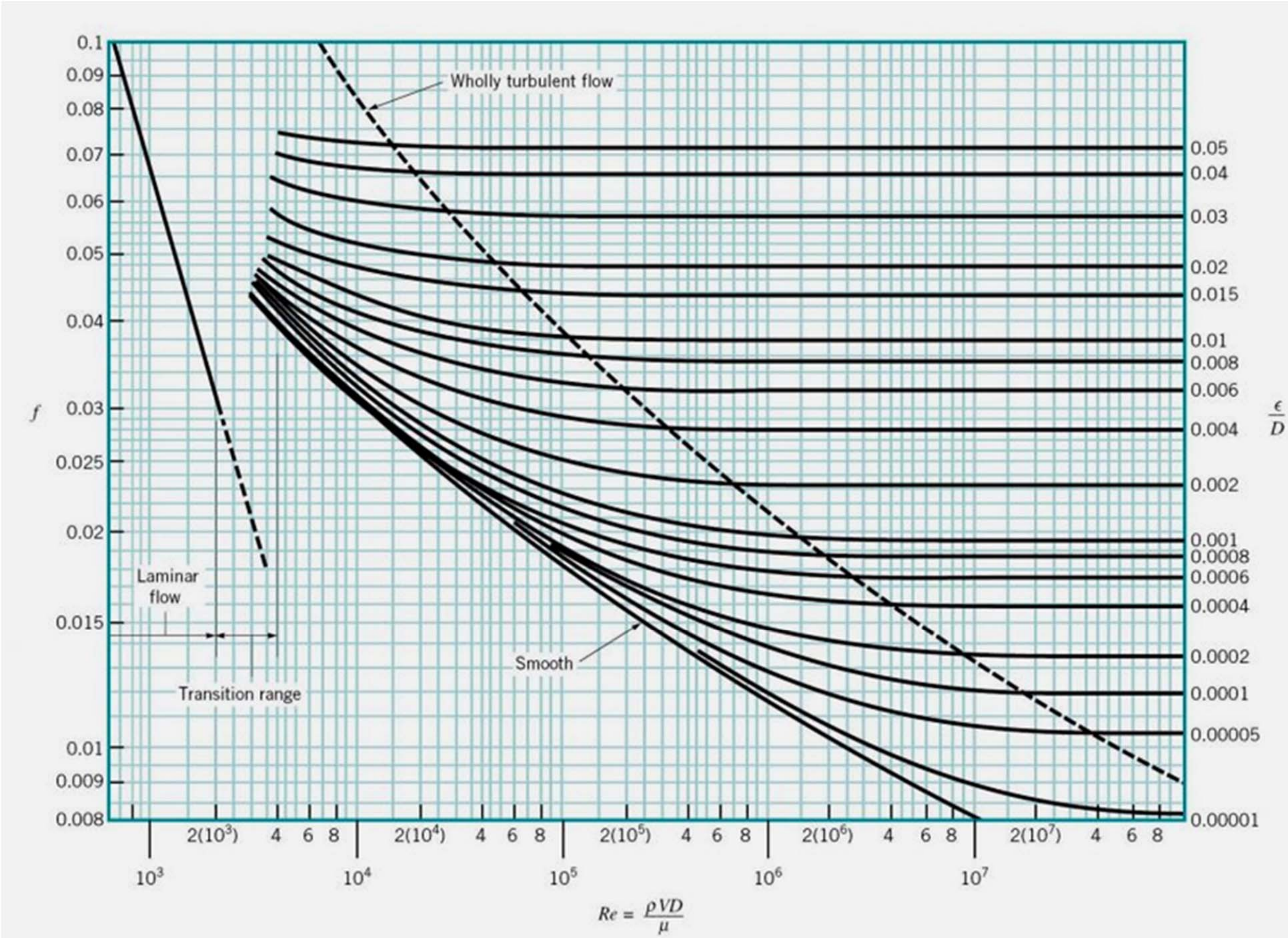
$$Re = \frac{\rho \bar{V} D}{\mu} \Rightarrow V = \frac{Q}{A} = 2.17\ m/s$$

$$\cancel{Re = 3} \rightarrow Re = 3.1 \times 10^5$$

$$f = 0.017$$

$$h_{IN} = K_{IN} \frac{V^2}{2} \qquad h_{EL} = f \frac{L_{eq}}{D} \frac{V^2}{2}$$

$$h_{EX} = K_{EX} \frac{V^2}{2}$$



Moody Diagram (to calculate major losses)

$$\frac{p_1}{\rho} = gz_2 + f\frac{L}{D}\frac{V^2}{2} + K_{IN}\frac{V^2}{2} + 3f\frac{L_{eq}}{D}\frac{V^2}{2} + K_{ex}\frac{V^2}{2}$$

$$\underline{L = 212\ m}$$

$Case\ \ iii)\quad \Delta P, L, D \quad known \quad Q\ \ unknown$

- Combine Eq. (A) with (B and/or C)
- Results in an expression of V (or Q) in terms of f
- Assume f, based on flow entirely in the rough region
- Calculate $\overline{V}$, recalculate f
- As f is a weak function of Re, two iterations are sufficient

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\overline{V_1^2}}{2} + g Z_1\right) = \left(\frac{p_2}{\rho} + \alpha_2 \frac{\overline{V_2^2}}{2} + g Z_2\right) + h_{LT} \qquad (A)$$

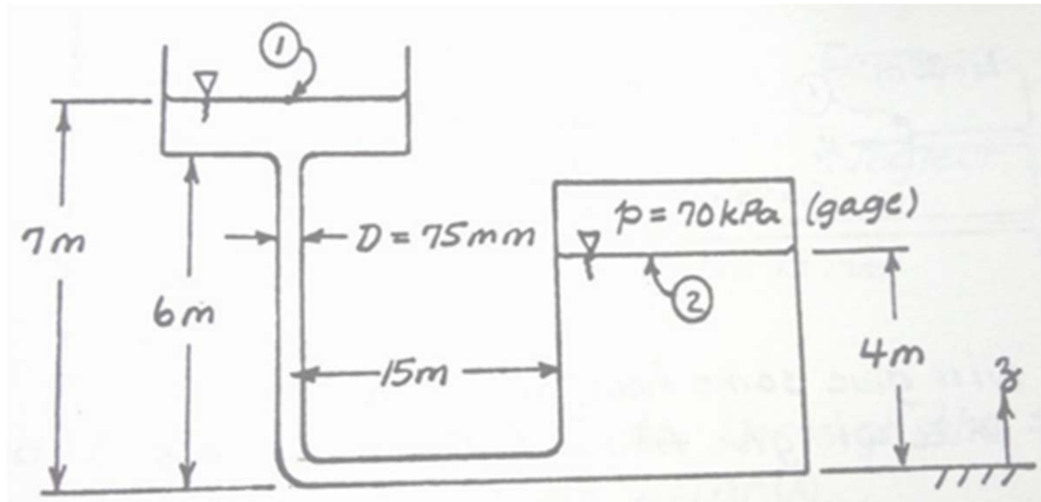
$$h_L = f \frac{L}{D}\frac{\overline{V^2}}{2}, \quad major\ head\ loss, \qquad (B)$$

$$f = {}^{64}\!/\!_{\mathrm{Re}} - Lamninar \quad OR\ Moody\ diagram - Turbulent$$

$$h_{LM} = K \frac{\overline{V^2}}{2} \quad (C1) \qquad h_{LM} = f \frac{L_e}{D}\frac{\overline{V^2}}{2} \quad (C2)$$

$$K = Loss\ coefficient \qquad L_e\ Equiv.\ length\ of\ straight\ pipe$$

The adjoining figure shows two large reservoirs containing water connected by a constant area, galvanized iron pipe (ε/D = 0.002) that has one right angle bend. The flow can be assumed to be in the fully rough region of the Moody diagram.



The surface pressure at the upper reservoir (1 in figure) is atmospheric whereas the pressure (absolute) at the lower reservoir (2 in the figure) surface is 171.3 KPa. The pipe diameter is 75 mm. Assume that the only significant losses occur in the pipe and the bend (Le/D for the bend is equal to 12). Determine the direction and magnitude of the volume flow rate of water ($\rho = 999$ kg/m^3, kinematic viscosity, $\nu = 1.1 \times 10^{-6}$ m^2/s).

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{\overline{V_1^2}}{2}+g\,Z_1\right)=\left(\frac{p_2}{\rho}+\alpha_2\frac{\overline{V_2^2}}{2}+g\,Z_2\right)+h_{LT} \qquad (A)$$

$$h_L = f\,\frac{L}{D}\frac{\overline{V^2}}{2}, \quad \textit{major head loss}, \qquad (B)$$

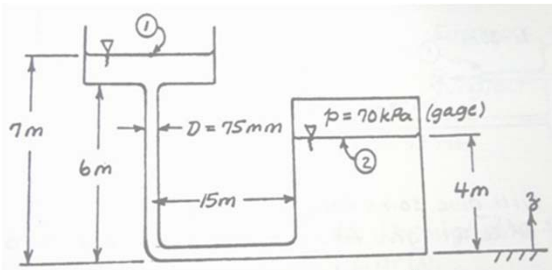
$$f = {}^{64}\!/_{\mathrm{Re}} - \textit{Lamninar} \quad \textit{OR Moody diagram} - \textit{Turbulent}$$

$$h_{LM} = K\frac{\overline{V^2}}{2} \qquad (C1) \qquad\qquad h_{LM} = f\,\frac{L_e}{D}\frac{\overline{V^2}}{2} \qquad (C2)$$

$K = \textit{Loss coefficient}$ $\qquad$ L_e *Equiv. length of straight pipe*

$\alpha = 1$

ASSUME FLOW FROM $1 \rightarrow 2$



$$h_{LT_{1-2}} = -70\times10^{3}\frac{N}{m^2} \times \frac{1}{10^3}\frac{m^3}{kg} + 9.81\frac{m}{s^2}(7-4)m$$

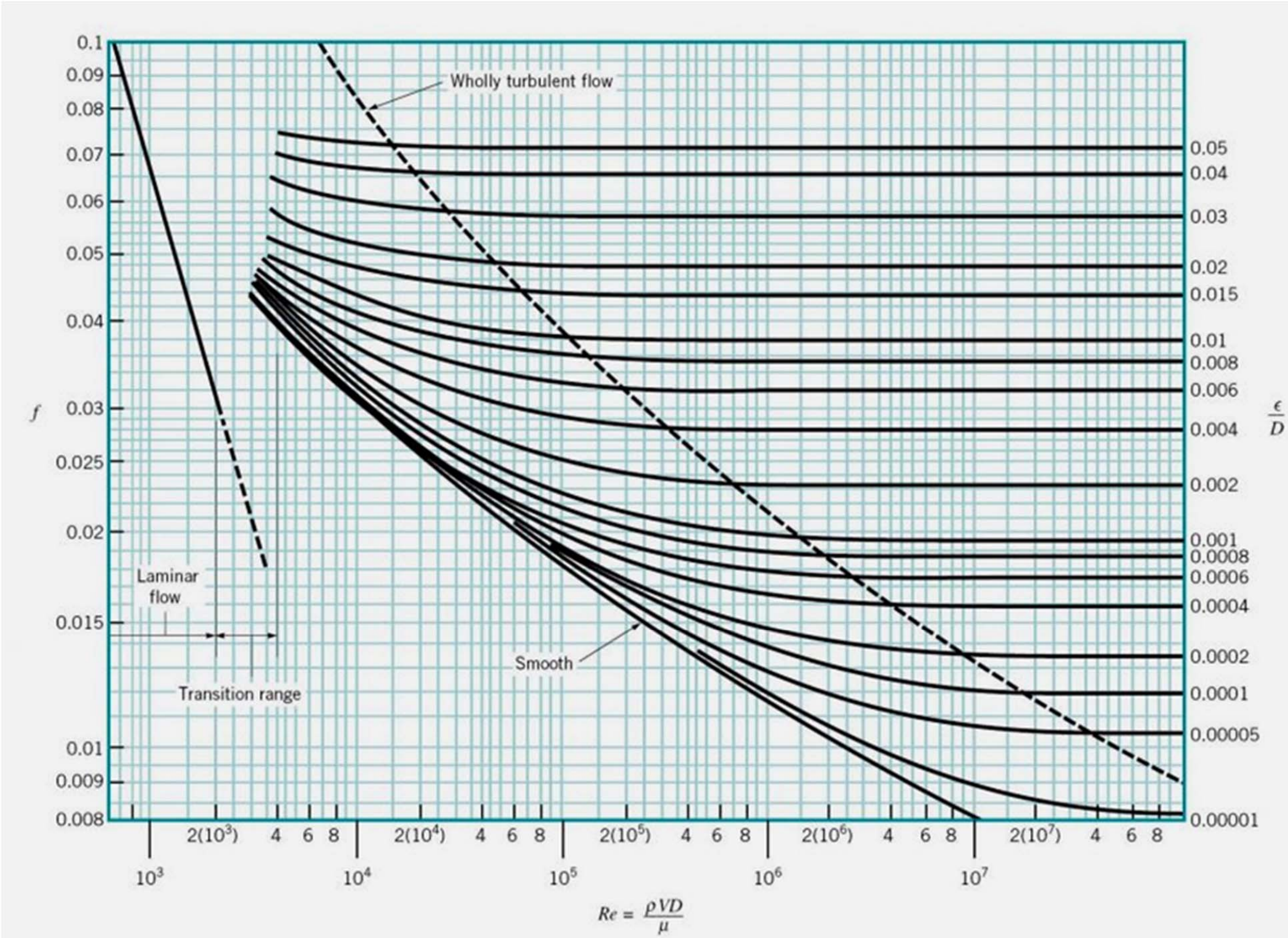
$$h_{LT_{1-2}} = -40.6\ m^2/s^2$$

h_{LT} <u>CANNOT</u> BE -ve.

FLOW 2 — 1 ✓

✓ $$h_{LT_{21}} = f\frac{L}{D}\frac{V^2}{2} + f\left(\frac{L_e}{D}\right)\frac{V^2}{2} = f\left(\frac{L}{D}+\frac{L_e}{D}\right)\frac{V^2}{2}$$

$$= 40.6\ m^2/s^2$$



Moody Diagram (to calculate major losses)

~~$L = 21\,m$, $L_e/D = 12$~~ ~~$\bar{V} = ?$~~

ITERATION NEEDED.

CHOOSE f. ~~$L_e/D = 0.002$~~ $\xrightarrow{\text{M Diagram}}$

$f \approx 0.023$ ✓

✓ $\bar{V} = \left[\dfrac{2 h_{LT}}{f\left(\frac{L}{D} + \frac{L_e}{D}\right)} \right]^{1/2} = 3.48\,m/s$

$\nu = ? \Longrightarrow Re = \dfrac{VD}{\nu} = 2.37 \times 10^5$

$f = 0.024 \longrightarrow$ RECALCULATE $\bar{V}$

$? = f \longleftarrow$ FIND Re

$Q = VA = 0.015\,m^3/s$