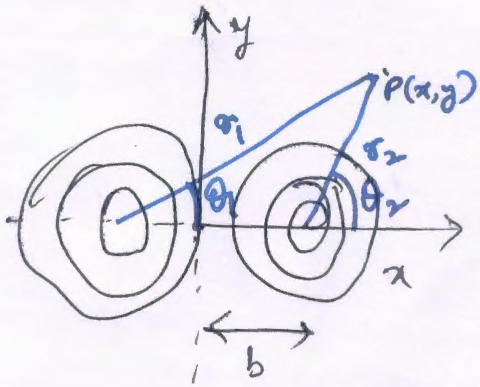
Q.4 Vortex Pair



$$F(z-b) = -i\frac{\Gamma}{2\pi}\ln(z-b)$$

$$F(z-(-b)) = -i\frac{(-\Gamma)}{2\pi}\ln(z-(-b))$$

$$\text{Superposed } F = -i\frac{\Gamma}{2\pi}\ln\frac{z-b}{z+b}$$

$$= -i\frac{\Gamma}{2\pi}\ln\frac{(x+iy-b)}{(x+iy+b)}$$

$$= -i\frac{\Gamma}{2\pi}\ln\left[\frac{(x-b)+iy}{(x+b)+iy}\right]$$

$$= -i\frac{\Gamma}{2\pi}\ln\frac{r_2 e^{i\theta_2}}{r_1 e^{i\theta_1}}$$

$$= -i\frac{\Gamma}{2\pi}\ln\left(\frac{r_2}{r_1}\right) - i\frac{\Gamma}{2\pi}i[\theta_2-\theta_1]$$

$$= -i\frac{\Gamma}{2\pi}\ln\left(\frac{r_2}{r_1}\right) + \frac{\Gamma}{2\pi}[\theta_2-\theta_1]$$

$$= \Phi + i\psi.$$

$$\Phi = \frac{\Gamma}{2\pi}[\theta_2-\theta_1]$$

$$\psi = -\frac{\Gamma}{2\pi}\ln\left(\frac{r_2}{r_1}\right)$$

$$\Phi = \frac{\Gamma}{2\pi}\left[\tan^{-1}\frac{y}{x-b} - \tan^{-1}\frac{y}{x+b}\right]$$

$$v_x = \frac{\partial\Phi}{\partial x} = \frac{\Gamma}{2\pi}\left[\frac{1}{\cdots y^2}\cdot\frac{(-y)}{(x-b)^2}\right]$$

$$v_y = \frac{\partial \phi}{\partial y} = \frac{\Gamma}{2\pi}\left[\frac{1}{1+\frac{y^2}{(x-b)^2}} \cdot \frac{1}{(x-b)} - \frac{1}{1+\frac{y^2}{(x+b)^2}} \cdot \frac{1}{x+b}\right]$$

$$= \frac{\Gamma}{2\pi}\left[\frac{x-b}{(x-b)^2+y^2} - \frac{x+b}{(x+b)^2+y^2}\right]$$

$$\left| \, v_y \right|_{\substack{@\, x=b \\ y=0}} = \frac{\Gamma}{4\pi b}$$

and @ $y=0$
$x=-b$

(4) Problem of baloon

constant temperature

$$\rho = \frac{PM}{RT} = \frac{P \times 0.029}{8.314 \times (273+30)}$$

$$dP = -\rho g \, dz = -\left[\frac{0.029 \times 9.8}{8.314 \times 303}\right] P \, dz$$

$$\Rightarrow \ln \frac{P}{1.013 \times 10^5} = -\left[\quad\right] H$$

$$\ln \left[\frac{\left(\frac{0.6}{\pi}\right) 8.314 \times 303}{0.029 \times 1.013 \times 10^5}\right] = -\left[\quad\right] H$$

$$\Rightarrow H = \qquad ?$$

Density of balloon

$$= \frac{0.1 \text{ kg}}{\frac{\pi}{6}(1)^3} = \frac{0.6}{\pi} \frac{\text{kg}}{\text{m}^3}$$

$$\Downarrow$$

Corresponding pressure

$$P = \frac{\rho RT}{M}$$

$$= \frac{\left(\frac{0.6}{\pi}\right) 8.314 \times 303}{0.029}$$

$= 16590.37$ Pa.

$= 0.1659 \times 10^5$ Pa.

(ii) Temperature decreases with height at a rate of $0.0065\ K\ m^{-1}$

$$\Rightarrow \quad T = 303 - 0.0065\, z$$

$$\rho = \frac{P \times 0.029}{8.314\left[303 - 0.0065 z\right]}$$

$$dP = -\rho g\, dz = \frac{-P \times 0.029\, g}{8.314} \frac{dz}{303 - 0.0065 z}$$

$$\ln \frac{P}{P_0} = -\frac{0.029 g}{8.314} \left[\ln (303 - 0.0065 z)\left(-0.0065\right)\right]_{z=0}^{z=z}$$

$$= \frac{0.029 \times 9.81 \times 0.0065}{8.314} \ln \left(\frac{303 - 0.0065 z}{303}\right)$$

$\ln\left(\frac{P}{P_0}\right)$ remains same as previous problem.

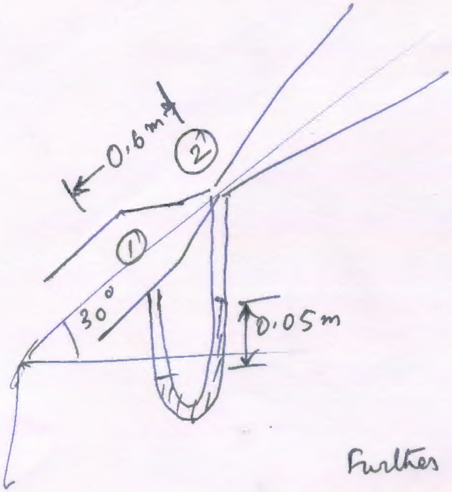
Calculate z.

$$T = T_0 - \beta z$$

$$\frac{dP}{dz} = -\rho g = -g \frac{PM}{RT} = -g \frac{PM}{R(T_0 - \beta z)}$$

$$\frac{dP}{P} = -g \frac{M}{R} \frac{dz}{(T_0 - \beta z)} \quad \Rightarrow \quad \frac{P}{P_0} = \left[\frac{T_0 - z\beta}{T_0}\right]^{\frac{gM}{R\beta}}$$

Find z



$$\frac{P_1}{\rho} + z_1 g + \frac{V_1^2}{2} = \frac{P_2}{\rho} + z_2 g + \frac{V_2^2}{2}$$

$$\Rightarrow \left(\frac{P_1}{\rho} + z_1 g\right) - \left(\frac{P_2}{\rho} + z_2 g\right) + \frac{V_1^2}{2} - \frac{V_2^2}{2} = 0 \quad \text{----- (1)}$$

Further $\cancel{P_1 + z_1 \rho g}$

$$P_1 - \left[P_2 + (z_2 - z_1)\rho_{oil} g\right] = 0.05(\rho_{mano} - \rho_{oil})g$$

~~Given~~

$\cancel{z_2 - z_1 = 0.6}$

$$\Rightarrow \left(\frac{P_1}{\rho_{oil}} + z_1 g\right) - \left(\frac{P_2}{\rho_{oil}} + z_2 g\right) = \frac{0.05(\rho_{mano} - \rho_{oil})}{\rho_{oil}} g$$

$$\left.\begin{aligned} D_1 &= 0.4 \\ D_2 &= 0.2 \end{aligned}\right\}$$

$A_1 = \frac{\pi}{4}(0.4)^2$

$A_2 = \frac{\pi}{4}(0.2)^2$

$\frac{A_1}{A_2} = 4$

$V_2 = \frac{A_1}{A_2} V_1$

$= 4 V_1$

Putting the above in Eqn. (1)

$$\frac{0.05(13600 - 700)g}{700} + \frac{V_1^2}{2} - \frac{16 V_1^2}{2} = 0$$

$$\Rightarrow \frac{15 V_1^2}{2} = \frac{0.05(13600 - 700)g}{700}$$

$$\Rightarrow V_1 =$$

where $C_v = 0.98$

$$Q = V_1 \frac{\pi}{4}(0.4)^2 C_v$$

$$\approx 0.13 \ m^3/s.$$