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Process Heat Transfer.

A system is only defined when we define its boundary.
control surface → control mass.] – two approaches.
control volume – fixed volume

Energy advection.

→ c. surface.
Energy in → CV → Energy out
Energy in due to mass in → CV → Energy out due to mass out
open system.

$(\Delta E)_{system}$ for isolated system $= 0$.
$(\Delta E)_{closed\ system} = Q - W$.

→ Energy conservation (can be written in a time interval).

$E_{in} - E_{out} + E_{gen} = E_{accumulated}$

or $\dfrac{dE_{acc}}{dt} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen}$

$E_{generated}$ = volumetric phenomena.
$E_{accumulated}$ = mass phenomena = depends on the mass of the system.
E_{in} & E_{out} → depends on surface of system.

E
- Mechanical E
  - P.E
  - K.E
- Internal E
  - Thermal E . . .
  - Chemical E
  - nuclear E.

Modes of heat transfer (heat enters to system through its boundary)

(i) Convection (ii) Conduction (iii) Radiation.

For convection - there has to be a bulk motion of liquid.

Conduction. (for any stationary system, liq, gas or solid, it becomes prominent).

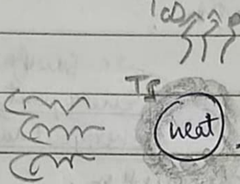
steel wool around a furnace works on principles of conduction.

heat flux = $q = Q/A$ (in Watt/m^2)

heat Rate = Q. (in watt).

convection.

Radiation.



→ conduction in area close to heat source are prominent.

By Fourier's Law of Heat Conduction.

$$q_{conduction} = -k\frac{dT}{dx}$$

Convection = Newton's Law of cooling (for moving systems).

$$q = h(T_S - T_\infty)$$

Conduction = Fourier's law $q = -k\dfrac{dT}{dx}$ (at normal temp.)

Radiation - Stefan Boltzmann Law (at high temp.)

$$q = \epsilon\sigma(T_S^4 - T_\infty^4).$$

equilibrium → no change / transfer all points at same temp.

steady state - transfer takes place but no change wrt. time.

2nd Law of thermodynamics governs ~~temp~~ heat transfer from high temp to low temperature.

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Heat transfer by conduction.

Q_{cond} [heat rate] by conduction (W) $\propto A$

$\propto \frac{\partial T}{\partial x}$

$Q_{cond,x} = -kA\frac{\partial T}{\partial x}$ [Fourier law → phenomological law]

$Q_{cond,x} \propto A\frac{\partial T}{\partial x}$.

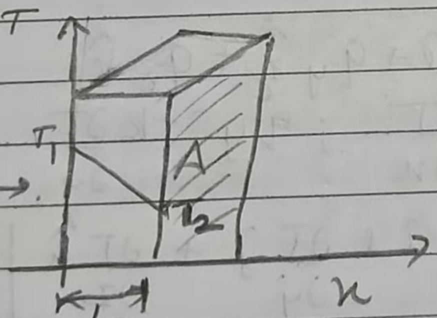
k = thermal conductivity = constt of proportionality

↓

just like μ ← It is a flow property, a material property.

k is related to KE of molecules

A = ⊥ to dir. of heat transfer.



$k_{gas} = f(T)$

at higher Temp k_{gas} is higher

$\neq f(P)$

no change in kgas with change in pressure.

Heat T. in Solids by conduction

1) Lattice energy

2). Flow of electrons (reason why metals are good conductor).

exception - Diamond - good conductor of ~~electricity~~ thermal heat, not of electricity.

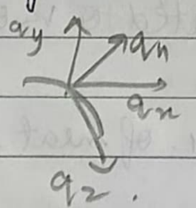
Thermal conductivity of fibrous / porous materials is high ~~less~~ due to air being trapped in it ; but they are good insulators of electricity.

Fourier's Law

Assuming : 1) Heat is a 1-dimensional flow
2) Steady state.

$$\Rightarrow Q_{\text{conduction}, x} = -kA \frac{\partial T}{\partial x}.$$

<u>Heat flux is a vector quantity</u> ($\because T = f(x,y) \Rightarrow$ we will have Q_x & Q_y & thus q_x & q_y)

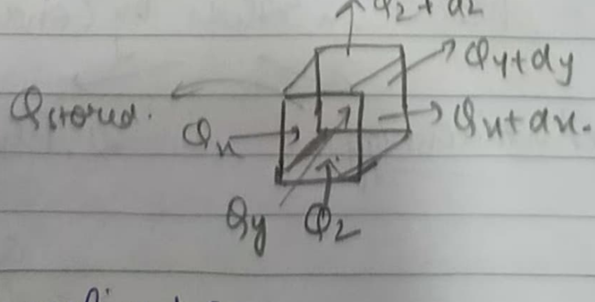


$$q_n = q_x \hat{i} + q_y \hat{j} + q_z \hat{k}$$

$$q_x = k \frac{\partial T}{\partial x} \; ; \; q_y = -k \frac{\partial T}{\partial y} \; ; \; q_z = -k \frac{\partial T}{\partial z}.$$

$$q = k \left[\frac{\partial T}{\partial x} \hat{i} + \frac{\partial T}{\partial y} \hat{j} + \frac{\partial T}{\partial z} \hat{k} \right].$$

1) Assumption - k is constant is all dir. i.e $k_x = k_y = k_z$
isotropic materials materials for which $k_x = k_y = k_z$.



For generalized case.

$$\Delta E_{stored} = E_{in} - E_{out} + E_{gen}.$$

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Assumption 1). homogeneous; const ρ, C_p, etc.
2) Isotropic $k_x = k_y = k_z = k$
3) ~~stionary~~ stationary.

$Q_x = -k\, dy\, dz \frac{\partial T}{\partial x}$ $Q_{x+dx} = Q_x + \frac{\partial Q_x}{\partial x} dx$.

$Q_x - Q_{x+dx} = +k\, dx\, dy\, dz \frac{\partial^2 T}{\partial x^2}$

$Q_y - Q_{y+dy} = k\, dx\, dy\, dz \frac{\partial^2 T}{\partial y^2}$

$Q_z - Q_{z+dz} = k\, dx\, dy\, dz \frac{\partial^2 T}{\partial z^2}$.

$Q_{gen} = \dot{q}_{gen}\, dx\, dy\, dz$.
(in W/m^3)

a volumetric phenomena
$\dot{q}_{gen}$ = Rate of heat generated / unit volume.

Q_{stored} = a mass phenomena.
$= \rho C_p\, dx\, dy\, dz \frac{\partial T}{\partial t}$

$Q_{stored} = Q_{in} - Q_{out} + Q_{gen}$.

$\rho C_p\, dx\, dy\, dz \frac{\partial T}{\partial t} = k\, dx\, dy\, dz \left(\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} + \frac{\partial T}{\partial z} \right)$
$+ \dot{q}_{gen}\, dx\, dy\, dz$.

$C_p \rho \frac{\partial T}{\partial t} = k \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] + \dot{q}_{gen}$

heat flow is in a dir.

generalised heat eq^n for conduction $T = f(x, y, z, t)$

$$\boxed{\rho C_p \frac{\partial T}{\partial t} = k \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] + \dot{q}_{gen}}$$

$$\frac{\text{mass diffusivity}}{\text{momentum diffusivity}} = \text{Smith no.}$$

heat flux vector is in a dir. normal to isothermal surface

$$\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \frac{\dot{q}_{gen}}{k} = \frac{\rho C_p}{k}\frac{\partial T}{\partial t}$$

heat diffusion equation in cartesian coordinates.

$\frac{\rho C_p}{k}$ has dimension of m^2/s.

$\frac{\rho C_p}{k} = \frac{1}{\alpha} = 1/\text{thermal diffusivity}$.

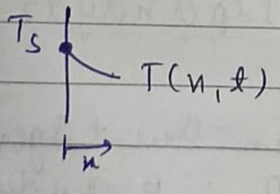
$\alpha = \frac{k}{\rho C_p}$ (in m^2/s) = thermal diffusivity. * It tells how fast heat is getting diffused.

* lower C_p - less heat absorbed more heat conducted

→ Boundary conditions for heat diffusion eq^n at surface ($x=0$)

1) at any particular surface the temp. of surface - constant.

1) Constant surface temperature

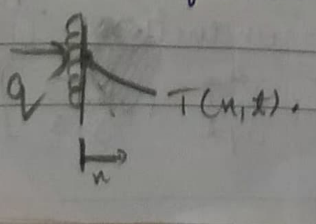


→ can happen if surface is in contact with phase changing material.

$T(0,t) = T_s$.

Boundary Condition of 1st kind → Dirichlet B.C → temp. is fixed

2). ~~Convection surface condition~~ constant surface heat flux.



↳ when there is a electrical wiring with constant voltage or current

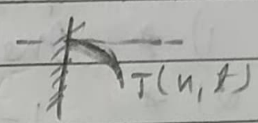
Finite heat flux.

$$-k\frac{dT}{dx}\Big|_{x=0} = q_s$$

2^{nd} Type of B.C → Neumann boundary condⁿ.

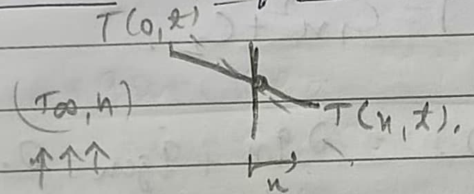
For insulated / Adiabatic surface.

$$\frac{dT}{dx}\Big|_{x=0} = 0.$$



Special type of Neumann boundary condⁿ.

3). Convection surface condition.



Convective flux = conductive flux

$$\cancel{-k}\cdot h[T_\infty - T(0,t)] = -k\frac{dT}{dx}\Big|_{x=0}$$

T_∞ = ambient temp. at infinite distance from surface.

C.V conduction analysis.

~~Special~~ $\dfrac{\partial^2 T}{\partial x^2} + \dfrac{\partial^2 T}{\partial y^2} + \dfrac{\partial^2 T}{\partial z^2} + \dfrac{\dot{q}_{gen}}{K} = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial t}.$

1 – D steady state heat conduction with no heat generation.

$\Rightarrow \dfrac{\partial^2 T}{\partial x^2} = 0. \Rightarrow \dfrac{dT}{dx} = c_1 x; T = c_1\dfrac{x^2}{2} + c_2.$

B.C to use $x=0,\ T=T_0$; (if given)
$x=L,\ T=T_L$

Our goal is to increase the heat transfer or reduce the heat loss.

Special Cases

1) * One dimensional S.S, heat conduction with heat generation
2) * 1-D, S.S, heat conduction with no heat generation
3) * 2-D, S.S., heat conduction with no heat generation.
4) * 1-D unsteady state, heat conduction with no heat generation.

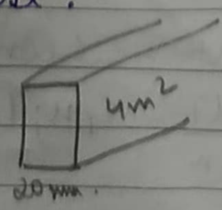
2) $\frac{\partial^2 T}{\partial x^2} = 0 \Rightarrow T = C_1 x^2 + C_2$.

1) $\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}_{gen}}{k} = 0. = \frac{\partial q_x}{\partial x} + \frac{\dot{q}_{gen}}{k} = 0.$

3) $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$

4) $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$

→ Q. 20 mm thickness of cross sectional area (2m×2m)
$K = 0.029$ W/m (K).
Temp. distributions in the wall at any instant of time is $T(x) = a + bx$ $a = 1400K$ $b = -1000 K/m$.
determine q ; Q. at $x=0$ & $x=20$ ~~mm~~. Do S.S. conditions exist.



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$\cancel{Q} = q_x = -kA \frac{\partial T}{\partial x}$

$Q_x = -0.029 \times 4 \times b$

$Q_x|_{x=0} = -0.029 \times 4 \times (-1000).$
$= 116\ W.$

$q_x|_{x=0} = \frac{116}{4} = 29\ W/m^2.$

$Q_x|_{x=20} = 116W$
$Q_x|_{x=0} = 29\ W/m^2$

∴ steady state exists ∵ Q & q does not change with time and dir.

$T = a + bx + cx^2$ $\quad c = -300$

$\dot{q}_{gen} = 1000 W/m^3$ $\quad \rho = 1600 \frac{kg}{m^3}$ $\quad k = 0.029 \frac{W}{mK}$

$c_p = 4 \frac{kJ}{kg/K}.$

Determine $\dot{q}_{in} - q_{out}$.

$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}_{gen}}{k} = \frac{\rho c_p}{k} \frac{\partial T}{\partial t}.$

$q_{in} - q_{out} = \cancel{k V \frac{\partial^2 T}{\partial x^2}}$ $\quad kA \frac{\partial T}{\partial x}$

$= \cancel{0.029 \times 2 \times 2 \times 20 \times 10^{-3} \times {}^\circ C}$

$q_x|_{x=0} - q_x|_{x=20 \times 10^{-2}}$
$= kA [b + 2cx - b - 2cx].$

$= 0.029 \times 4 \{2 \times (-300) \times 0 - 2 \times (-300) \times 20 \times 10^{-3}\}$

$= 0.029 \times 4 \times 2 \times 300 \times 20 \times 10^{-3}$

$= \underline{1.392}$

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$\dot{q}_{in})_{x=0} = 0.029 \times 4 \times [1000 + 500(0)]$.
$= 116\ W/m^2$.
$\dot{q}_{in}|_{x=L} = 117.392\ W/m^2$.

(ii). ~~$\dot{q}_{in} - \dot{q}_{out} + \dot{q}_{gen} = \dot{q}_{acc}$~~.

check again

$(\dot{q}_{in} - \dot{q}_{out})A + \dot{q}_{gen} \cancel{V} = \dot{Q}_{acc}$.
$= 1.392 \times 4 + 1000 \times 4 \times 2.0 \times 10^{-3} = \dot{Q}_{acc}$
$= \underline{13.568\ W}$.

(iii). $\dfrac{\partial^2 T}{\partial x^2} + \dfrac{\dot{q}_{gen}}{k} = \dfrac{c_p \rho}{k}\dfrac{\partial T}{\partial t}$.

$\cancel{2C}\ \dfrac{\partial T}{\partial t} = \dfrac{\dot{q}_{gen}}{k}\ \dfrac{\cancel{k}}{c_p \rho}$

$= -\dfrac{300}{4 \times 1600}$

$2C + \dfrac{\dot{q}_{gen}}{k} = \dfrac{c_p \rho}{k}\dfrac{\partial T}{\partial t}$.

$\left(-600 + \dfrac{1000}{0.029}\right) = \dfrac{4 \times 1600}{0.029}\dfrac{\partial T}{\partial t}$.

$\boxed{\dfrac{\partial T}{\partial t} = 0.15353\ K/sec}$

→ $$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{q_{gen}}{K}=\frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

<u>Case 1</u> → 1d S.S; no heat generation

$$K\frac{\partial^2 T}{\partial x^2}=0.$$

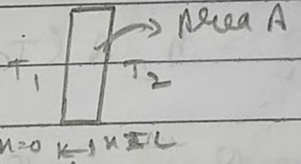
$$\Rightarrow \frac{d}{dx}\left(K\frac{dT}{dx}\right)=0.$$

$$\frac{dq_x}{dx}=0$$

$$\Rightarrow q_x \neq f(x).$$

Heat transfer does not ~~dif~~ change in dir. of heat transfer or ⊥ to it.

∴ Temperature profile = $T(x)=C_1x+C_2$



$$T(0)=T_1=C_2$$
$$T(\Delta x)=T_2=C_1\Delta x+C_2$$
$$\frac{T_2-T_1}{\Delta x}=C_1$$
$$\therefore T(x)=\frac{T_2-T_1}{\Delta x}x+T_1.$$

For $\Delta x=L$

$$\Rightarrow \boxed{T(x)=\left(\frac{T_2-T_1}{L}\right)x+T_1}$$

K = constant. (assumption).

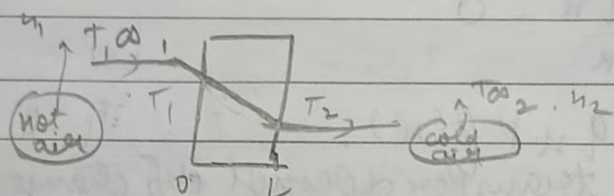
If $K=K_0(1+\beta T)$.

$$k\frac{dT}{dx} = C_1$$

$$k_0(1+\beta T)\frac{dT}{dx} = C_1$$

$$\int_{T_1}^{T} k_0(1+\beta T)\,dT = \int_{0}^{x} C_1\,dx$$

$$\boxed{k_0(T-T_1) + \frac{\beta}{2}(T^2-T_1^2) = C_1 x}$$



$$Q_{x\,conv_1} = h_1 A(T_{\infty_1} - T_1).$$

$$Q_{x\,cond} = \frac{kA(T_1 - T_2)}{L}$$

$$Q_{x\,conv_2} = h_2 A(T_{\infty_2} - T_2).$$

$Q_{x\,conv_1} = Q_{x\,cond} = Q_{x\,conv_2}$ (due to steady state).
(1D $\therefore Q_x$).

$$h_1 A(T_{\infty_1} - T_1) = \frac{kA(T_1 - T_2)}{L} = h_2 A(T_{\infty_2} - T_2).$$

$$h_1(T_{\infty_1} - T_1) = \frac{k(T_1 - T_2)}{L} = h_2(T_{\infty_2} - T_2).$$

If h_1 & h_2 are not known

$$h_1 = f(v, \rho, \mu, C_p, k)$$

Its value is obtained using empirical correlation.

$Nu = fn(Re, Pr)$.

$$\frac{hD}{k} = C\left(\frac{D \cdot v\rho}{\mu}\right)^{0.8}\left(\frac{C_p\mu}{k}\right)^{0.3}.$$

$$Q_x = Q_{x,conv_1} = Q_{x,cond} = Q_{x,conv_2}$$

$$T_{\infty_1} - T_1 = \frac{Q_x}{h_1 A}$$

$$+ \quad T_1 - T_2 = \frac{Q_x}{\frac{KA}{L}}$$

$$+ \quad T_2 - T_{\infty_1} = \frac{Q_x}{h_2 A}.$$

$$T_{\infty_1} - T_{\infty_2} = Q_x\left[\frac{1}{h_1 A} + \frac{L}{KA} + \frac{1}{h_2 A}\right]$$

$$\frac{T_{\infty_1} - T_{\infty_2}}{Q_x} = \frac{1}{h_1 A} + \frac{L}{KA} + \frac{1}{h_2 A}$$

$$\frac{T_{\infty_1} - T_{\infty_2}}{Q_x} = \frac{1}{A}\left[\frac{1}{h_1} + \frac{L}{K} + \frac{1}{h_2}\right].$$

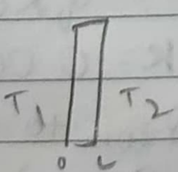
$T_{\infty_1} - T_{\infty_2} \rightarrow$ heat transfer Driving force or Driving potential

$Q_x \Rightarrow$ ~~conc~~ heat flow rate.

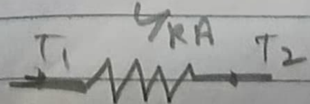
Driving potential / heat flow rate = Resistance

$$\boxed{R_{th,q} = \text{Resistance to heat flow rate} = \left[\frac{1}{h_1} + \frac{L}{K} + \frac{1}{h_2}\right]\frac{1}{A}}$$

$\rightarrow$

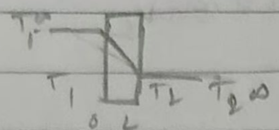


$\Rightarrow$

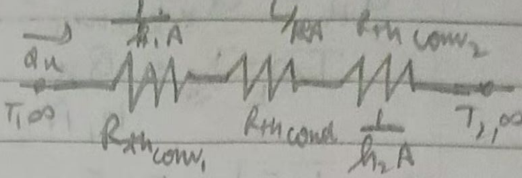


equivalent circuit.

$$Q_x = K\frac{A}{L}\Delta T.$$

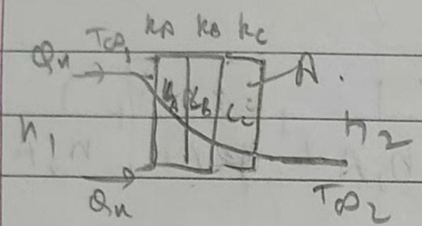


$\Rightarrow$

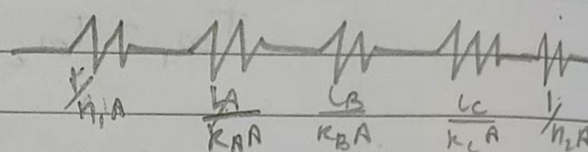


$$R_{electric} = \frac{L}{\sigma A} \quad \Leftrightarrow \quad R_{th} = \frac{L}{kA}$$

σ = Electrical conductivity $\qquad$ k = thermal conductivity.

→ 

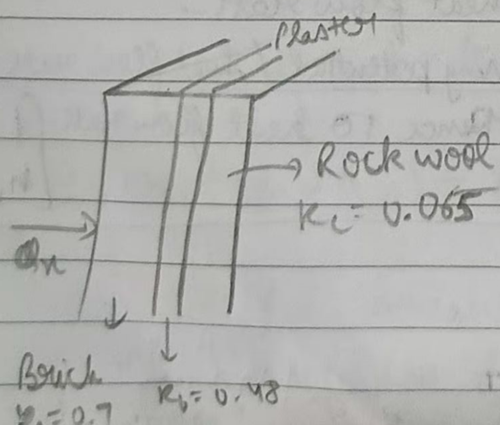
$\Longrightarrow$



$$Q_x = \frac{\Delta T_{ov}}{R_{th\,ov}} \qquad ov = overall.$$

$$= \frac{T_{\infty_1} - T_{\infty_2}}{R_{th\,ov}}$$

$$R_{th\,ov} = \frac{1}{h_1 A} + \sum_{i=A}^{C} \frac{L_i}{k_i A} + \frac{1}{h_2 A}.$$

Q. 

20cm layer of brick wall [$k = 0.7 W/m(C)$]
5cm layer of plaster [$k = 0.48 W/m(C)$]
L = ? of Rockwool. [$k = 0.065 W/m(C)$]
Q_{loss} to be reduced by 60%.

$$Q = \frac{T_1 - T_2}{R_{th}} \qquad R_{th} = \frac{20 \times 10^{-2}}{0.7 \times A} + \frac{5 \times 10^{-2}}{0.48 A} + \frac{L}{0.065 A}.$$

$$q' = \frac{T_1 - T_2}{R_{th}'} \quad ; \quad R_{th} = \frac{20\times10^{-2}}{0.7A} + \frac{5\times10^{-2}}{0.48A} + \frac{L'}{0.065A}.$$

$$\frac{q'}{q} = \frac{1}{0.4} = \frac{R_{th}}{R_{th}'}$$

$$= \frac{\frac{20\times10^{-2}}{0.7} + \frac{5\times10^{-2}}{0.48} + \frac{L}{0.065}}{\frac{20\times10^{-2}}{0.7} + \frac{5\times10^{-2}}{0.48} + \frac{L'}{0.065}} = \cancel{0.4}\ \frac{1}{0.4}$$

$$= \frac{20\times10^{-2}}{0.7}(1-0.4) + \frac{5\times10^{-2}}{0.48}(1-0.4)$$

$$+ \frac{0.4L - \cancel{0.4L'}}{0.065} = 0.$$

Initially $L = 0$.

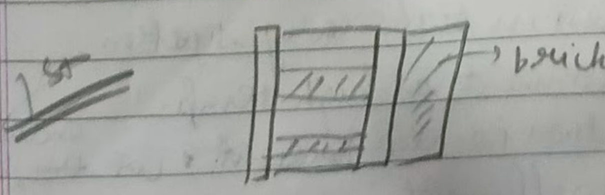
$$\Rightarrow \frac{20\times10^{-2}}{0.7}(0.6) + \frac{5\times10^{-2}}{0.48}(0.6) = \frac{\cancel{0.4}\ L'}{0.065}$$

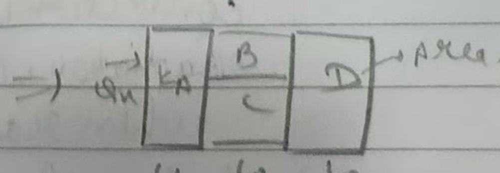
$$\Rightarrow L' = \underline{0.015}$$

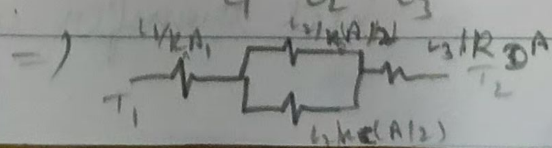
$$\Rightarrow \underline{\underline{L' = 1.5\,cm}}$$

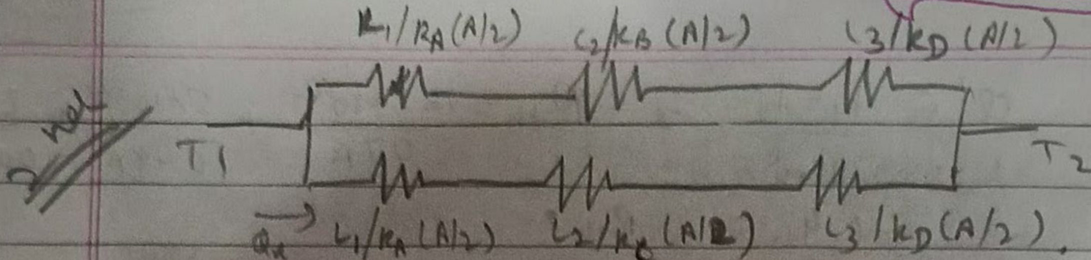
→ Q. thickness = 5 cm $k = 0.69\ W/m(°C)$ of brick wall.
insulation [$k = 0.05\ W/m(°C)$].
gypsum sheets [$k = 0.9\ W/m(°C)$] → 4 cm thickness.
insulation is fixed to brick wall by (2×4) wooden studs.
(4.13 × 9.21 cm) $k = 0.15\ W/m(°C)$.
Estimate heat loss & overall heat transfer coefficient of wall. Determine R value of wall.

Sol









1st circuit → assumption → across H.T surfaces are isothermal
isothermal surface. $\to Q_x$

2nd circuit → assumption → along H.T surfaces are adiabatic
$\to Q_x$ adiabatic.

as $|k_B - k_C|$ value ↑ R_{th_1} ~~for~~ becomes more different for R_{th_2}.

$|k_B - k_C| \uparrow$ → multidimensional H.T ↑.
non-adiabatic → secondary H.T.
Primary H.T → $T_1 \to T_2$

This different circuit is used for Multi dimensional H.T.

$Q_x = h A \Delta t$

~~conduction~~ convective heat transfer coefficient. = Local heat transfer coefficient.

$$Q_x = UA(\Delta T)_{ov} = \Delta T_{ov} \Big/ \left[\frac{1}{h_1 A} + \sum \frac{L_i}{k_i A} + \frac{1}{h_2 A} \right]$$

$$\boxed{\frac{1}{UA} = \frac{1}{h_1 A} + \sum \frac{L_i}{k_i A} + \frac{1}{h_2 A}}$$ – ~~Overall heat transfer coefficient.~~

convection is less in stationary system.
Radiation is prominent at high temp.
when radiation is also present; we ~~use~~ include radiation heat transfer coefficient

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heat exchanger → used to give heat → a tubular / circular structure.

$$R_{th,overall} = \frac{1}{UA}$$

$\frac{1}{U}$ = overall heat transfer coefficient

$$\boxed{\frac{1}{U} = \frac{1}{h_1} + \sum \frac{L_i}{k_i} + \frac{1}{h_2}}$$

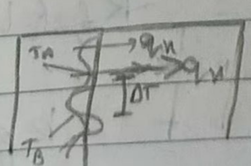
R – value = $\frac{1}{U}$ = Resistance value of insulators.
= used to define effective of insulators.

Imp: Assumption used in above problems → smooth walls i.e. heat coming into it = heat going out of it.

→ Temp. drop due to thermal contact Resistance
contact Resistance arises due to rough wall i.e. not in contact with all points but only at few points and the ~~the~~ gap is filled with fluid of low thermal conductivity.

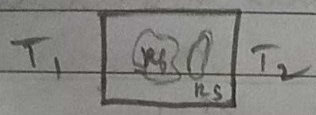
$$R_{th,contact} = \frac{T_A - T_B}{q_x} = \text{contact resistance.}$$

to calculate q_x we will use parallel ~~to~~ connection



This contact Resistance must be included in calculations. If not included it must be mentioned (otherwise use Thermal Resistance of different solid-solid interfaces).

Porous solids



→ It must have some eff. thermal conductivity (k_{eff})

$$Q_x = k_{eff} A \Delta T.$$

$\Delta T = T_1 - T_2$).

$T_1 > T_2$

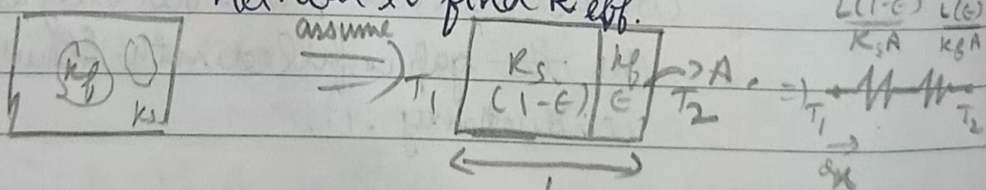
$$\boxed{\epsilon = \frac{\text{void volume}}{\text{total volume}}}$$

$k_{eff} = f(\epsilon, k_s, k_f, \text{geometric arrangement})$

→ saturated porous solid – all pores filled with a particular material

→ unsaturated porous solid – pores ~~all~~ are filled with diff. filler material.

Method to find k_{eff}.

① 

$Q_x = \cancel{Q_x} = k_{eff} A (\Delta T)/L = k$

$$\Rightarrow \frac{\Delta T}{Q_x} = \frac{L}{k_{eff} A} = R_{th} = \frac{L(1-\epsilon)}{k_s A} + \frac{L\epsilon}{k_f A}$$

$$\cancel{\therefore k_{eff} = k_f(L - L\epsilon) + k_s L\epsilon}$$

$$\frac{L}{k_{eff}} = \frac{k_f(L - L\epsilon) + k_s L\epsilon}{k_f k_s}$$

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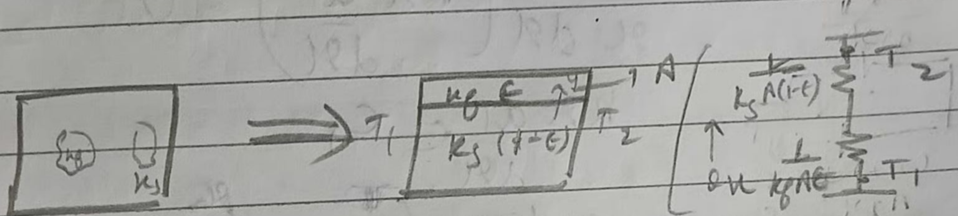
$$\frac{k_{eff}}{\cancel{x}} = \frac{k_s k_f}{\cancel{x}(k_f - k_f\epsilon + k_s\epsilon)}$$

$$k_{eff} = \frac{k_s k_f}{k_f(1-\epsilon)+k_s}$$

$$\frac{1}{k_{eff}} = \left(\frac{1-\epsilon}{k_s}\right) + \frac{\epsilon}{k_f}.$$

(2)



$$\cancel{\frac{L}{k_{eff}A} = R_{th} = \left(\frac{L\epsilon}{k_f A\epsilon}\right)^{-1} + \left(\frac{L(1-\epsilon)}{k_s A}\right)^{-1}}$$

$$\frac{L}{k_{eff}A} = R_{th} = \frac{L}{k_s A(1-\epsilon)} + \frac{L}{k_f A\epsilon}.$$

$$\boxed{k_{eff} = k_s(1-\epsilon) + k_f\epsilon}$$

$\cancel{\frac{1}{k_{eff}}}$

with change in direction k changes as seen above ⇒ this violates isomorphic ~~iso~~ assumption

The above series gives two extreme values of effective thermal resistivity.

→ Heat exchanger. — Double pipe — common in industry.
— Shell and tube —

For cylindrical coordinates.

If $\because l > r$

~~for~~ More heat loss in radial direction then we can assume 1d, S.S, no heat gen.

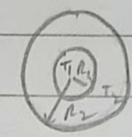
q_r ; $T(r)$ to be found.

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\Rightarrow \frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right) = 0.$$

$$kr\frac{dT}{dr} = C_1$$

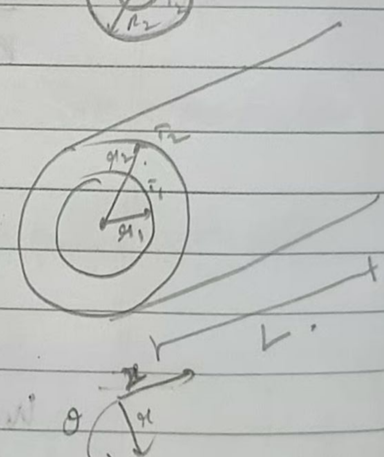
$$\cancel{r\,dT} \quad \frac{k}{C_1}\int_{T_1}^{T} dT = \int_{R_1}^{r} \frac{dr}{r}$$



$$\frac{k}{C_1}(T - T_1) = \ln \frac{r}{R_1}$$

at $r = R_2$, $T = T_2$

$$\frac{k}{C_1}(T_2 - T_1) = \ln \frac{R_2}{R_1}$$



$$C_1 = \frac{k(T_2 - T_1)}{\ln\left(\frac{R_2}{R_1}\right)}.$$

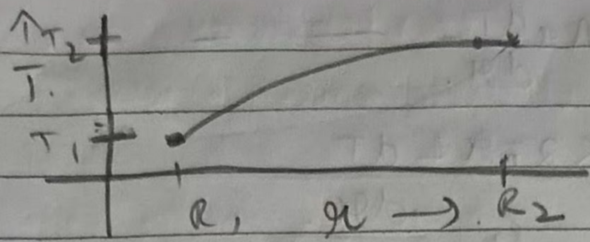
ΔT in radial dir >> ΔT in azimuthal or axial dir.

$$\ln \frac{r}{R_1} = \frac{\cancel{k}(T - T_1)}{\cancel{k}(T_2 - T_1)} \ln\left(\frac{R_2}{R_1}\right)$$

$$\ln\left(\frac{r}{R_1}\right) = \left(\frac{T - T_1}{T_2 - T_1}\right) \ln\left(\frac{R_2}{R_1}\right)$$

$$T(r) = \cancel{T_2} \; T_1 + (T_2 - T_1)\frac{\ln(r/R_1)}{\ln(R_2/R_1)}$$

$$\boxed{T(r) = T_1 + (T_2 - T_1)\frac{\ln(r/R_1)}{\ln(R_2/R_1)}}$$

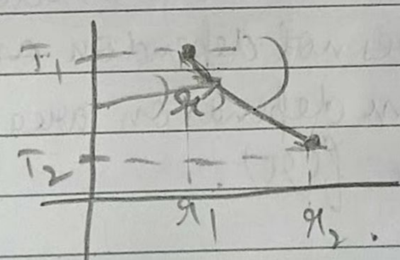


$$Q_r = -K A_r \frac{dT}{dr}$$

$\cancel{A_r = 2\pi r\, dr}$. area of heat transfer is not constt. it keeps on changing.

$$A_r = 2\pi r L$$

temp. profile is linear for wall but logarithmic for cylindrical.



$$q(r) = -K\, 2\pi r L \times \frac{(T_2 - T_1)}{\ln(R_2/R_1)} \times \frac{\cancel{R_1}}{\cancel{R_1}} \; \cancel{\ln} \; \frac{1}{\cancel{R_1}}$$

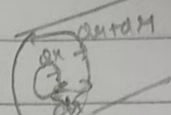
$$\frac{dT}{dr} = \frac{(T_2 - T_1)}{\ln(R_2/R_1)} \; \frac{\cancel{R_1}}{r} \times \frac{1}{\cancel{R_1}}$$

$$q(r) = -K\, \frac{2\pi L}{} \times \frac{(T_2 - T_1)}{\ln(R_2/R_1)}$$

(1) $$q(r) = -K \left(\frac{2\pi L}{\ln(R_2/R_1)}\right) (T_2 - T_1).$$

$$\boxed{\text{madam } q(r) = \frac{K(2\pi L)}{\ln(R_2/R_1)} (T_1 - T_2).}$$

$Q_r = -KA_r \dfrac{dT}{dr}$.



$= -K\, 2\pi r L \dfrac{dT}{dr}$.

$\dfrac{Q_r}{K(2\pi L)} \displaystyle\int_{R_1}^{R_2} \dfrac{dr}{r} = \int_{T_1}^{T_2} -dT$

$\dfrac{Q_r}{K(2\pi L)} \ln\left(\dfrac{R_2}{R_1}\right) = \cancel{\ln}\left((T_1 - T_2)\right)$.

$\Rightarrow Q_r = (T_1 - T_2)\dfrac{K(2\pi L)}{\ln(R_2/R_1)}$. $\quad q(r) \neq 0$

$q_r = f(r)$.

But $Q_r \neq f(r)$.

$\Rightarrow$ By Fourier Law the rate of heat flow is constant and does not depend on ~~axial~~ radius.

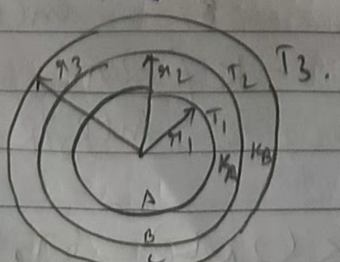
∵ heat flux depends on area ⊥ to heat ~~flowrate~~ transfer.

thus $q_r = f(r)$.

$q_r = \dfrac{Q_r}{A_r}$.

$\boxed{\therefore R_{th}\text{, conductivity} = \dfrac{\ln(R_2/R_1)}{2\pi L K}.}$

∴ Steam carrying pipe contains insulation.



$\Rightarrow$

$Q_r = \dfrac{(T_1 - T_2)2\pi L K_A}{\ln(R_2/R_1)} = \dfrac{(T_2 - T_3)\,2\pi L K_B}{\ln(R_3/R_2)}$ (∵ Q_r is const).

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$$T_1 - T_2 = \frac{Q_r \ln(R_2/R_1)}{2\pi L K_A}$$

$$T_2 - T_3 = Q_r \frac{\ln(R_3/R_2)}{2\pi L K_B}$$

$$T_1 - T_3 = \frac{Q_r}{2\pi L}\left[\frac{\ln(R_2/R_1)}{K_A} + \frac{\ln(R_3/R_2)}{K_B}\right]$$

$$\therefore \boxed{Q_r = (T_1 - T_3)(2\pi L)\left[\frac{K_A K_B}{K_B \ln(R_2/R_1) + K_A \ln(R_3/R_2)}\right]}$$

$$\therefore R_{th,\,overall} = \left(\frac{K_B \ln(R_2/R_1) + K_A \ln(R_3/R_1)}{K_A K_B\, 2\pi L}\right).$$

Assumption
Isotropic material.

$$\boxed{R_{th,\,overall} = \frac{\ln\left[\left(\frac{R_2}{R_1}\right)^{K_B} \times \left(\frac{R_3}{R_2}\right)^{K_A}\right]}{2\pi L K_A K_B}}$$

Application → Insulation of a copper pipe carrying steam.

For n radial systems.

$$R_{th,\,overall} = \frac{\ln\left[\left(\frac{R_2}{R_1}\right)^{K_{n-1}} \times \left(\frac{R_3}{R_2}\right)^{K_{n-2}} \times \cdots \left(\frac{R_n}{R_{n-1}}\right)^{K_1}\right]}{2\pi L (K_1 \ldots K_n)}.$$

$$\therefore R_{th,\,overall} = \frac{1}{2\pi L} \sum \frac{\ln(R_{i+1}/R_i)}{K_i}.$$

$$\frac{Q_r}{L} = \frac{\Delta T_{ov}}{2\pi} \sum_{i=1}^{N} \frac{\ln(R_{i+1}/R_i)}{K_i}$$

→ Q. SS $k = 19\,W/m(C)$ of 4 cm ID and 10 cm OD
Asbestos - 5 cm layer $k = 0.2\,W/m(C)$.
$T_{inside} = 400°C$ $T_{outside} = 37°C$.

Find ~~a~~ heat loss per meter ~~of~~ of tube length

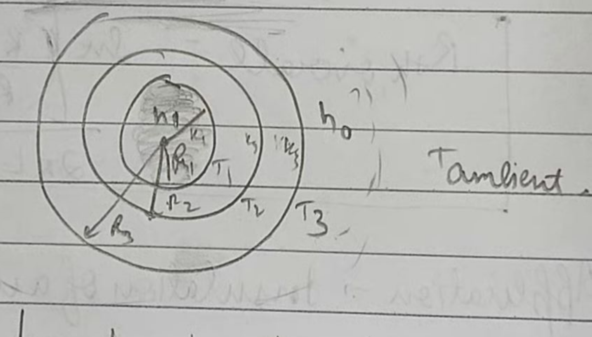
b) temp. at tube insulation interface.

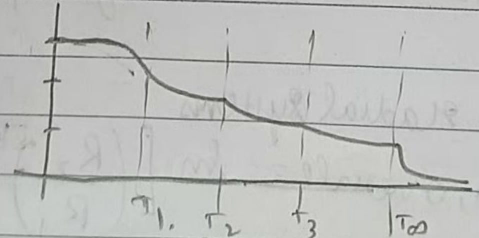
$$\frac{Q}{\cancel{L}} = \frac{\Delta T}{\frac{1}{K_{ss}} \ln\left(\frac{r_2}{r_1}\right) + \frac{1}{K_I} \ln\left(\frac{r_3}{r_2}\right)}$$

$$\frac{Q_r}{L} = \frac{T_1 - T_2}{\frac{1}{2\pi K_I} \ln\left(\frac{r_3}{r_2}\right)}.$$

Conduction of heat in 1st tube 1D SS.

h_i = convection ~~coeff.~~ ~~conductivity of~~ of of steam.





convection at R_1 (by hot ~~liq.~~ fluid) & at R_n ~~to~~ ambient & environment.

$$\frac{Q_r}{L} = \frac{\Delta T_{ov}}{\frac{1}{h_1 A_1} + \frac{1}{2\pi L}\sum_{i=1}^{n} \frac{1}{k_i} \ln\left(\frac{R_{i+1}}{R_i}\right) + \frac{1}{h_o A_o}}.$$

$$R_{th,ov} = \frac{1}{h_1 2\pi R_1 L} + \frac{1}{2\pi L} \sum_{i=1}^{n} \frac{1}{K_i} \ln\left(\frac{R_{i+1}}{R_i}\right) + \frac{1}{h_o 2\pi R_o L}$$

$$Q_r = h A \Delta T$$

$$= U A \Delta T$$

~~U = Ov~~ $\frac{1}{U}$ = Overall heat transfer coefficient

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$R_{th} = \frac{1}{UA}$

Overall heat transfer coefficient depends on total temp difference taking both conduction & convection

- $\frac{1}{U} = \frac{1}{AR_{th}}$

$\therefore R_{th} = \frac{1}{UA}$

$\frac{1}{U}$ = Overall heat transfer coefficient.

$\frac{1}{U} = \int R_{th}\, dA$ $\because$ A is variable we can't simply define overall H-T coefficient.

$\therefore Q_i = Q_o$

$\therefore U_i A_i = U_o A_o$

but $U_i \neq U_o$

$\because A_i \neq A_o$.

$\therefore$ we ~~can~~ thus define overall heat transfer coefficient over an area.

$$\therefore \frac{1}{U_o A_o} = \frac{1}{h_i A_i} + \sum \frac{\ln(R_{i+1}/R_i)}{2\pi L K_i} + \frac{1}{h_o}.$$

$$\Rightarrow \frac{1}{U_o} = \frac{A_o}{h_i A_i} + \sum \frac{\ln(R_{i+1}/R_i)}{2\pi L K_i} + \frac{\cancel{A_o}}{h_o \cancel{A_o}}$$

= Overall heat transfer coefficient over an area.

$$\text{'lly } \frac{1}{U_i} = \frac{A_i}{h_i A_i} + \sum \frac{\ln(R_{i+1}/R_i)}{2\pi L K_i} + \frac{A_i}{h_o A_o}$$

- for a single pipe, there are two heat transfer coefficient that can be defined for two convection taking place.

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Q) T of $H_2O = 80°C$, tube $= 2.5cm$ I.D.

5 mm = wall thickness.

Tube's $k = 16 W/m(°C)$ $h_i = 3500 W/m^2(°C)$.

Estimate overall heat transfer coefficient & heat loss per unit length. $h_o = 7.6 W/m^2(°C)$.

$T_\infty = 30°C$.

→ Spherical coordinates

$$\frac{1}{r^2}\left(\frac{\partial^2(rT)}{\partial r^2}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) + \frac{1}{r^2}\frac{1}{\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\cdot\frac{\partial T}{\partial t}$$

1D, S.S.

$$\Rightarrow \frac{1}{r^2}\left(\frac{\partial^2(rT)}{\partial r^2}\right) = 0.$$

$$\frac{\partial^2(rT)}{\partial r^2} = 0.$$

$$\frac{\partial(rT)}{\partial r} \neq f(r)$$

$$\frac{\partial(rT)}{\partial r} = C_1.$$

$$rT = C_1 r + C_2$$

$$\cancel{T_2 - T_1} = C_1 + \frac{C_2}{r}.$$

$$Q_r = -kA_r\frac{dT}{dr}$$

$$A_r = 4\pi r^2.$$

$$Q_r = Q_{r+dr}.$$

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$$\frac{Q_r}{K4\pi r}\cdot\int_{R_1}^{R_2}\frac{dr}{r^2}=-\int_{T_1}^{T_2}dT$$

$$\Rightarrow R_{th\ cond}=\frac{1}{4\pi K}\left(\frac{1}{R_1}-\frac{1}{R_2}\right).$$

→ Find R_{th} overall for composite material derive temp. profile for sphere.