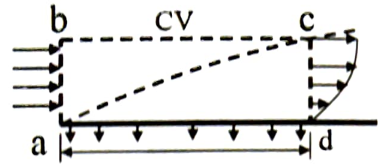
Consider the steady flow of water past a porous plate with a constant suction velocity of 0.2 mm/s (in the negative y direction, i.e., V = - 0.2j mm/s). A thin boundary layer grows over the flat plate and the velocity profile at section cd is $\frac{u}{U_\infty} = \frac{3}{2}\frac{y}{\delta} - 2\left(\frac{y}{\delta}\right)^{1.5}$, where U_∞ is the velocity of approach at section ab and is equal to 3 m/s and u is the x-component of velocity. Find the mass flow rate across section bc. Given: width of the plate = 1.5m, length = 2m, δ at CD = 1.5 mm.



Apply conservation of mass using the cv.

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V}\cdot d\vec{A}$$

Steady, incompressible flow, $\vec{V} = -v_0 \hat{j}$ along ad

$$0 = \int_{CS} \rho \vec{V}\cdot d\vec{A} = \int_{ab} \rho \vec{V} d\vec{A} + \dot{m}_{bc} + \int_{cd} \rho \vec{V}\cdot d\vec{A} + \int_{da} \rho \vec{V}\cdot d\vec{A}$$

$$0 = -\rho U_\infty W \delta + \dot{m}_{bc} + \int_0^\delta \rho U_\infty \left[\frac{3}{2}\frac{y}{\delta} - 2\left(\frac{y}{\delta}\right)^{1.5}\right] W dy + \rho v_0 W L \quad (1)$$

$$\therefore \dot{m}_{bc} = \rho U_\infty W \delta - \rho U_\infty W \delta \int_0^1 \left[\frac{3}{2}\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^{1.5}\right] d\left(\frac{y}{\delta}\right) - \rho v_0 W L$$

$$\dot{m}_{bc} = \rho W \left[U_\infty \delta - U_\infty \delta \left\{\frac{3}{4}\left(\frac{y}{\delta}\right)^2 - \frac{2}{2.5}\left(\frac{y}{\delta}\right)^{2.5}\right\}_0^1 - v_0 L\right] \quad (1)$$

$$= \rho W \left[U_\infty \delta - U_\infty \delta \left(\frac{3}{4} - \frac{2}{2.5}\right) - v_0 L\right]$$

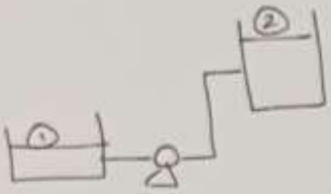
$$= \rho W \left[1.05 U_\infty \delta - v_0 L\right].$$

(1) $$= 999 \frac{kg}{m^3} \times 1.5\,m \left(1.05 \times 3\frac{m}{s} \times 0.0015\,m - 0.0002\frac{m}{s} \times 2\,m\right)$$

$\dot{m}_{bc} = 6.48 \frac{kg}{s}$ (Since $\dot{m} > 0$, flow is out of CV)

Calculate the power requirement to pump water (μ = 0.01 Pa s) at 60 L/s from a supply tank through a 100 mm diameter and 25 m long pipeline into the storage tank. The liquid level of storage tank is 9 m above that of the supply tank. Four 90^0 elbows, two fully open gate valves are present in the pipeline. The average thickness of the surface roughness of the pipe is 0.25 mm. Following head loss coefficient (K) are available - entrance loss: 0.5; exit loss:1.0; 90^0 elbow:1.8; fully open gate valve: 0.3. The friction factor can be calculated from:

$$f = 0.25\left[\log\left(\frac{\varepsilon / D}{3.7} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^{-2}$$



$$\frac{p_1}{\rho} + \frac{v_1^2}{2} + gz_1 + P_{pump} = \frac{p_2}{\rho} + \frac{v_2^2}{2} + gz_2 + h_L$$

$$P_{ump} = h_L + g(z_2 - z_1)$$

$$h_L = h_{L\,major} + h_{L\,minor}$$

$$f\frac{L}{D}\frac{v^2}{2}$$

$$\underset{(\text{entry})}{\frac{0.5v^2}{2}} + \underset{(\text{exit})}{\frac{v^2}{2}} + \underset{(\text{elbow})}{\frac{4\times1.8v^2}{2}} + \underset{\text{valve})}{\frac{2\times0.3v^2}{2}}$$

$$h_{L\,minor} = \frac{9.3v^2}{2} = 4.65v^2 \quad 1$$

$Q = 60\times10^{-3}\ m^3/s$

$D = 0.1m$

$$\frac{\pi D^2}{4}\times v = 60\times10^{-3}$$

$$1 \quad v = \frac{60\times4\times10^{-3}}{\pi\times(0.1)^2} = 7.64 m/s \qquad Re = \frac{0.1\times7.64\times10^3}{1\times10^{-2}} = 76433$$

$$1 \quad f = 0.25\left[\log\left(\frac{0.25\times10^{-3}}{0.1\times3.7} + \frac{5.74}{(76433)^{0.9}}\right)\right]^{-2} = 0.027$$

$$h_{L\,major} = 0.027\times\frac{25}{0.1}\times\frac{v^2}{2} = 3.375v^2 \quad 1$$

$$\therefore \text{Total frictional loss} = (3.375 + 4.65)v^2 = 8.025\times(7.64)^2 = 468.42$$

$$g(z_2 - z_1) = 9.81\times9 = 88.29 \quad 1$$

$$\therefore \text{Total loss (h)} = 88.29 + 468.42 = 556.71 \quad 1$$

$$\text{Power of the pump} = h\rho Q = 556.71\times10^3\times60\times10^{-3}\ W$$

$$P = 33.4\ kW \quad 1$$

$$h\rho Q = \frac{m^2}{s^2}\cdot\frac{kg}{m^3}\cdot\frac{m^3}{s} = \frac{kg\,m^2}{s^3}$$

$$W = \text{Force}\times\frac{\text{dist}}{\text{time}} = kg\cdot\frac{m}{s^2}\cdot\frac{m}{s} = \frac{kg\,m^2}{s^3}$$