INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR
Mid-Autumn Semester Examination 2023-24

<u>Q1.</u> In a game of air hockey, the ball (called puck, which are slim discs made of resin) floats above a thin film of air on a table. The tables will typically have some sort of machinery that produces a cushion of air on the play surface through tiny holes, with the purpose of reducing friction and increasing play speed. The idea is to hit the puck hard such that it travels fast towards the opponent's goal while floating on air. The mass of one such puck is 30 g, with a diameter of 100 mm. The air film (of viscosity 1.75 x 10^{-5} N.s/m^2) under the puck is 0.1 mm thick. Calculate the time required after impact for the puck to lose 10 percent of its initial speed. Assume that the other (top) surface of the puck does not contribute to its slowing down. (**Marks = 4)**

There is a very thin layer of air beneath the puck which makes this situation similar to pure (no applied pressure gradient) Couette flow.

Couette flow is characterized by a linear velocity profile. If the top plate velocity is V and the gap is equal to H,

$$v_x = \frac{Vy}{H} \Rightarrow \tau = \mu \frac{dv_x}{dy} = \frac{\mu V}{H} \qquad \mathbf{1}$$

Using Newton's second law

$$ma = m\frac{dv}{dt} = -\tau A; \quad -mdv = \frac{\mu V}{H} A\, dt \qquad 1$$

$$\int_V^{0.9V} dv = -\frac{\mu V}{mH} A \int_0^t dt \qquad 1$$

$$0.1 = \frac{\mu A}{mH} t \Rightarrow t = 0.1 {mH}/{\mu A} = \frac{0.1 \times 0.03 \times 1 \times 10^{-4}}{1.75 \times 10^{-5} \times \frac{\pi}{4}\left(100 \times 10^{-3}\right)^2} \qquad 1$$

$$t = 2.18s$$

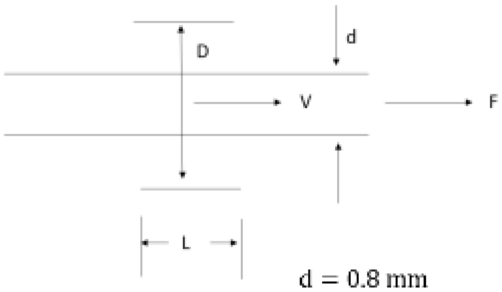
Q2. Magnet wire is to be coated with varnish for insulation by drawing it through a circular die of 0.9 mm in diameter. The wire diameter is 0.8 mm and it is centered in the die. The varnish (μ = 20 centipoise) completely fills the space between the wire and the die for a length of 20 mm. The wire is drawn through the die at a speed of 50 m/s. Determine the force required to pull the wire. **(Marks = 4)**

Find: Force required to pull the wire

$F = \tau A$

$$\rho\left(\cancelto{0}{\frac{\partial v_z}{\partial t}} + \cancelto{0}{v_r}\frac{\partial v_z}{\partial r} + \cancelto{0}{\frac{v_\theta}{r}}\left(\frac{\partial v_z}{\partial \theta}\right) + v_z \cancelto{0}{\frac{\partial v_z}{\partial z}}\right) = \cancelto{0}{\rho g_z} - \cancelto{0}{\frac{\partial P}{\partial x}} + \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)\right) + \frac{1}{r^2}\cancelto{0}{\frac{\partial^2 v_z}{\partial \theta^2}} + \cancelto{0}{\frac{\partial^2 v_z}{\partial z^2}}$$

IF RADIAL COORDINATES ARE CHOSEN



$d = 0.8$ mm
D=0.9 mm
L=20 mm
$\mu = 20$ cp
$v = 50\frac{m}{s}$

$\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) = 0$ **1**

$r\frac{\partial v_z}{\partial r} = 0$

$v_z = C_1 \ln r + C_2$

$v_z = V$ at $r = r_1$ $\left(r_1 = \frac{d}{2}\right)$ **0.5**

$v_z = 0$ at $r = r_2$ $\left(r_2 = \frac{D}{2}\right)$ **0.5**

$V = C_1 \ln r_1 + C_2$

$0 = C_1 \ln r_2 + C_2$

$V = C_1 \ln\frac{r_1}{r_2} \Rightarrow C_1 = \frac{V}{\ln\frac{r_1}{r_2}}$, $C_2 = -C_1 \ln r_2 \Rightarrow \frac{V \ln r_2}{\ln\frac{r_1}{r_2}}$

$\tau_{rz} = \mu\frac{C_1}{r} = \mu\frac{1}{r}\frac{V}{\ln\frac{r_1}{r_2}} = \frac{\mu V}{r \ln\frac{r_1}{r_2}}$

$\tau_{rz}|_{r=r_1} = \frac{\mu V}{r_1 \ln\frac{r_1}{r_2}} = \frac{20*10^{-2}*10^{-1}*50}{0.4*10^{-3}\ln\frac{0.8}{0.9}} = -2.12*10^4\,\frac{N}{m^2}$ **1**

Force = $2\pi r_1 L*(-\tau_{rz}) = 2\pi*0.4*10^{-3}*20*10^{-3}*2.12*10^4 = 1.06$ N **1**

IF COUETTE FLOW APPROXIMATION ARE ADOPTED

$d = 0.8$ mm, $D = 0.9$ mm, $L = 20$ mm
$\mu = 20$ cP, $V = 50$ m/s
Force required to pull the wire

Since V_{wire} = const., applied force must be sufficient to balance friction force F **1**

$F = \tau A$, $\tau = \mu\frac{du}{dr}$ and $A = \pi d L$

Since the gap is too small, a linear velocity profile (Couette flow) can be assumed. **1**

$\tau = \mu\frac{V_{D/2} - V_{d/2}}{\frac{D}{2} - \frac{d}{2}} = -\frac{\mu V}{(D-d)/2}$ **1**

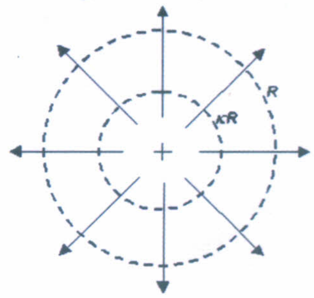
(negative stress in the $-x$ direction)

$F = \tau A = \mu\frac{2V}{(D-d)}\pi D L$ **1**

$F = 20\,cP \cdot \frac{gm}{100\,cm\,s\,cP} \times \frac{2\pi \times 50\frac{m}{s}}{} \frac{\times 0.8\times10^{-3}\,m \times 20\times10^{-3}\,m}{0.1\times10^{-3}} \times \frac{1}{10^3}$

$F = 1.01$ N

An isothermal, incompressible fluid of density ρ flows radially outward owing to a pressure difference between two vertical, fixed porous, concentric spherical shells of radii κR and R (Top view of the assembly is provided). Note that the velocity is not zero at the solid surfaces. Assume negligible end effects and steady laminar flow in the region $\kappa R \le r \le R$.



a) Use the equation of continuity to obtain a functional form for v_r

b) Simplify the equation of motion for a Newtonian fluid of viscosity μ.

c) Obtain the pressure profile $P(r)$ in terms of P_R and v_R, the pressure and velocity at the outer sphere of radius R respectively.

(Marks = 2+2+3=7)

a) Since the steady laminar flow is directed radially outward, the only nonzero components of velocity, in spherical coordinate system will be v_r.

Conti. eq^n $\Rightarrow$ $\frac{1}{r}\frac{\partial}{\partial r}(r^2 v_r) = 0$

$\therefore$ $\boxed{r^2 v_r = \text{constant} = C.}$ 2

b) The eq^n of motion is

$$\rho \frac{Dv}{Dt} = -\nabla p - \nabla \cdot T + \rho g.$$

For incompressible Newtoninan fluid $-\nabla\cdot T = \mu\nabla^2 v$ and as $r^2 v_r$ = constant, the NS eq^n in spherical coordinate becomes

$$\rho v_r \frac{\partial v_r}{\partial r} = -\frac{\partial p}{\partial r} - \rho g \cos\theta$$

Defining a modified pressure $P = p + \rho g h$
$= p + \rho g r \cos\theta$.

$$\rho\left(v_r \frac{\partial v_r}{\partial r}\right) = -\frac{\partial P}{\partial r}. \quad — 2$$

c) From θ comp. and ϕ comp. of NS eq^n

$\frac{\partial P}{\partial \theta} = 0$, $\frac{\partial P}{\partial \phi} = 0$ $\Rightarrow$ P is a f^n of r only

$P = P(r)$, $v_r = v_r(r)$ 1

$$\therefore \frac{dP}{dr} = -\rho v_r \frac{dv_r}{dr}$$

Substituting $v_r = \frac{c}{r^2}$

$$\frac{dP}{dr} = 2\rho \frac{c^2}{r^5}$$

$$P = -\frac{\rho c^2}{2r^4} + C_1$$

As $P = P_R$ and $v_r = v_R$ at $r = R$ (B.C.) ①

$$C_1 = P_R + \frac{\rho c^2}{2R^4}, \quad c = R^2 v_R.$$

$$\therefore P = P_R + \frac{\rho c^2}{2R^4}\left[1 - \left(\frac{R}{r}\right)^4\right].$$

$$P = P_R + \frac{1}{2}\rho v_R^2\left[1 - \left(\frac{R}{r}\right)^4\right].$$ ①