→ Energy is conserved at every instant,

$$E_{in} - E_{out} + E_{gen} = E_{acc}$$

or,
$$\frac{d\bar{E}_{acc}}{dt} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen}$$

Energy

mechanical energy   Internal energy

P.E   K.E

Modes of heat transfer

i) Convection (bulk motion required)

(ii) Conduction.

iii) Radiation.

heat flux = q (watt/m²)

heat rate = Q (watt)

Fourier' law of heat conduction gives,

$$q = -k\frac{dT}{dx}.$$

Convection: Newton's law of cooling, $q = h(T_s - T_\infty)$ [moving sys]

Conduction: Fourier's law, $q = -k\frac{dT}{dx}$ [at ~~high~~ normal temp]

Radiation: Stefan Boltzmann law, [at high temp]

$$q = \varepsilon\sigma(T_s^4 - T_\infty^4)$$

Perpendicular to the dir. of H.T

$$Q_{Cond,x} = -kA\frac{\partial T}{\partial x}$$

thermal conductivity (related to KE of molecules)

$\therefore k_{gas} = f(T)$ (at high temp ; $k_{gas} \neq f(P)$)

Heat in solids transported by conduction:

i) Lattice energy

ii) Flow of electrons

• Fourier's Law:

<u>assumption:</u>

i) Heat is a 1-D flow

ii) Steady state

$$Q_{conduction,x} = -kA\frac{\partial T}{\partial x}$$

<u>Heat flux is a vector quantity:</u>

$$q = q_x \hat{i} + q_y \hat{j} + q_k \hat{k}$$

$$q = k\left[\frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j} + \frac{\partial T}{\partial z}\hat{k}\right]$$

assumed isotropic material $k_x = k_y = k_z = k$

$\Delta E_{Stored} = E_{in} - E_{out} + E_{gen}$

$$Q_x = -k\, dy\, dz \frac{\partial T}{\partial x}$$

$$Q_{x+dx} = Q_x + \frac{\partial Q_x}{\partial x} dx.$$

$$Q_{x+dx} - Q_x = -\frac{\partial Q_x}{\partial x} dx.$$

$$Q_{x+dx} - Q_x = -k\, dx\, dy\, dz \frac{\partial^2 T}{\partial x^2}$$

$$Q_{y+dy} - Q_y = -k\, dx\, dy\, dz \frac{\partial^2 T}{\partial y^2}$$

$$Q_{z+dz} - Q_z = -k\, dx\, dy\, dz \frac{\partial^2 T}{\partial z^2}$$

So. $Q_{gen} = \dot{q}_{gen} \cdot (dx\, dy\, dz)$

$$Q_{stored} = \rho C_p\, dx\, dy\, dz \frac{\partial T}{\partial t}$$

So.

$$\rho C_p\, dx\, dy\, dz \frac{\partial T}{\partial t} = k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + \dot{q}_{gen}.$$

So,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}_{gen}}{k} = \frac{\rho C_p}{k} \frac{\partial T}{\partial t}$$

→ this is heat diffusion equation.

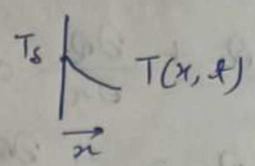
$$U\left[\frac{\rho C_p}{k}\right] = m^2/s$$

$$\frac{\rho C_p}{k} = \frac{1}{\alpha} = 1/\text{thermal diffusivity}$$

Boundary conditions:

i) Dirichlet Boundary condition:

$$T(0,t) = T_s$$



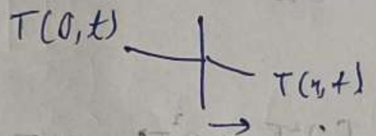
ii) Neumann boundary condition:

$$-k \left.\frac{\partial T}{\partial x}\right|_{x=0} = q_s.$$

for insulated surface,

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0$$

iii) Convection Surface condition:



$$-\cancel{k}.\, h\,(T_\infty - T(0,t)) = -k \left.\frac{\partial T}{\partial x}\right|_{x=0}$$

Special cases.

1) $\frac{\partial^2 T}{\partial x^2} = 0$ [1-D steady state, no heat gen]

2) $\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}_{gen}}{k} = 0$ [1-D steady state, ~~no~~ heat gen]

3) $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$ [2-D steady state, no heat gen]

4) $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$ [1-D unsteady state, no. heat gen]

Case-1

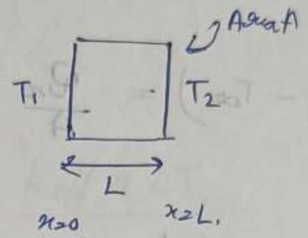
$$k \frac{\partial^2 T}{\partial x^2} = 0$$

$$\frac{d}{dx}\left(k \frac{\partial T}{\partial x}\right) = 0$$

$$\frac{dq_x}{dx} = 0$$

$\therefore$ q_x = Constant.

So. $T = C_1 x + C_2$



So $T = \frac{T_2 - T_1}{L} x + T_1$

[for constant k.]

if,

$$k = (1 + \beta T) k_0$$

$$(1 + \beta T) k_0 \frac{\partial T}{\partial x} = C_1$$

$$T + \frac{1}{2}\beta T^2 = \frac{C_1}{k_0} x + C_2$$

at; $x = 0$, $T = T_1$

$x = L$, $T = T_2$.

$$C_2 = \left(T_1 + \frac{1}{2}\beta T_1^2\right)$$

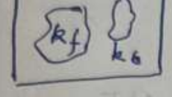
$$\frac{C_1}{k_0} = \left[(T_2 - T_1) + \frac{\beta}{2}(T_2^2 - T_1^2) \right] \frac{1}{L}.$$

So.

$$T\left(1 + \frac{\beta}{2} T\right) = \left[\frac{(T_2 - T_1)}{L} + \frac{\beta}{2L}(T_2^2 - T_1^2) \right] x + \left(T_1 + \frac{\beta}{2} T_1^2 \right).$$

this contact resistance must be included in calculations.

Porous solids:

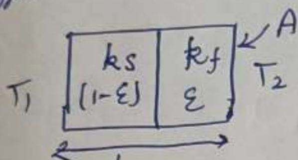


$$Q_x = k_{eff}\,(T_1 - T_2)$$

$$k_{eff} = f(\varepsilon, k_s, k_f, \text{geometry})$$

ε ← void fraction

Suppose.



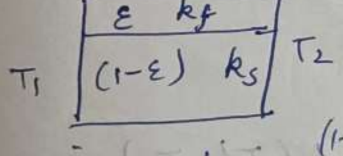
$$Q_x = \frac{k_{eff}\,A\,\Delta T}{L}$$

$$R_{th} = \frac{L(1-\varepsilon)}{k_s A} + \frac{L\,\varepsilon}{k_f A}$$

$$\frac{L}{A k_{eff}} = \frac{L}{A}\left[\frac{1-\varepsilon}{k_s} + \frac{\varepsilon}{k_f}\right]$$

$$\therefore \frac{1}{k_{eff}} = \frac{1-\varepsilon}{k_s} + \frac{\varepsilon}{k_f}$$

Suppose.



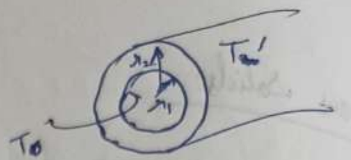
$$Q_x = \frac{k_{eff}\,A\,\Delta T}{L}$$

$$\frac{1}{R_{th}} = \frac{(1-\varepsilon)k_s A}{L} + \frac{\varepsilon k_f A}{L}$$

$$\therefore k_{eff} = (1-\varepsilon)k_s + \varepsilon k_f$$

we can't assume isomorphic in such scenario.

for cylindrical coordinates,



If, $l > r$,

radially heat loss,

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\Rightarrow \frac{1}{r}\frac{d}{dr}\left(k r \frac{\partial T}{\partial r}\right) = 0 \qquad [T = T(r)$$

$$\Rightarrow dT = \frac{C_1}{kr}\, dr$$

$$\therefore T - T_0 = \frac{C_1}{k}\ln\left(\frac{r}{r_1}\right)$$

$$T = T_0 + \frac{C_1}{k}\ln\left(\frac{r}{r_1}\right)$$

$$\frac{(T' - T_0)\, k}{\ln\left(\frac{r_2}{r_1}\right)} = C_1$$

$$\therefore T = T_0 + \frac{(T' - T_0)}{\ln\left(\frac{r_2}{r_1}\right)} \ln\left(\frac{r}{r_1}\right)$$

So.

$$Q_r = k\, 2\pi r L \cdot \frac{1}{kr} \cdot \frac{k(T' - T_0)}{\ln\left(\frac{r_2}{r_1}\right)}$$

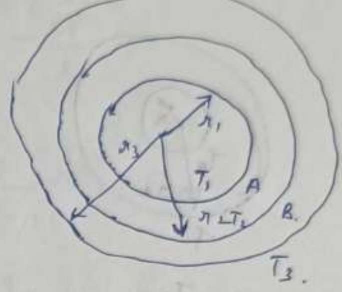
$$\therefore Q_r = \frac{k\, 2\pi L\, (T' - T_0)}{\ln\left(\frac{r_2}{r_1}\right)}$$

so $q_r = f(r)$

but, Q_r is not a function of r.

$$q_r = \frac{Q_r}{A_r}$$

$\therefore$ $R_{th,\text{conductivity}} = \dfrac{\ln(R_2/R_1)}{2\pi k L}$



now,

$$Q_r = \frac{(T_1 - T_2)\, 2\pi L\, k_A}{\ln(r_2/r_1)}.$$

$$T_1 - T_2 = Q_r \frac{\ln(r_2/r_1)}{2\pi L k_A}$$

$$T_2 - T_3 = Q_r \frac{\ln(r_3/r_2)}{2\pi L k_B}.$$

$$\therefore\ T_1 - T_3 = \frac{Q_r}{2\pi L}\left[\frac{\ln(r_2/r_1)}{k_A} + \frac{\ln(r_3/r_2)}{k_B}\right]$$

So

$$R_{th,\text{cond}} = \frac{1}{2\pi L}\sum_{i=1}^{n}\frac{1}{k_i}\ln\left(\frac{r_{i+1}}{r_i}\right)$$

now.

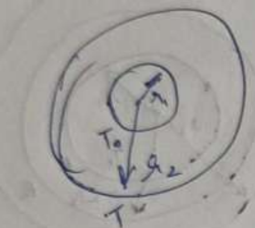
$$\frac{1}{U_0 A_0} = \frac{1}{h_i A_i} + \sum_{i=1}^{n}\frac{\ln(R_{i+1}/R_i)}{k_i\, 2\pi L} + \frac{1}{h_0 A_0}$$

$$\therefore\ \frac{1}{U_0} = \frac{A_0}{h_i A_i} + \sum_{i=1}^{n}\frac{\ln(R_{i+1}/R_i)}{k_i\, 2\pi L} + \frac{1}{h_0}.$$

Spherical coordinates:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(k r^2 \frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial \phi}\left(k \frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(k \sin\theta \frac{\partial T}{\partial \theta}\right)$$

$$+\dot{q} = \rho C_p \frac{\partial T}{\partial t}$$



$T < T_0$

$$\frac{\partial}{\partial r}\left(k r^2 \frac{\partial T}{\partial r}\right) = 0.$$

$$\Rightarrow r^2 \frac{\partial T}{\partial r} = \frac{C_2}{k}$$

$$\Rightarrow \frac{\partial T}{\partial r} = \frac{C_2}{k r^2}$$

$$\Rightarrow T = T_0 + \frac{C_2}{k}\left[\frac{1}{r} + \frac{1}{r_1}\right]$$

$$\Rightarrow (T_2 - T_0)\, k \frac{r_2 r_1}{r_2 - r_1} = C_2.$$

so.

$$Q_r = -k\, 4\pi \cancel{r^2} \frac{C_2}{\cancel{k r^2}}$$

$$\therefore Q_{r_1} = -4\pi C_2$$

Q. $h = 2W/m^2C$, $k = 0.04 W/m°C$.

$$\frac{1}{r^2}\frac{\partial^2(rT)}{\partial r^2} + \frac{\dot{q}}{k} = 0$$

$$\Rightarrow \frac{\partial^2(rT)}{\partial r^2} = -\frac{\dot{q}}{k}r^2$$

$$\Rightarrow \frac{\partial}{\partial r}(rT) = -\frac{\dot{q}}{3k}r^3 + C_1$$

$$\Rightarrow rT = -\frac{\dot{q}}{12k}r^4 + C_1 r + C_2$$

$$\Rightarrow T = -\frac{\dot{q}}{12k}r^3 + C_1 + \frac{C_2}{r}$$

$\dot{q} = 0$.

$$\Rightarrow T = \left(\frac{C_2}{r} + C_1\right)$$

$T(R_1) = \frac{C_2}{R_1} + C_1$

$$\Rightarrow \frac{\partial T}{\partial r} = -\frac{C_2}{r^2}$$

$T(R_2) = \frac{C_2}{R_2} + C_1$

$T(R_2) - T(R_1) = C_2\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$

$Q_r = -k.A_r$

PTO.

Q.''

$$R_{th} = \frac{r_2 - r_1}{4\pi k\, r_1 r_2}$$

$$R_{th} = \frac{1}{4\pi k}\left[\frac{1}{r_1} - \frac{1}{r}\right]$$

So

$$R_{th}\Big|_{Total} = \frac{1}{4\pi k}\left[\frac{1}{r_1} - \frac{1}{r}\right] + \frac{1}{hA_0} + \frac{1}{hA_r}$$

$$\therefore \frac{\partial R}{\partial r} = \frac{1}{4\pi k}\left(\frac{1}{r^2}\right) - \frac{2}{4\pi h\, r^3}$$

$$= \frac{1}{4\pi r^2}\left(\frac{1}{k} - \frac{2}{rh}\right) = 0.$$

So

$$\frac{2}{r_c h} = \frac{1}{k}.$$

$$\therefore r_c = \frac{2k}{h} = \frac{2\times 0.04}{2} = 0.04\,m.$$

So $r_c = 4\,cm$

Now,

$\cancel{Q = \frac{1}{kA_0}}$

$$R_{th}\Big|_{Total} = \frac{100}{4\pi\times 0.04}\left[\frac{1}{1.5} - \frac{1}{4}\right] + \frac{10^4}{4\pi\times 2\times 1.5^2}$$

$$+ \frac{10^4}{4\pi\times 2\times 4^2}$$

Q

$R_{th}|_{Total} = 176.83 + 82.89 + 24.867$

$\doteq 284.587.$

now.

$Q_r = \dfrac{(T_2 - T_1)}{R}$

$= \dfrac{150}{284.587} = 0.527\ W$

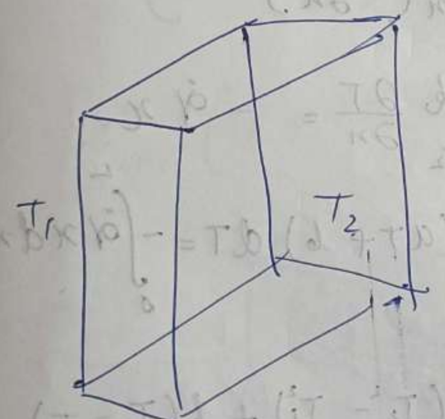
old,

$Q_r = 150 \times 4\pi \times 1.5^2 \times 10^{-4} \times 2$

$Q_r = 0.848\ W.$

So heat loss reduced.

Q.



$\dfrac{\partial^2 T}{\partial x^2} + \dfrac{\dot{q}}{k} = 0$

$\dfrac{\partial^2 T}{\partial x^2} = -\dfrac{\dot{q}}{k}$

$\dfrac{\partial T}{\partial x} = -\dfrac{\dot{q}}{k} x + C_1$

$T = -\dfrac{\dot{q}}{2k} x^2 + C_1 x + C_2$

$T = -\dfrac{\dot{q}}{2k} x^2 + C_1 x + T_{0_1}$

$\dfrac{(T_2 - T_1)}{L} + \dfrac{\dot{q}}{2k} L = C_1$

$\dot{q} = 5 \times 10^8, \ k = 20.$

$L = 10^{-2}\ m$

now.

$$\Rightarrow \frac{100}{10^{-2}} + \frac{5\times10^{8}\times10^{-2}}{40} = C_2$$

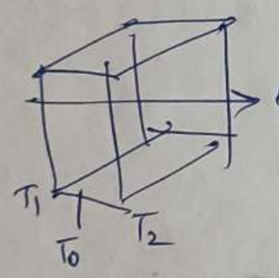
$\therefore C_1 = 1.35\times10^{5}$

$$T = -1.25\times10^{7}x^2 + 1.35\times10^{5}x + 373$$

and

$$T(L/2) = 462.5^\circ C.$$

Q



$\dot{Q} = 2\,kW \qquad k = (aT+b)$

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{q} = 0.$$

$$\Rightarrow k\frac{\partial T}{\partial x} = -\dot{q}x$$

$$\Rightarrow \int_{T_1}^{T_2}(aT+b)\,dT = -\int_0^L \dot{q}x\,dx$$

$$\Rightarrow \frac{a}{2}(T_2^2 - T_1^2) + b(T_2 - T_1) = -\frac{\dot{q}L^2}{2}$$

and

$$\frac{a}{2}(T_0^2 - T_1^2) + b(T_0 - T_1) = -\frac{\dot{q}L^2}{8}$$

assuming steady state, 1-D flow of heat.

or

$$\frac{aT^2}{2} + bT - \frac{aT_1^2}{2} - bT_1 = -\frac{\dot{q}x^2}{2}$$

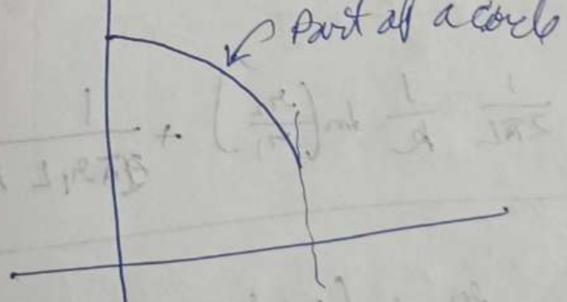
$$\frac{a}{2}T^2 + 2\sqrt{\frac{a}{2}}\,T\cdot\frac{b}{\sqrt{\frac{a}{2}}} + \frac{2b^2}{a} - \frac{2b^2}{a} + \frac{\dot{q}x^2}{2} = \frac{aT_1^2}{2} + bT_1$$

$$\therefore \left(\sqrt{\frac{a}{2}}\,T + 2b\sqrt{\frac{2}{a}}\right)^2 + \frac{\dot{q}x^2}{2} = \frac{a}{2}T_1^2 + bT_1 + \frac{2b^2}{a}$$

$$\frac{a}{2}\left(T + \frac{2b}{a}\right)^2 + \frac{\dot{q}x^2}{2} = \frac{a}{2}T_1^2 + bT_1 + \frac{2b^2}{a}$$

$$\therefore \left(T + \frac{2b}{a}\right)^2 + \frac{\dot{q}}{a}x^2 = T_1^2 + \frac{2b}{a}T_1 + \frac{4b^2}{a^2}$$

$$\therefore \left(T + \frac{2b}{a}\right)^2 + \frac{\dot{q}}{a}x^2 = \left(T_1 + \frac{2b}{a}\right)^2$$



for constant k, it's linear.

5.

~~[illegible]~~

$$R_{th} = \frac{1}{4\pi k}\left[\frac{1}{r_1} - \frac{1}{r}\right] + \frac{1}{hA_0} + \frac{1}{hA_r}.$$

$$Q_r = \frac{(T_r - T_0)}{\frac{1}{4\pi k}\left[\frac{1}{r_1} - \frac{1}{r_0}\right] + \frac{1}{hA_0} + \frac{1}{hA_r}}$$

$$T_0 = T_r - Q_r A_0\left[\frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_0}\right) + \frac{1}{hA_0} + \frac{1}{hA_r}\right]$$

$$T_0 = T_r - q_r A_0\left[\frac{1}{4\pi k r_1} + \frac{1}{4h\pi r_1^2} + \frac{1}{hA_0} - \frac{1}{4\pi k r_1}\right]$$

$$= T_r - \frac{q_r}{h}$$

Q.

$r_1 = \frac{25\,mm}{2}$, $r_2 = \frac{27\,mm}{2}$ $k = 20$

$h_1 = 100$, $h_2 = 12$.

$$R_{th} = \frac{1}{2\pi L}\frac{1}{k}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{2\pi r_1 L h_1} + \frac{1}{2\pi r_2 L h_2}$$

$$\boxed{\frac{1}{U_0} = \frac{r_1}{k}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{h_1} + \frac{r_1}{r_2 h_2}}$$

#

$$\frac{1}{r^2}\frac{\partial^2(rT)}{\partial r^2} = -\frac{\dot{q}}{k}$$

$$\Rightarrow \frac{\partial^2(rT)}{\partial r^2} = -\frac{\dot{q}}{k} r^2$$

$$\Rightarrow \frac{\partial(rT)}{\partial r} = -\frac{\dot{q}}{3k} r^3 + C_1$$

$$\therefore rT = -\frac{\dot{q}}{12k} r^4 + C_1 r + C_2$$

$$\therefore T = -\frac{\dot{q}}{12k} r^3 + C_1 + \frac{C_2}{r}$$

$$\frac{\partial T}{\partial r} = -\frac{\dot{q}}{4k} r^2 - \frac{C_2}{r^2}$$

$$4\pi r^2 \frac{\partial T}{\partial r} = -\frac{\pi \dot{q}}{4k} - C_2$$

$$Q = kA\frac{\partial T}{\partial r} = -\frac{\pi}{4}\dot{q} - C_2 k \qquad kC_2 = -\frac{\pi}{4}\dot{q} - 4\pi R^2 \dot{q}$$

So $hA(T_s - T_0) = -\frac{\pi}{4}\dot{q}$

$$\Rightarrow T_0 = T_s + \frac{\pi}{4h}\dot{q}$$

#

$$CO + S \underset{k_{-A}}{\overset{k_A}{\rightleftharpoons}} CO.S$$

rate of attachment = $k_A P_{CO} C_v$

rate of detachment = $k_{-A} C_{CO.S}$

$$r_A = k_A P_{CO} C_v - k_{-A} C_{CO.S}$$

$$\therefore r_A = k_A \left[P_{CO} C_v - \frac{C_{CO.S}}{K_A} \right] \qquad K_A = \frac{k_A}{k_{-A}}$$

at equilibrium,

$$r_A = 0$$

$$C_{CO.S} = K_A P_{CO} C_v$$

now, site balance,

$$C_t = C_v + C_{CO.S}$$

$$C_t = \frac{C_{CO.S}}{K_A P_{CO}} + C_{CO.S}$$

$$\therefore \frac{1}{C_{CO.S}} = \frac{1}{K_A P_{CO} C_t} + \frac{1}{C_t}.$$

or

$$CO + 2S \underset{k_{-A}}{\overset{k_A}{\rightleftharpoons}} C.S + O.S$$

$$r_A = k_A P_{CO} C_v^2 - k_{-A} C_{C.S} C_{O.S}$$

$$r_A = k_A \left[P_{CO} C_v^2 - \frac{C_{C.S} C_{O.S}}{K_A} \right]$$

$C_t = C_v + C_{C.S} + C_{D.S}$

$C_t = C_v + 2C_{C.S}$

Q.

$A + S \underset{k_{-A}}{\overset{k_A}{\rightleftharpoons}} A.S$

$B + S \underset{k_{-B}}{\overset{k_B}{\rightleftharpoons}} B.S$

$A.S + B.S \underset{k_{-S}}{\overset{k_S}{\rightleftharpoons}} C.S + D.S$

$\frac{k_A}{k_{-A}}$

$\therefore r_A = k_A \left[P_A C_v - \frac{C_{A.S}}{K_A} \right]$

$r_B = k_B \left[P_B C_v - \frac{C_{B.S}}{K_B} \right]$

$r_S = k_S \left[C_{C.S} C_{D.S} - \frac{C_{A.S} C_{B.S}}{K_S} \right]$

or $C.S \underset{k_{-D_C}}{\overset{k_{DC}}{\rightleftharpoons}} C + S$

$\therefore r_{DC} = k_{DC} \left[C_{C.S} - \frac{P_C \cdot C_v}{K_{DC}} \right]$

$\therefore D.S \underset{k_{-DD}}{\overset{k_{DD}}{\rightleftharpoons}} D + S$

$\therefore r_{DD} = k_{DD} \left[C_{D.S} - \frac{P_D \cdot C_v}{K_{DD}} \right]$

* $C \longrightarrow B + P$

$$C + S \underset{k_{-A}}{\overset{k_A}{\rightleftharpoons}} C \cdot S$$

$$C \cdot S \underset{k_{-S}}{\overset{k_S}{\rightleftharpoons}} B \cdot S + P$$

$$B \cdot S \underset{k_D}{\overset{k_D}{\rightleftharpoons}} B + S.$$

Now

$$r_A = k_A \left[P_C C_v - \frac{C_{C \cdot S}}{K_A} \right]$$

$$r_S = k_S \left[C_{C \cdot S} - \frac{P_P \cdot C_{B \cdot S}}{K_S} \right]$$

$$r_D = k_D \left[C_{B \cdot S} - \frac{P_B \cdot C_v}{K_D} \right]$$

now $\frac{r_S}{k_S} >> \frac{r_D}{k_D}$. $\frac{r_S}{k_S} >> \frac{r_A}{k_A}$.

Now.

$$C_{B \cdot S} = \frac{P_B \, C_v}{K_D}.$$

and,

$$C_{C \cdot S} = P_C . K_A C_v.$$

and.

$$C_t = C_v + C_{C \cdot S} + C_{B \cdot S}$$

$$C_t = C_v \left[1 + P_C K_A + \frac{P_B}{K_D} \right]$$

$$r_s = k_s \left[P_C K_A - \frac{P_P \cdot P_B}{K_D K_s} \right] \frac{C_t}{1 + P_C K_A + \frac{P_B}{K_D}}$$

2.

$$\left.\begin{array}{l} I + S \underset{k_{-AI}}{\overset{k_{AI}}{\rightleftharpoons}} I.S \\ W + S \underset{k_{-AW}}{\overset{k_{AW}}{\rightleftharpoons}} W.S \end{array}\right\} \text{adsorbed}$$

$$I.S + W.S \underset{k_{-s}}{\overset{k_s}{\rightleftharpoons}} TBA.S + S \} \text{ reaction.}$$

$$TBA.S \underset{k_{-D}}{\overset{k_D}{\rightleftharpoons}} TBA + S \} \text{ desorption.}$$

now.

$$r_{AI} = k_{AI} \left[P_I . C_v - \frac{C_{I.S}}{K_{AI}} \right] \quad K_{AI} = \frac{k_{AI}}{k_{-AI}}$$

$$r_{AW} = k_{AW} \left[P_W C_v - \frac{C_{W.S}}{K_{AW}} \right] . \quad K_{AW} = \frac{k_{AW}}{k_{-AW}}.$$

$$r_s = k_s \left[C_{I.S} C_{W.S} - \frac{C_v C_{TBA.S}}{K_S} \right]$$

$$r_D = k_D \left[C_{TBA.S} - \frac{P_{TBA} . C_v}{K_D} \right]$$

so $\frac{r_s}{k_s} >> \frac{r_D}{k_D}, \frac{r_{AI}}{k_{AI}}, \frac{r_{AW}}{k_{AW}} \simeq 0.$

so. $C_{I.S} = P_I K_{AI} C_v$, $C_{W.S} = P_W K_{AW} C_v$.

$C_{TBA.S} = P_{TBA} K_{A\,TBA} C_v$

So. $C_t = C_v \left[1 + P_I K_{AI} + P_W K_{AW} + P_{TBA} K_{A\,TBA} \right]$.

So.

$$r_s = k_s \left[P_I P_W K_{AI} . K_{AW} - \frac{P_{TBA} K_{A\,TBA}}{K_s} \right] C_v^2$$

(b)
$$I + S \underset{k_{-A}}{\overset{k_A}{\rightleftharpoons}} I.S$$
$$I.S + W \underset{k_{-s}}{\overset{k_s}{\rightleftharpoons}} TBA + S.$$

$$r_A = k_A \left[P_I C_v - \frac{C_{I.S}}{K_A} \right]$$

$$r_s = k_s \left[P_W C_{I.S} - \frac{P_{TBA} C_v}{K_S} \right]$$

$$\frac{r_s}{k_s} >> \frac{r_A}{k_A}$$

so. $C_{I.S} = P_I K_A C_v.$

$$C_t = C_v + C_{I.S}$$

$$C_t = C_v [1 + P_I K_A]$$

$$r_s = k_s \left[P_I P_W K_A - \frac{P_{TBA}}{K_S} \right] \left(\frac{C_t}{1 + P_I K_A} \right)$$

(c)
$$I + S_1 \underset{k_{-AI}}{\overset{k_{AI}}{\rightleftharpoons}} I.S_1$$
$$W + S_2 \underset{k_{-AW}}{\overset{k_{AW}}{\rightleftharpoons}} W.S_2$$
$$I.S_1 + W.S_2 \underset{k_{-s}}{\overset{k_s}{\rightleftharpoons}} TBA + S_1 + S_2$$

$$r_{AI} = k_{AI} \left[P_I C_{v_1} - \frac{C_{I.S_1}}{K_{AI}} \right]$$

$$r_{AW} = k_{AW} \left[P_W C_{v_2} - \frac{C_{W.S_2}}{K_{AW}} \right]$$

$$r_s = k_s \left[C_{I.S_1} C_{W.S_2} - \frac{P_{TBA} C_{v_1} C_{v_2}}{K_s} \right]$$

no.

$$\frac{r_s}{k_s} >> \frac{r_{AI}}{k_{AI}} \simeq \frac{r_{AW}}{k_{AW}} \simeq 0$$

no.

$$C_{I.S_1} = P_I K_{AI} \cdot C_{v_1}$$

$$C_{W.S_2} = P_W K_{AW} C_{v_2}$$

So

$$C_t = C_{v_1} + C_{v_2} + P_I K_{AI} C_{v_1} + P_W K_{AW} C_{v_2}.$$

So

$$r_s = k_s \left[P_I P_W K_{AI} K_{AW} - \frac{P_{TBA}}{K_s} \right] C_{v_1} C_{v_2}$$

$$C_t = C_{v_1}(1 + P_I K_{AI}) + C_{v_2}(1 + P_W K_{AW})$$

$$C_t^2 \approx 2 C_{v_1} C_{v_2} (1 + P_I K_{AI})(1 + P_W K_{AW})$$

So

$$r_s = k_s \left[P_I P_W K_{AI} K_{AW} - \frac{P_{TBA}}{K_s} \right] \frac{C_t^2}{2(1 + P_W K_{AW})(1 + P_I K_{AI})}$$

$C_{v_1} \approx C_{v_2} = C_v$

$$C_t = C_v \left[2 + P_W K_{AW} + P_I K_{AI} \right]$$

So

$$r_s = k_s \left[P_I P_W K_{AI} K_{AW} - \frac{P_{TBA}}{K_s} \right] \frac{C_t^2}{(2 + P_W K_{AW} + P_I K_{AI})^2}$$

Q. $\frac{1}{r}\frac{\partial^2}{\partial r^2}(rT) = -\frac{\dot{q}}{k}$

$\frac{\partial}{\partial r}(rT) = -\frac{\dot{q}}{k}r + C_1$

$rT = -\frac{\dot{q}}{2k}r^2 + C_1 r + C_2$

$10^6\left[\frac{x \times 10^{-2}}{2 \times 20} - \frac{1}{1?}\right]$

$\therefore rT = \left[-\frac{\dot{q}}{2k}r^2 + C_1 r + C_2\right]$

$= \frac{10^9}{20} - \frac{10^6}{15}$

$\lim_{\substack{r \to 0 \\ T \to T_0}} rT = 0 \Rightarrow C_2 = 0$

So,

$T = -\frac{\dot{q}}{2k}r + C_1$

at $r = 0$, $T = T_0$

$T = \left[T_0 - \frac{\dot{q}}{2k}r\right]$

Now..

$\frac{\partial T}{\partial r} = -\frac{\dot{q}}{2k}$

$T_s = T_0 - \frac{\dot{q}}{2k}R$

So.

$Q_r = hA(T_\infty - T_s)$

So.

$\frac{\dot{q}}{h} = T_\infty - T_s$

$\frac{\dot{q}}{h} = T_\infty - \left(T_0 - \frac{\dot{q}}{2k}R\right)$

$\Rightarrow T_0 = \frac{\dot{q}}{2k}R + T_\infty - \frac{\dot{q}}{h}$

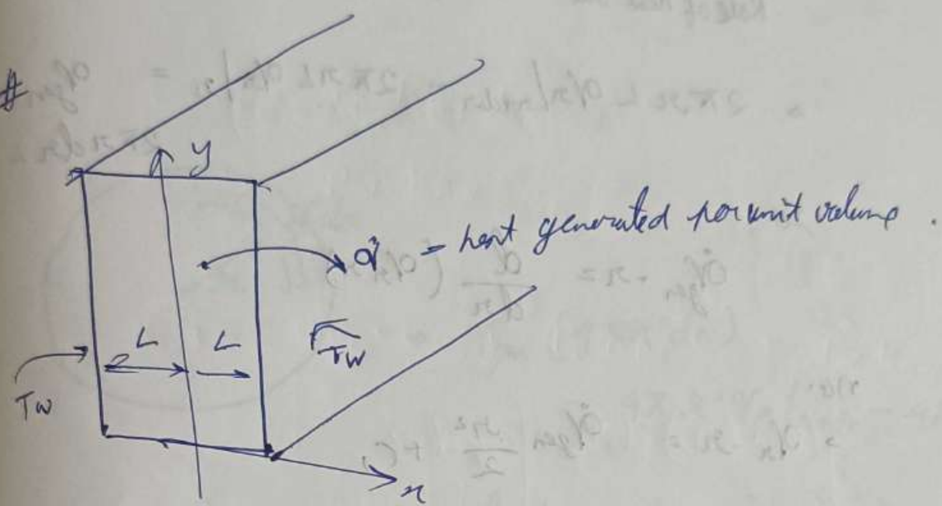
$= T_\infty + \dot{q}\left[\frac{R}{2k} - \frac{1}{h}\right]$

$\cancel{q \ast k (\Delta T)}$

now.

$$-h(T_\infty - T_s) = -k \frac{\partial T}{\partial r} \Big|$$

#



$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{\rho C_p}{k} \frac{\partial T}{\partial t}$$

so

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = 0$$

$$\Rightarrow \frac{\partial^2 T}{\partial x^2} = -\frac{\dot{q}}{k}$$

$$\Rightarrow \frac{\partial T}{\partial x} = -\frac{\dot{q}}{k} x + C_1$$

$$\Rightarrow T = -\frac{\dot{q}}{2k} x^2 + C_1 x + C_2$$

now.

$$T_W = -\frac{\dot{q}}{2k} L^2 + C_1 L + C_2$$

$$T_W = -\frac{\dot{q}}{2k} L^2 - C_1 L + C_2$$

$$2C_2 = \frac{\dot{q}}{k} L^2 + 2T_W$$

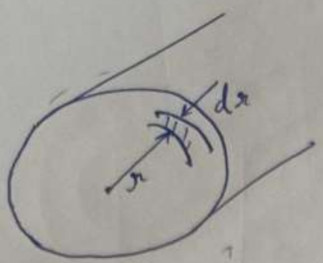
$$C_2 = \frac{\dot{q}}{2k} L^2 + T_W$$

$2C_1 L = 0$

So.

$$T = \frac{\dot{q} L^2}{2k}\left(1 - \left(\frac{x}{L}\right)^2\right) + T_W$$

Q.



Rate of heat out − Rate of heat in = Rate of heat generation.

$\Rightarrow 2\pi r L \, q_r|_{r+dr} - 2\pi r L \, q_r|_r = \dot{q}_{gen} \cdot 2\pi r \, dr \, L$.

so

$$\dot{q}_{gen} \cdot r = \frac{d}{dr}(q_r r)$$

n.a.

$$\Rightarrow q_r \cdot r = \dot{q}_{gen} \frac{r^2}{2} + C_1$$

$$\Rightarrow q_r = \dot{q}_{gen} \frac{r}{2} + \frac{C_1}{r}$$

$$\Rightarrow k \frac{dT}{dr} = \dot{q}_{gen} \frac{r}{2} + \frac{C_1}{r}$$

$$\therefore \; T = \frac{\dot{q}_{gen}}{4k} r^2 + C \ln r + C_1$$

or. $C = 0$

so.

$$T = \frac{\dot{q}_{gen}}{4k} r^2 + C_1$$

at center T_0

so.

$$\therefore \; T = T_0 + \frac{\dot{q}_{gen}}{4k} r^2$$

this is negative in sign

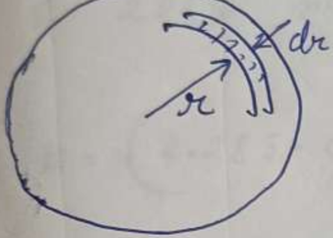
See.

$$q_{cond} = \frac{\dot{q}_{gen}}{2} r$$

so $q_{cond} = \frac{Q}{2\pi r L}$

$$Q = \frac{\dot{q}}{k}\pi r^2 L$$

flux is not constant for such system.



$$\Rightarrow q_{gen}(4\pi r^2 dr) = 4\pi r^2 q_r\big|_{r+dr} - 4\pi r^2 q_r\big|_r$$

$$\Rightarrow \frac{d}{dr}(r^2 q_r) = \dot{q}_{gen} r^2$$

$$\Rightarrow r^2 q_r = \frac{\dot{q}_{gen}}{3} r^3 + C_1$$

$$\Rightarrow q_r = \frac{\dot{q}_{gen}}{3} r + \frac{C_1}{r^2}$$

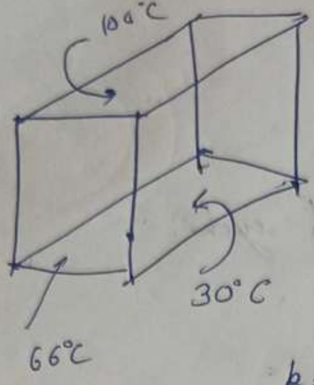
$$\Rightarrow k\frac{\partial T}{\partial r} = \frac{\dot{q}_{gen}}{3} r + \frac{C_1}{r^2}$$

$$\Rightarrow T = \frac{\dot{q}_{gen}}{6k} r^2 + \frac{C_1}{r} + C_2$$

so $C_1 = 0$

so $\boxed{T = \frac{\dot{q}_{gen}}{6k} r^2 + T_0}$

Q.



$A = 0.2\,m^2$

$d = 2.5\,cm$

$k = \cancel{\quad}\,(a + bT)$

no.

$$\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) = 0.$$

so.

$$\int_{T_1}^{T_2} (a + bT)\, dT = \int_{L_1}^{L_2} C\, dx$$

no.

$$a(T_2 - T_1) + \frac{b}{2}(T_2 - T_1)(T_2 + T_1) = C(L_2 - L_1)$$

so

$$a\, 70 + \frac{b}{2}\, 70\,(130) = C(0.025)$$

$$70a + 4550\,b = 0.025\,C.$$

or.

$$36a + 1728\,b = 0.0125\,C$$

$$34a + 2822\,b = 0.0125\,C$$

or.

$$aT + \frac{b}{2}T^2 = C_1 x + C_2$$

$$\frac{\partial T}{\partial x}(a + bT) = C_1$$

$\Rightarrow -\frac{\partial T}{\partial x}(a+bT) = -C_1$

$\Rightarrow \cancel{-2000(a+bT) = C_1}$

$\cancel{a+bT = -\frac{1}{2000}C_1}$

So $C_1 = 2.$

$a = 2.285$ or $b = -0.0186$

So. $k = (2.285 - 0.0186\,T)$

$= k_0(1 - aT)$

$T = T_0 - \frac{\dot{q}_{gen}}{6k} r^2$

& $\frac{\partial T}{\partial r} = \frac{\dot{q}_{gen}}{3k} r$

So $k\frac{\partial T}{\partial r} = \frac{\dot{q}_{gen}}{3} r.$

So $h(T_s - T_\infty) = \frac{\dot{q}_{gen}}{3} r.$

$\Rightarrow T_s = T_\infty + \frac{\dot{q}_{gen}}{3h} r$

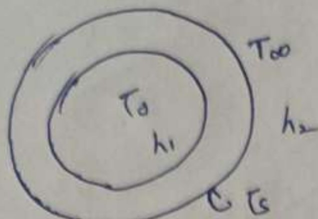
$= 30 + \frac{10^6}{45} \times 0.02 = 474.44\,°C$

So. $T_0 = 474.44 + \frac{10^6}{6\times 20} \times 0.02^2$

$\boxed{T_0 = 477.77\,°C}$

$L_2 - L_1)$

Q.



$h_1 = 100\, W/m^2 (°C)$

$h_2 = 12\, W/m^2 (°C)$

$d = 0.025\, m$

now.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0.$$

So.

$$r\frac{\partial T}{\partial r} = C$$

$$\frac{\partial T}{\partial r} = \frac{C}{r}$$

$$\int_{T_0}^{T} dT = C\int_{r_1}^{r} \frac{dr}{r}$$

$$T = T_0 + C \ln\left(\frac{r}{r_1}\right)$$

$$T_s = T_0 + C \ln\left(\frac{r_2}{r_1}\right)$$

so

$$T = T_0 + \frac{T_s - T_0}{\ln\left(\frac{r_2}{r_1}\right)} \ln\left(\frac{r}{r_1}\right)$$

$$\frac{\partial T}{\partial r} = \frac{T_s - T_0}{r \ln\left(\frac{r_2}{r_1}\right)}$$

$$\Rightarrow -\frac{T_s - T_0}{r_2 \ln\left(\frac{r_2}{r_1}\right)} k = h(-T_\infty + T_s)$$

$$\Rightarrow h T_s - h T_\infty = -\frac{T_s}{r_2 \ln\left(\frac{r_2}{r_1}\right)} k + \frac{T_0}{r_2 \ln\left(\frac{r_2}{r_1}\right)} k$$

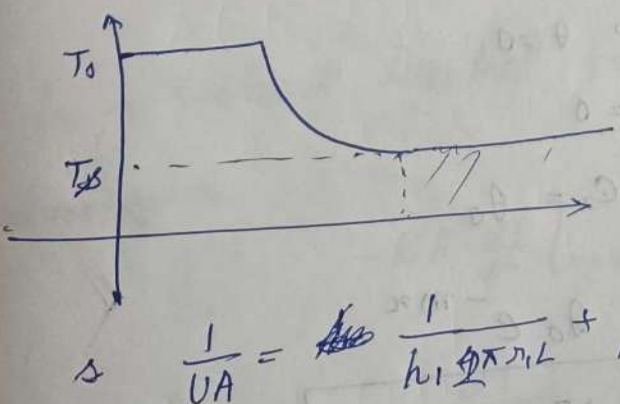
$$\Rightarrow T_s\left[h + \frac{k}{r_2 \ln\left(\frac{r_2}{r_1}\right)}\right] = h T_\infty + \frac{T_0 k}{r_2 \ln\left(\frac{r_2}{r_1}\right)}$$

$$\therefore T_s = \frac{h T_\infty + \frac{T_0 k}{r_2 \ln\left(\frac{r_2}{r_1}\right)}}{\left[h + \frac{k}{r_2 \ln\left(\frac{r_2}{r_1}\right)}\right]}$$

$$-Ak\frac{\partial T}{\partial r} = \frac{(T_0 - T_s)\, 4\pi r L k}{r \ln\left(\frac{r_2}{r_1}\right)}$$

$$Q = \frac{\cancel{4}\pi k L}{\ln\left(\frac{r_2}{r_1}\right)}(T_0 - T_s)$$

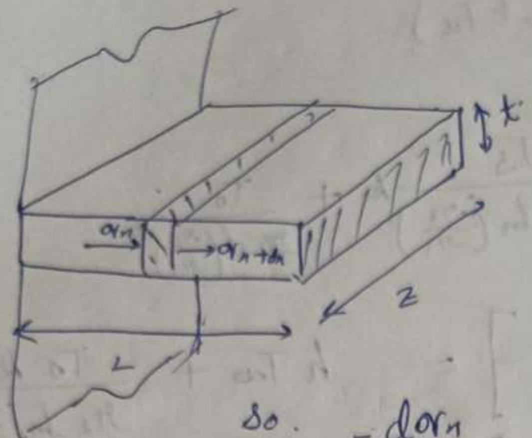
$$R_{th} = \cancel{\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k L}}\ \frac{\ln(r_2/r_1)}{2\pi k L}$$



$$\frac{1}{UA} = \frac{1}{h_1 \cancel{2}\pi r_1 L} + \frac{1}{h_2 \cancel{2}\pi r_2 L} + \frac{\ln(r_2/r_1)}{\cancel{2\pi k L}}$$

$$\frac{1}{U_0} = \left[\frac{1}{h_1} + \frac{1}{h_2}\left(\frac{r_1}{r_2}\right) + \frac{\ln(r_2/r_1)\, r_1}{k}\right]$$

#



so. $-\dfrac{dq_x}{dx} = \dfrac{hP}{A}(T-T_\infty)$

$\therefore \quad k\dfrac{d^2T}{dx^2} = \dfrac{hP}{A}(T-T_\infty)$

$\theta = T - T_\infty$

$\theta_0 = T_0 - T_\infty$

so. $\dfrac{d^2T}{dx^2} - \dfrac{hP}{kA}(T-T_\infty) = 0$

$\therefore \quad \dfrac{d^2\theta}{dx^2} - \dfrac{hP}{kA}\theta = 0$

so.

$\theta = C_1 e^{-mx} + C_2 e^{mx} \qquad m = \sqrt{\dfrac{hP}{kA}}$

B.C: 1, $x=0$, $T=T_0$, $\theta=\theta_0$

Case-1

B.C. $x=\infty$, $\theta=0$

$C_2 = 0$

so. $C_1 + C_2 = \theta_0$

so. $\theta = \theta_0 e^{-mx}$

$\therefore \quad \boxed{\dfrac{T-T_\infty}{T_0-T_\infty} = e^{-mx}}$

Case-II .. end is insulated,

$x = L, \quad \frac{dT}{dx} = 0.$

So $\left.\frac{d\theta}{dx}\right|_{x=L} = 0$

$\therefore \; C_2 e^{mL} - C_1 e^{-mL} = 0$

$C_1 + C_2 = \theta_0$

$C_1 = C_2 e^{2mL}$

$C_2(1 + e^{2mL}) = \theta_0$

So $\theta = \frac{\theta_0}{1+e^{2mL}} e^{mx} + \frac{e^{2mL}\theta_0}{1+e^{2mL}} e^{-mx}$

$= \frac{\theta_0}{e^{mL}+e^{-mL}} e^{-m(L-x)} + \frac{\theta_0 \, e^{m(L-x)}}{e^{mL}+e^{-mL}}.$

$\therefore \; \theta = \frac{\theta_0}{\cosh(mL)} \cosh(m(L-x))$

$\therefore \; T - T_\infty = (T_0 - T_\infty) \frac{\cosh[m(L-x)]}{\cosh(mL)}$

Case-III : fin is finite and losses heat from convection at end

$-kA \left.\frac{\partial T}{\partial x}\right|_{x=L} = hA(T - T_\infty).$

$\therefore \; \left.\frac{\partial T}{\partial x}\right|_{x=L} = -\frac{h}{k}(T - T_\infty)$

2. $\theta = C_1 e^{-mx} + C_2 e^{mx}$

at, $x=0$, $\theta = \theta_0$

$\therefore \theta_0 = C_1 + C_2$

or. $\frac{\partial \theta}{\partial x} = -mC_1 e^{-mL} + mC_2 e^{mL}$

$-\frac{h}{mk}(\theta_L) = -C_1 e^{-mL} + C_2 e^{mL}$

$$\theta_0 e^{-mL} = (C_1 + C_2) e^{-mL}$$

$$\theta_0 e^{-mL} - \frac{h}{mk}\theta_L = C_2 (e^{mL} + e^{-mL})$$

$$\therefore C_2 = \frac{\theta_0 e^{-mL} - \frac{h}{mk}\theta_L}{(e^{mL} + e^{-mL})}$$

or.

$$C_1 = \theta_0 - \frac{\theta_0 e^{-mL} - \frac{h}{mk}\theta_L}{e^{mL} + e^{-mL}}$$

$$= \frac{\theta_0 e^{mL} + \frac{h}{mk}\theta_L}{e^{mL} + e^{-mL}}$$

So

$$\theta = \frac{\theta_0 e^{m(L-x)} + \frac{h}{mk}\theta_L e^{-mx}}{e^{mL}+e^{-mL}} + \frac{\theta_0 e^{-m(L-x)} - \frac{h}{mk}\theta_L e^{mx}}{e^{mL}+e^{-mL}}$$

So.

$$\theta_L = \frac{\theta_0 + \frac{h}{mk}\theta_L e^{-mL} + \theta_0 - \frac{h}{mk}\theta_L e^{mL}}{e^{mL}+e^{-mL}}$$

$$\Rightarrow \theta_L e^{mL} + e^{-mL}\theta_L = 2\theta_0 + \frac{h}{mk}\theta_L\left(e^{mL}-e^{-mL}\right)$$

$$\Rightarrow \theta_L \frac{\left(e^{mL}+e^{-mL}\right)}{2} = \frac{2\theta_0}{2} + \frac{h}{mk}\theta_L \frac{\left(e^{mL}-e^{-mL}\right)}{2}$$

$$\therefore \theta_L \cosh(mL) = \theta_0 - \frac{h}{mk}\theta_L \sinh(mL)$$

$$\theta_L\left(\cosh(mL) + \frac{h}{mk}\sinh(mL)\right) = \theta_0$$

So.

$$\frac{\theta}{\theta_0} = \cancel{\frac{\left[e^{m(L-x)} + \left(\frac{h}{mk}\right)\frac{e^{-mx}}{\cosh(mL) - \frac{h}{mk}\sinh(mL)}\right]}{2\cosh(mL)}}$$

$$= \frac{\cosh m(L-x) + \frac{h}{mk}\sinh m(L-x)}{\cosh(mL) + \frac{h}{mk}\sinh mL}$$

So.

$$\boxed{\frac{\theta}{\theta_0} = \frac{\cosh m(L-x) + \frac{h}{mk}\sinh m(L-x)}{\cosh(mL) + \frac{h}{mk}\sinh mL}}$$

Fin Efficiency: η_f = actual heat transferred / Heat which would be transferred if the entire thing is at T_0.

Case-I

$$\theta = \theta_0 e^{-mx}$$

$$\frac{d\theta}{dx} = \theta_0(-m)e^{-mx}$$

at $d\dot{Q} = \theta_0(-m)e^{-mx}\, k\, dA$

$$\eta_f = \frac{\sqrt{hPkA_c}\,\theta_0}{hPL\theta_0}$$

$$\therefore \eta_f = \frac{1}{L}\sqrt{\frac{kA_c}{hP}} = \frac{1}{mL}$$

$e^{mx} + e^{-mx}$

Case-II

$$\theta = \theta_0 \frac{\cosh(m(L-x))}{\cosh mL}$$

now $\frac{\partial^2\theta}{\partial x^2} = \frac{\cosh(m(L-x))}{\cosh mL}$

$$\eta = \frac{\theta_0 \tanh(mL)\sqrt{hPKA}}{hPL\theta_0}$$

$$\eta = \tanh(mL)\frac{\sqrt{hPK_c}}{hPL}$$

$$\therefore \eta = \frac{\tanh(mL)}{mL}$$ for small L, $\eta_{f,max} = 1$

Effectiveness

$$\epsilon = \frac{Q_{fin}}{Q_{without\ fin}}$$

ex-I

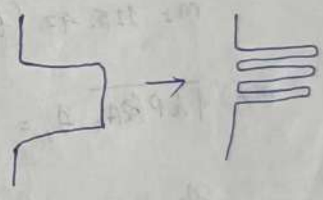
$$\epsilon = \frac{\eta_f A_f h \theta_0}{h A_c \theta_0}$$

ie $$\epsilon = \frac{1}{mL}\cancel{\frac{A_f}{A_c}}\sqrt{\frac{kP}{hA_c}}$$

ex-II

$$\epsilon = \frac{\tanh(mL)}{mL}\cancel{\frac{A_f}{A_c}}\sqrt{\frac{kP}{hA_c}}.$$

$\frac{P}{A_c}$ needs to be large. (Closely spaced thin fins)

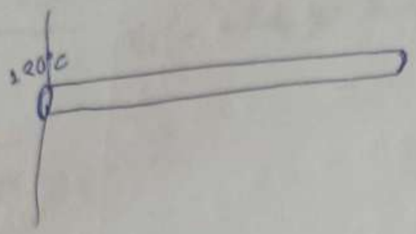


only good for, $\epsilon_f \geq 2$

$$\frac{kP}{hA_c} \geq 2$$

\# compare the heat transfer with and without fin for a SS pin-fin ($k = 16$ W/m·K), $L = 10$ cm, $d = 1$ cm ($h = 5000$ W/m²K)

Q



$d = 2mm = 0.002m$

$T_\infty = 35°C$

$h = 100 W/m^2 K.$

$k_1 = 400 W/m.K.$

$k_2 = 15 W/mK.$

$\theta = \theta_0 e^{-mx}$

$m = \sqrt{\frac{hP}{kA_c}}$ ← cross sectional area

$= \sqrt{\frac{h}{k}\frac{\pi D}{\frac{\pi D^2}{4}}}$

$= \sqrt{\frac{4h}{kD}}$

$m = 22.36$ (for copper)

$m = 115.47$ (for alloy)

$\sqrt{hPkA_c}\,\theta_0 = \sqrt{hPkA_c}\tanh mL\,\theta_0.$

So. $\tanh mL = 1$

$\Rightarrow 2\sigma(2mL) - 1 = 1$

$\Rightarrow \cancel{\sigma(2mL) = 1}$

$2\sigma(2mL) \geq 1.99$

$\sigma(2mL) \geq 0.995.$

So.

$1 + e^{-2mL} \leq \frac{1}{0.995}$

$e^{-2mL} \leq 5.025 \times 10^{-3}$

$-2mL \leq -5.29.$

$2m$

$K.$

1.99

0.995

$\leq \frac{1}{0.995}$

$\leq 5.025 \times 10^{-3}$

$$\therefore L \geq \frac{2.65}{m} = \frac{\ln\sqrt{199}}{m}$$

for copper, $L_{min} = 0.1185\ m = 118.5\ mm$

for alloy, $L_{min} = 0.0229\ m = 22.9\ mm$

for heat transffer find the L_∞:

Q. $h A_c (T_L - T_\infty) = h (P) (T_L - T_\infty) \cdot \Delta L$

S. $h \frac{\pi D^2}{4} (T_L - T_\infty) = h\, \Delta L\, (\pi D)(T_L - T_\infty)$.

$$\Rightarrow \frac{\pi D^2}{4} = \Delta L\, \pi D$$

$$\therefore \Delta L = \frac{D}{4} = \frac{R}{2} \quad \text{and.} \quad L_\infty = \frac{\ln\sqrt{199}}{\sqrt{\frac{4h}{kD}}}$$

$$\Rightarrow \cancel{L_\infty = (\sqrt{k}\ \ln\sqrt{199})\sqrt{\frac{D}{4h}}}$$

$$\cancel{L_\infty = C\sqrt{\Delta L}}$$

$$\cancel{\therefore \boxed{\Delta L = \frac{L_\infty^2}{C^2}}}$$

Q.

$$m \frac{\sinh mL + \frac{h}{mk}\cosh mL}{\cosh mL + \frac{h}{mk}\sinh mL} \geq m(0.99)$$

for L_∞

$\Rightarrow \tanh(mL) + \frac{h}{mk} \geq 0.99\left(1 + \frac{h}{mk}\tanh mL\right)$

$\Rightarrow \left(1 - 0.99\frac{h}{mk}\right)\tanh mL \geq \left(0.99 - \frac{h}{mk}\right)$

$\Rightarrow 2\sigma(2mL) - 1 \geq \dfrac{0.99 - \frac{h}{mk}}{1 - 0.99\frac{h}{mk}}$

$\Rightarrow \dfrac{2}{1+e^{-2mL}} \geq \dfrac{1 - 0.99\frac{h}{mk} + 0.99 - \frac{h}{mk}}{1 - 0.99\frac{h}{mk}}$

$\Rightarrow \dfrac{2\left(1 - 0.99\frac{h}{mk}\right)}{1.99\left(1 - \frac{h}{mk}\right)} \geq 1 + e^{-2mL_\infty}$

$\Rightarrow e^{-2mL_\infty} \leq \dfrac{2 - 1.98\frac{h}{mk} - 1.99 + 1.99\frac{h}{mk}}{1.99\left(1 - \frac{h}{mk}\right)}$

$\Rightarrow -2mL_\infty \leq \ln\left(\dfrac{0.01\left(1 + \frac{h}{mk}\right)}{1.99\left(1 - \frac{h}{mk}\right)}\right)$

$\Rightarrow 2mL_\infty \geq \ln\left(\dfrac{199\left(1 - \frac{h}{mk}\right)}{\left(1 + \frac{h}{mk}\right)}\right)$

$L_\infty \geq \frac{1}{2m}\ln(199) + \frac{1}{2m}\ln\left(\dfrac{1 - \frac{h}{mk}}{1 + \frac{h}{mk}}\right)$

Unsteady state:

$$T = T(x, y, z, t)$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

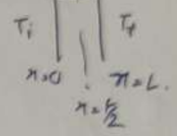
for 1-D,

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$t = 0, \; T = T_i \quad 0 \le x \le L$

$x = 0, \; T = T_1, \; t > 0$

$x = L, \; T = T_1 \quad t > 0$



$T = T_1 + \theta$: so

at: $t = 0, \; \theta = \theta_i \; , \; 0 \le x \le L$

$x = 0, \; \theta = 0 \; : t > 0$

$x = L, \; \theta = 0 \; : t > 0$

$$\theta(x, t) = X(x)\,\tau(t)$$

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{\alpha}\frac{\partial \theta}{\partial t}$$

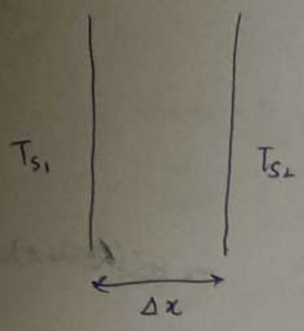
so. $\frac{\partial^2 \theta}{\partial x^2} = \tau(t) \cdot \frac{\partial^2 X}{\partial x^2}$

so, $\tau(t)\frac{\partial^2 X}{\partial x^2} = \frac{1}{\alpha}\frac{\partial \tau}{\partial t} X(x)$

$$\Rightarrow \frac{1}{X(x)}\frac{\partial}{\partial x}\left(\frac{\partial X}{\partial x}\right) = \frac{1}{\alpha}\frac{1}{\tau(t)}\frac{\partial(\tau)}{\partial t}$$

so $\Rightarrow$

$$\frac{\theta}{\theta_i} = \boxed{\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}e^{-\left(\frac{n\pi}{2L}\right)^2\alpha t}} \sin\frac{n\pi x}{2L}$$



$$Q = \frac{T_{S1} - T_{S2}}{\Delta x / kA} = \frac{T_{S2} - T_\infty}{1/hA}$$

So.
$$\frac{T_{s_1} - T_{s_2}}{T_2 - T_\infty} = \frac{h\,\Delta x}{k}.$$

$$\frac{\Delta x}{k} << \frac{1}{h} \quad \text{so} \quad \frac{h\Delta x}{k} << 1$$

Biot number $(B_i) = \dfrac{h_f\,\Delta x}{k_s}$ for fluid.

for solid

$Q_{in} = Q_{store}$

$-Q_{out} = Q_{stored}$

$$\Rightarrow -hA_s\,(T_s - T_\infty) = \rho C_p\, V \frac{dT}{dt}.$$

$$\theta = T_s - T_\infty.$$

$$\Rightarrow -hA_s\theta = \rho C_p\, V \frac{d\theta}{dt}.$$

$$\Rightarrow \int_{\theta_0}^{\theta} \frac{d\theta}{\theta} = -\frac{hA_s}{\rho C_p V} \int_0^t dt$$

$$\boxed{\theta = \theta_0\, e^{-\frac{hA_s}{\rho C_p V} t}}.$$

Fuiyoh!

$$\therefore \theta = \theta_0 e^{-t/\tau}$$

$$\tau = \frac{\rho C_p V}{h A_s}$$

time constant for system

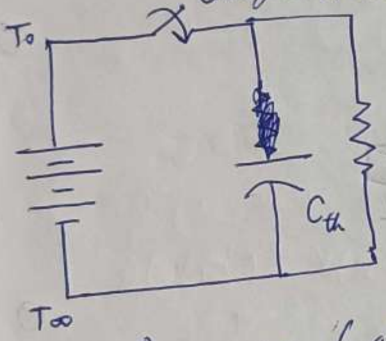
$$\therefore \tau = \frac{1}{h A_s} \cdot \rho C_p V$$

$$\therefore \tau = R_{th} C_{th}$$

lumped thermal capacity.

Resistance to heat transfer.

charged capacitor then again disconnected.



$$\frac{\theta}{\theta_i} = 0.368 \quad (\text{at } t = \tau)$$

now

$$\frac{\theta}{\theta_i} = \exp\left(\frac{-hA_s}{\rho C_p V}\right) t$$

$$= \exp\left(\frac{-h t}{\rho C_p L_{cr}}\right)$$

no.

$$\frac{h_{conv} L_{cr}}{K_s} \; \frac{K_s}{\rho C_p} \; \frac{t}{L_{cr}^2}$$

$$= \frac{B_i \alpha t}{L_{cr}^2}$$

$\dfrac{\alpha t}{L_{cr}^2}$ is known as Fourier's number (F_o)

$$\underbrace{\frac{\theta}{\theta_i} = \exp\left(-B_i F_o\right) = \exp\left(-\frac{t}{\tau}\right)}_{\text{Lumped capacitance model.}} \qquad \text{valid for, } B_i << 1$$

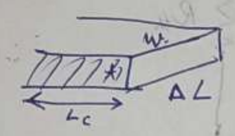
$$L_{eff} = \frac{V}{A_s}$$

→ for a conservative calculation we usually take where is the maximum temperature gradient occurs.

Q. Fig 1

$$mL_c = \sqrt{\frac{hP}{kA_c}}\, L_c$$

for rectangular,



$$w >> t \qquad P \simeq 2W \qquad \frac{P}{A_c} = \frac{2}{t}$$

$$mL_c = \sqrt{\frac{2h}{kt}}\, L_c = \sqrt{\frac{2h}{ktL_c}}\, L_c^{3/2}$$

$$= \sqrt{\frac{2h}{kA_m}}\, L_c^{3/2}$$

Ex.

$$L_c = \left(L + t/2\right) \qquad m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{2h(w+t)}{k\, wt}} = \sqrt{\frac{2h}{k}}\sqrt{\frac{1}{w}+\frac{1}{t}}$$

$$-\frac{1}{2m}\ln\left(\frac{1-\frac{h}{mk}}{1+\frac{h}{mk}}\right)$$

$$\simeq -\frac{1}{2m}\; \frac{\frac{2h}{mk}}{1-\frac{h}{mk}} = \frac{h}{mk}\left(\frac{1}{m-\frac{h}{k}}\right)$$

i << 1

re is the.

$\sqrt{\frac{2h(w+t)}{k\,wt}}$

$\sqrt{\frac{2h}{k}}\sqrt{\frac{1}{w}+\frac{1}{t}}$

$\frac{\cancel{h}\,1}{m-\frac{h}{k}}\Big)$

error $< 8\%$ if $\sqrt{\frac{ht}{2k}} \le \frac{1}{2}$.

So.

$$\frac{h}{mk} = \frac{h}{\sqrt{\frac{2h}{k}}\sqrt{\frac{1}{w}+\frac{1}{t}}\;k}$$

$$= \sqrt{\frac{h}{2k}}\cdot\frac{1}{\sqrt{\frac{1}{w}+\frac{1}{t}}}$$

if w is high.

Si.

$$\frac{h}{mk} \approx \sqrt{\frac{ht}{2k}} \le \frac{1}{2}.$$

$$\ln\left(\frac{1-\frac{h}{mk}}{1+\frac{h}{mk}}\right)$$

$$= \ln\left(\frac{\frac{1}{2}}{\frac{3}{2}}\right) = \frac{-\ln 3}{2m}$$

$$= -\frac{\ln\sqrt{3}}{\sqrt{\frac{2ht}{k}}}$$

$$= -\frac{0.549}{\sqrt{\frac{2h}{k}}\sqrt{t}}$$

$D_i = 2.5\,cm$
$D_o = 3\,cm$

Q.

$$\frac{1}{U_o} = \frac{1}{h_i}\frac{A_o}{A_i} + \frac{1}{h_o} + \frac{A_o}{2\pi k L}\ln\left(\frac{D_o}{D_i}\right)$$

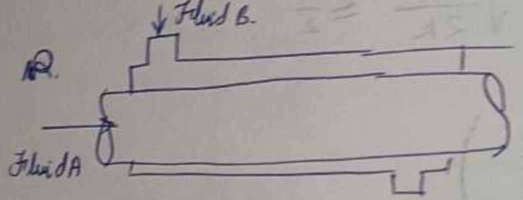
$$= \frac{1}{h_i}\left(\frac{D_o}{D_i}\right)^2 + \frac{1}{h_o} + \frac{D_o^2}{8kL}\ln\left(\frac{D_o}{D_i}\right)$$

for unit length $L = 1$

$$\frac{1}{U_o} = \frac{1}{3500}\left(\frac{3}{2.5}\right)^2 + \frac{1}{7.6} + \frac{0.03}{8\times 16}\ln\left(\frac{3}{2.5}\right)$$

$\therefore U_o = 7.573\ W/m^2\cdot{}^\circ C$.

Q.



$Q = UA\Delta T_m$

$$\Delta T_m = \frac{(T_{h,out} - T_{c,out}) - (T_{h,in} - T_{c,in})}{\ln\left(\frac{T_{h,out} - T_{c,out}}{T_{h,in} - T_{c,in}}\right)}$$

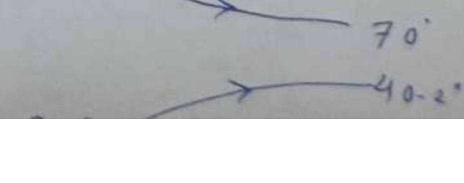
Q.

$120^\circ C \rightarrow 70^\circ C$. water is at $30^\circ C$.

So.

$\Rightarrow 0.25 \times 4178 \times (T - 30) = 2131 \times 0.1\,(50)$

$\therefore T_{c,out} = 40.2^\circ C$.



$D_i = 2.5cm$

$D_o = 3cm$

Q.

$$\frac{1}{U_o} = \frac{1}{h_i}\frac{A_o}{A_i} + \frac{1}{h_o} + \frac{A_o}{2\pi k L}\ln\left(\frac{D_o}{D_i}\right)$$

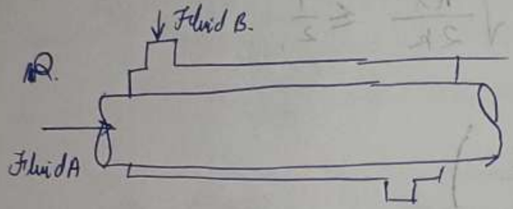
$$= \frac{1}{h_i}\left(\frac{D_o}{D_i}\right)^2 + \frac{1}{h_o} + \frac{D_o^2}{8kL}\ln\left(\frac{D_o}{D_i}\right)$$

for unit length $L = 1$

$$\frac{1}{U_o} = \frac{1}{3500}\left(\frac{3}{2.5}\right)^2 + \frac{1}{7.6} + \frac{0.03}{8\times 16}\ln\left(\frac{3}{2.5}\right)$$

$\therefore U_o = 7.573\ W/m^2\cdot{}^\circ C$

Q.



$Q = UA\Delta T_m$

$$\Delta T_m = \frac{(T_{h,out} - T_{c,out}) - (T_{h,in} - T_{c,in})}{\ln\left(\frac{T_{h,out} - T_{c,out}}{T_{h,in} - T_{c,in}}\right)}$$

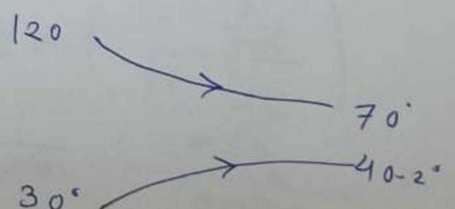
Q.

$120^\circ C \rightarrow 70^\circ C.$ water is at $30^\circ c.$

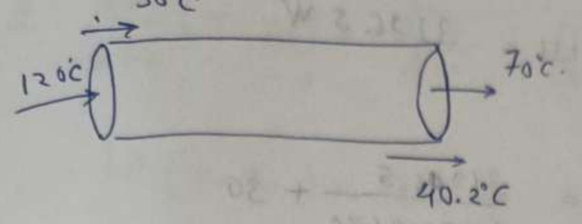
Sol.

$\Rightarrow 0.25 \times 4178 \times (T - 30) = 2131 \times 0.1\ (50)$

$\therefore T_{c,out} = 40.2^\circ C.$



now.
for co-current:



now.

$$\Delta T_m = \frac{90 - 29.8}{\ln\left(\frac{90}{29.8}\right)} = 54.46\,°C.$$

now.

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} = \frac{1}{\frac{1}{2280} + \frac{1}{38.8}} = 38.14\ W/m^2 K$$

now,

$$\Rightarrow Q = 0.25 \times 4178 \times 10.2$$
$$= 10653.9$$

now.

$$10653.9 = 38.14 \times 54.46 \times \pi \times (0.025)\,L$$

$$\therefore \boxed{L = 65.307\,m}$$

for counter current:

$$\Rightarrow 10653.9 = \Delta T_m L\,(\pi \times 38.14 \times 0.025)$$

$$\therefore L = \frac{3556.624}{\Delta T_m}$$

and

$$\Delta T_m = \frac{(120 - 40.2) - (70 - 30°)}{\ln\left(\frac{120 - 40.2}{70 - 30}\right)} = \frac{79.8 - 40}{\ln\left(\frac{79.8}{40}\right)}$$

$$= 57.627\,°C$$

$$\therefore \boxed{L = 61.7\ m}$$

Q.

$$Q = 15 \times 0.1 \times 2131 = 3196.5 \text{ W}$$

now,

$$T_{w,o} = \frac{3196.5}{0.25 \times 4178} + 30$$

$$\therefore T_{w,o} = 33.06^\circ C$$

Q.

$\Rightarrow$ assuming Co-current:

$$\therefore \Delta T_m = \frac{(85-30)-(70-33.06)}{\ln\left(\frac{85-30}{70-33.06}\right)} = \frac{55 - 36.94}{\ln\left(\frac{55}{36.94}\right)}$$

$$= 45.37^\circ C$$

Q.

$$\Rightarrow 3196.5 = 45.37 \times 38.14 \times \pi \times 0.025 \times L$$

$$\therefore \boxed{L = 23.52 \text{ m}}$$

Q.

$$Q = \sum Q_j = \sum U_j A_j \, \Delta T_{LMTD,j}$$

$\hookrightarrow$ use it when U varies with temperature way too much.

$$dQ_{conv} = Q_x\big|_x - Q_x\big|_{x+dx}$$

$$\Rightarrow \frac{hP}{kA_c}(T-T_\infty) = \frac{d^2T}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx}$$

$$\Rightarrow hP(T-T_\infty) = k\left[\frac{d}{dx}\left(A_c\frac{dT}{dx}\right)\right]$$

$$\therefore \frac{d}{dx}\left(A_c\frac{dT}{dx}\right) = \frac{hP}{k}(T-T_\infty)$$

$$\therefore \frac{d^2T}{dx^2} + \frac{1}{A_c}\frac{dA_c}{dx}\left(\frac{dT}{dx}\right) - \frac{hP}{kA_c}(T-T_\infty) = 0.$$

$$\therefore \boxed{\frac{d^2\theta}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{d\theta}{dx} - m^2\theta = 0}$$

– for all fins.

for uniform cross-section: $\frac{dA_c}{dx} = 0.$

$$\therefore \frac{d^2\theta}{dx^2} - m^2\theta = 0$$

So. effectiveness: $\frac{Q_{with\ fin}}{Q_{without}} = \frac{\eta_f A_f h\theta_0}{hA_c\theta_0}$

$$E = \eta_f\left(\frac{A_f}{A_c}\right)$$

and

$$\eta_f = \frac{Q_{actual}}{Q_{Total\ fin\ at\ \theta_0}}.$$

36.94

55

6.94

7°C

too much.

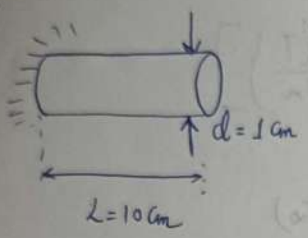
$$\eta_f = \frac{Q_f}{h A_f \theta_0}$$

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{5000 \times \pi D}{16 \times \frac{\pi D^2}{4}}} = \sqrt{\frac{5000 \times 4}{16 \times D}} = \sqrt{\frac{5000 \times 4}{16 \times 0.01}} = \frac{2.65}{m}$$

$L_\infty = 7.43\,mm$

$L_\infty < L$

Q.

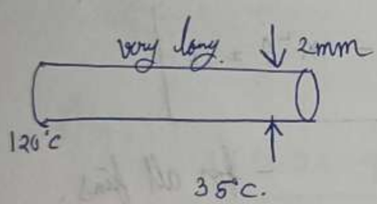


$h = 5000\ W/m^2 \cdot K$,

$k = 16\ W/m \cdot K$

$$\varepsilon_f = \sqrt{\frac{kP}{hA_c}} = \sqrt{\frac{16 \times \pi \times 10^{-2}}{5000 \times \pi (0.1)^2} \times 4} = 0.113$$

Q.



$$\frac{\theta}{\theta_0} = e^{-mx}$$

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{h \pi D \, 4}{k \pi D^2}} = \sqrt{\frac{4h}{kD}} = \sqrt{\frac{4 \times 100}{k \times 0.002}} = \frac{447.2}{\sqrt{k}}$$

$\theta_0 = 85°C$

$$\theta = 85\, e^{-\frac{447.2}{\sqrt{k}} x}$$

for copper:

$$\theta = 85°C\, e^{-(22.36\ m^{-1})\, x}$$

Al. alloy:

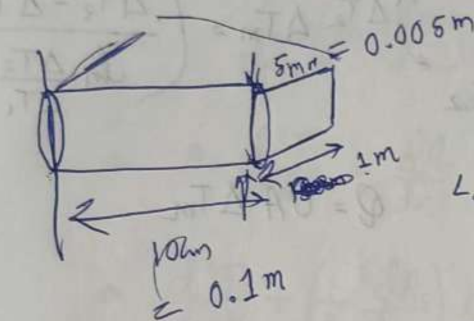
$$\theta = 85°C\, e^{-33.33x}$$

~~[illegible]~~

$$L_\infty = \frac{2.65}{m}$$

for copper, $L_\infty = 0.1185\ m$

for Alloy, $L_\infty = 0.0795\ m$



$$L_\infty = \frac{2.65}{m}$$

$$m = \sqrt{\frac{10}{200} \cdot \frac{2 \times 1.005}{0.05}} = 2.01$$

$$L_\infty = 1.86\ m$$

$$Q = \sqrt{hPkA_c}\,\theta_0 \left(\frac{\tanh mL + \frac{h}{mk}}{1 + \frac{h}{mk} \tanh(mL)} \right)$$

$$L_c = 0.1 + \frac{0.005}{2}$$

$$= 0.1025 \qquad A_m = t L_c = 5.125 \times 10^{-4}$$

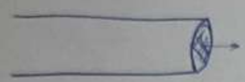
$$L_c^{3/2} \left(\frac{h}{kA_m} \right)^{1/2} = 0.1025^{3/2} \times \left(\frac{10}{200 \times 5.125} \times 10^{4} \right)^{1/2}$$

$$= 0.324$$

$$\eta_f \simeq 0.92$$

$$Q_f = Q \cdot 0.92 = 10 \times 2 \times 1.005 \times 0.1 \times 200 \times 0.92$$

$$\boxed{Q_f = 369.84}$$



Heat Exchanger:

$$\Delta T_m = \left(\frac{\Delta T_2 - \Delta T_1}{\ln \frac{\Delta T_2}{\Delta T_1}} \right)$$

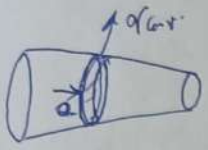
$$Q = UA\,\Delta T_m$$

$$U = \frac{1}{\sum \frac{1}{h_j}}$$

and. $A \rightarrow$ area between two.

$$Q = \frac{A\left[U_2 \Delta T_2 - U_1 \Delta T_1\right]}{\ln\left(\frac{U_2 \Delta T_2}{U_1 \Delta T_1}\right)}$$

$$Q = \sum U_j A_j \,\Delta T_{LMTD,j}.$$



$$Q_x = q_{conv} + Q_{x+dx}$$

$$Q_x - Q_{x+dx} = hP\,dx\,(T - T_\infty)$$

$$\Rightarrow -\frac{d}{dx}(Q_x) = hP(T - T_\infty)$$

$$\Rightarrow -\frac{d}{dx}\left(-kA_c\frac{dT}{dx}\right) = hP(T - T_\infty)$$

$$\Rightarrow \frac{d}{dx}\left(A_c\frac{dT}{dx}\right) = \frac{hP}{k}(T - T_\infty)$$

$$\Rightarrow A_c\frac{d^2T}{dx^2} + \frac{dA_c}{dx}\frac{dT}{dx} - \frac{hP}{k}(T - T_\infty) = 0$$

$$\Rightarrow \frac{d^2T}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx} - \frac{hP}{kA_c}(T - T_\infty) = 0$$

$$\therefore \frac{d^2\theta}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{d\theta}{dx} - m^2\theta = 0$$

and for uniform crossection.

$$\therefore \frac{d^2\theta}{dx^2} - m^2\theta = 0$$

Case-I $\quad \theta = \theta_0 e^{-mx} \quad$ (inf)

Case-II $\quad \theta = \theta_0 \dfrac{\cosh(m(L-x))}{\cosh(mL)}$

Case-III $\quad \theta = \theta_0 \dfrac{\cosh(m(L-x))}{\cosh(mL)} \left(\dfrac{\frac{h}{mk}\tanh(m(L-x)) + 1}{1 + \frac{h}{mk}\tanh(mL)}\right)$

$$Q_f = M \frac{\tanh mL + \left(\frac{h}{mk}\right)}{1 + \frac{h}{mk} \tanh mL}$$

$$m = \sqrt{\frac{hP}{kA_c}} \qquad M = \sqrt{hPkA_c}\,\theta_0.$$

$\cancel{\frac{1}{U_i} = \frac{1}{h_i} + \frac{1}{h_o}}$

Q

$$\cancel{\frac{\partial^2 T}{\partial r^2}}$$

$$\frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right) = 0$$

$R_{fa} = \frac{1}{\eta_f A_f h}$

$$R_w = \frac{\Delta x}{k_w A_o}$$

$$dQ = -m\, C_{p,h}\, dT_h$$

$$= m C_{p,c}\, dT_c$$

S. $dT_h - dT_c = -\left(\frac{dQ}{\dot m_h c_{ph}} + \frac{dQ}{\dot m_c c_{pc}}\right)$

$\therefore d(T_h - T_c) = -dQ\left(\frac{1}{\dot m_h c_{ph}} + \frac{1}{\dot m_c c_{pc}}\right)$

$= \cancel{dQ}$

$\cancel{dQ} = -U\,dA\left(\frac{1}{\dot m_h c_{p,h}} + \frac{1}{\dot m_c c_{pc}}\right)$

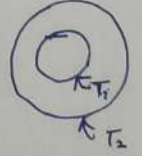
$=$

$$\varepsilon_f = \frac{\eta_f A_f h \theta_0}{h A_b \theta_0} = \sqrt{\frac{kP}{hA_c}} \quad L\to\infty$$

$$\eta_f = \frac{Q_f}{h A_f \theta_0} = \frac{\tanh mL}{mL}.$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0$$

$$\therefore r\frac{\partial T}{\partial r} = C_1$$



$$\frac{\partial T}{\partial r} = \frac{C_1}{r}$$

$$\therefore T = C_1 \ln r + C_2$$

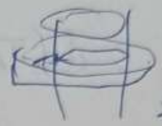
$$T_1 = C_1 \ln r_1 + C_2$$
$$T_2 = C_1 \ln r_2 + C_2$$
$$C_1 \ln\left(\frac{r_2}{r_1}\right) = \Delta T$$

$$\therefore T = \frac{\Delta T}{\ln\frac{r_2}{r_1}} \ln r \,;\quad A k \frac{\partial T}{\partial r} = k \frac{T_2 - T_1}{\ln\frac{r_2}{r_1}} \frac{A}{r}$$

$$Q \cdot \left\{\ln\frac{r_2}{r_1} \frac{r}{kA}\right\} = \Delta T$$

$$\ln\frac{r_2}{r_1}\left(\frac{r}{kA}\right) = \frac{1}{4\pi}$$

$\frac{D}{4}$



$\frac{t}{2}$

#

~~A_{Total}~~ , r_2

$$r_{2c} = \left(r_2 + \frac{t}{2}\right)$$

$$A_f = 2\left(\pi r_{2c}^2 - \pi r_1^2\right) = 2\pi\left(r_{2c}^2 - r_1^2\right)$$

$$NA_f = \underbrace{2\pi N\left(r_{2c}^2 - r_1^2\right)}$$

$$\boxed{A_t = 2\pi r_1(H - Nt) + 2\pi N\left(r_{2c}^2 - r_1^2\right)}$$

~~$Q = A_t$~~

$$T_1 = T_\infty + \frac{Q}{hA_t}$$

$$T_2 = T_\infty + \frac{Q}{hA'}$$

$$\Delta T = \frac{Q}{h}\left(\frac{1}{A_t} - \frac{1}{A'}\right)$$

G = Irradiation

$$\frac{G_{ref}}{G} = \rho$$

$$\frac{G_{abs}}{G} = \alpha \text{ (absorptivity).}$$

$$\frac{G_{tr}}{G} = \tau$$

$$\rho + \alpha + \tau = 1 \text{ (semitransparent body)}$$

$$\alpha + \rho = 1 \text{ (Opaque).}$$

$$\alpha = 1 \text{ (black body).}$$ (South Indians and africans)

Diffuse Emitter, $E \neq E(\theta)$, $E = E(\lambda, T)$

→ All black bodies are diffuse emitter.

$$G = G_{abs} = E_b$$

$$G_{abs} = \alpha G.$$

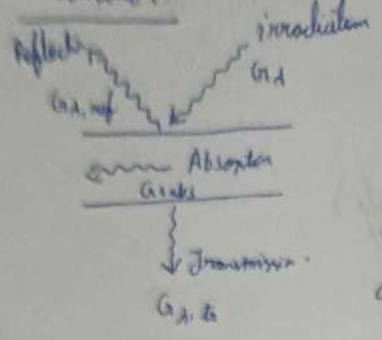
$$G_{abs} = \alpha G = \alpha E_b$$

So. $\alpha = \dfrac{E}{E_b} = \varepsilon$

↖ one

$$\varepsilon_b = 1, \quad \alpha = \varepsilon_b = 1.$$

Radiation



$$G_\lambda = G_{\lambda,ref} + G_{\lambda,abs} + G_{\lambda,tr}$$

Emmisitive power: Rate at which radiation is emitted from a surface per unit surface area per unit time.

Radiosity → all radiant energy leaving surface.

Net radiative flux (W/m^2) = outgoing radiation − Incoming radiation.

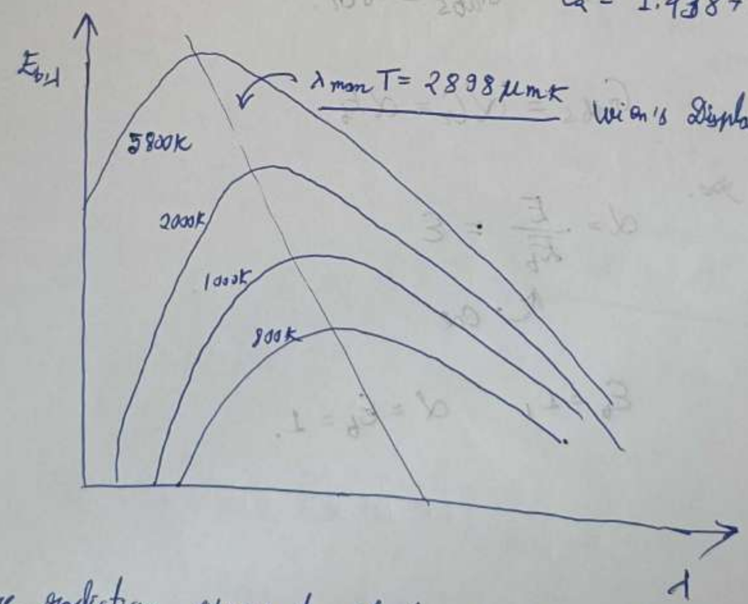
→ Radiation emitted by a black body is a function of λ and T but independent of θ.

for black body.

$$E_{b,\lambda} = \frac{C_1}{\lambda^5\left(e^{C_2/\lambda T} - 1\right)}$$

$C_1 = 3.743 \times 10^8\ W.\mu m^4/m^2$

$C_2 = 1.4387 \times 10^4\ \mu m.K$



Wien's Displacement law.

→ more radiation appears at shorter λ at higher T.

→ $T \le 800K$, ⇒ radiation is infrared.

→ $T = 5800K$ ⇒ Sun is a black body.

Stefan Boltzmann Law:

$$\int_0^\infty E_{b,\lambda}\, d\lambda = \sigma T^4$$

$$I = \int_0^\infty \frac{C_1}{\lambda^4 \left(e^{C_2/\lambda T} - 1\right)}\, d\lambda \quad \text{let, } \frac{1}{\lambda} = t$$

$$= \int_0^\infty \frac{C_1 t^3\, dt}{\left(e^{\frac{C_2 t}{T}} - 1\right)}$$

$$= \int_0^\infty C_1 t^3 e^{-\frac{C_2 t}{T}} \left(1 + e^{-\frac{C_2}{T}t} + e^{-\frac{2C_2}{T}t} + \dots\right) dt$$

$$= \int_0^\infty C_1 \sum_{n=1}^{\infty} t^3 e^{-\frac{nC_2}{T}t}\, dt$$

$$= \sum_{n=1}^{\infty} C_1\, \mathcal{L}\{t^3\}\Big|_{\frac{nC_2}{T}}$$

$$= \sum_{n=1}^{\infty} C_1 \frac{6}{\left(\frac{nC_2}{T}\right)^4} = \frac{6C_1}{C_2^4} T^4 \sum_{n=1}^{\infty} \frac{1}{n^4}$$

$$= \frac{6C_1}{C_2^4} \frac{\pi^4}{90} T^4$$

So, $$I = \left(\frac{C_1 \pi^4}{15 C_2^4}\right) T^4 = \sigma T^4$$

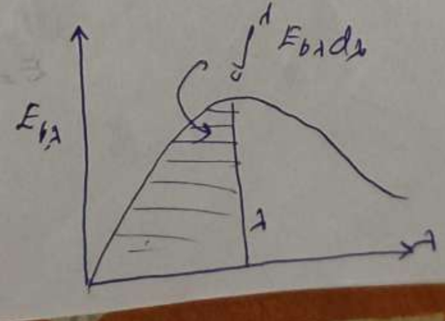
$\sigma \rightarrow 5.67 \times 10^{-8}\ W/m^2.K^4$

characteristics of Black Body (Isothermal)

- → Absorbs all irradiation
- → at a Particular λ and T, black body emmits maximum energy.
- → Diffuse emitter. $\quad E_b = \sigma T^4$
- → Diffuse irradiation of interior surface.
- → for ideal emitter and absorber $\varepsilon = 1$, $\alpha = 1$

Band Emission of black body:

$$\frac{E_{b,0-\lambda}}{E_{b,0-\infty}} = f_{0-\lambda} = \frac{\int_0^\lambda E_{b\lambda}\, d\lambda}{\int_0^\infty E_{b\lambda}\, d\lambda} = \frac{\int_0^\lambda E_{b\lambda}\, d\lambda}{\sigma T^4}$$



$f_{0-\lambda}$ is a function of λT only. (Wavelength – Temperature product)

→ Glass is essentially transparent to visible light but totally opaque to thermal radiation emitted at ordinary room temperature.

Q. Square glass plate: 30 cm

$\tau = 0.5$ from $0.2 - 3.5\ \mu m$

$\tau = 0$ elsewhere.

T furnace $= 4273\ K$ → given | $\epsilon = 0.3 \quad \lambda < 3.5\ \mu m$

T surface $= 2273\ K$ | $\epsilon = 0.9 \quad \lambda > 3.5\ \mu m$

↳ find for -

nw.. $G_1 = 5.67 \times 10^{-8} \times (4473)^4\ W/m^2$

$= 22.7\ MW/m^2$

and $G_2 = 5.67 \times 10^{-8}\ (2273)^4\ W/m^2$

$G_2 = 1.51\ MW/m^2$

Gray body:

Monochromatic emissivity of body is independent of wavelength.

$$\varepsilon_\lambda = \frac{E_\lambda}{E_{b\lambda}}$$

For diffuse emitter $E = E(\lambda, T)$

$$E_b = \int_0^\infty E_{b,\lambda}\, d\lambda$$

$$E = \int_0^\infty \epsilon_\lambda(T)\, E_{b\lambda}\, d\lambda$$

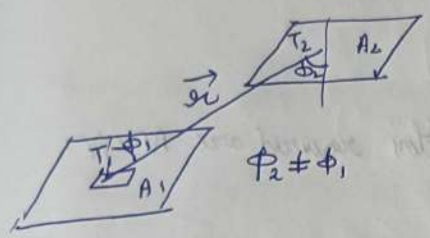
Heat interaction b/w two radiating bodies depends on view factor F_{m-n}.

which depends on-

i) shape of two bodies m and n.

ii) distance b/w two bodies m and n.

iii) orientation of two bodies wrt. each other.



$E_{b1} = f(\lambda, T_1)$

$E_{b2} = f(\lambda, T_2)$

$\phi_2 \neq \phi_1$

Total energy radiated $= E_{b1} A_1$

for normal body → all energy incident is not transmitted or absorbed.

F_{m-n} → fraction of energy leaving surface m and reaching surface n.

↳ view factor.

Q_{12} = net Radiant energy exchange b/w 1 and 2.

$= E_{b1} A_1 F_{12} - E_{b2} A_2 F_{21}$

if; $T_2 > T_1 \Rightarrow$ then 1 heats up

if. $T_1 > T_2 \Rightarrow$ then 2 heats up.

if both bodies are at same temp.

$Q_{12} = 0.$

& $E_{b1} = E_{b2} = \sigma T^4$

$\boxed{A_1 F_{12} = A_2 F_{21}}$

$F_{12} \neq F_{21}$

— Reciprocity relationship.

for ~~an~~ a surface within an enclosure the sum of energies radiation from i and falling on all object near it.

$$\sum_{\forall j} F_{ij} = 1$$

for plane surface $F_{ii} = 0$.

for convex surface $F_{ii} = 0$.

for concave surface $F_{ii} \neq 0$.

→ for N materials no. of view-factors required are $N \times N$.

$$N \times N - N(\text{sum}) - \frac{N(N-1)}{2} \text{ (reciprocity)}$$

$$= N\left[N - 1 - \frac{N-1}{2}\right] = \frac{N(N-1)}{2} \text{ (actual no. of ~~shape fac~~ view factors)}$$

→ due to energy transfer both bodies come at same temperature.

$$\text{Intensity} = E_b \cdot dA_1 = I_b \, dA_1 \cos\phi_1$$

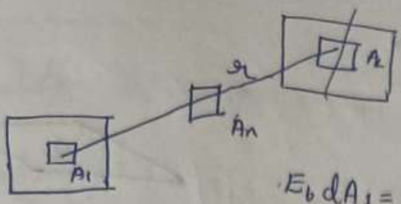
→ If there is a dA_n body in between the two bodies then out of the total intensity radiated from 1, $E(\text{total}) \times \frac{dA_n}{r^2}$

i.e, solid angle subtended by A_1 on A_n is the energy incident on A_n.

$$E_b \, dA_1 = I_b \, dA_1 \cdot \cos\phi \, \frac{dA_n}{r^2}$$

$$dA_n = r^2 \sin\phi \cdot d\theta \, d\phi.$$

Emissive Power in terms of radiation from A_2.



$$E_b\, dA_1 = I_b\, dA_1 \int_0^{2\pi}\int_0^{\pi/2} \sin\phi_1 \cos\phi_1\, d\phi_1\, d\theta$$

$$E_b\, dA_1 = \pi I_b\, dA_1$$

$$\therefore \boxed{E_b = \pi I_b}$$

$$\because dA_n = dA_2 \cos\phi_2$$

Energy leaving dA_1 and reaching dA_2,

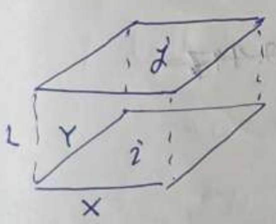
$$dQ_{12} = E_{b1} \cos\phi_1 \frac{\cos\phi_2\, dA_2}{\pi r^2} dA_1$$

$$dQ_{21} = E_{b2} \cos\phi_2 \frac{\cos\phi_1\, dA_2}{\pi r^2} dA_1$$

$$dQ_{net} = dQ_{12} - dQ_{21}$$

$$= (E_{b1} - E_{b2}) \frac{\cos\phi_1 \cos\phi_2}{\pi r^2} dA_1\, dA_2$$

$$Q_{net} = (E_{b1} - E_{b2}) \underbrace{\int_{A_1}\int_{A_2} \frac{\cos\phi_1 \cos\phi_2}{\pi r^2} dA_1\, dA_2}_{A_1F_{12} \text{ or, } A_2F_{21}}$$

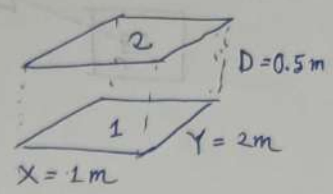


$$\bar{X} = \frac{X}{L}, \quad \bar{Y} = \frac{Y}{L}$$

$$F_{ij} = \frac{2}{\pi \bar{X} \bar{Y}} \left\{ \ln\left[\frac{(1+\bar{X}^2)(1+\bar{Y}^2)}{1+\bar{X}^2+\bar{Y}^2} \right]^{1/2} + \bar{X}(1+\bar{Y}^2)^{1/2} \tan^{-1} \frac{\bar{X}}{(1+\bar{Y}^2)^{1/2}} + \bar{Y}(1+\bar{X}^2)^{1/2} \tan^{-1} \frac{\bar{Y}}{(1+\bar{X}^2)^{1/2}} - \bar{X} \tan^{-1}\bar{X} - \bar{Y} \tan^{-1}\bar{Y} \right\}$$

Q.

$D = 0.5\,m$. , $X = 1\,m$, $Y = 2\,m$



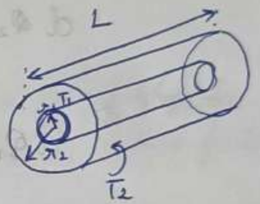
so $\bar{X} = 2$, or $\bar{Y} = 4$

$F_{12} = 0.5$ $\quad A_1 = A_2 = 2\,m^2$.

so $A_1 F_{12} = A_2 F_{21} = 1$

now
$$Q_{net} = \sigma\left(1873^4 - 1073^4\right)$$
$$= 5.67 \times 10^{-8}\left(1873^4 - 1073^4\right)$$
$$= 622.64\ kW$$

Q.



$A_1 F_{12} = A_2 F_{21}$

$F_{12} = \dfrac{A_2}{A_1} F_{21} = \dfrac{r_2}{r_1} F_{21}$. $\quad r_2 > r_1$

$\dfrac{r_2}{r_1} = 2$ $\qquad F_{11} + F_{12} + \cancel{F_{1A}+F_{1B}} = 1$

$\qquad F_{22} + F_{21} = 1 \quad \therefore F_{21} = \cancel{\ } 0.42$

$\dfrac{L}{r_2} = 2$, $\cancel{F_{22} = 0.33}$, $\cancel{F_{11} = 0}$ $\qquad F_{11} = 0$

So.
$\qquad F_{22} = 0.33$

$\Rightarrow \cancel{0.33 + 3F_{21} = 1}$

$\Rightarrow \cancel{3F_{21} = 1 - 0.33}$

$\therefore \cancel{F_{21} = 0.223}$ $\quad \cancel{F_{12} = 0.447}$

now. $E_{b1} = \sigma T_1^4$, $\quad E_{b2} = \sigma T_2^4$

so.
$$Q_{net} = \sigma\left(T_2^4 - T_1^4\right) A_1 F_{12}$$
$$= 5.67 \times 10^{-8}\left(2273^4 - 1073^4\right) \times \pi \times 0.35 \times 0.3 \times 0.42$$
$$= \cancel{90.822\ kW} = 170.805\ kW$$

Relations between view factors:

i) Reciprocity $A_1 F_{12} = A_2 F_{21}$

ii) Summation. $\sum_{\forall j} F_{ij} = 1$ (for an enclosure)

(iii) $F_{i-j} = \sum_{k=1}^{N} F_{i-k}$

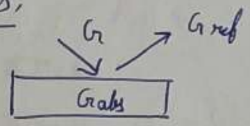
k = sub-parts of receiving body.

$$A_i F_{i-j} = A_i \sum_{k=1}^{N} F_{i-k}$$

$$A_j F_{j-i} = \sum_{k=1}^{N} A_k F_{k-i}$$

(iv) $$\therefore \quad F_{j-i} = \frac{\sum_{k=1}^{N} A_k F_{k-i}}{A_j}$$

for opaque bodies,



$G_{ref} = \rho G$. $G_{abs} = \alpha G$.

$J = \epsilon E_b + \rho G$ = Radiosity

net energy radiated from source $= J - G$

$$= \epsilon E_b + \rho G - G$$

$$= \epsilon E_b - (1-\rho) G$$

$$\therefore \quad Q_{net} = \epsilon E_b - \alpha G$$

$G = E \Rightarrow \alpha G = \epsilon E_b \Rightarrow \alpha = \epsilon.$

$$= \epsilon (E_b - G)$$

So. $J = \epsilon E_b + \rho G = \epsilon E_b + (1-\epsilon) G$

$$= G + \epsilon (E_b - G)$$

$$\therefore \quad G = \frac{J - \epsilon E_b}{1-\epsilon}$$

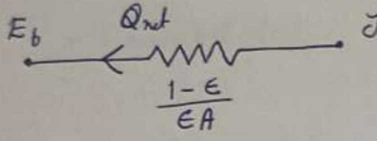
$J = G + q_{net}$

$q_{net} = J - \frac{J - \epsilon E_b}{1-\epsilon} = \frac{\epsilon E_b - \epsilon J}{1-\epsilon}$

$$\boxed{q_{net} = \frac{\epsilon (E_b - J)}{1-\epsilon}}$$

So. $Q_{net} = \frac{\epsilon A (E_b - J)}{1-\epsilon} = \frac{E_b - J}{\left(\frac{1-\epsilon}{\epsilon A}\right)}$

So. $E_b - J =$ driving force. $\frac{1-\epsilon}{\epsilon A} =$ Surface resistance.



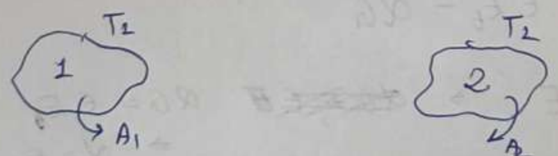
$Q_{net\,1\text{-}2} =$ Energy leaving from 1 and reaching 2. − Energy leaving from 2 and reaching 1.

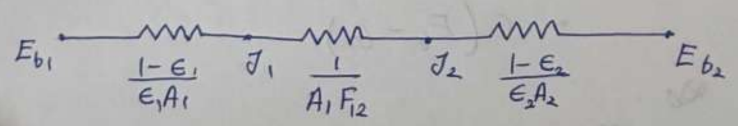
1 to 2 $= J_1 A_1 F_{12}$

2 to 1 $= J_2 A_2 F_{21}$

$$\boxed{Q_{net\,1-2} = (J_1 - J_2) A_1 F_{12}}$$

for black body driving force is only E_b.

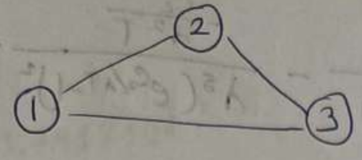




So.

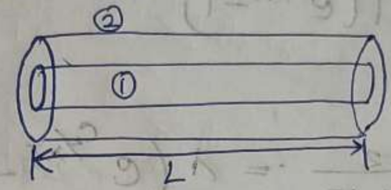
$$Q_{net_{12}} = \frac{E_{b_2} - E_{b_1}}{\frac{1-\epsilon_1}{\epsilon A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\epsilon_2}{\epsilon_2 A_2}}$$

for 3 body system:



$$Q_{net} = \frac{E_{b2} - E_{b1}}{\frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\epsilon}{\epsilon A_2}} + \frac{E_{b3} - E_{b2}}{\frac{1-\epsilon_2}{\epsilon_2 A_2} + \frac{1}{A_2 F_{23}} + \frac{1-\epsilon_3}{\epsilon_3 A_3}} + \frac{E_{b1} - E_{b3}}{\frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1 F_{13}} + \frac{1-\epsilon_3}{\epsilon_3 A_3}}$$

for two concentric cyllinders:



$F_{11} = 0, \quad F_{12} = 1$

$$Q_{net} = \frac{\sigma A_1 (T_1^4 - T_2^4)}{\frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1} + \frac{1-\epsilon_2}{\epsilon_2 A_2}}$$

$A_1 = \pi r_1 L$

$A_2 = \pi r_2 L$

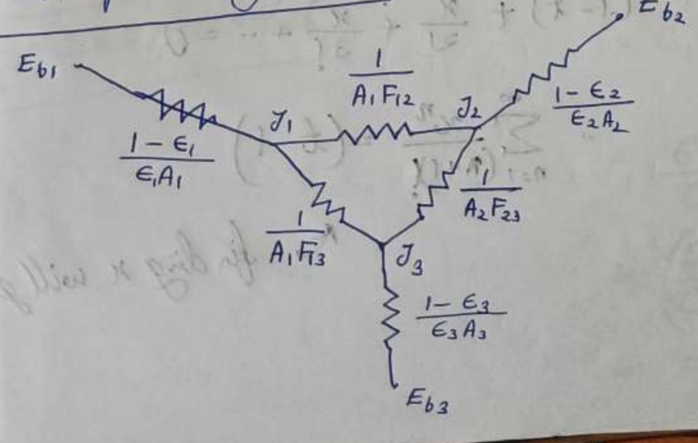
$$= \frac{\sigma A_1 (T_1^4 - T_2^4)}{\frac{1-\epsilon_1}{\epsilon_1} + 1 + \frac{1-\epsilon_2}{\epsilon_2}\frac{A_1}{A_2}}$$

Infinite Parallel Plate.

$F_{11} = 0, \quad F_{22} = 0, \quad A_1 = A_2 = A$

$$Q_{net} = \frac{\sigma (T_1^4 - T_2^4) A}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

circuit for 3 body system:



Solve the circuit using KVL and KCL

$$\frac{dE_{b,\lambda}}{d\lambda} = -\frac{5C_1}{\lambda^6(e^{C_2/\lambda T}-1)} - \frac{-\frac{C_2}{\lambda^2 T}}{\lambda^5(e^{C_2/\lambda T}-1)^2} = 0.$$

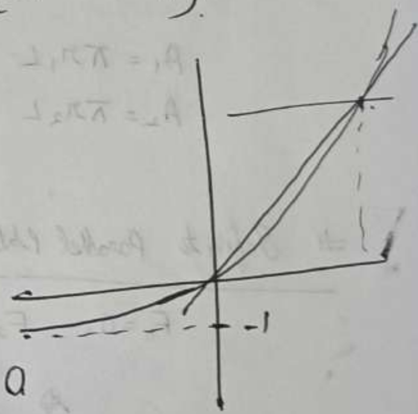
$$\Rightarrow \frac{C_2}{\lambda^7 T(e^{C_2/\lambda T}-1)^2} - \frac{5C_1}{\lambda^6(e^{C_2/\lambda T}-1)} = 0.$$

$$\Rightarrow \frac{C_2}{\lambda T(e^{C_2/\lambda T}-1)} - 5C_1 = 0.$$

$$\Rightarrow \frac{C_2}{5C_1 T} = \lambda\left(e^{C_2/\lambda T}-1\right).$$

$$\Rightarrow \frac{1}{5C_1} = \frac{\lambda T}{C_2}\left(e^{C_2/\lambda T}-1\right).$$

$$\frac{e^x-1}{x} = t.$$



$$\boxed{tx = e^x - 1}$$

$$\Rightarrow x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = 0$$

$$\Rightarrow (1-t)x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = 0.$$

$$\therefore (1-t) + \frac{x}{2!} + \frac{x^2}{3!} + \cdots = 0.$$

$$\therefore \sum_{n=1}^{\infty} \frac{x^n}{(n+1)!} = (t-1)$$

← finding x will give T.

σ.

$-1) = 0$.

-1

will give T.

$$\int_0^\infty \frac{C_1\, d\lambda}{\lambda^5 \left(e^{C_2/T\lambda}-1\right)}\ \cancel{d\lambda}$$

$c_1 = 3.743\times10^{8}$

$c_2 = 1.4387\times10^{4}$

Let, $\frac{1}{\lambda} = t$

$$\frac{d\lambda}{\lambda^2} = -dt$$

$$= C_1\int_0^\infty \frac{t^3\, dt}{\left(e^{C_2/T\, t}-1\right)}$$

$$= C_1\int_0^\infty t^3 \left(e^{C_2/T\, t}-1\right)^{-1} dt$$

$$= C_1\int_0^\infty t^3 e^{-\frac{C_2}{T}t}\underbrace{\left(1-e^{-C_2/T\, t}\right)^{-1}}_{\text{expand it.}} dt$$

$$0 < e^{-C_2/T\, t} < 1$$

$$-\infty < -\frac{C_2}{T}t < 0.$$

$$\infty > t > 0.$$

$$= C_1\int_0^\infty t^3 e^{-C_2/T\, t}\left(1+e^{-C_2/T\, t} + e^{-\frac{2C_2}{T}t} + \dots\right)$$

$$= C_1\int_0^\infty t^3 \sum_{n=1}^{\infty} e^{-\frac{nC_2}{T}t}\, dt$$

$$= C_1 \sum_{n=1}^{\infty}\int_0^\infty t^3 e^{-\frac{nC_2}{T}t}\, dt$$

$$= C_1 \sum_{n=1}^{\infty} \mathcal{L}\{t^3\}\left(\frac{nC_2}{T}\right)$$

$$= C_1 \sum_{n=1}^{\infty} \frac{6}{\left(\frac{nC_2}{T}\right)^4}$$

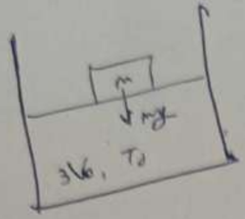
$$= \frac{6T^4}{C_2^4} C_1 \sum_{n=1}^{\infty}\frac{1}{n^4}$$

$$= \left(\frac{6C_1}{C_2^4}\frac{\pi^4}{90}\right) T^4$$

$$= \sigma T^4$$

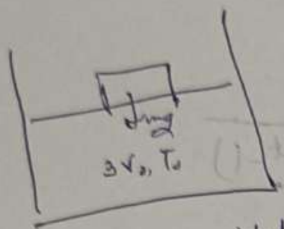
$$\sigma = \frac{C_1 \pi^4}{15 C_2^4}$$

$$\boxed{\sigma = 5.67\times10^{-8}\ W/m^2K^4}$$



Sol: $w = mg\,\frac{2V_0}{A}$, $P_i = \frac{mg}{A}$

and



$w = mg\,\frac{2V_0}{A}$ $P_f = \frac{mg}{A}$

but; $P_i = \frac{2mg}{A}$

$\boxed{P_0 = \frac{mg}{A}}$ $PV = nRT$

Case-I assuming ideal gas

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{V_0}{T_0} = \frac{3V_0}{T_2}$$

$$\boxed{T_2 = 3T_0}$$

$$Q = nC_P\,\Delta T$$

$$Q = \frac{P_0 V_0}{R T_0} C_P\, 2T_0$$

$$\boxed{Q = \frac{2P_0 V_0}{R} C_P}$$

Case-II

$$\frac{\frac{2mg}{A} V_0}{T_0} = \frac{\frac{mg}{A} 3V_0}{T_2}$$

$$\boxed{T_2 = \frac{3T_0}{2}}$$

$$Q = \frac{2P_0 V_0}{R T_0} C_P \frac{T_0}{2}$$

$$\boxed{Q = \frac{P_0 V_0}{R} C_P}$$

$$e^{m(L-x)} + e^{-m(L-x)}$$

Q.

$$\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) = -\dot{q}\, r^2$$

$$kr^2\frac{\partial T}{\partial r} = -\frac{1}{3}\dot{q}\, r^3 + C_1$$

$$\therefore \frac{\partial T}{\partial r} = -\frac{1}{3}\dot{q}\,\frac{r}{k} + \frac{C_1}{k r^2}$$

$$\int_{T_0}^{T} dT = -\frac{1}{3}\dot{q}\frac{1}{k}\int_{r_0}^{r} r\, dr + \underbrace{\frac{C_1}{k} f(r)}_{0.}$$

$$C_1 = 0$$

$$T = T_0 - \frac{1}{3}\dot{q}\frac{1}{k}\frac{r^2}{2}$$

$$= T_0 - \frac{1}{6}\frac{\dot{q} r^2}{k}$$

$$(2\times10^{-2})^2 = 4\times10^{-4}.$$

$$4\pi d^2 \dot{q} = h(T_S - T_\infty)$$

$$16\pi \times 10^{6} = 15(T_S - 30^\circ)$$

$$\therefore T_S = 365\,^\circ C$$

So,

$F_d = C_f A \left[\frac{\rho u^2}{2}\right]$

Q. $u_\infty = 7 m/s$, $T = 293K$, $C_p = 1.007 \ kJ/kg.K$,

$\rho = 1.204 \ kg/m^3$ $L = 3m$

$$Re_L = \frac{1.204 \times 3 \times 7}{1.81 \times 10^{-5}} = 13.96 \times 10^5$$

$Pr = 0.7309$, $A = 6 m^2$.

$$Nu = \frac{hL}{k}$$

ans: $$0.86 = C_f \times 6 \times \frac{1.204 \times 49}{2} \times 2$$

$$\therefore C_f = 4.85 \times 10^{-3} \times \frac{1}{2} = 2.43 \times 10^{-3}$$

$$St = \frac{h}{\rho C_p V} = \frac{1}{2} \times 4.85 \times 10^{-3}$$

$\cancel{h = 1.204 \times 1.007 \times 10^3 \times 7 \times 4.85 \times 10^{-3}}$

$\therefore h =$

$$T = -\frac{\mu}{2k}\left(\frac{v}{L}\right)^2 \cdot y^2 + C_1 y + T_0.$$

$$L C_1 = (T_1 - T_0) + \frac{\mu}{2k}\left(\frac{v}{L}\right)^2 \cancel{L^2}$$

$$C_1 = \frac{(T_1 - T_0) + \frac{\mu}{2k} v^2}{L.}$$

$T_1 = T_0.$

$$\cancel{T = \frac{\mu}{2k}(}$$

$$C_1 = \frac{\mu}{2kL} v^2$$

$$T = \frac{\mu v^2}{2kL}\left(y - \frac{y^2}{L}\right) + T_0$$

$$T = \frac{\mu v^2}{2kL}\, y\left(1 - \frac{y}{L}\right) + T_0$$

$$\boxed{T = \frac{\mu v^2}{2k}\left(\frac{y}{L}\right)\left[1 - \left(\frac{y}{L}\right)\right] + T_0}$$

$273 + 100$
$= 373 + 20$
$= 393\,K$

<u>iter-1</u> $$T = \frac{\mu v^2}{8k} + T_0$$

$$= \frac{0.8 \times 12^2}{8 \times 0.148} + T_0$$

$$= \frac{144}{1.48} + T_0$$

$$\cong 393\,K = 120\,^\circ C$$

Q.

$$\frac{dR_{th,ov}}{dR_2} = \frac{1}{2\pi k \cdot R_2} - \frac{1}{h_0 R_2^2 2\pi} = 0.$$

$$\frac{1}{k} - \frac{1}{R_2 h_0} = 0.$$

$$\boxed{R_2 = \frac{k}{h_0}}$$

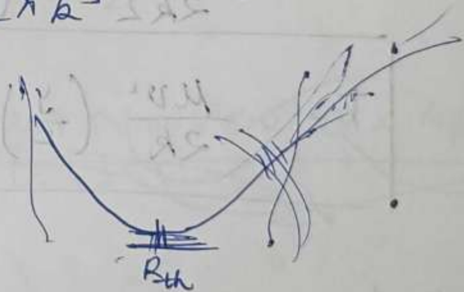
$$R_2 = \frac{k}{h_0}$$

$$\frac{d^2 R_{th}}{dR_2^2} = \frac{1}{2\pi}\left[\frac{1}{k}\left(-\frac{1}{R_2^2}\right) + \frac{2}{h_0 R_2^3}\right]$$

$$= \frac{1}{2\pi}\left[\left(\frac{2 h_0^2}{k^3} - \frac{h_0^2}{k^3}\right)\right]$$

$$= \frac{h_0^2}{2\pi k^3} > 0.$$

$L_c = L + D/4$



Q. reduce heat loss.

$$r|_a = \frac{0.17}{2}\, m = 0.085\ m = 8.5cm$$

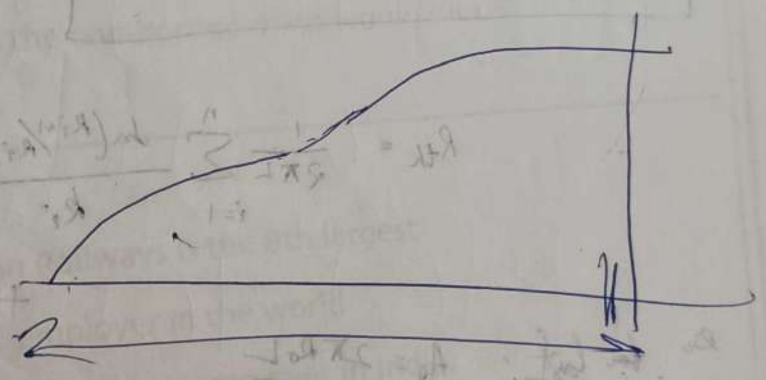
$$r|_f = \frac{0.04}{2} = 0.02\ m = 2cm$$

$R_1 > R_c$.

$$Q = \frac{T_2 - T_1}{\frac{\ln(R_2/R_1)}{2\pi k L R_2} + \frac{1}{2\pi R_2 h_0}} = \frac{220}{\quad}$$

$$\frac{Q_r}{4\pi k}\left[\frac{1}{R_1}-\frac{1}{R_2}\right] = (T_1 - T_2)$$

$$\therefore\ Q_r = \frac{4\pi k\,\Delta T}{\left[\frac{1}{R_1}-\frac{1}{R_2}\right]}$$



L

$$\frac{m}{V} = \rho$$

$$\frac{m}{\rho} = V$$

$$\frac{dV}{dt} = -\frac{m}{\rho^2}\frac{d\rho}{dt}$$

$$= -\frac{V}{\rho}\frac{d\rho}{dt}$$

$$\Rightarrow \frac{1}{V}\frac{dV}{dt} = -\frac{1}{\rho}\frac{d\rho}{dt}$$

Q.

$$T_1 - T_3 = Q_r \left[\frac{\ln R_3/R_2}{2\pi L k_B} + \frac{\ln (R_2/R_1)}{2\pi L k_A} \right]$$

$$\Rightarrow$$

$$\therefore \cancel{Q_r = \frac{(T_1 - T_3)\, 2\pi L}{\left[\frac{\ln R_3/R_2}{k_B} + \frac{\ln R_2/R_1}{k_A} \right]}}$$

$$\boxed{Q_r = \frac{\Delta T \, 2\pi L}{\sum_{i=1}^{n} \frac{\ln (R_{i+1}/R_i)}{k_i}}}$$

$$\therefore R_{th} = \frac{1}{2\pi L} \sum_{i=1}^{n} \frac{\ln (R_{i+1}/R_i)}{k_i} + \frac{1}{h_i (2\pi R_i L)} + \frac{1}{h_o (2\pi R_o L)}$$

So for last; $A_o = 2\pi R_o L$

$$U \, 2\pi R_o L = \frac{2\pi L}{\sum_{i=1}^{n} \frac{\ln (R_{i+1}/R_i)}{k_i} + \frac{1}{h_i R_i} + \frac{1}{h_o R_o}}$$

$$\therefore U = \frac{1}{\frac{1}{h_o} + \frac{1}{h_i}\left(\frac{R_o}{R_i}\right) + \sum_{i=1}^{n} \frac{R_o}{k_i} \ln \left(\frac{R_{i+1}}{R_i} \right)}$$

$$\therefore \boxed{\frac{1}{U} = \frac{1}{h_o} + \left(\frac{R_o}{R_1}\right) \frac{1}{h_i} + \sum_{i=1}^{n} \frac{R_o}{k_i} \ln \left(\frac{R_{i+1}}{R_i} \right)}$$