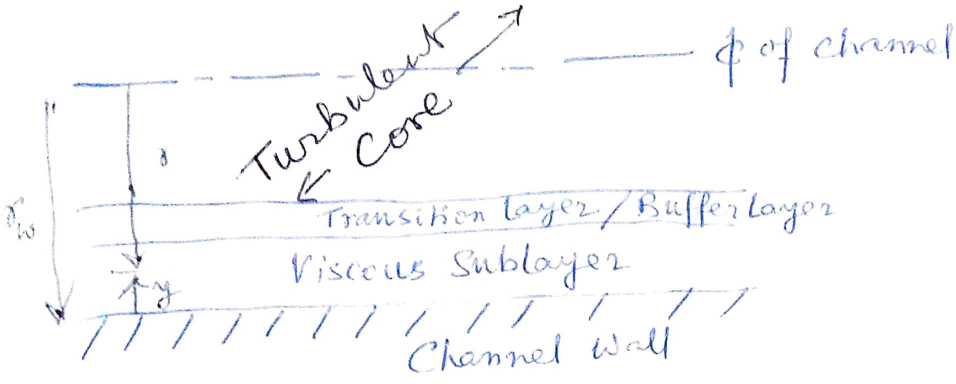
TURBULENT FLOW & UNIVERSAL VELOCITY DISTRIBUTION



* Difference between average velocity and maximum velocity is less.
* Eddy diffusion is insignificant in viscous sublayer. Instead, viscous shear driven sliding of layers is present.
* velocity gradient is zero at the centre-line.
* Most of the kinetic energy of the eddies lies in the buffer zone.

Dimensionless variables

Friction Velocity $u^* \equiv \bar{V}\sqrt{\frac{f}{2}} = \sqrt{\frac{\tau_w}{\rho}}$

Dimensionless Velocity

$$u^+ = \frac{u}{u^*}$$

Dimensionless Distance

$$y^+ = \frac{y u^* \rho}{\mu}$$

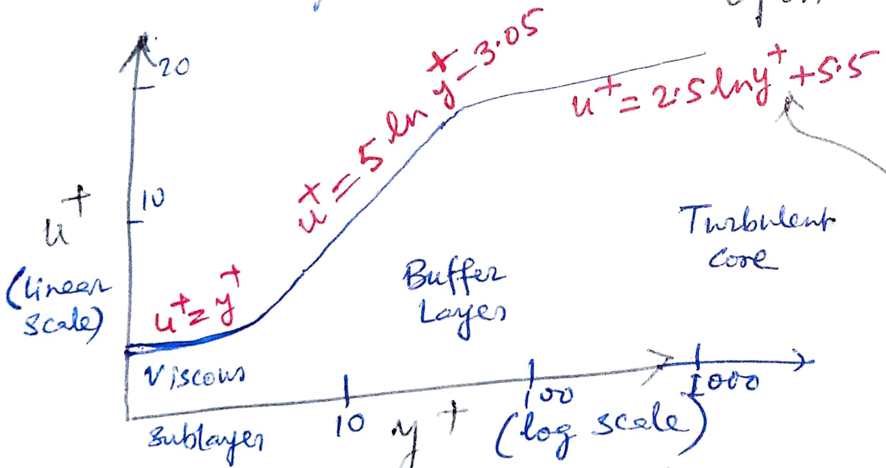
For Viscous sublayer

$$r_w = r + y \approx r$$

Since the sublayer is very thin

If $\tau_w = -\mu \frac{du}{dr} \Rightarrow \frac{du}{dy} = \frac{\tau_w}{\mu} \Rightarrow \frac{du^+}{dy^+} = 1$

Upon integration $u^+ = y^+$



y^+ from 0 to 5 (Viscous sublayer)

y^+ from 5 to 30 (buffer layer)

from 30 to centre of pipe (turbulent core)

this relation primarily applies to the outer part of the turbulent core, as the velocity gradient has to be zero at the centre of the pipe

CALCULATION OF FRICTION FACTOR

Ignore the contribution of viscous sublayer and buffer layer.

At the centre of the pipe $\quad u_c^+ = 2.5 \ln y_c^+ + 5.5$

At intermediate location $\quad u^+ = 2.5 \ln y^+ + 5.5$

Upon subtraction $\quad u^+ - u_c^+ = 2.5 \ln \dfrac{y^+}{y_c^+}$

$$\bar{V} = \frac{1}{\pi r_w^2}\int_0^{r_w} u\,(2\pi r\,dr) = \frac{2}{r_w^2}\int_{r_w}^{0} u\,(r_w - y)\,dy$$

$r = r_w - y$

$dr = -dy$

$\left.\begin{array}{l} r=0 \Rightarrow y = r_w \\ r = r_w \Rightarrow y = 0 \end{array}\right\}$

$$= \frac{5\left(\frac{\mu}{\rho}\right)^2}{r_w^2\, u^*}\int_0^{y_c^+}\left(0.4\,u_c^+ + \ln\frac{y^+}{y_c^+}\right)\left(y_c^+ - y^+\right)dy$$

$$\Rightarrow \frac{\bar{V}}{u^*} = u_c^+ - 3.75 \Rightarrow u_c^+ = \frac{1}{\sqrt{f/2}} + 3.75$$

Also,

$$y_c^+ = \frac{r_w \bar{V}}{\left(\frac{\mu}{\rho}\right)}\sqrt{\frac{f}{2}} = \frac{D\bar{V}}{2\left(\frac{\mu}{\rho}\right)}\sqrt{\frac{f}{2}} = \frac{Re}{2}\sqrt{\frac{f}{2}}$$

Since $u_c^+ = 2.5 \ln y_c^+ + 5.5$

$$\Rightarrow \frac{1}{\sqrt{f/2}} = 2.5 \ln\left(Re\sqrt{\frac{f}{8}}\right) + 1.75$$

This eqn. predicts friction factor for smooth tube for $10^4 < Re < 10^6$ within 2% of Moody's Plot