

**Mass Transfer – I (CH21202)**  
**Solutions of Tutorial Sheet No.: MT-I/NCP/2024/1**

1. A tray tower is to be designed to absorb SO<sub>2</sub> from an air stream by using pure water at 20°C. Approximately 180 m<sup>3</sup>/h (at 20°C and 1 atm) of gas is to be processed and the SO<sub>2</sub> content of the gas is to be reduced from 20 mol% to 2 mol%. Determine (a) the minimum water rate and (b) the number of real trays required for a water rate 1.2 times the minimum. Assume an overall tray efficiency of 50%.

**Equilibrium Data:**

x	0.0	5.640 x 10 <sup>-4</sup>	8.420 x 10 <sup>-4</sup>	1.403 x 10 <sup>-3</sup>	1.965 x 10 <sup>-3</sup>	2.790 x 10 <sup>-3</sup>	4.200 x 10 <sup>-3</sup>	6.980 x 10 <sup>-3</sup>
y	0.0	0.0112	0.0185	0.0342	0.0513	0.0775	0.121	0.212

**Solution:**

The given equilibrium data were converted to mole ratio concentrations as follows:

X	0.0	5.640 x 10 <sup>-4</sup>	8.420 x 10 <sup>-4</sup>	1.403 x 10 <sup>-3</sup>	1.965 x 10 <sup>-3</sup>	2.790 x 10 <sup>-3</sup>	4.22 x 10 <sup>-3</sup>	0.007
Y	0.0	0.0113	0.0189	0.0354	0.0541	0.0840	0.1376	0.269

The equilibrium curve was drawn.

Given,  $y_{N+1} = 0.20$ ,  $Y_{N+1} = 0.25$ ;  $y_1 = 0.02$ ,  $Y_1 = 0.0204$ ;  $x_o = X_o = 0.0$ ,  $X_N = ?$

Inlet gas rate = 180 m<sup>3</sup>/h = (180 x 273)/(293 x 22.4) = 7.487 kmol/h ( $G_{N+1}$ )

Therefore,  $G_S = G_{N+1} (1 - y_{N+1}) = 5.989 \approx 6.0$  kmol air/h

The operating line equation was written for minimum liquid rate as under:

$$G_S (Y_{N+1} - Y_1) = L_{S, \min} (X_{N, \max} - X_o).$$

From the plot,  $X_{N, \max}$  was found to be 0.00655.

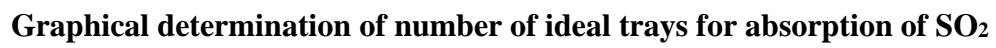
Therefore,  $L_{S, \min} = 6.0 \times (0.25 - 0.0204)/0.00655 = 210.3$  kmol/h = **3786 kg/h**

Now,  $L_S = 1.2 \times L_{S, \min} = 252.36$  kmol/h.

From the operating line equation,  $X_N = 6.0 \times (0.25 - 0.0204)/252.36 = 0.00546$

Now, the actual operating line was drawn and the number of equilibrium trays were determined to be 5.

Therefore, the number of actual trays = 5/0.5 = **10**.



### Graphical determination of number of ideal trays for absorption of SO<sub>2</sub>

2. A tray tower is to be designed to absorb CO<sub>2</sub> from a flue gas stream by scrubbing into an aqueous amine solution at 25°C. Approximately, 180 m<sup>3</sup>/h (at 25°C and 1 atm) of gas is to be processed and the CO<sub>2</sub> content of the gas is to be reduced from 15 mol% to 2 mol%. The scrubbing liquid, which is recycled from a stripper, will contain 0.058 mol CO<sub>2</sub>/mol solution. Determine (i) the minimum liquid rate, kmol/h and (ii) the number of real trays required for a liquid rate 1.2 times the minimum. Assume an overall tray efficiency of 53%.

**Equilibrium Data:**

Mole CO <sub>2</sub> /mole amine solution	0.058	0.060	0.062	0.064	0.066	0.068
$p_{\text{CO}_2}$ (mm Hg)	5.6	12.8	29.0	56.0	98.7	155.0

**Solution:**

The given equilibrium data were converted to mole ratio concentrations as follows:

X	0.0616	0.0638	0.0661	0.0684	0.0706	0.073
Y	0.0074	0.0171	0.0396	0.0795	0.1492	0.2562

Given,  $y_{N+1} = 0.15$ ,  $Y_{N+1} = 0.1765$ ;  $y_1 = 0.02$ ,  $Y_1 = 0.0204$ ;  $x_o = 0.058$ ,  $X_o = 0.0616$ ,  $X_N = ?$

Inlet gas rate = 180 m<sup>3</sup>/h = (180 x 273)/(298 x 22.4) = 7.36 kmol/h ( $G_{N+1}$ )

Therefore,  $G_S = G_{N+1} (1 - y_{N+1}) = 6.25$  kmol/h

The equilibrium curve was drawn and the point ( $X_o$ ,  $Y_1$ ) was located. The operating line will start from P ( $X_o$ ,  $Y_1$ ) and for minimum liquid/gas ratio will cut the equilibrium line at an ordinate of  $Y_{N+1}$ .

The material balance equation can be written as,  $G_S (Y_{N+1} - Y_1) = L_S (X_N - X_o)$ .

(i) For minimum liquid/gas ratio, this becomes

$$\frac{L_{S,\min}}{G_S} = \frac{Y_{N+1} - Y_1}{X_{N,\max} - X_o}$$

From the plot,  $X_{N,\max} = 0.0713$

$$\therefore \frac{L_{S,\min}}{G_S} = \frac{0.1765 - 0.0204}{0.0713 - 0.0616} = 16.1$$

$$\therefore L_{\min} = 16.1 \times G_S (1 + X_o) = 16.1 \times 6.25 \times 1.0616 = 106.82$$

Therefore, minimum liquid rate is **106.82 kmol/h**.

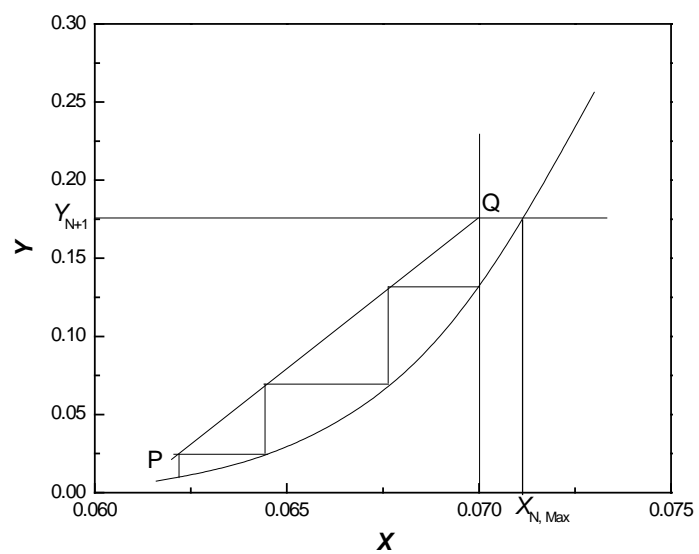
(ii)  $L_S/G_S = 1.2 \times L_{S,\min}/G_S = 1.2 \times 16.1 = 19.32$

From the material balance equation,

$$X_N = X_o + \frac{Y_{N+1} - Y_1}{L_S / G_S} = 0.0616 + \frac{0.1765 - 0.0204}{19.32} = 0.07$$

Now, the operating line was drawn between points ( $X_o$ ,  $Y_1$ ) and ( $X_N$ ,  $Y_{N+1}$ ) and the number of theoretical stages was found out to be **3.2**.

No. of real trays = 3.2/0.53 = **6.0**



**Graphical determination of number of ideal trays for absorption of  $\text{CO}_2$**

3. The hydrotreater off gas of a refinery processing sour crude contains 20% H<sub>2</sub>S. The gas is sent to the amine treating unit of the refinery where it is blown into a sieve-tray tower operated at 1.0 std atm, 30°C and scrubbed with a 25 wt% diethanol amine (DEA) solution in water. The scrubbing liquid, which is recycled from a stripper, contains 0.01 mol H<sub>2</sub>S/mol solution. The gas leaving the absorber is to contain 1.4% H<sub>2</sub>S. Assuming isothermal operation, determine (i) the minimum liquid/gas ratio, mol/mol; (ii) the number of theoretical trays required for a liquid/gas ratio 1.2 times the minimum. The equilibrium partial pressures of H<sub>2</sub>S over aqueous solutions of diethanol amine (25 wt%) are given below:

Moles H <sub>2</sub> S/mole DEA solution (x)	0.0225	0.0366	0.052	0.063	0.067	0.071	0.076
P <sub>H<sub>2</sub>S</sub> (mm Hg)	14	35	72	110	133	152	175

### Solution:

The equilibrium data were converted to mole ratio coordinates as follows:

X		0.023	0.038	0.055	0.067	0.072	0.076	0.082
Y		0.0187	0.048	0.104	0.17	0.212	0.25	0.30

From the given condition:

$$y_{N+1} = 0.20 \text{ Or, } Y_{N+1} = 0.25; y_1 = 0.014 \text{ Or, } Y_1 = 0.0142; x_o = 0.01 \text{ Or, } X_o = 0.0101$$

The equilibrium data were plotted and point P (X<sub>o</sub>, Y<sub>1</sub>) was located. The operating line will start from P and for minimum liquid/gas ratio will cut the equilibrium line at an ordinate of Y<sub>N+1</sub>.

The material balance equation can be written as, G<sub>S</sub> (Y<sub>N+1</sub> - Y<sub>1</sub>) = L<sub>S</sub> (X<sub>N</sub> - X<sub>o</sub>).

For minimum liquid/gas ratio, this becomes

$$\frac{L_{S,\min}}{G_S} = \frac{Y_{N+1} - Y_1}{X_{N,\max} - X_o}$$

From the plot, X<sub>N, max</sub> = 0.076

$$\therefore \frac{L_{S,\min}}{G_S} = \frac{0.25 - 0.0142}{0.076 - 0.0101} = 3.578$$

$$\therefore \frac{L_{\min}}{G} = \frac{L_{S,\min}(1 + X_o)}{G_S(1 + Y_{N+1})} = \frac{3.578 \times 1.0101}{1.25} = 2.89$$

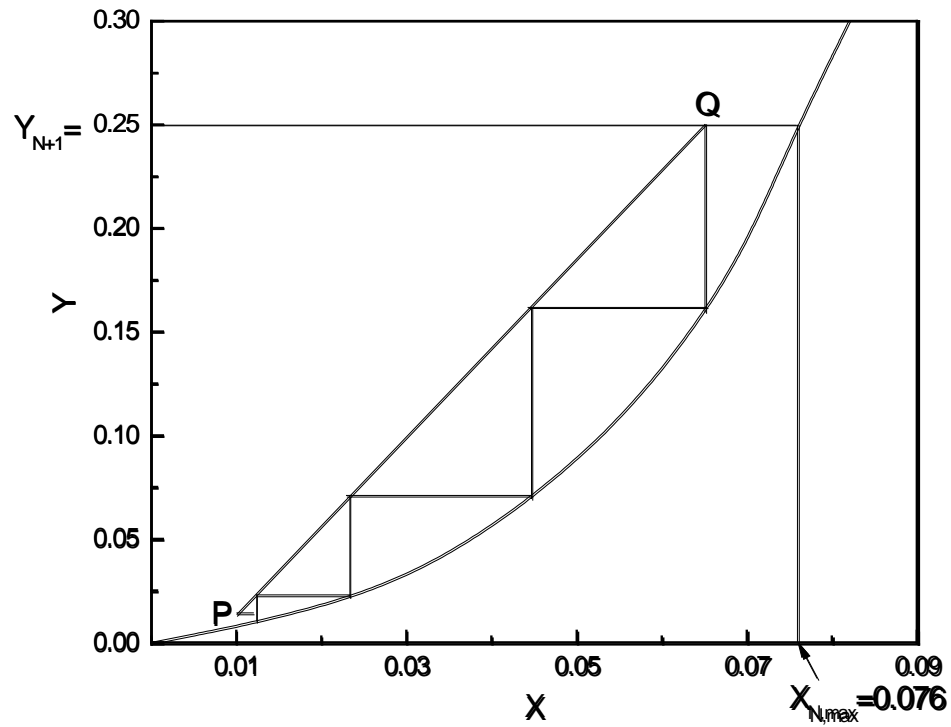
Therefore, minimum liquid to gas ratio is **2.90 mol/mol**.

$$(b) L_S/G_S = 1.2 \times L_{S,\min}/G_S = 1.2 \times 3.578 = 4.294$$

From the material balance equation,

$$X_N = X_o + \frac{Y_{N+1} - Y_1}{L_S / G_S} = 0.0101 + \frac{0.25 - 0.0142}{4.294} = 0.065$$

Now, the operating line was drawn between points P ( $X_0, Y_1$ ) and Q ( $X_N, Y_{N+1}$ ) and the number of theoretical stages was found out to be 3.6.



**Graphical determination of number of ideal trays for absorption of  $H_2S$**

4. Carbon dioxide evolved during the production of ethanol by fermentation contains 0.01 mole fraction of alcohol vapour. It is proposed to remove the alcohol by absorption into water in a bubble-cap plate tower. Absorption may be assumed to occur isothermally at 30°C and 1 atm pressure. The water for absorption is supplied from the subsequent distillation step for alcohol recovery and may be assumed to contain 0.0001 mole fraction alcohol. To be processed are 240 kmol/h of gas per hour. Over the conditions of operations, the solubility of alcohol in water may be approximated by the relation  $y = 1.0682 x$  (where  $y$  and  $x$  are the gas phase and liquid phase mole fractions).
- (a) Calculate the minimum water rate for 98% absorption of the alcohol vapour.  
 (b) Calculate the number of theoretical plates required for 98% absorption at a water rate twice the minimum.  
 (c) Calculate the percentage absorption which would be obtained in one equilibrium stage at the flow rates of part (b).

**Solution:**

The composition of the gas leaving the tower is obtained from a material balance as follows:

Total gas entering = 240 kmol/h

Alcohol entering =  $240 \times 0.01 = 2.4$  kmol/h

Alcohol removed =  $2.4 \times 0.98 = 2.352$  kmol/h

Alcohol leaving the tower =  $2.4 - 2.352 = 0.048$  kmol/h

Inert gas flow rate,  $G_s = 240 - 2.4 = 237.6$  kmol/h

Therefore, mole fraction of alcohol in the leaving gas,

$$y_1 = 0.048 / (237.6 + 0.048) = 2.02 \times 10^{-4}$$

(a) As the gas phase is very dilute, the operating line in the  $x$ - $y$  plot can be considered as straight line. At the minimum liquid rate, the operating line will intersect the equilibrium line at  $y_{N+1} = 0.01$ . Therefore,

$$x_{N,\max} = \frac{y_{N+1}}{1.0682} = 9.36 \times 10^{-3}$$

By material balance,

$$G_s(y_{N+1} - y_1) = L_{s,\min} (x_{N,\max} - x_o)$$

$$\Rightarrow 237.6 (0.01 - 2.02 \times 10^{-4}) = L_{s,\min} (9.36 \times 10^{-3} - 0.0001)$$

$$\Rightarrow L_{s,\min} = \underline{\underline{251.4 \text{ kmol/h}}}$$

(b) Actual water flow rate,  $L_s = 2.0 \times 251.4 = 502.8$  kmol/h

$$\text{Absorption factor, } A = L_s / m G_s = 502.8 / (1.0682 \times 237.6) = 1.981$$

Number of theoretical plates required is calculated from Kremser equation as follows:

$$N = \log \left[ \frac{y_{N+1} - m x_o}{y_1 - m x_o} \left( 1 - \frac{1}{A} \right) + \frac{1}{A} \right] / \log A = \log(51.976) / \log(1.981) = \underline{\underline{5.78}}$$

(c) For single stage absorption, by material balance we get,

$$G_s (y_1 - y_2) = L_s (x_2 - x_1)$$

$$\Rightarrow 237.6 (0.01 - y_2) = 502.8 (x_2 - 0.0001)$$

$$\Rightarrow 2.376 - 237.6 y_2 = 502.8 \times y_2 / 1.0682 - 0.05028 = 470.7 y_2 - 0.05028$$

$$\Rightarrow y_2 = 2.428 / 708.3 = 3.425 \times 10^{-3}$$

$$\text{Alcohol absorbed} = 237.6 (0.01 - 3.425 \times 10^{-3}) = 1.5622 \text{ kmol/h}$$

$$\text{Percentage absorption in one equilibrium stage} = (1.5622 / 2.4) \times 100 = \underline{\underline{65\%}}$$



5. A rich absorption oil containing 8% propane is being stripped by direct superheated steam in a tray tower to reduce the propane content to 0.5%. A total of 30 kmol of direct steam is used for 600 kmol of total entering liquid. The vapour-liquid equilibrium may be represented by  $y = 26x$ , where  $y$  is the mole fraction of propane in the steam and  $x$  is the mole fraction of propane in the oil. Steam can be considered as inert gas and will not condense. Determine the number of theoretical trays needed for the stripping operation.

**Solution:**

$$x_o = 0.08, x_N = 0.005; G_s = 30 \text{ kmol}$$

$$L_s = L_o (1 - x_o) = 600 (1 - 0.08) = 552 \text{ kmol non-volatile absorption oil}$$

Operating line equation is:

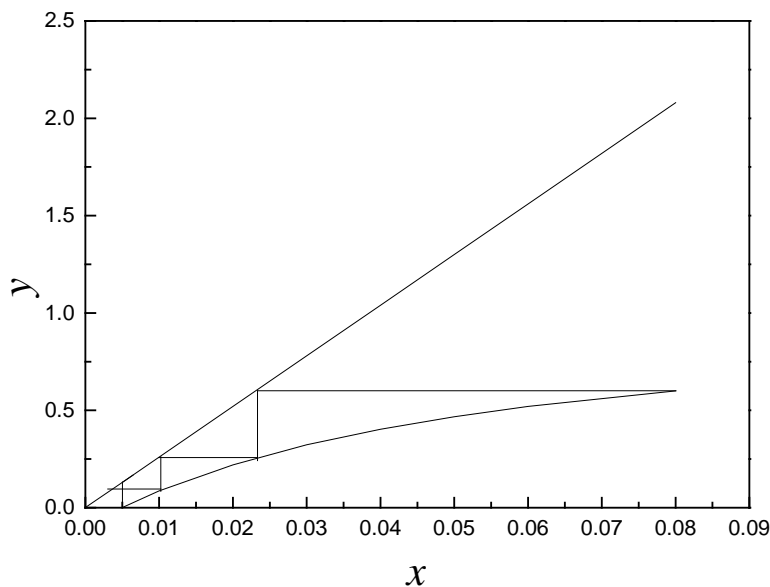
$$30 \left( \frac{y}{1-y} \right) = 552 \left( \frac{x}{1-x} - 0.005 \right)$$

From the operating line equation, the data points for the line were generated as follows:

$x$	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.005
$y$	0.60	0.56	0.52	0.467	0.403	0.323	0.22	0.086	$4.62 \times 10^{-4}$

The equilibrium line equation is given as:  $y = 26x$ .

Both the operating and equilibrium lines were drawn and stepping down from tower top, the number of theoretical trays were determined to be **2.8**.



**Graphical determination of number of ideal trays for stripping of propane**

6. A relatively nonvolatile hydrocarbon oil containing 3 mol% benzene is being stripped at the rate of 260 kmol/h by direct superheated steam in a packed tower to reduce the benzene content to 0.2%. The vapour-liquid equilibrium may be represented by  $y = 22.5 x$ , where  $y$  is the mole fraction of benzene in the steam and  $x$  is the mole fraction of benzene in the oil. Steam can be considered as inert gas and will not condense. Determine the height of the packing if the diameter of the tower is 1.5 m. The steam rate is 360 kg/h and  $K_x a$  is 150 kmol/m<sup>3</sup> h ( $\Delta x^*$ ).

**Solution:**

Steam rate,  $G = 360/18 = 20$  kmol/h

For dilute system, we can assume that the gas and liquid rates remain substantially constant. To calculate the outlet concentration of the gas, the material balance equation may be written as

$$G (y_2 - y_1) = L (x_2 - x_1)$$

$$\text{Or, } 20 \times (y_2 - 0.0) = 260 \times (0.03 - 0.002) \text{ Or, } y_2 = 0.364$$

For dilute systems, the number of transfer units may be expressed as

$$N_{iOL} = \frac{x_2 - x_1}{(x - x^*)_M}$$

In the present case,  $x_1^* = 0.0$ ,  $x_2^* = y_2/m = 0.364/22.5 = 0.016$

$$\therefore (x - x^*)_M = \frac{(0.03 - 0.016) - (0.002 - 0.0)}{\ln[(0.03 - 0.016)/(0.002 - 0.0)]} = 6.166 \times 10^{-3}$$

$$N_{iOL} = \frac{(0.03 - 0.002)}{0.006166} = \underline{\underline{4.54}}$$

$$H_{iOL} = \frac{L'}{K_x a} = \frac{260 / (3.14 \times 1.5^2 / 4)}{150} = 0.98 \text{ m}$$

$$\text{Depth of the packing} = 0.98 \times 4.54 = \underline{\underline{4.45 \text{ m}}}$$