Storage of Solids

Lecture-2

Dr. Swambabu Varanasi

Review of previous class

- Mass flow and funnel flow
- Segregation
- Arching or Bridging

Influence of External Variables on Powder flow

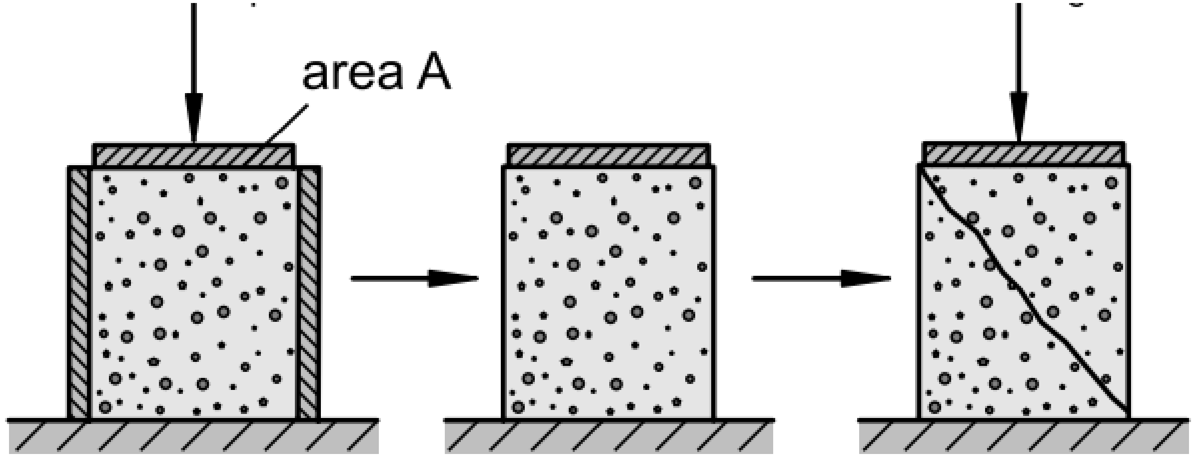
Source: https://www.labtube.tv/video/MTA0Nzc3

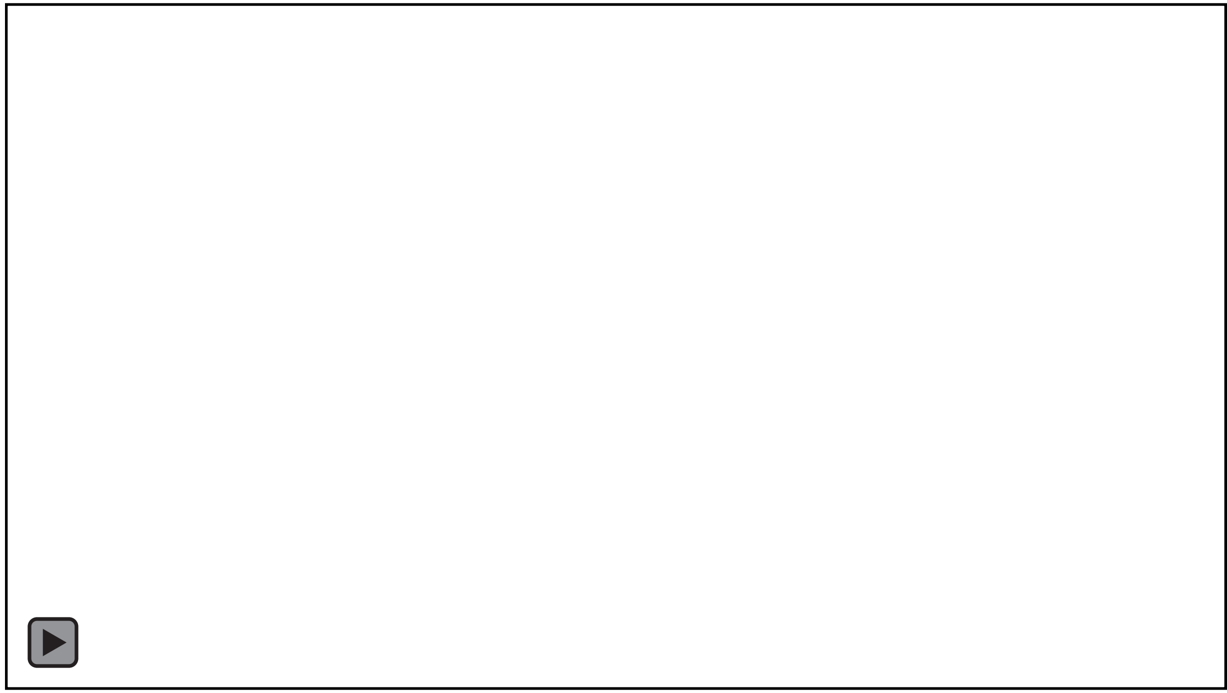
Uniaxial Compression Test

[Source:https://www.labtube.tv/video/MTA2MDgw](https://www.labtube.tv/video/MTA2MDgw)

Compact stress (σ_C)

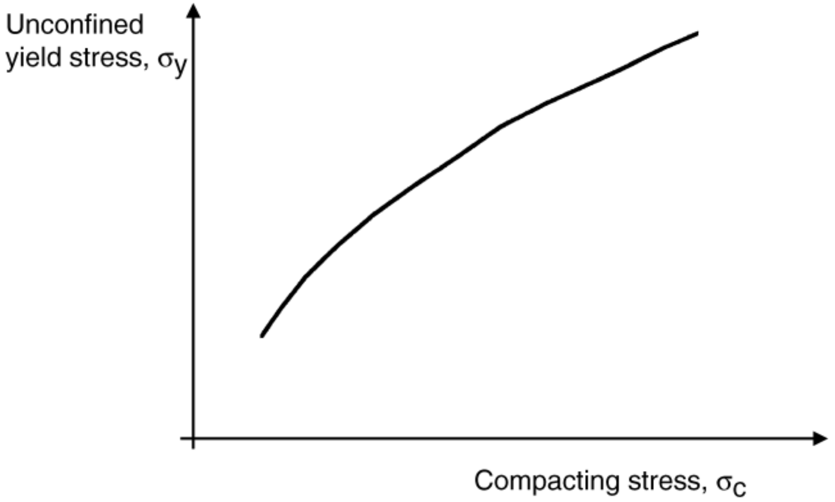
Unconfined yield stress (σ_y)





- σ_y is compressive strength or cohesive strength or unconfined yield strength.
- At failure consolidated bulk specimen starts to flow

Powder flow function : $\sigma_y = f\,(\sigma_C)$



Flow – No Flow criterion

- Gravity flow of a solids occurs :

  stress developed by the solids (σ_D) > compacting stresses in arch (σ_C)

$$ff = \frac{\sigma_C}{\sigma_D} = \frac{\text{compacting stress in the hopper}}{\text{stress developed in the powder}}$$

- Higher the ff, lower the flowability
- ff depends on:
  - The nature of the solid
  - The nature of the wall material
  - The slope of the hopper wall.

- Gravity flow of a solids occurs :

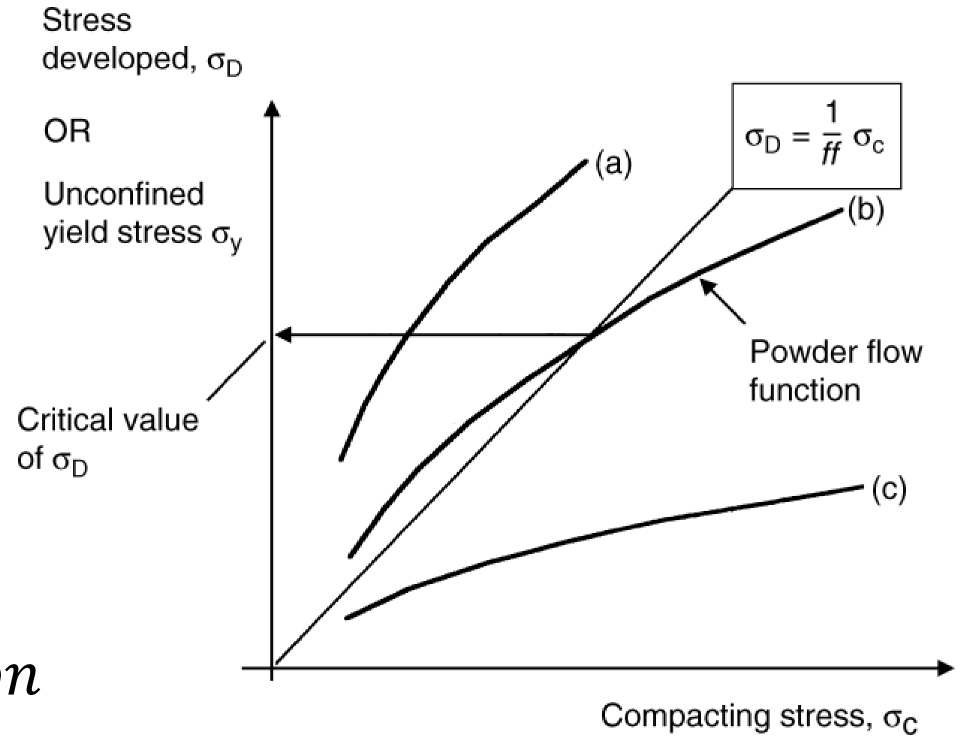
stress developed by the solids (σ_D) > compacting stresses in arch (σ_C)

If $\sigma_D >= \sigma_y$ -> Powder will flow

$$ff = \frac{\sigma_C}{\sigma_D} \Rightarrow \frac{\sigma_C}{ff} > \sigma_y$$

Limiting or Critical condition is

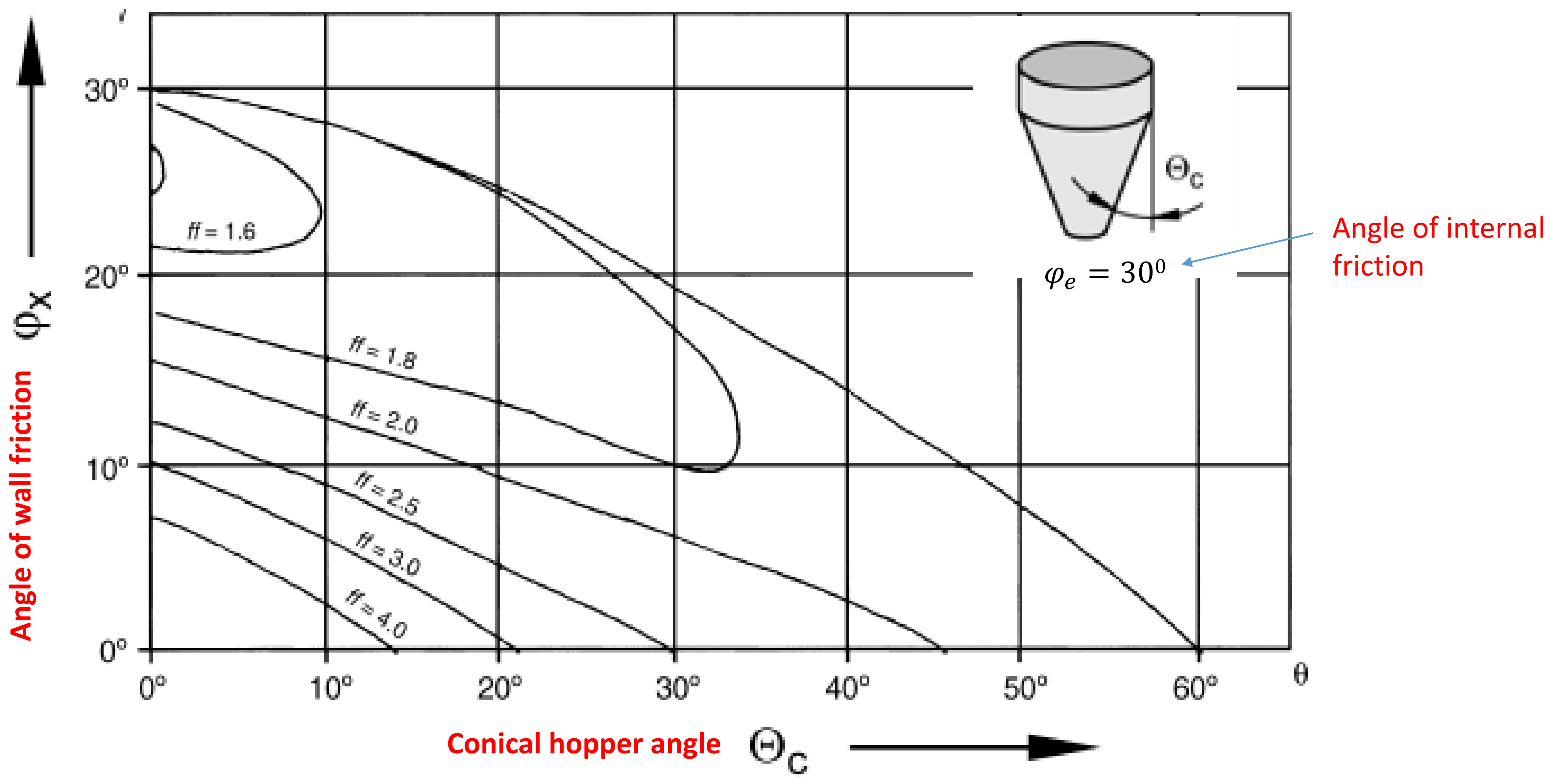
$$\frac{\sigma_C}{ff} = \sigma_y \Rightarrow \frac{\sigma_c = f(\sigma_y)}{ff} = \sigma_y$$



$$\sigma_{crit} \text{ is defined the } \sigma_y \text{ at the critical condition}$$

If actual stress developed $< \sigma_{crit} \Rightarrow$ no flow

If actual stress developed $> \sigma_{crit} \Rightarrow$ flow



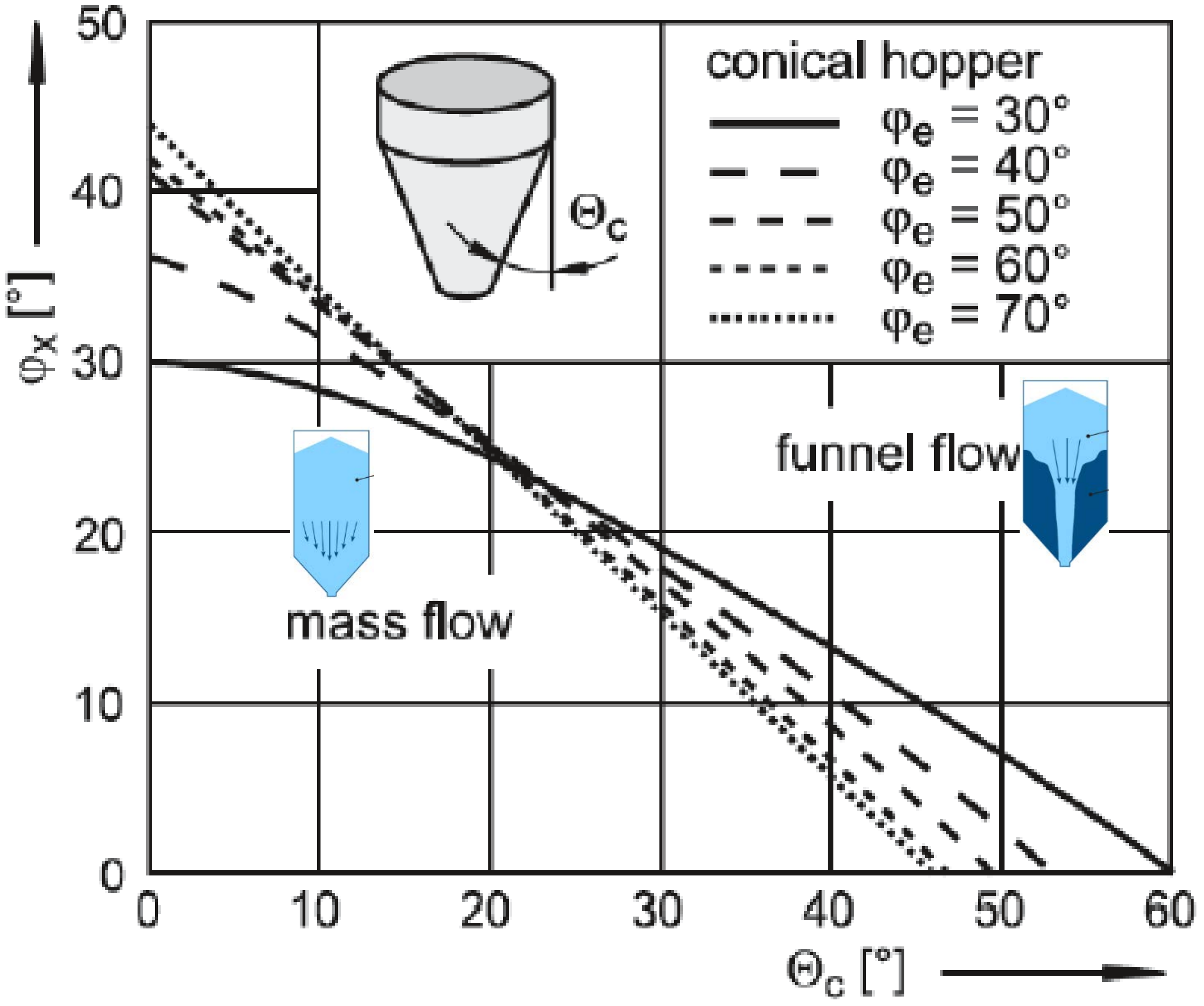
Angle of wall friction
φ_x
30°
20°
10°
0°
ff = 1.6
ff = 1.8
ff = 2.0
ff = 2.5
ff = 3.0
ff = 4.0
Θ_c
$\varphi_e = 30^0$
Angle of internal friction
0°
10°
20°
30°
40°
50°
60°
θ
Conical hopper angle Θ_c

JENIKE's Flow Phase diagram for a hopper

$$\sigma_1 = rg\rho_b s(\Theta', \Theta, \varphi_x, \varphi_e).(1+\sin\varphi_e)$$

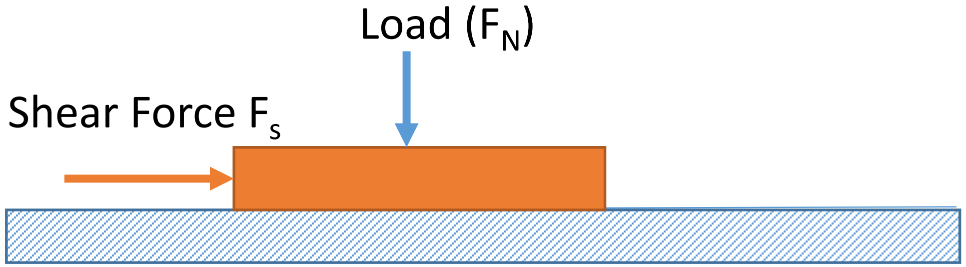
φ_e : Angle of Internal Friction

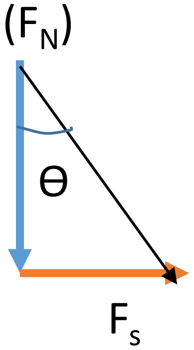
φ_x : Angle of Wall Friction



Powders and Bulk Solids. Behavior, Characterization, Storage and Flow. Authors: **Schulze**, Dietmar, **Springer**

Angle of internal friction





$$\sum F = 0$$

Θ = friction angle

tan Θ = τ / σ

τ = σ tan Θ

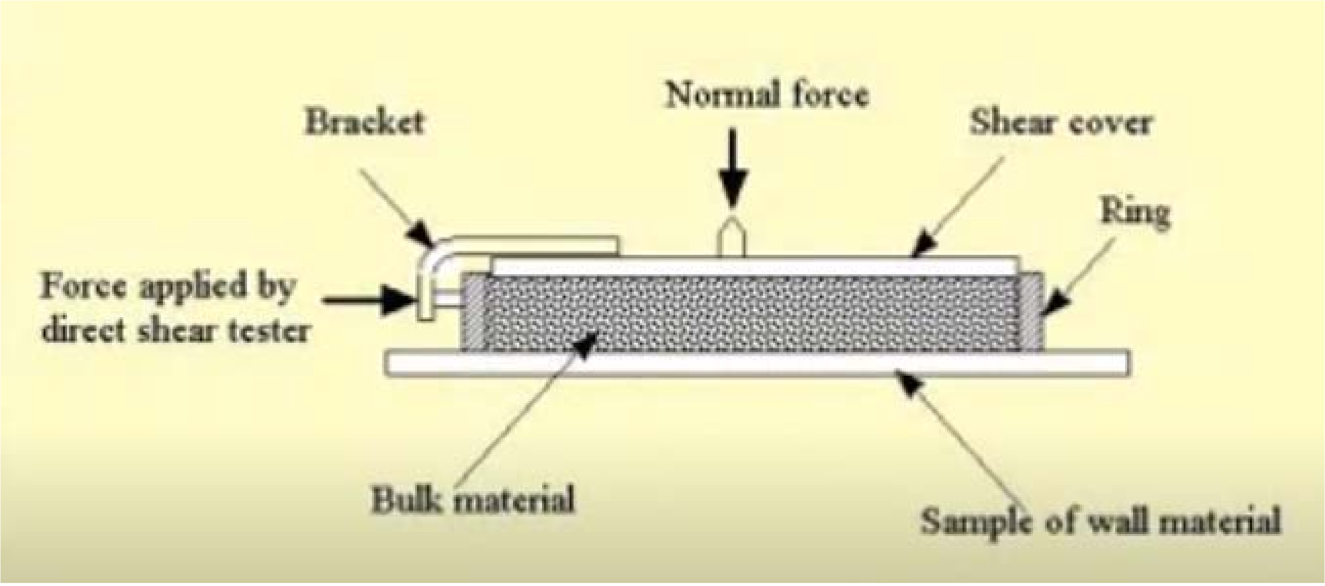
$F_S \propto F_N$

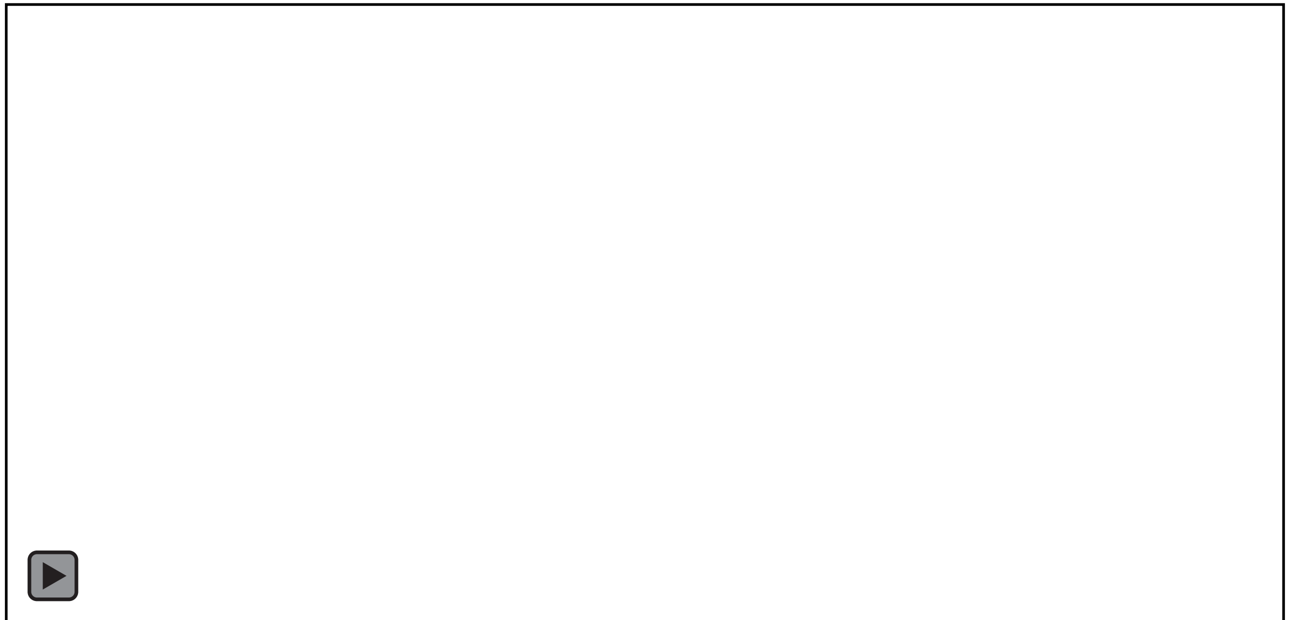
$F_s = \mu F_N$

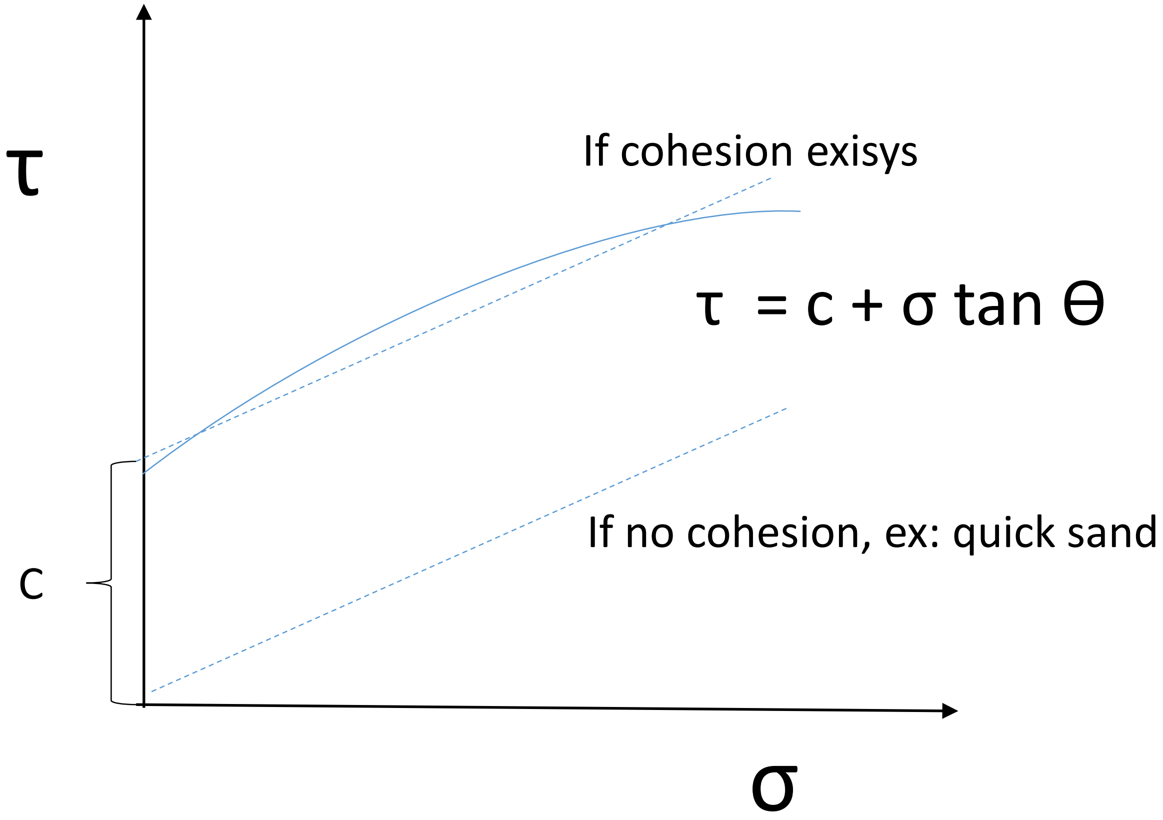
$\tau = \mu \sigma$

μ = frictional coefficient

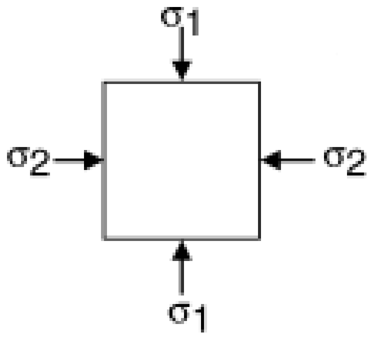
Jenike shear tester







$$\tau = c + \sigma \tan \Theta$$



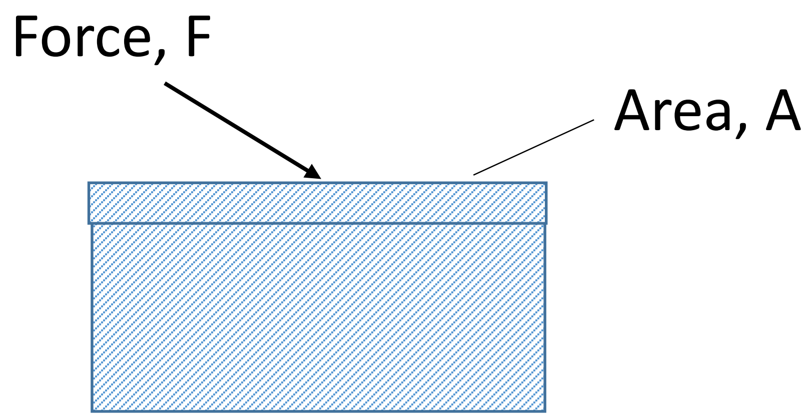
$$\frac{\sigma_1}{\sigma_2} = \frac{1 + sin\theta}{1 - sin\theta}$$

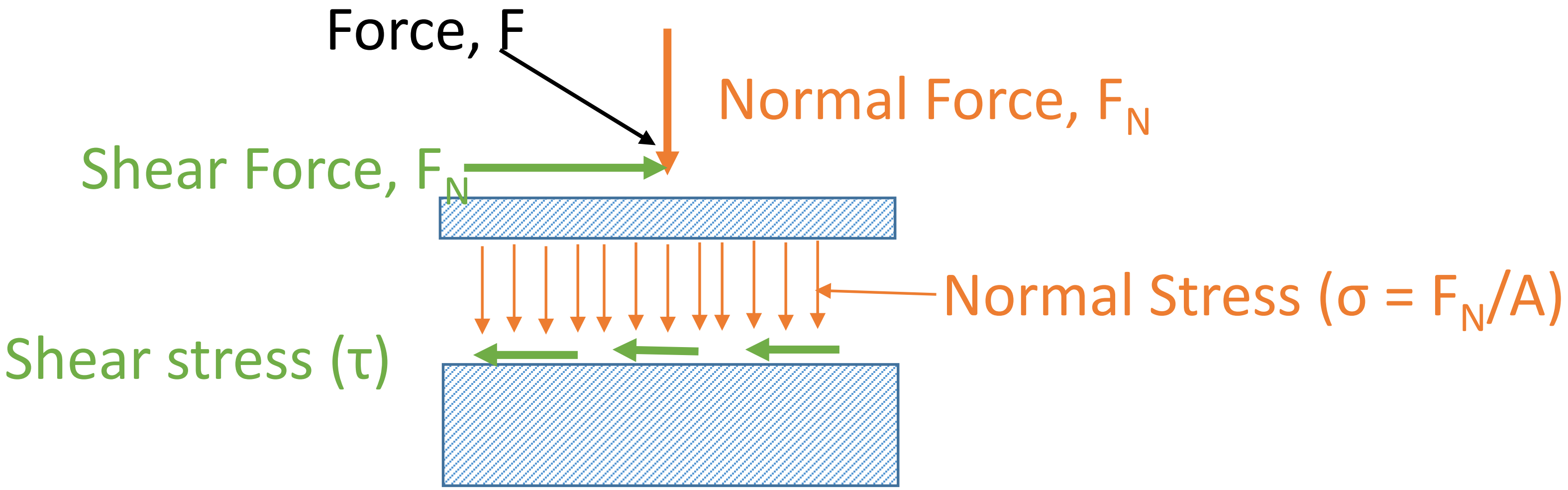
ϴ = effective angle of internal friction

This comes of Mohr's stress circle or Mohr circle

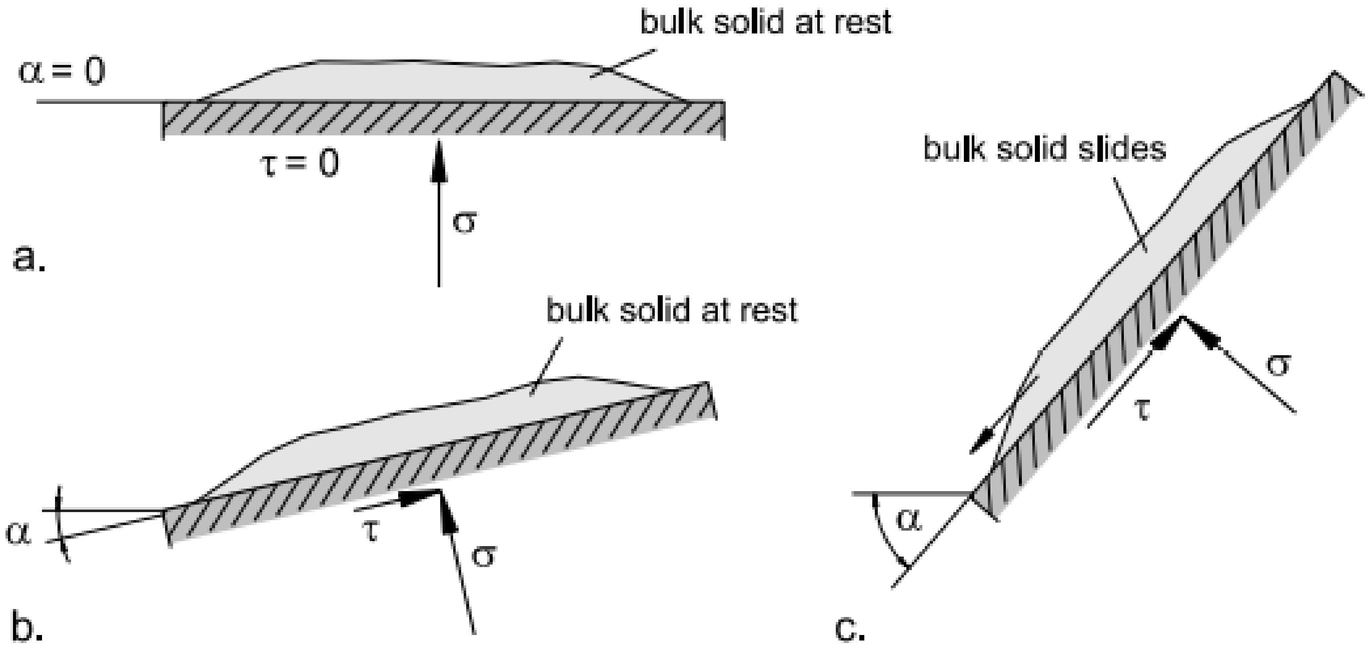
Similar manner angle of internal friction between powder and hopper wall is determined

Forces in Bulk Solids

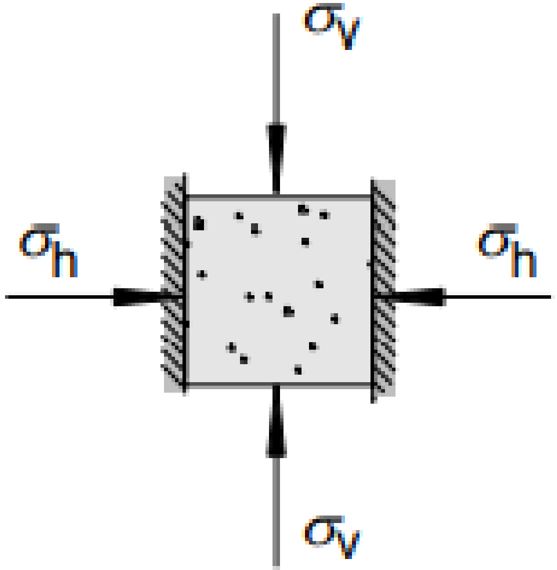




- Shear Stress emerges due to frictional effects.
- It is dependent on the friction between the bulk solid and the plane surface.
- Rough surface will have high transferable shear stress



Stresses in Bulk Solids

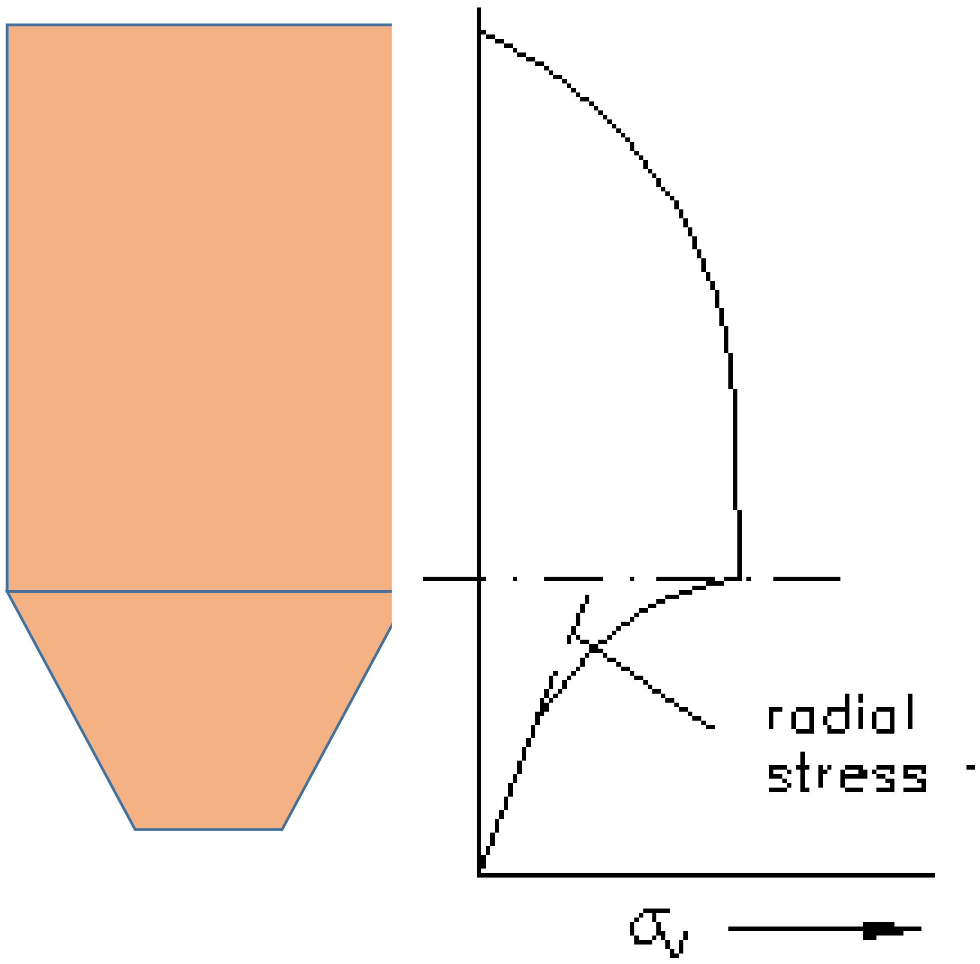


- In Vertical direction, positive normal stress (σ_v) also known as compressive stress.
- Horizontal stress (σ_h) acts due to σ_v

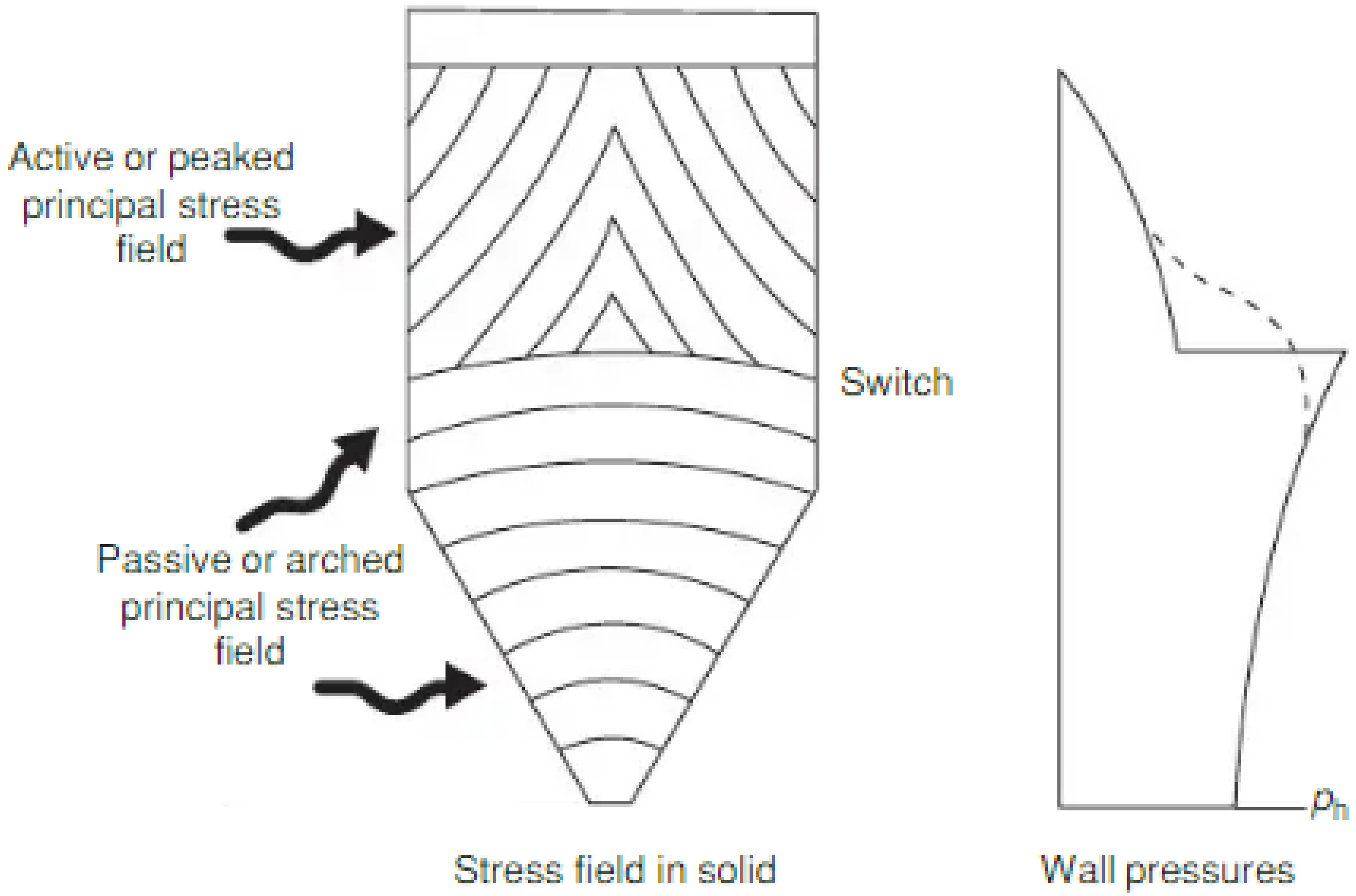
- Lateral stress ratio: K = λ = $\frac{\sigma_h}{\sigma_v}$

  K values typically varies from 0.3 to 0.6.

Stress profile



During Storage



During the Flow

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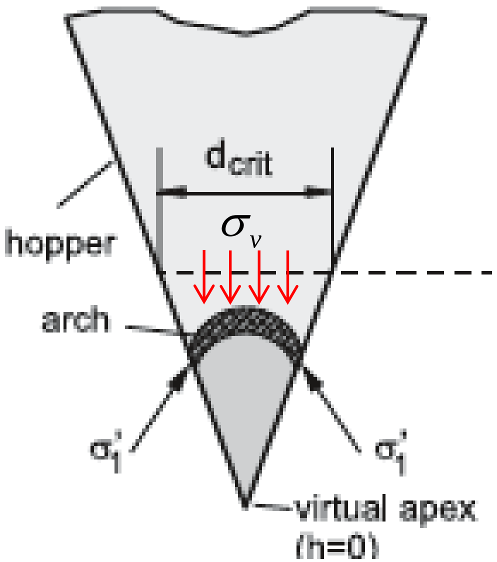
Critical Diameter to avoid Arching



In a hopper, as we go near mouth, the vertical stress decreases, whereas cohesive forces start dominating.

Hence arch formation usually takes place near the mouth of the hopper.

If the **vertical stress** is able to **overcome** the **yield stress of Arch** formed due to cohesive forces, no arch formation will take place.

There exists a critical diameter, above which no arch formation is possible.

Assumptions:

- σ_v plays a role in an indirect way.
- Weight of material above arch is very small as compared the total weight of the material kept in cylindrical section.
- The major normal stress experienced by arch is σ_1'.

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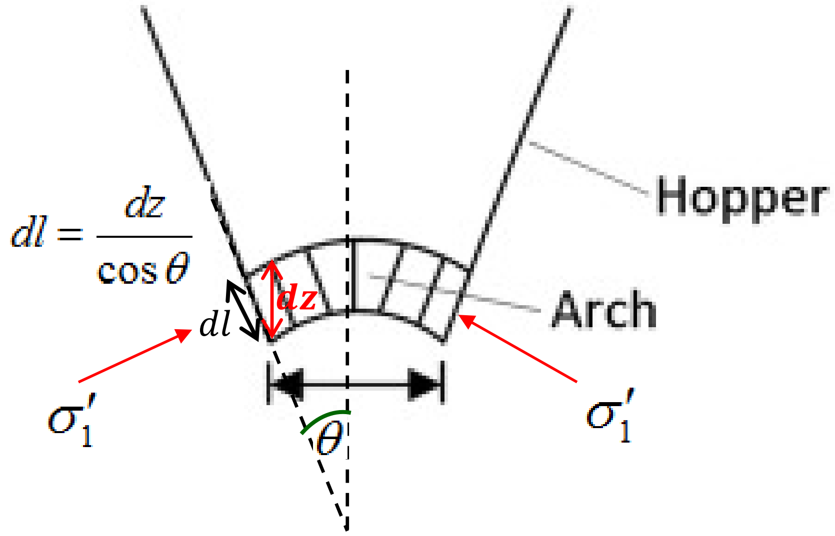
Critical Diameter to avoid Arching



Force Balance:

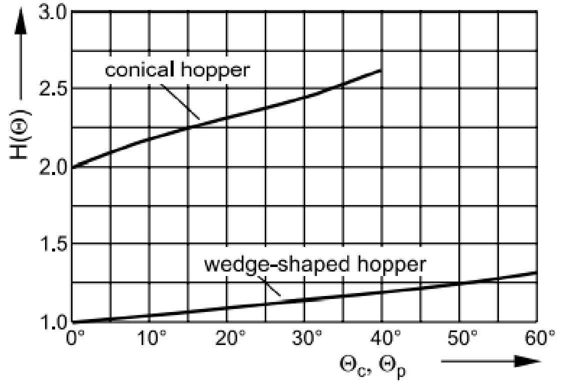
$$\boxed{\sigma_1'(2\pi r)\,dl\sin\theta = \pi r^2\rho_b g\,dz \quad \Rightarrow \sigma_1' = \frac{r\rho_b g}{2\tan\theta}}$$

$$\boxed{\begin{aligned}&\frac{r_{cri}\rho_b g}{2\tan\theta} = \sigma_1' = \sigma_{yc}\\ &\Rightarrow r_{cri} = \frac{2\sigma_{yc}\tan\theta}{\rho_b g}\end{aligned}}$$

$$\boxed{D_{cri} = \frac{4\sigma_{yc}\tan\theta}{\rho_b g}}$$

minimum outlet dimension, $B = \dfrac{H(\theta)\sigma_{\text{crit}}}{\rho_{\text{B}}g}$

For conical Hopper $H(\theta) = 2.0 + \dfrac{\theta}{60}$



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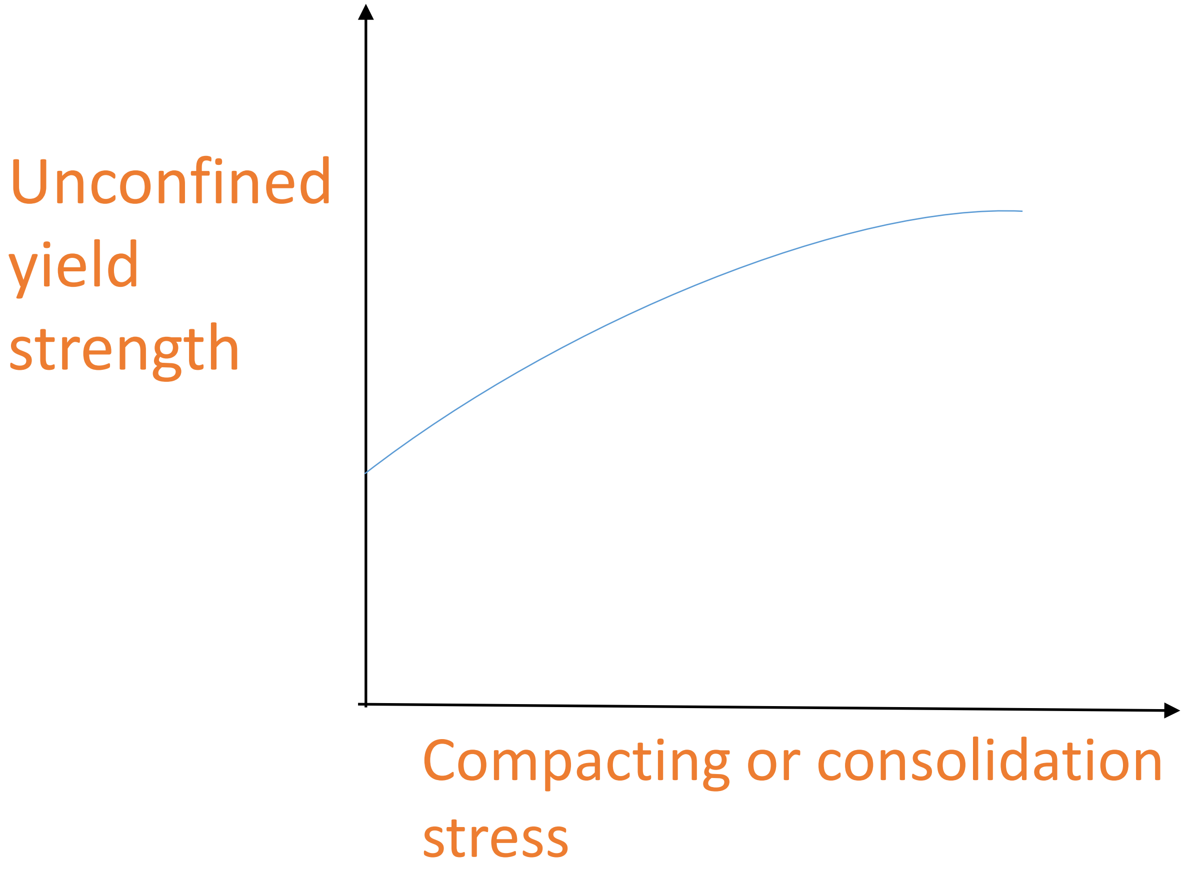
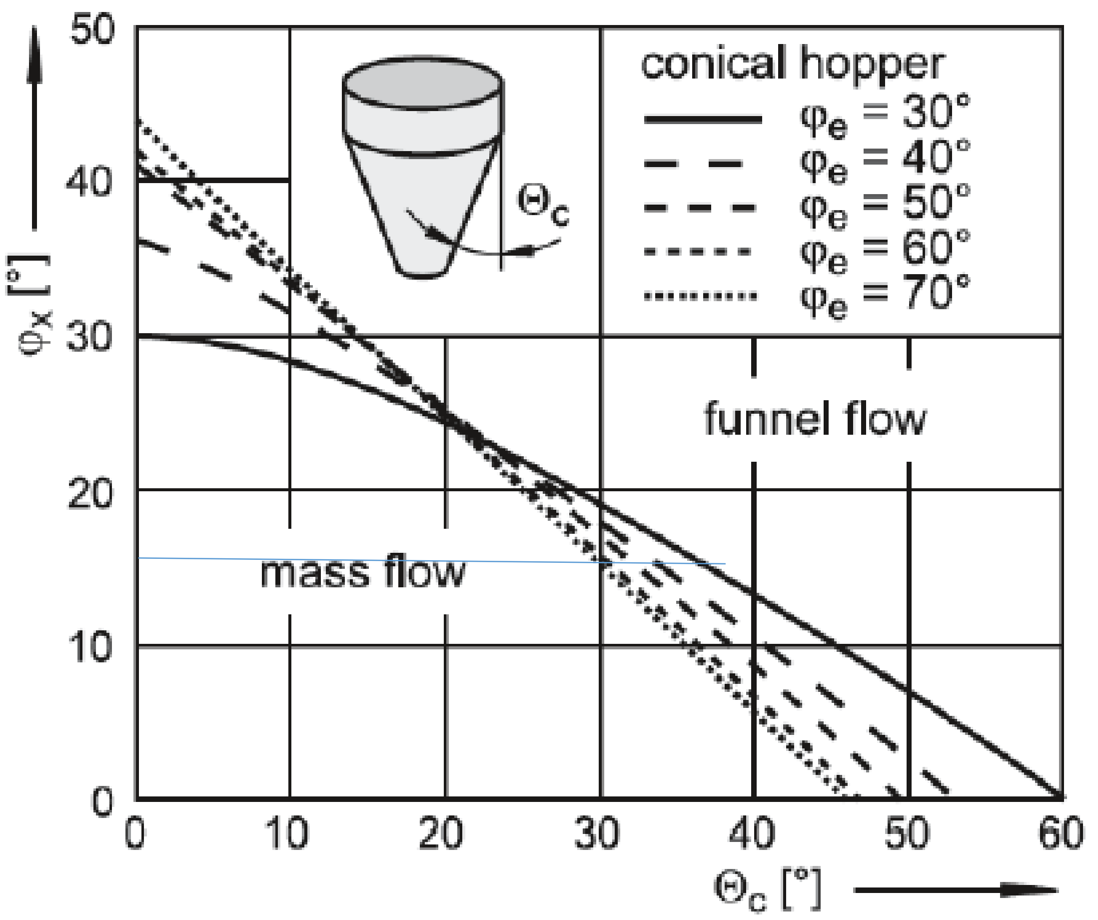
Design Procedure for mass flow hopper

1. Flow function from uniaxial compression test

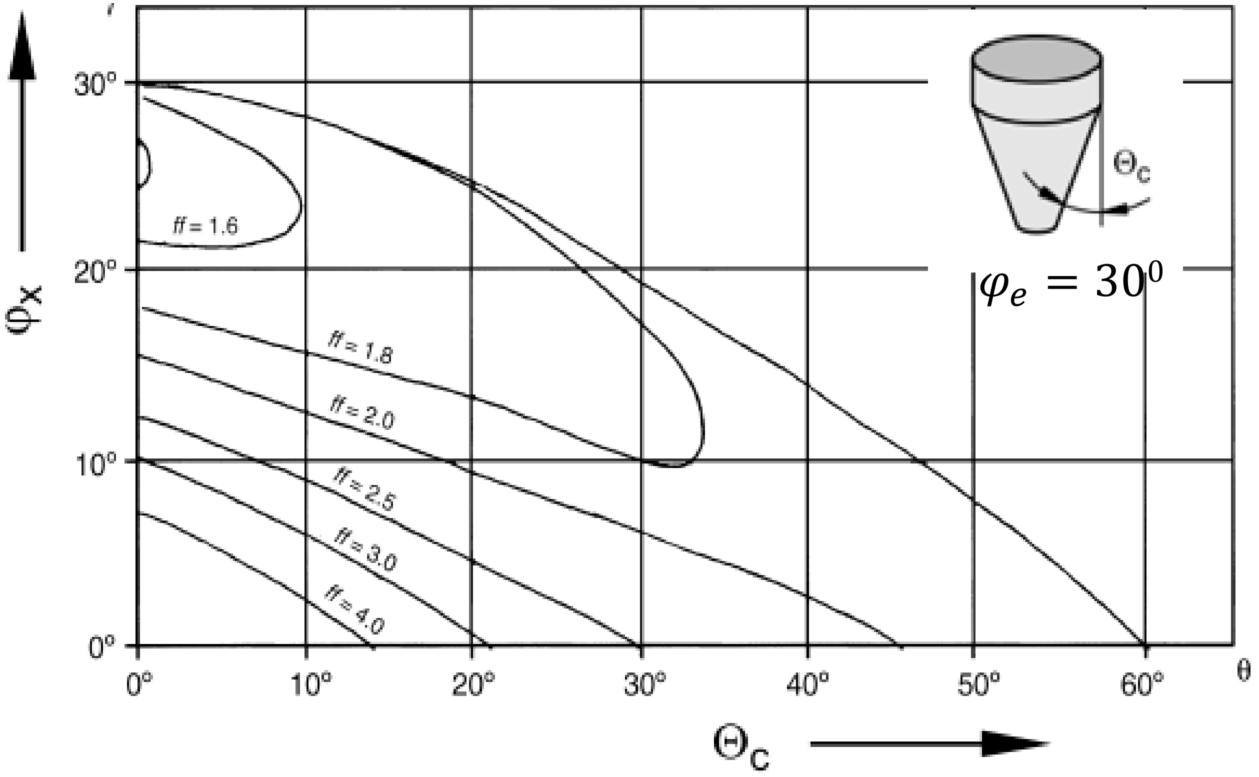


2. Determine Angle of internal friction (φ_c) and wall friction (φ_x) – Jenike Shear tester

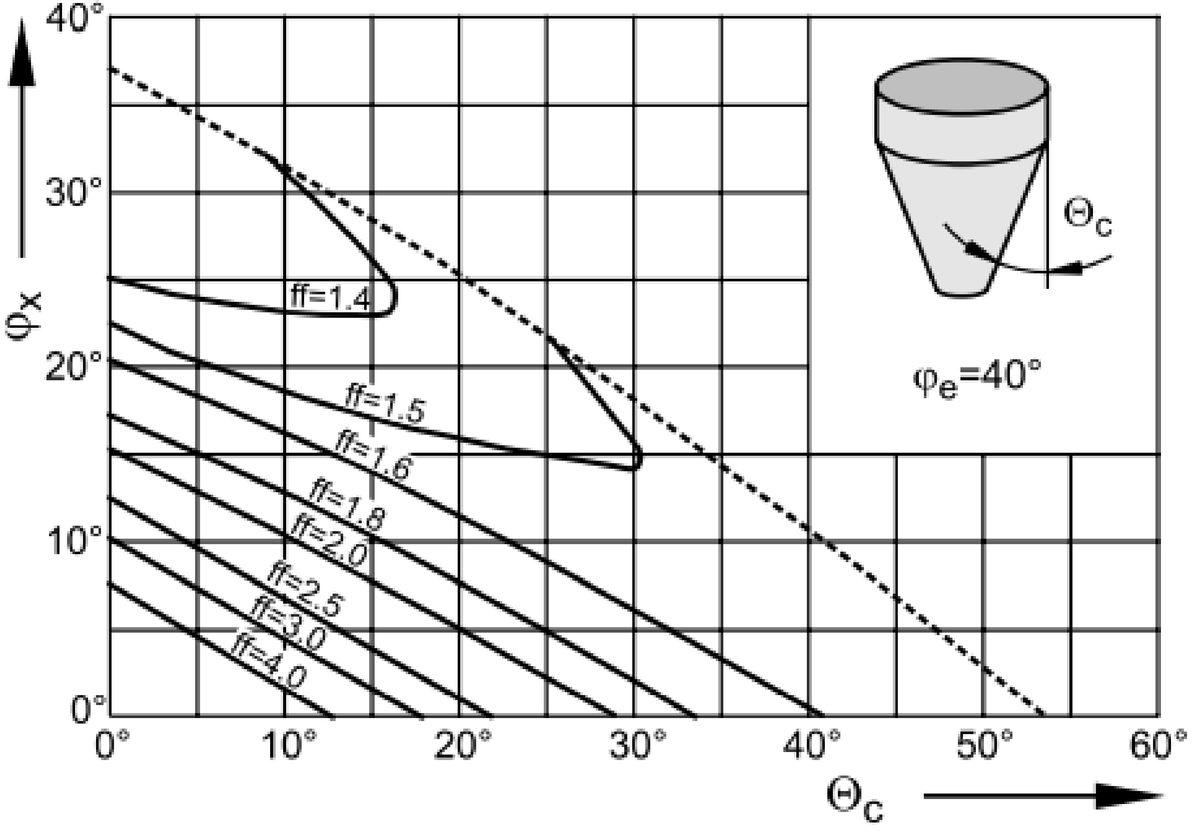
3. Determine hopper angle



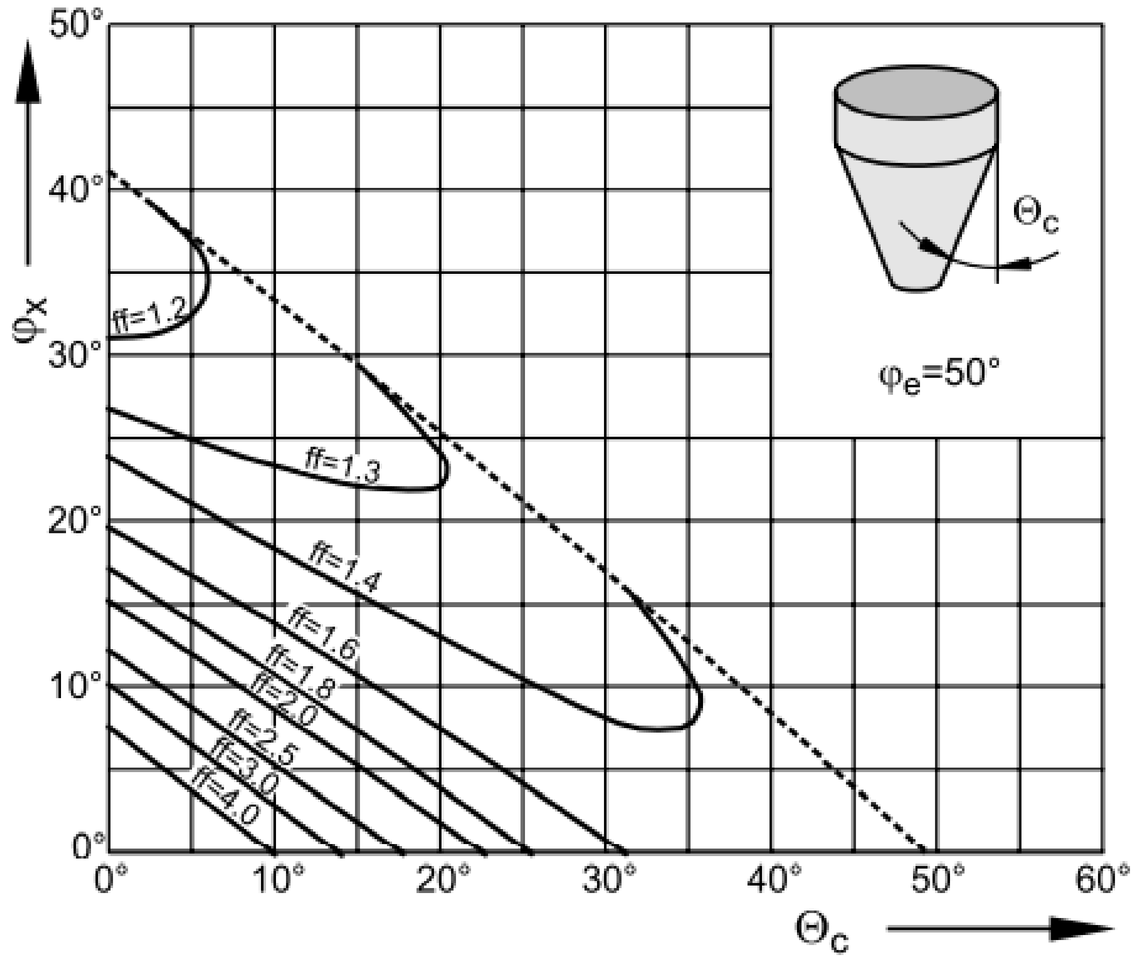
4.Determine ff_c value

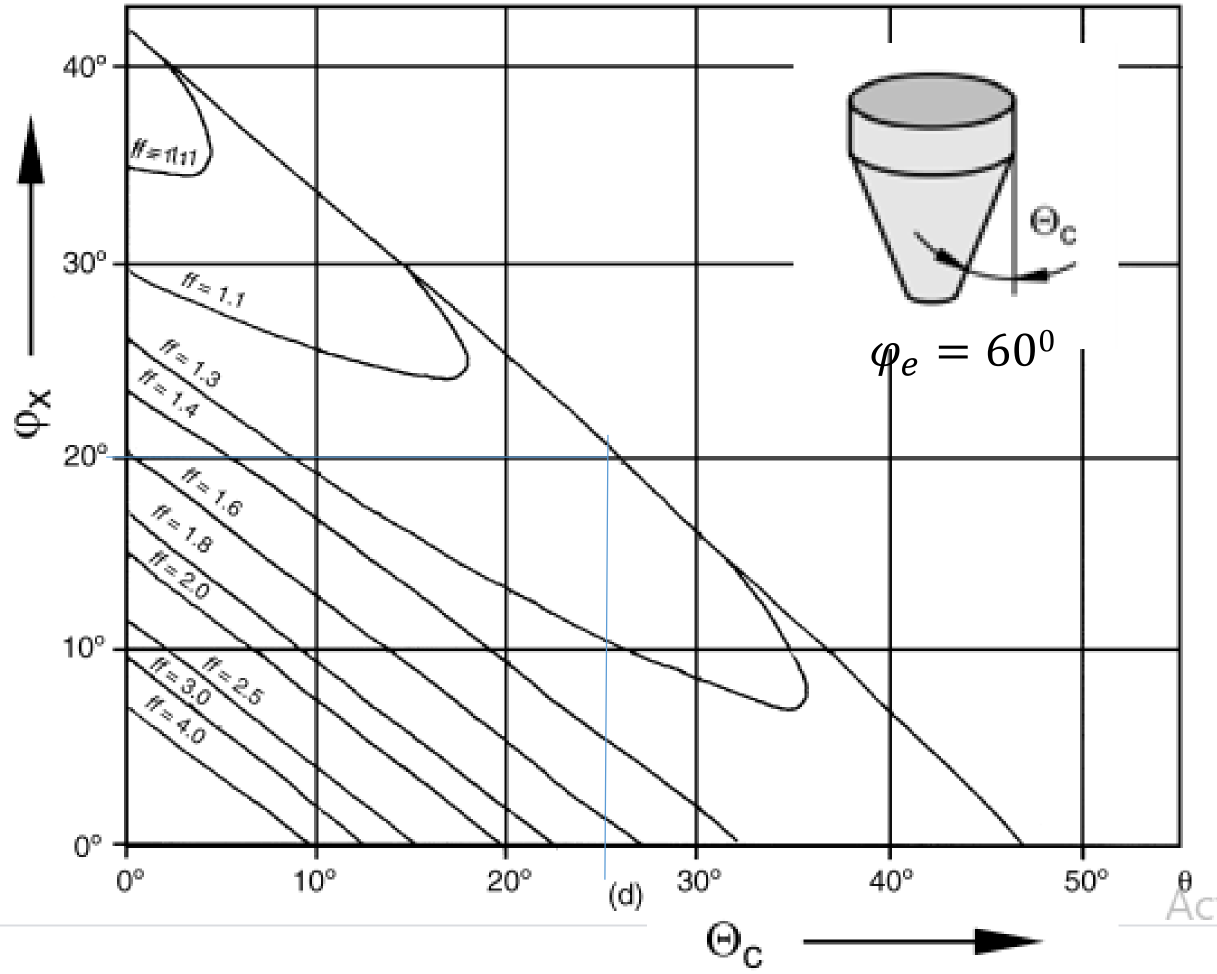


φx
30°
20°
10°
0°
ff = 1.6
ff = 1.8
ff = 2.0
ff = 2.5
ff = 3.0
ff = 4.0
0°
10°
20°
30°
40°
50°
60°
θ
Θc
$\varphi_e = 30^0$

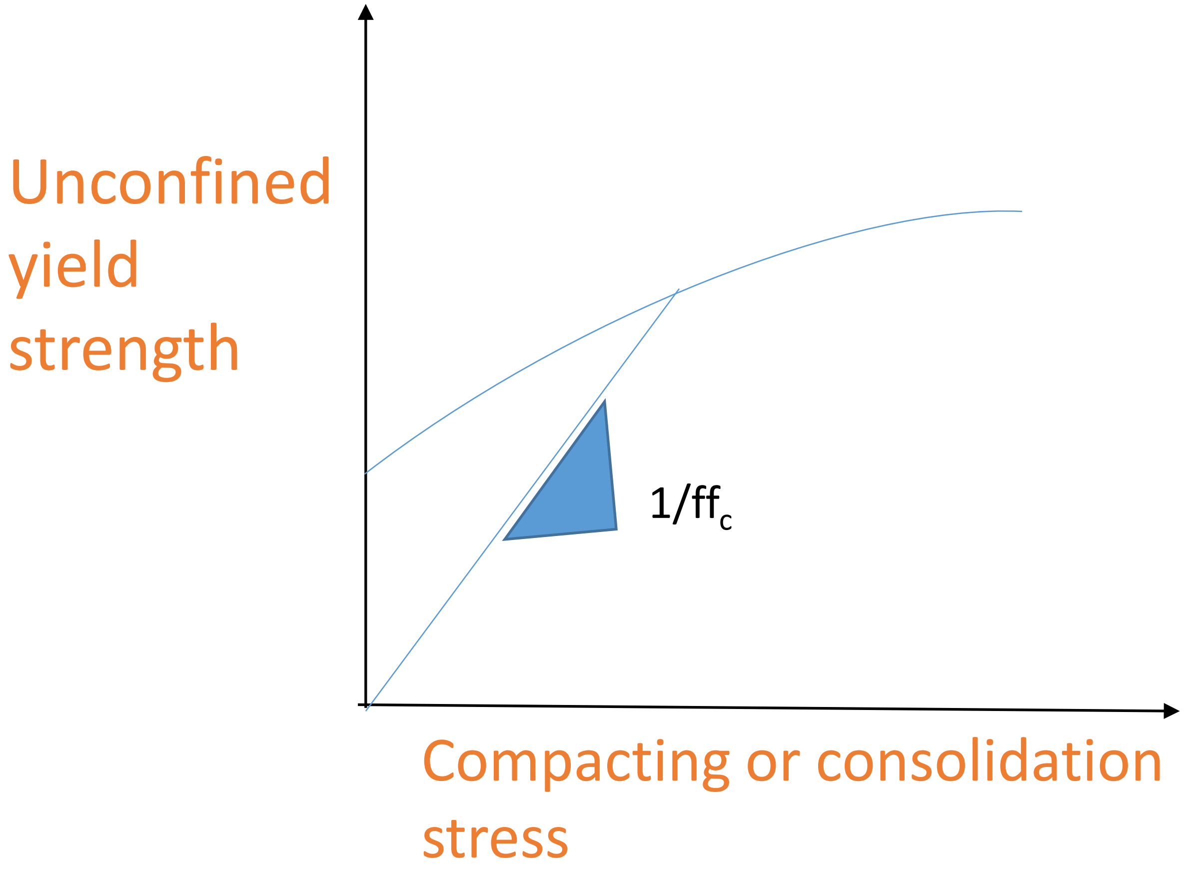


φx
40°
30°
20°
10°
0°
ff=1.4
ff=1.5
ff=1.6
ff=1.8
ff=2.0
ff=2.5
ff=3.0
ff=4.0
0°
10°
20°
30°
40°
50°
60°
Θc
φe=40°

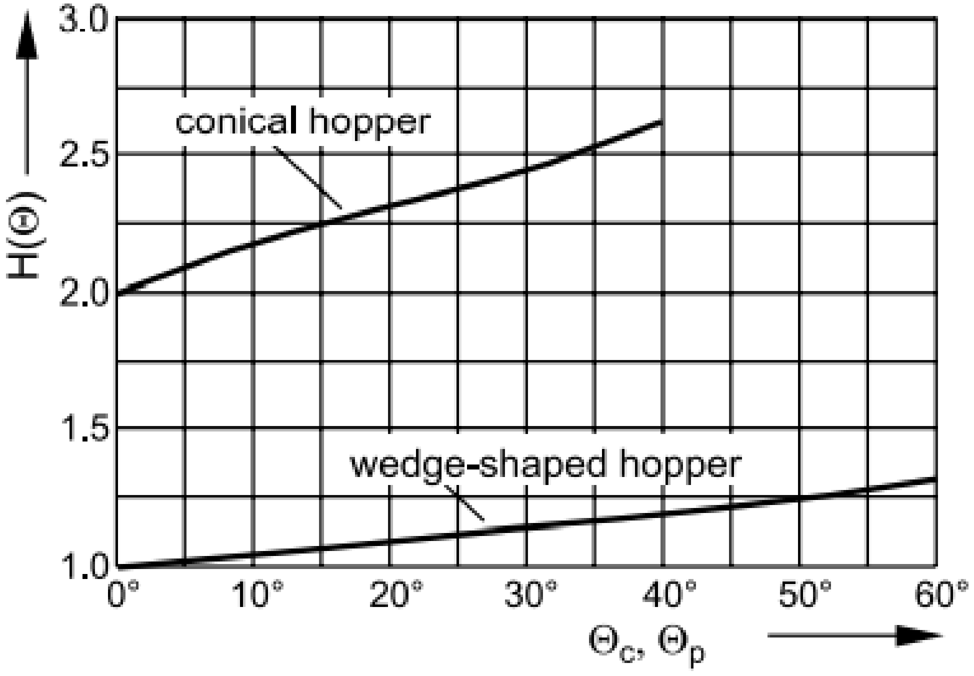

φe=50°
Θc
φx
ff=1.2
ff=1.3
ff=1.4
ff=1.6
ff=1.8
ff=2.0
ff=2.5
ff=3.0
ff=4.0
50°
40°
30°
20°
10°
0°
0°
10°
20°
30°
40°
50°
60°


$\varphi_e = 60^0$
Θc
φx
ff = 1.1
ff = 1.3
ff = 1.4
ff = 1.6
ff = 1.8
ff = 2.0
ff = 2.5
ff = 3.0
ff = 4.0
40°
30°
20°
10°
0°
0°
10°
20°
(d)
30°
40°
50°
θ

6. Determine critical σ_c

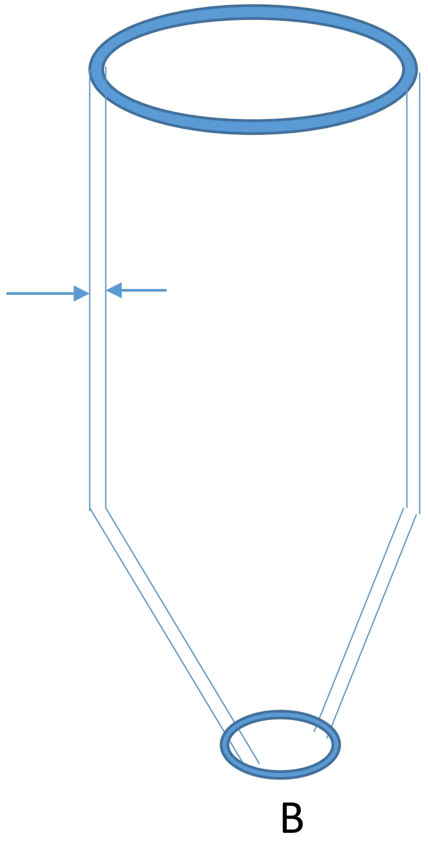


7. Estimate H(Θ) value



8. Estimate Outlet diameter of hopper $B = \frac{H(\theta)\sigma_{crit}}{\rho_B g}$

9. Estimate the diameter and height of hopper
based on mass or volume of powder
to be stored.



10. Thickness of silo construction material -
(estimated from stress profile)

11. Mass flow rate of powder

$$M_p = \frac{\pi}{4}\sqrt{2}\rho_B g^{0.5} h^{0.5} B^2$$

Design Problem

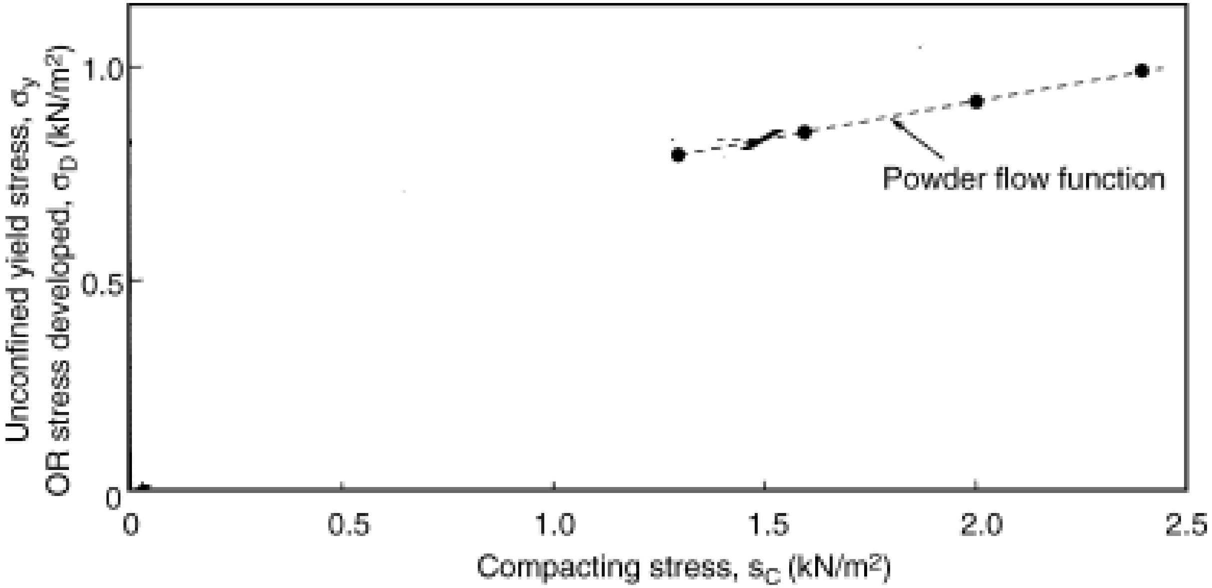
- Powder having the angle of internal friction of 30^0 and angle of friction on SS is 19^0 needed to be stored in a silo. Bulk density of powder is 1300 kg/m^3. Design the conical hopper SS silo.

Uniaxial compression test results

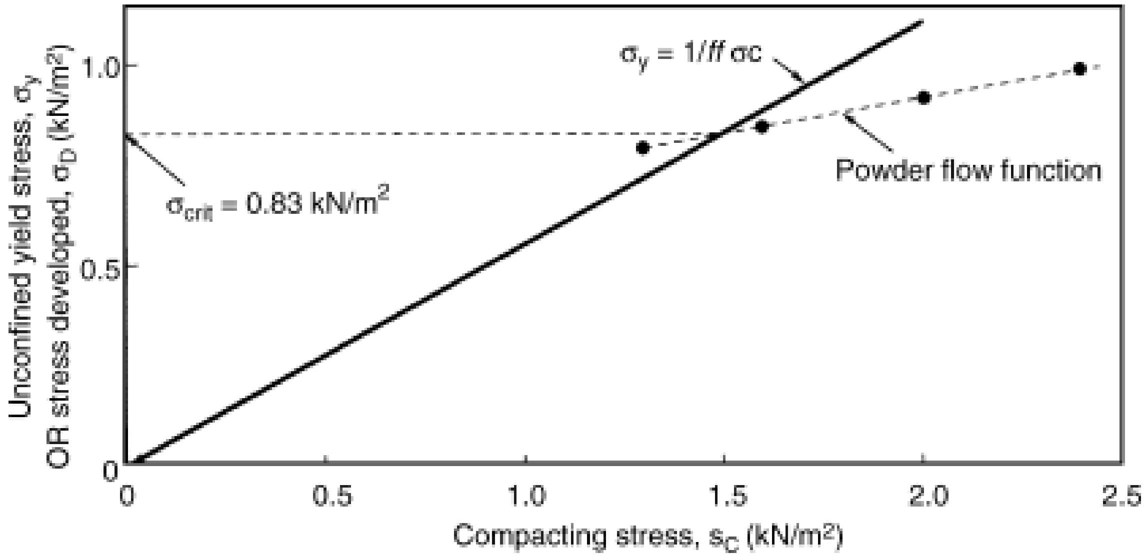
Unconfined yield strength	2.4	2.0	1.6	1.3
Consolidation stress	0.97	0.91	0.85	0.78

Solution

1. Flow function



2. Angle of internal friction $(\varphi_c) = 30^0$
   Angle of wall friction (θ_c) = 19^0



3. Hopper angle = 27.5^0

4. ff_c = 1.8

5. Critical stress = 0.83

- H (ϴ) = 2.46
- Outlet diameter $B = \dfrac{2.46 \times 0.83 \times 10^3}{1300 \times 9.81} = 0.160\,\mathrm{m}$

Thank you

- https://www.youtube.com/watch?v=ZYifIAtqusM