Statistical Enquiry into Modern Portfolio Theory

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Abstract

The Harry Markowitz framework for the construction of Portfolio based on optimization of risk and return put the foundation of Modern Portfolio Theory.The aim of this paper is to provide an analytical perspective of Modern Portfolio Theory by applying statistical methods on various assumptions of the theory and test how these assumptions interact with each other in order to test the significance of these theory in modern financial system.

Introduction

Markowitz introduced the mean-variance Portfolio selection theory in 1952. Since then the Markowitz model for portfolio optimization has become a cornerstone in the field of modern finance. The Modern Portfolio theory quantifies the relationship between risk and return in accordance with the behaviour of a rational investor.

Harry Markowitz (1952) devised a risk-return model based on mean return of the portfolio and the variance associated with the return. In his model Markowitz suggested the as the number of securities in the portfolio increases the idiosyncratic risk associated with the portfolio decreases. The reason for the decrease as suggested by the model is due to imperfect correlation between the securities in the portfolio.

Furthermore, he also hypothesized that there can be an infinite number of portfolios based on infinite possible combination of weights of different securities but the investor will choose an efficient portfolio (maximizing the utility of investor) over an inefficient portfolios.

The aim of this paper is to perform statistical analysis in order to verify the significance of assumptions of Modern Portfolio Theory in the context of US stock market and shed some light on the validity of these assumptions.

Data

Twelve stock quoted on the New York Stock Exchange (NYSE) are used for the purpose of analysis. The data (from Yahoo Finance) for the given stock ranges from 1st January 2008 to 31st December 2017 which implies 2518 daily observations spanning over a period of 10 years. The stocks are dividend adjusted. Summary of the stocks is presented in Table 1.

Company	Symbol
Apple Inc	AAPL
American Express Co	AXP
Boeing Co	BA
General Electric	GE
Johnson & Johnson	JNJ
JPMorgan Chase & Co	JPM
Coca-Cola Co	KO
McDonald's Corp	MCD
Microsoft Corp	MSFT
Verizon Communications Inc	VZ
Walmart Inc	WMT
Exxon Mobil Corp	XOM

Logarithmic Returns

The methodology employed to calculate the daily returns of the stocks is by using the first difference of there natural logarithm.

$$\mathbf{R_{asset} = \ln FV - \ln PV}$$

Where

FV is the present value of adjusted close price for an individual stock X

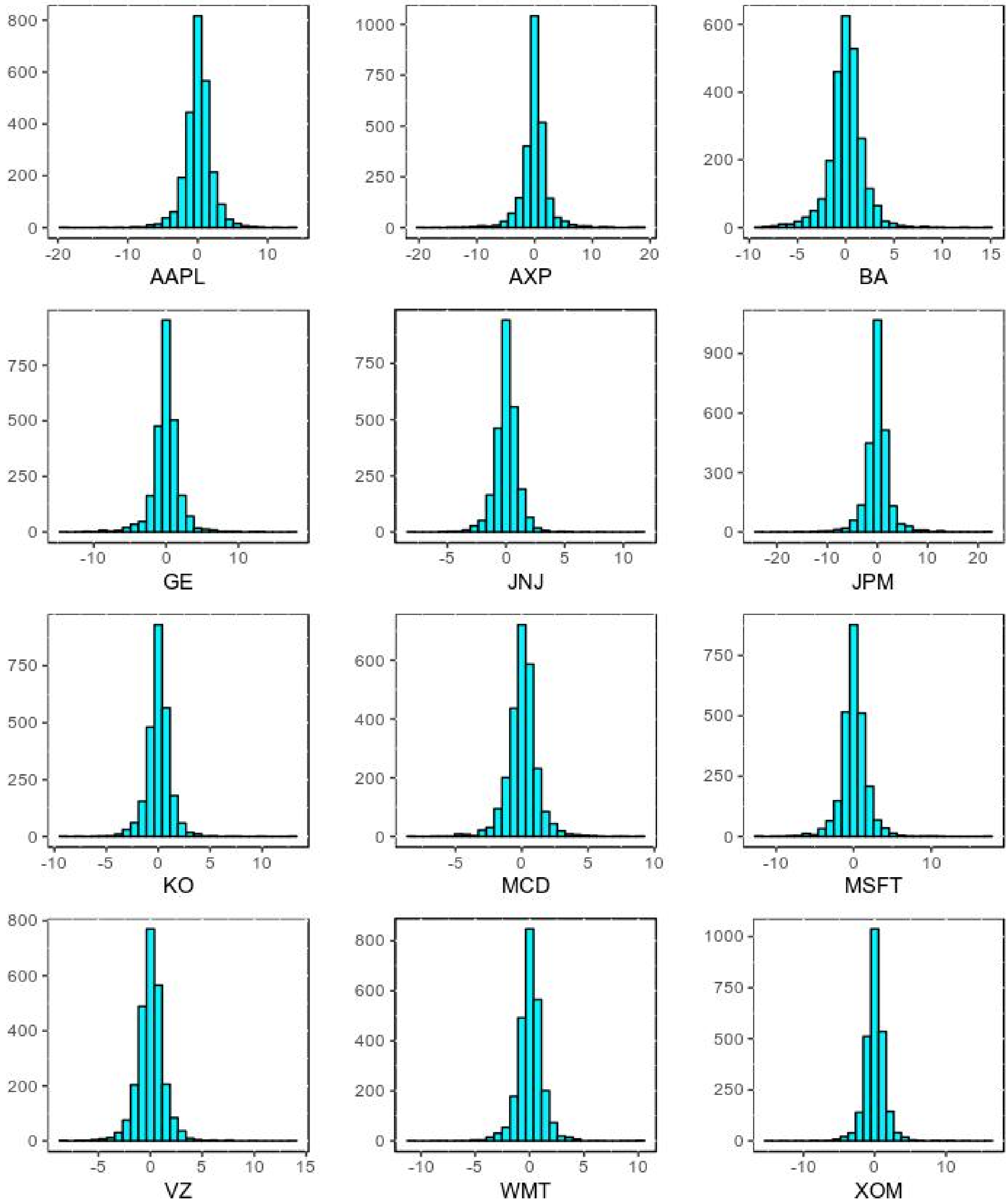
PV is the previous day adjusted close price for an individual stock X

R_{asset} is the logarithmic daily return of stock X

There are several advantages of using this method, first is the yield obtained from the change in logarithmic price of the security represents the continuous compounding from the holding day. Other one is the change of less than +/- 15 percent is approximately equal to the percentage change in the price which makes the data convenient for analysis.

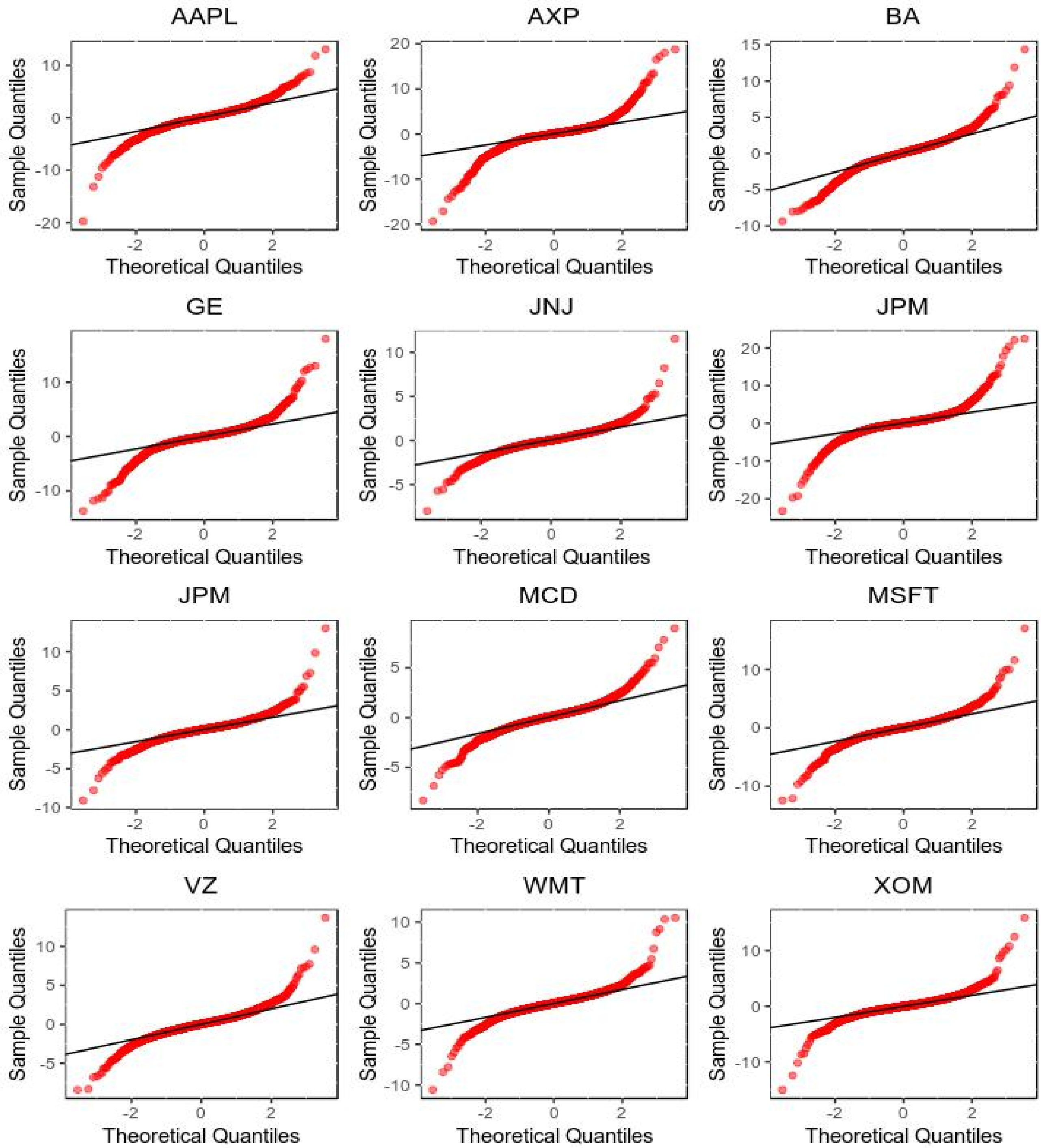
The returns of the stock are then collected and represented in the form of histogram given in Figure A. in order to check the nature of distribution of returns. In our analysis we find that the returns for most of the stocks closely resemble normally distribution but they are not perfectly normal in nature.

Figure A



By visualizing the Q-Q plot method we found that all the stocks deviate from the 45 degree slope in the graph which represents the perfect normal distribution, these deviations can be attributed to the presence of fat tails in the distribution of returns, these fat tails are due to some of extreme deviations of returns from the mean return value which is not a characteristic of a normal distribution.

Figure B



From Table 2 we can see that all the assets highly leptokurtic in nature (K >3) . This leptokurtic behaviour is due to the risk associated with the security which we will discuss in later section. As for skewness we find most of the assets are positively skewed but the effect of skewness is not that impact full on the distribution. From this we conclude that kurtosis play very significant role in determining the riskiness of the distribution of returns.

TABLE 2

Company	Mean Daily Return	Skewness	Kurtosis
AAPL	0.086	-0.507	8.208
AXP	0.032	0.023	11.946
BA	0.059	0.030	5.327
GE	-0.014	0.018	10.758
JNJ	0.041	0.469	12.205
JPM	0.045	0.310	14.655
KO	0.033	0.439	12.832
MCD	0.055	0.094	6.682
MSFT	0.044	0.144	10.016
VZ	0.030	0.370	9.275
WMT	0.038	0.151	11.634
XOM	0.007	0.142	16.915

Risk

In the context of finance the risk can be defined as the uncertainty in the market. Mathematically it can defined as the deviation away from the expected historical returns during a particular time period [1]. A more preferred definition in our context of our analysis is the degree of making loss on a potential investment.

There are several statistical methods use to measure financial risk, Standard Deviation is the most commonly employed method for calculating financial risk. It can be expressed as.

$$SD_{asset} = [1/N-1\ \Sigma\ (R_{market} - R_{mean})^{\wedge}2]^{\wedge}(1/2)$$

Where

N is the total number of daily returns observed

R_{market} is daily observed return

R_{mean} is mean return observed during N days.

TABLE 3

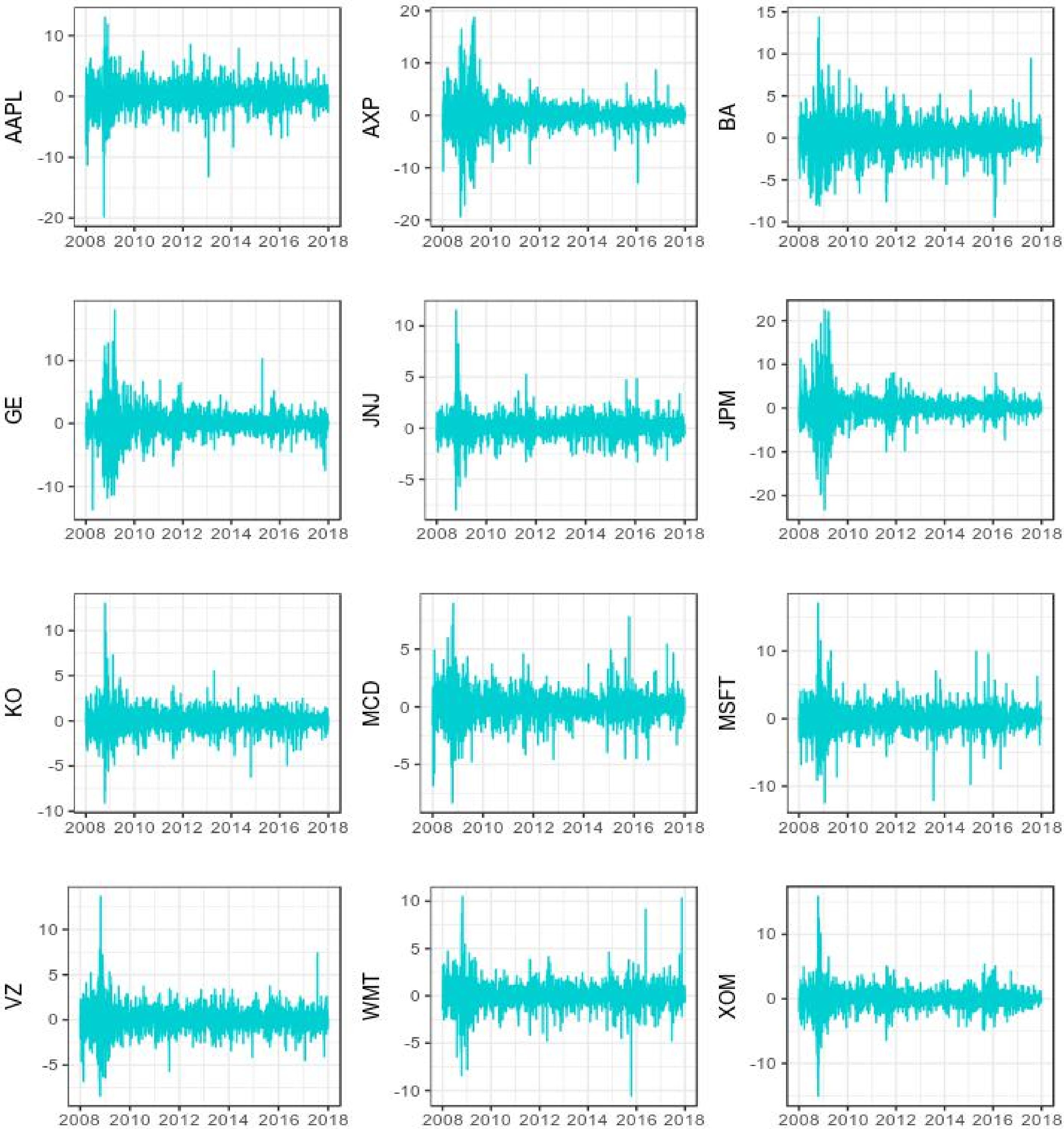
Company	Standard Deviation (%)
AAPL	1.977
AXP	2.452
BA	1.809
GE	1.991
JNJ	1.032
JPM	2.733
KO	1.170
MCD	1.168
MSFT	1.745
VZ	1.359
WMT	1.228
XOM	1.533

The greater the Standard Deviation of the security the more risky the asset is (in relative terms). The Standard Deviation calculates the historical volatility of the security, greater the dispersion from the mean value indicates more volatile asset. From the table we can observe that JNJ is least risky asset with standard deviation of 1.0321% while JPM is the most risky asset with standard deviation of 2.733%.

The other method which is more preferable is using the daily volatility clustering technique which helps in visualizing the volatility at specific periods in time.

From Figure C we observe that the period between the year 2008 to 2010 was extremely volatile period due to rapid change in the stock prices from their mean value, the contributing factor for this rapid movement was in response to the subprime mortgage crises in US due to which their was a large scale financial panic in the market. After the US government intervened in the financial markets to calm down the rapid volatility in the market, the price movement started to become less intense.

FIGURE C



Markowitz (1952) argued that the individual risk pertaining to an asset is not as much important as the contribution of risk from each asset to the aggregate portfolio. He categories the financial risk into two major categories.

a) Systematic Risk - It is a macro level risk and it affects all the assets to one degree or another (). It can be contributed by several factors such as interest rates, inflation, unemployment levels, exchange rates etc. The systematic risk can not be diversified.

b) Unsystematic Risk - It is micro level risk and it only affects a single asset or a group of asset due to individual performance of the company or the sector, it includes the profitability of the company, brand image, sector outlook etc. This type of risk can be easily diversified and investor should took active advantage of this effect.

Markowitz pointed out that the riskier the asset the greater the required potential return. In the financial markets only those risk are compensated that cannot be avoided [2].

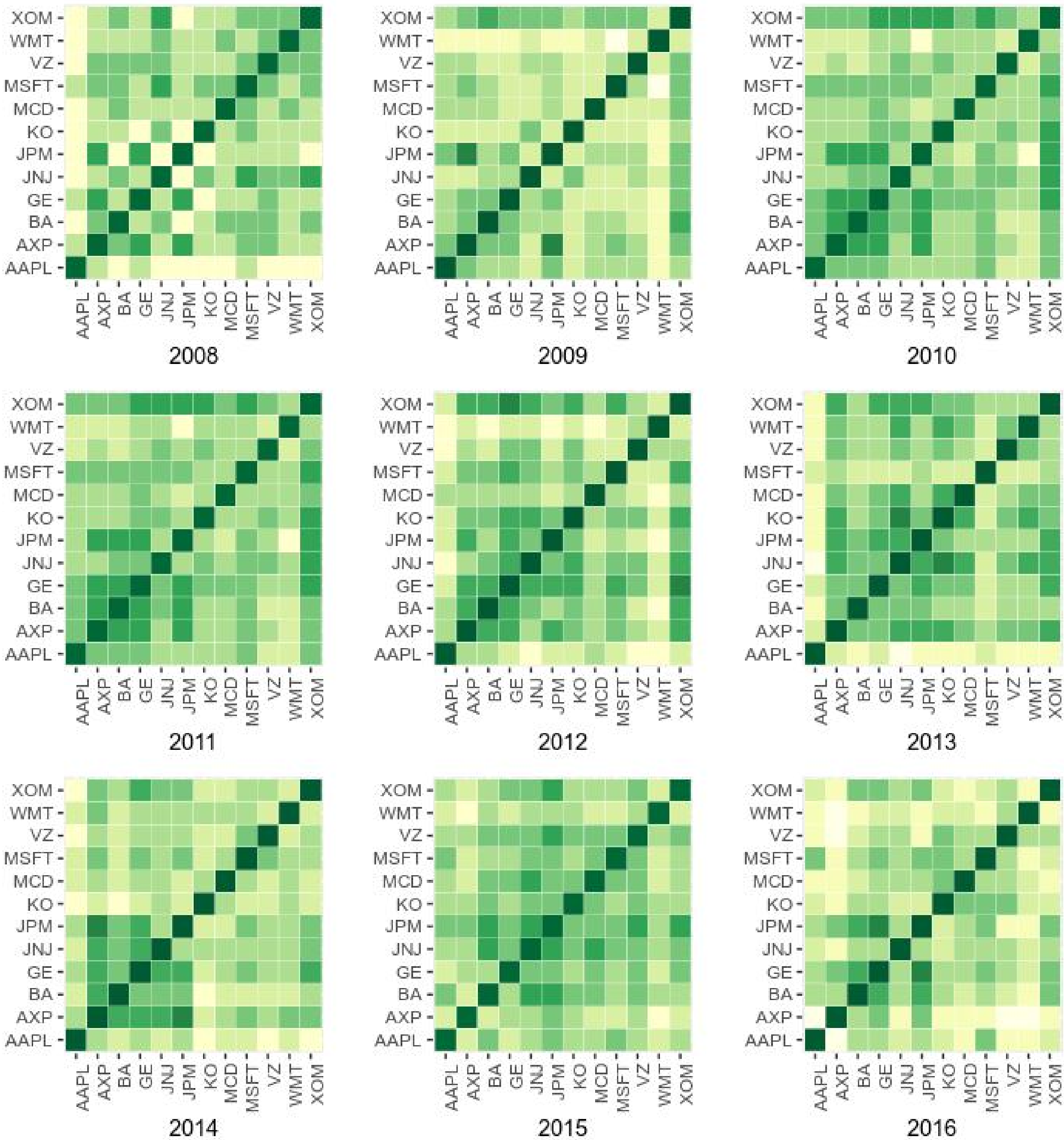
Correlation among assets

In the financial market no two financial assets are either completely correlated or completely independent of each other, to some degree each asset is interrelated to the movement of the prices of another asset , some assets have very high degree of co-movement while some have less exposure to each other. The Modern Portfolio Theory attempts to analyze the interrelationship between different investments. It utilizes statistical measures such as correlation to quantify the diversification effect on portfolio [3].

The extent to which the risk of the portfolio can be reduced largely depends on the covariance between different assets. Portfolio assets which have low correlation coefficients are considered to be less risky than pairs with high coefficients[4].

Within the portfolio of N assets there are N variances and N(N-1) covariance. As the number of assets in the portfolio increases the number of covariance increases rapidly. This large number of covariance are very important towards determining the risk of the portfolio.

FIGURE D



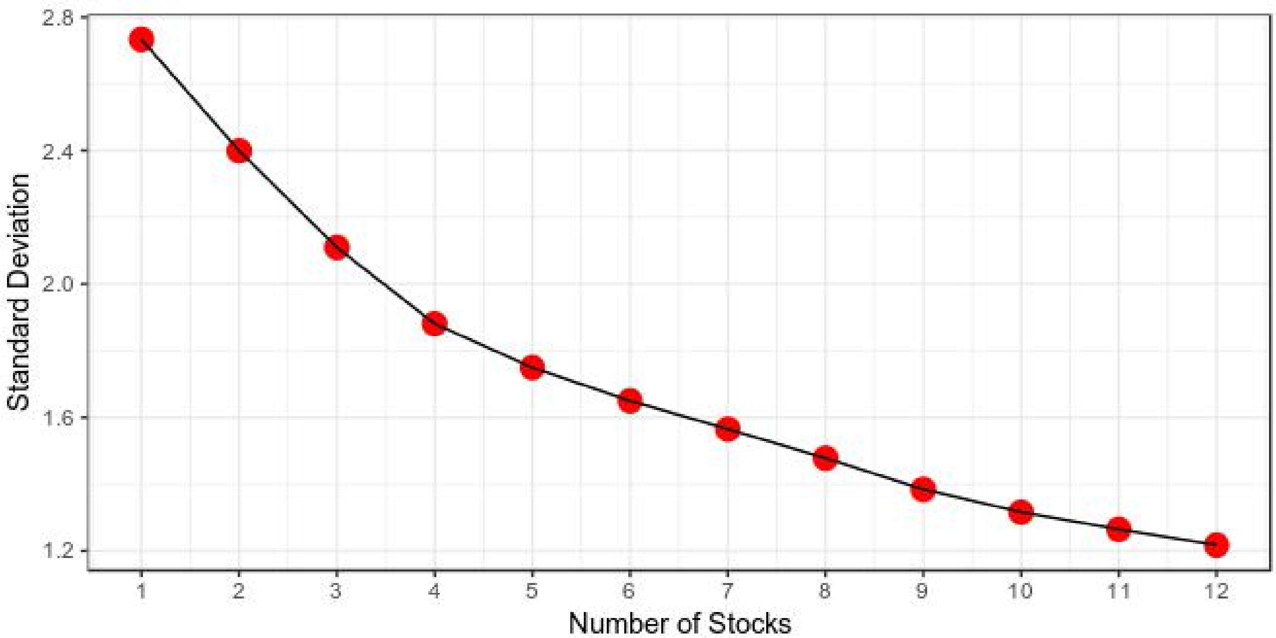
In the Figure D we can observe that at the start of the year 2008 the correlation among the assets were low as the crises took of , the correlation start to become stronger between the various pairs of asset. In the year between 2009 and 2013 we can see a high correlation among the assets which can be attributed to the increase in systematic risk due to panic in the US financial industry. After 2013 the correlation among the pairs of asset start to decline due to decrease in systematic risk.

During the time of financial stress the correlation among the pairs of the portfolio becomes very high due to intense volatility in the market.

Diversification Effect

The effect of diversification is of central importance to Modern Portfolio Theory. The process of diversification suggests the investor to invest in multiple assets across different asset classes and industries. As the number of securities in the portfolio increases the portfolio become less risk inherent.

FIGURE E

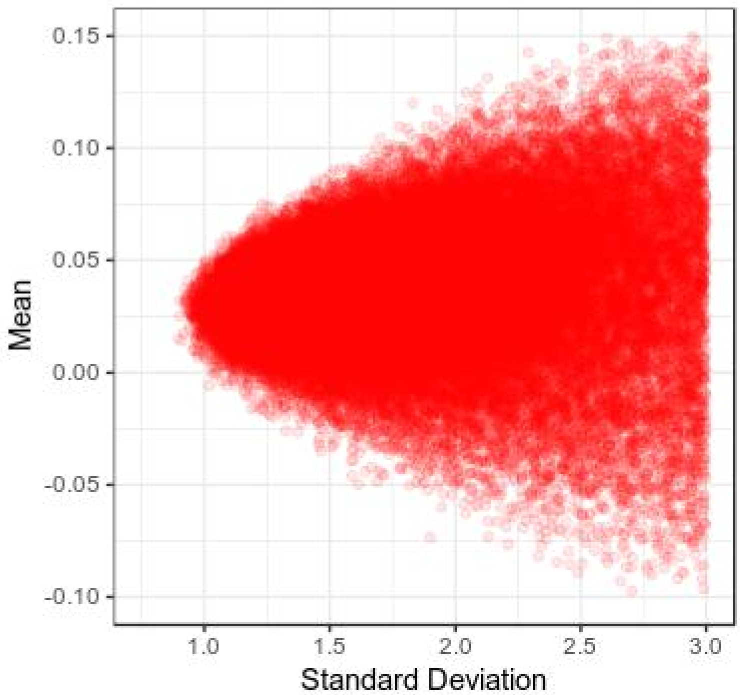


But the decreases is non linear in nature. As in the Figure E we can observe that as the number of assets increase (equal weighted portfolio) the standard deviation decreases at a diminishing rate, after adding a certain number of assets if we add more assets to the portfolio it does not have much impact in decreasing the portfolio risk because some risk will always remain in the form of systematic risk which cannot be reduced.

Markowitz concluded that out of infinite portfolio that can be constructed by assigning different weights to the portfolio (of the 12 stocks) there will be one efficient portfolio which gives an investor maximum return for a given amount of risk. The investor would always prefer the portfolio over the other inefficient portfolios because it maximizes his utility[5].

Using computer simulation we have generated more than 50,000 randomly weighted portfolios out of the 12 stocks and plotted their expected return and standard deviation.

FIGURE F



From the Figure F we can observe that these random portfolios constitutes a parabolic curve. At the edge of these curve lie the efficient portfolios which investor prefer over other portfolios based on their utility.

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