

# Syntax Analysis

## [Chapter 4 - Part 4]

### Lecture 10

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# LR( $k$ ) Parsing

- The most common type of **bottom-up** parser is based on **LR( $k$ ) parsing**, where  $k$  is the number of input symbols of lookahead (usually 1 or 0).
- LR(0) are grammar rules with a special **dot** added somewhere in the right-hand side.

# LR( $k$ ) Parsing

- LR parsers are table driven.
- **LR parser generator** is usually used to construct an LR parser, because it is too much work to build an LR parser by hand.

# LR( $k$ ) Parsers' Items

- LR parser maintains **states** to make shift-reduce decisions.
- Each **state** represents **a set of items**
- Each **item** represents a **production** of a grammar with a **dot** that indicates how much of a production we have seen at a given point in the parsing process.

# LR( $k$ ) Parsers' Items - Example

**Example:** The production  $A \rightarrow XYZ$  produces 4 items:

$$A \rightarrow .XYZ$$

$$A \rightarrow X.YZ$$

$$A \rightarrow XY.Z$$

$$A \rightarrow XYZ.$$

The item  $A \rightarrow X.YZ$  indicates that we have seen on the input a string derivable from  $X$  and we expect to see string(s) derivable from  $Y Z$

# Defining Transitions in DFA

- The **GOTO(I,X)** function is used to define the **transitions** in the deterministic finite automata (DFA) called (**LR(0) automation**) for a grammar, where **I** is a transition state and **X** is a grammar symbol.
- **GOTO(I, X)** specifies the **transition from the state I** under production **X**.

# GOTO (I, X) Function Definition

- GOTO (I, X) is defined as the closure of the set of all items generated from a production  $[A \rightarrow aX \cdot \beta]$  such that  $[A \rightarrow a \cdot X\beta]$  in I.

# GOTO Function - Example

Given the following grammar:

$$E \rightarrow E + T$$

$$E \rightarrow T$$

$$T \rightarrow T * F$$

$$T \rightarrow F$$

$$F \rightarrow ( E )$$

$$F \rightarrow \mathbf{id}$$

If **I** is the set of two items  $\{[E' \rightarrow E.], [E \rightarrow E. + T]\}$ ,  
then what are the items generated by **GOTO(I, +)** ?



# Solution

**Step 1:** Compute  $\text{GOTO}(I, +)$  by examining  $I$  for items with  $+$  immediately to the right of the dot.

So,

$E' \rightarrow E \cdot$  is not such an item, but  $E \rightarrow E \cdot + T$  is.

**Step 2:** Move the dot over the  $+$  to get  $E \rightarrow E + \cdot T$  and then took the closure of this singleton set.

$$E \rightarrow E + \cdot T$$

$$T \rightarrow \cdot T * F$$

$$T \rightarrow \cdot F$$

$$F \rightarrow \cdot (E)$$

$$F \rightarrow \cdot \text{id}$$

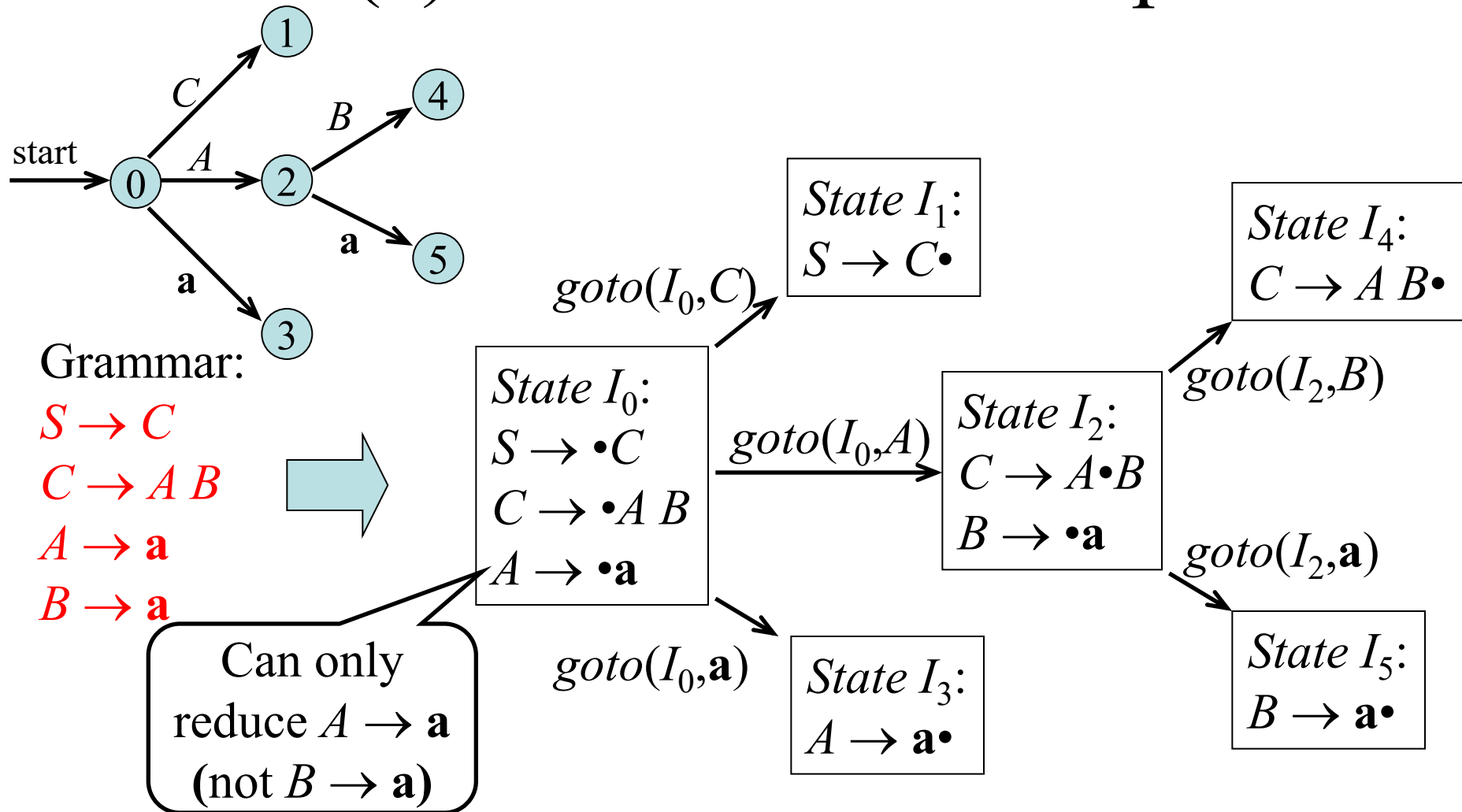
# LR( $\emptyset$ ) Sets Computation

Computation of the canonical collection of sets of LR( $\emptyset$ ) items

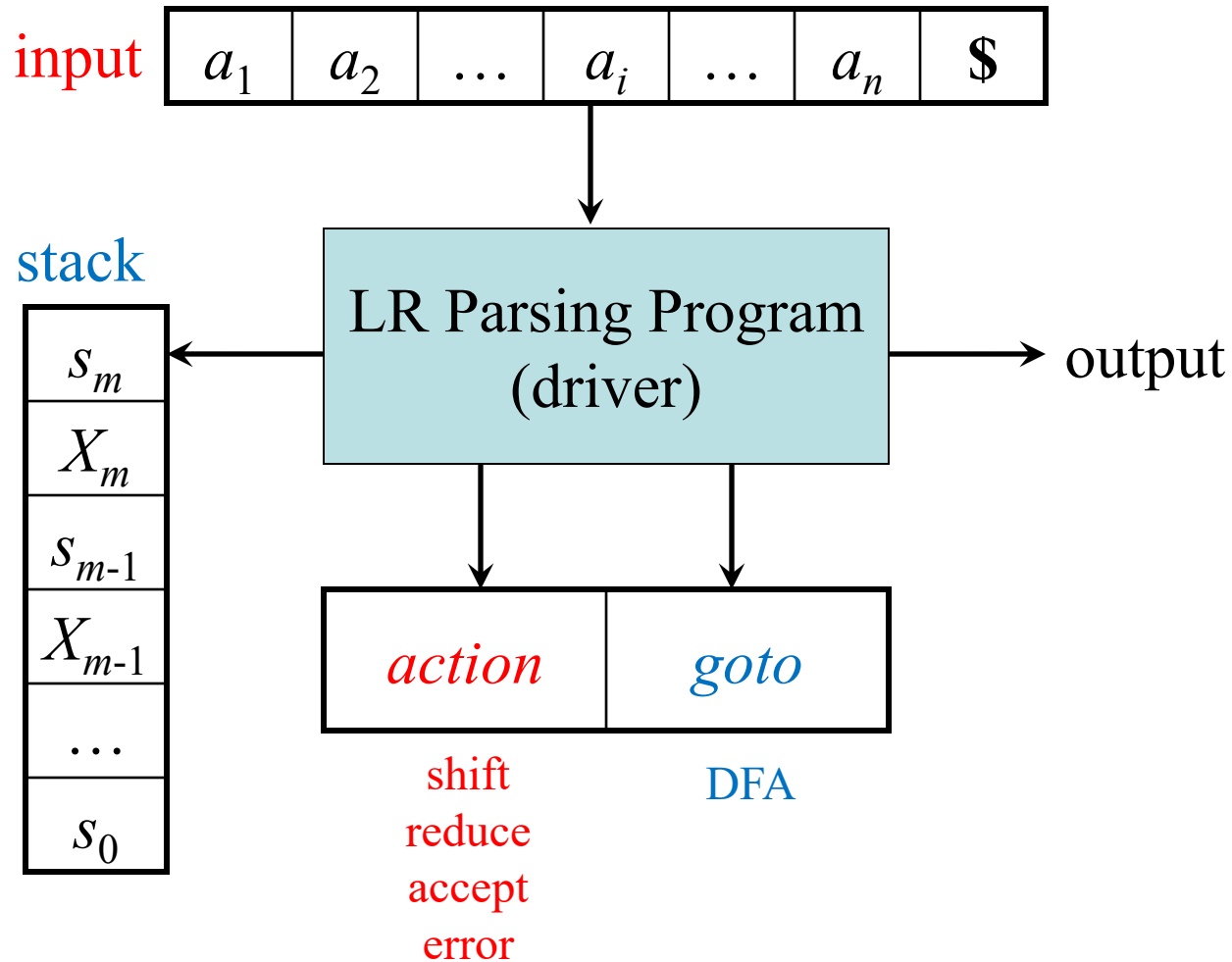
```
void items( $G'$ ) {  
     $C = \text{CLOSURE}(\{[S' \rightarrow \cdot S]\});$   
    repeat  
        for ( each set of items  $I$  in  $C$  )  
            for ( each grammar symbol  $X$  )  
                if (  $\text{GOTO}(I, X)$  is not empty and not in  $C$  )  
                    add  $\text{GOTO}(I, X)$  to  $C$ ;  
    until no new sets of items are added to  $C$  on a round;  
}
```

# Using DFA for Shift/Reduce

## LR( $k$ ) Decisions - Example



# LR Parser Implementation



All transitions to state  $j$  must be for the same grammar symbol  $X$ .  
 State  $j$  must be associated with grammar symbol  $X$ .

# LR Parsing Mechanism

- The parser reads the current **input** symbol  $a_i$  and the **state** on the top of the stack  $s_m$ , and then consulting the entry  $action[s_m, a_i]$
- The states of the DFA are used to determine if a handle is on top of the stack.

# LR parser Configuration

$$\underbrace{(s_0 X_1 s_1 X_2 s_2 \dots X_m s_m)}_{\text{stack}}, \underbrace{a_i a_{i+1} \dots a_n \$}_{\text{input}}$$

The *action* function inputs are a state  $s_m$  and a terminal  $a_i$ :

- If  $\text{action}[s_m, a_i] = \text{shift } s$ , then push  $a_i$ , push  $s$ , and advance input:  
 $(s_0 X_1 s_1 X_2 s_2 \dots X_m s_m a_i s, a_{i+1} \dots a_n \$)$
- If  $\text{action}[s_m, a_i] = \text{reduce } A \rightarrow \beta$  and  $\text{goto}[s_{m-r}, A] = s$  with  $r=|\beta|$  (length of beta) then pop  $2r$  symbols, push  $A$ , and push  $s$ :  
 $(s_0 X_1 s_1 X_2 s_2 \dots X_{m-r} s_{m-r} A s, a_i a_{i+1} \dots a_n \$)$
- If  $\text{action}[s_m, a_i] = \text{accept}$ , then stop (parsing is completed)
- If  $\text{action}[s_m, a_i] = \text{error}$ , then call error recovery routine

# DFA for LR Parsing - Example 1

Implement the  $goto(I, X)$  function to check if the input *aa* is accepted by the following grammar.

$$S \rightarrow C$$

$$C \rightarrow A B$$

$$A \rightarrow a$$

$$B \rightarrow a$$

State  $I_0$ :

$$S \rightarrow \bullet C$$

$$C \rightarrow \bullet A B$$

$$A \rightarrow \bullet a$$

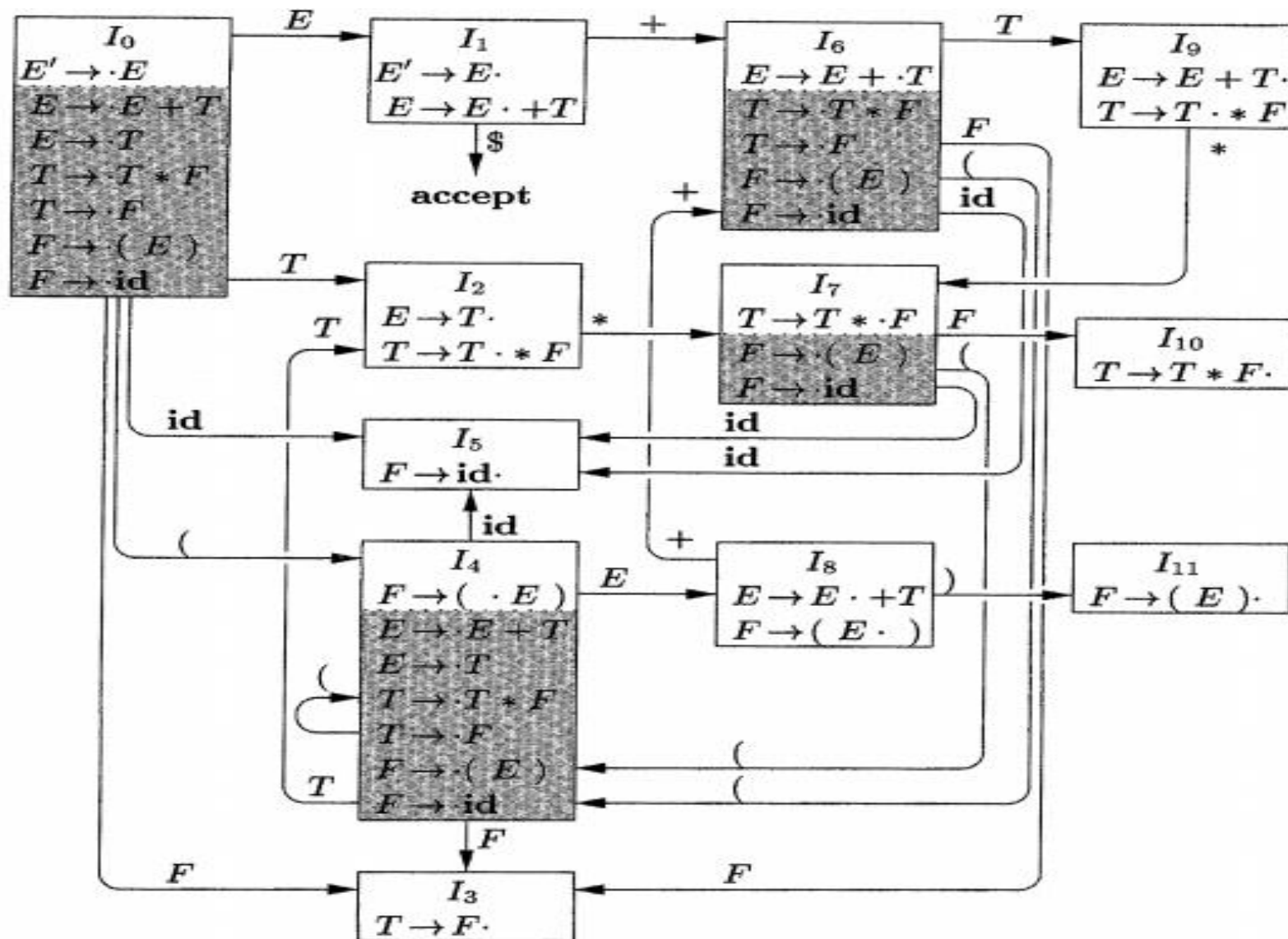
$goto(I_0, a)$

State  $I_3$ :

$$A \rightarrow a \bullet$$

Stack	Input	Action
\$ 0	<b>aa</b> \$	start in state 0
\$ 0	<b>aa</b> \$	shift (and goto state 3)
\$ 0 <u>a</u> 3	<b>a</b> \$	reduce $A \rightarrow a$ (goto 2)
\$ 0 A 2	<b>a</b> \$	shift (and goto state 5)
\$ 0 A 2 <u>a</u> 5	\$	reduce $B \rightarrow a$ (goto 4)
\$ 0 <u>A</u> 2 <u>B</u> 4	\$	reduce $C \rightarrow AB$ (goto 1)
\$ 0 <u>C</u> 1	\$	reduce $S \rightarrow C$
\$ 0 S 1	\$	accept

# LR Parsing – Example 2





# LR Parsing (Bottom-Up) - Example

Grammar:

1.  $E \rightarrow E + T$
2.  $E \rightarrow T$
3.  $T \rightarrow T * F$
4.  $T \rightarrow F$
5.  $F \rightarrow ( E )$
6.  $F \rightarrow \text{id}$

Stack	Input	Action
\$ 0	<b>id*id+id\$</b>	shift 6
\$ 0 <b>id</b> 6	<b>*id+id\$</b>	reduce 6 goto 3
\$ 0 <i>F</i> 3	<b>*id+id\$</b>	reduce 4 goto 2
\$ 0 <i>T</i> 2	<b>*id+id\$</b>	shift 7
\$ 0 <i>T</i> 2 * 7	<b>id+id\$</b>	shift 5
\$ 0 <i>T</i> 2 * 7 <b>id</b> 5	<b>+id\$</b>	reduce 6 goto 10
\$ 0 <i>T</i> 2 * 7 <i>F</i> 10	<b>+id\$</b>	reduce 3 goto 2
\$ 0 <i>T</i> 2	<b>+id\$</b>	reduce 2 goto 1
\$ 0 <i>E</i> 1	<b>+id\$</b>	shift 6
\$ 0 <i>E</i> 1 + 6	<b>id\$</b>	shift 5
\$ 0 <i>E</i> 1 + 6 <b>id</b> 5	<b>\$</b>	reduce 6 goto 3
\$ 0 <i>E</i> 1 + 6 <i>F</i> 3	<b>\$</b>	reduce 4 goto 9
\$ 0 <i>E</i> 1 + 6 <i>T</i> 9	<b>\$</b>	reduce 1 goto 1
\$ 0 <i>E</i> 1	<b>\$</b>	accept

# LR Parse Table - Example

Grammar:

1.  $E \rightarrow E + T$

2.  $E \rightarrow T$

3.  $T \rightarrow T * F$

4.  $T \rightarrow F$

5.  $F \rightarrow ( E )$

6.  $F \rightarrow \text{id}$

	state	action						goto		
		id	+	*	(	)	\$	E	T	F
	0	s6			s5			1	2	3
	1		s6				acc			
	2		r2	s7		r2	r2			
	3		r4	r4		r4	r4			
	4	s5			s4			8	2	3
	5		r6	r6		r6	r6			
	6	s5			s4				9	3
Shift & goto 5	7	s5			s4					10
	8		s6			s11				
	9		r1	s7		r1	r1			
Reduce & goto 1	10		r3	r3		r3	r3			
	11		r5	r5		r5	r5			