3. Using the master methods, solve the following recurrences:

a. $T(n) = 2T(n/4) + 1$

a) $T(n) = 2T(n/4) + 1$

$T(n) = aT(n/b) + f(n)$

$a=2, b=4, k=0$

$b^k = 4^0 = 1$

$a > b^k, \; T(n) = \Theta(n^{\log_b a})$

$= \Theta(n^{\log_4 2})$

$= \Theta(n^{\frac{1}{2}})$

$= \Theta(\sqrt{n})$ #

Case 1: If $\log_b a > k$ then, $T(n) = \Theta(n^{\log_b a})$

b. $T(n) = 2T(n/4) + \sqrt{n}$

b) $T(n) = 2T(n/4) + \sqrt{n}$

$T(n) = aT(n/b) + f(n)$

$a=2,\ b=4,\ k=\frac{1}{2},\ p=0$

$a=2 \quad b^k = 4^{\frac{1}{2}}$

$= 2$

$\therefore a = b^k$

since $p > -1$, then

$T(n) = \theta(n^{\log_a b} \log^{p+1} n)$

$= \theta(n^{\log_4 2} \log n)$

$= \theta(\sqrt{n} \log n)$ *

Case 2: If $\log_b a = k$ then

a) If $p > -1$, then, $T(n) = \Theta(n^k \log^{p+1} n)$

c. $T(n) = 2T(n/4) + n$

$c.\ T(n) = 2T(n/4) + n$

$T(n) = aT(n/b) + f(n)$, $f(n) = \theta(n^1 \log^0 n)$

$a = 2$, $b = 4$, $k = 1$, $p = 0$

$\log_b a = \log_4 2$

$= \frac{1}{2} < 1$

$\log_b a < k$, $p \geq 0$, $T(n) = \theta(n^1 \log^0 n)$

$= \theta(n)$

$$T(n) = aT\left({}^{n}/_{b}\right) + \Theta\left(n^k \log^p n\right)$$

f(n)

Where, $a \geq 1, b > 1, k \geq 0, p =$ real number

Case 3: If $\log_b a < k$ then

a) If $p \geq 0$, then, $T(n) = \Theta\left(n^k \log^p n\right)$

b) If $p < 0$, then, $T(n) = O\left(n^k\right)$

d. $T(n) = 2T(n/4) + n^2$

$d.\ T(n) = 2T(n/4) + n^2$

$T(n) = aT(n/b) + f(n)$, $f(n) = \theta(n^2 \log^0 n)$

$a = 2$, $b = 4$, $k = 2$, $p = 0$

$\log_b a = \log_4 2$

$= \frac{1}{2} < 2$

$\log_b a < k$, $p \geq 0$, $f(n) = \theta(n^2 \log^0 n)$

$= \theta(n^2)$

$$T(n) = aT\left({}^{n}/_{b}\right) + \Theta\left(n^k \log^p n\right)$$

f(n)

Where, $a \geq 1, b > 1, k \geq 0, p =$ real number

Case 3: If $\log_b a < k$ then

a) If $p \geq 0$, then, $T(n) = \Theta\left(n^k \log^p n\right)$

b) If $p < 0$, then, $T(n) = O\left(n^k\right)$

e. $T(n) = 3T(n/2) + n^2$

$$T(n) = 3T\left(\frac{n}{2}\right) + n^2$$

$a = 3$ $\quad b = 2$ $\quad f(n) = n^2$ $\quad k = 2$ $\quad p = 0$

$\log_b a = \log_2 3$

≈ 1.585

$1.585 < 2$

Since $\log_b a < k$, $p \geqslant 0$

$\therefore T(n) = \Theta(n^2)$

Case 3: If $\log_b a < k$ then

a) If $p \geq 0$, then, $T(n) = \Theta(n^k \log^p n)$

b) If $p < 0$, then, $T(n) = O(n^k)$

f. $T(n) = 4T(n/2) + n^2$

$$T(n) = 4T\left(\frac{n}{2}\right) + n^2$$

$a = 4$ $\quad b = 2$ $\quad f(n) = n^2$ $\quad k = 2$ $\quad p = 0$

$\log_b a = \log_2 4$

$= 2$

Since $\log_b a = k$, $p > -1$

$\therefore T(n) = \Theta(n^2 \log n)$

Case 2: If $\log_b a = k$ then

a) If $p > -1$, then, $T(n) = \Theta(n^k \log^{p+1} n)$

b) If $p = -1$, then, $T(n) = \Theta(n^k \log \log n)$

c) If $p < -1$, then, $T(n) = \Theta(n^k)$

g. $T(n) = T(n/2) + 2^n$

g. $a = 1$

$b = 2$

$f(n) = 2^n$

$n^{\log_b a} = n^{\log_2 1} = n^0 = 1$

$f(n) = 2^n >> n^0$

$f(n)$'s growth faster than $n^{\log_b a}$

$\Rightarrow T(n) = \theta(2^n)$

h. T (n) = 16T (n/4)+ n

h. $a = 16$

$b = 4$

$f(n) = n \quad (n^1 \Rightarrow k = 1)$

$n^{\log_b a} = n^{\log_4 16} = 2$

since $f(n) < \log_b a$

$\therefore$ case 1

$T(n) = \theta(n^2)$

i. T (n) = 2T (n/2)+ n log n

i. $a=2$

$b=2$

$f(n)=n\log n \quad (k=1, p=1)$

$\log_2^2 = 1$

since $\log_b^a = k > -1$

$\therefore$ case 2

$T(n)=\theta(n\log^2 n)$