Q1 : Given

$T(n) = n^2 + T(n-1)$, where $n > 0$

$T(n) = 1$, where $n = 0$

$T(n) = n^2 + T(n-1)$ —— ①

* Subs $n = n-1$ into ①

$T(n-1) = (n-1)^2 + T[(n-1)-1]$

$= (n-1)^2 + T(n-2)$ —— ②

Now, Subs ② into ①

$T(n) = n^2 + (n-1)^2 + T(n-2)$ — ③

* Subs $n = n-2$ into ①

$T(n-2) = (n-2)^2 + T[(n-2)-1]$

$= (n-2)^2 + T(n-3)$ —— ④

Now, Subs ④ into ③

$T(n) = n^2 + (n-1)^2 + (n-2)^2 + T(n-3)$

base case $T(0)$

$\therefore T(n) = n^2 + (n-1)^2 + (n-2)^2 + \ldots (n-k)^2 + T(n-(k+1))$ — ⑤

since $T(0) = T(n-(k+1))$, compare to find value of k

$n - k + 1 = 0$

$k = n-1$ #

Subs $k = n-1$ into e.q ⑤

$\therefore T(n) = n^2 + (n-1)^2 + (n-2)^2 + \ldots + (n-n-1)^2 + T(0)$

$= n^2 + (n-1)^2 + (n-2)^2 + \ldots + (-1)^2 + 1$

$= n^2 + (n-1)^2 + (n-2)^2 + \ldots 1^2 + 1$ #

Refer to Arithmetic-like series

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} = \frac{2n^{3} + 3n^2 + n}{6}$$

highest order term

$\therefore T(n) = O(n^3)$ #