

Tuto 1

[T-2005-1-Tutorial 1.pdf](#)

1.

```
input sequence of numbers array n
input value v

for every number in array n
    if value at index i equals v
        return index i
    else
        increment i
return null
```

2.

$\theta(n^3)$

3.

a. represents upper bound for a function to within a constant factor

b. when $n \Rightarrow 4$, $A = O(B)$.

the algorithm B is smaller in size when input is small but is larger when input is big

c. Algorithm A. Algo B is better for smaller dataset but Algo A is less complex with more dataset

d. it does not effect my preferences. As $A = O(B)$

e.

1. Input size. for small input sizes the runtime may not align with asymptotic analysis because analysis is made for large dataset.

2. Input sequence.

4.

a.

$$f(n) = 3n + 2$$

$$g(n) = n$$

$$f(n) \leq cg(n)$$

$$3n + 2 \leq c \cdot n$$

$$3 + 2/n \leq c$$

assume $n \rightarrow \text{infinity}$, $2/n \rightarrow 0$. So $3 \leq c$

thus $f(n) = O(g(n))$

$$g(n) \leq cf(n)$$

$$n \leq c \cdot 3n + 2$$

$$1 \leq c(3 + 2/n)$$

$$1/(3 + 2/n) \leq c$$

assume $n \rightarrow \text{infinity}$, $2/n \rightarrow 0$. So $1/3 \leq c$

$g(n) = O(f(n))$

so both cases are valid

b.

$$f(n) = (n^2 - n)/2$$

$$g(n) = 6n$$

$$f(n) \leq cg(n)$$

$$(n^2 - n)/2 \leq c \cdot 6n$$

$$(n^2 - n) \leq c \cdot 12n$$

$$n - 1 \leq c \cdot 12$$

assume $n \rightarrow \text{infinity}$, $n - 1 \rightarrow \text{infinity}$. So inequality is false

$f(n) \neq O(g(n))$

thus $g(n) = O(f(n))$

c.

$$f(n) = n + 2\sqrt{n}$$

$$g(n) = n^2$$

$$f(n) \leq cg(n)$$

$$n + 2\sqrt{n} \leq c \cdot n^2$$

$$1 + 2/n^{1/2} \leq c \cdot n$$

assume $n \rightarrow \text{infinity}$, $2/n^{1/2} \rightarrow 0$.

$$1 \leq c \cdot n$$

thus $f(n) = O(g(n))$

d.

$$f(n) = n^2 + 3n + 4$$

$$g(n) = n^3$$

$$f(n) \leq c g(n)$$

$$n^2 + 3n + 4 \leq c \cdot n^3$$

$$1 + 3/n + 4/n^2 \leq c \cdot n$$

assume $n \rightarrow \text{infinity}$, $3/n + 4/n^2 \rightarrow 0$.

$$1 \leq c \cdot n$$

thus $f(n) = O(g(n))$

5.

a. $O(n)$

b. $O(n^2)$

c. $O(n^{1/2})$

d.

outermost loop (i)

starting loop: $n/2$

end loop: n

increment: 1

total run:

end loop - starting loop

$$= n - n/2$$

$$= n/2$$

$$\Rightarrow \text{total run: } n/2$$

outermost loop (i) complexity: $O(n)$

middle loop (j)

```
starting loop:  $n/2$   
end loop:  $n$   
increment: 1  
total run:  
end loop - starting loop  
=  $n - n/2$   
=  $n/2$   
=> total run:  $n/2$   
  
middle loop (j) complexity:  $O(n)$ 
```

```
innermost loop (k)  
  
starting loop: 1  
end loop:  $n$   
increment:  $k * 2 = 2^m$   
total run:  
 $2^m \geq n$   
 $m \geq \log_2 n$   
=> total run:  $\log_2 n$   
  
innermost loop (k) complexity:  $O(\log_2 n)$ 
```

total time complexity:
= $O(n) O(n) O(\log_2 n)$
= $n^2 \log_2 n$
=> $O(n^2 \log_2 n)$