Assignment No. 5 **Numerical Computing**

Start Date: December 11, 2020

End Date: December 18, 2020

Solve the following problems through Picard Method of successive approximation by writinng code in Mathematica. Solve the problems upto 10th order.

(v) $\dfrac{dy}{dx} = e^x + y^2$, $y(0) = 0$

$$\left[Ans.\ y_1 = e^x - 1,\ y_2 = \frac{e^{2x}}{2} - e^x + x + \frac{1}{2},\ y_3 = \frac{e^{4x}}{16} - \frac{e^{3x}}{3} + \frac{xe^{2x}}{2} + \frac{1}{2}e^{2x} - 2xe^x + 2e^x + \frac{x^3}{3} + \frac{x^2}{2} + \frac{x}{4} - \frac{107}{48} \right]$$

(vi) Obtain $y(0.1)$, given $y' = \dfrac{y-x}{y+x}$ and $y(0) = 1$.

[$Ans.\ y(0.1) = 1.0906$]

(vii) Given $y' = \dfrac{x^2}{1+y^2}$ and $y(0) = 0$. Find $y(0.25)$, $y(0.5)$.

[$Ans.\ y(0.25) = 0.005,\ y(0.5) = 0.042$]

(viii) Solve $y' = x - y^2$, given $y(0) = 1$.

$$\left[Ans.\ y = 1 - x + \frac{5}{2}x^2 - 2x^3 + x^4 - \frac{x^5}{4} \right]$$

(ix) Solve $y' = x^2 + y^2$, given $y(0) = 0$.

$$\left[Ans.\ y = \frac{x^3}{5} + \frac{x^7}{63} + \frac{2x^{11}}{2079} + \ldots \right]$$

(x) Solve $y' = 2x - y$, with $y(1) = 3$. Find also $y(1.1)$.

$$\left[Ans.\ y = \frac{73}{12} - \frac{35}{6}x + \frac{7}{2}x^2 - \frac{5}{6}x^3 + \frac{x}{12},\ y(1.1) = 2.914508 \right]$$