Heaps

CS223: Data Structures

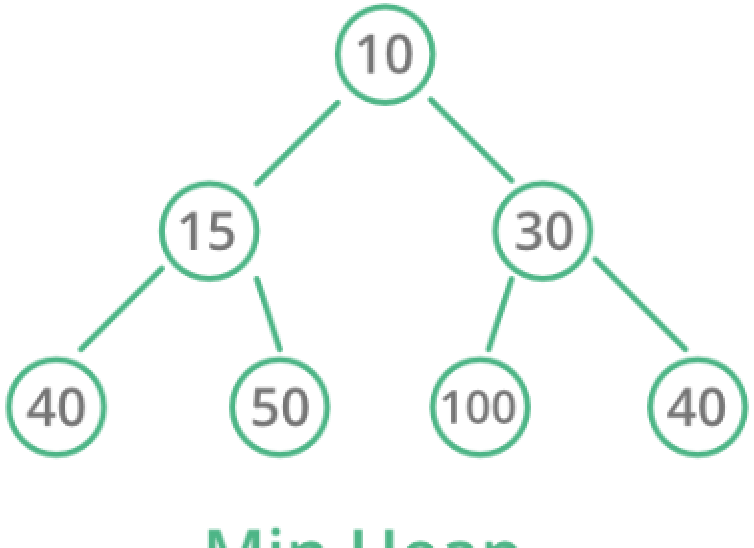
Binary Heaps

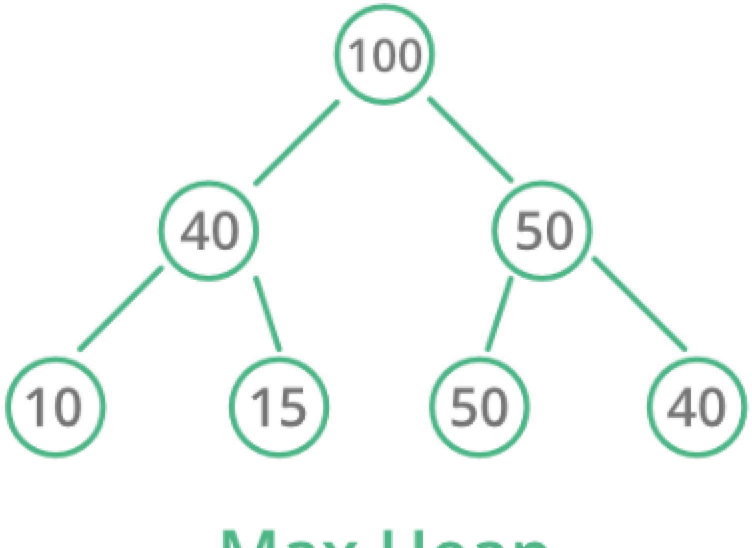
A binary heap is a special tree-based data structure in which the tree is a complete binary tree.

- Complete binary tree – binary tree that is completely filled, with the possible exception of the bottom level, which is filled left to right

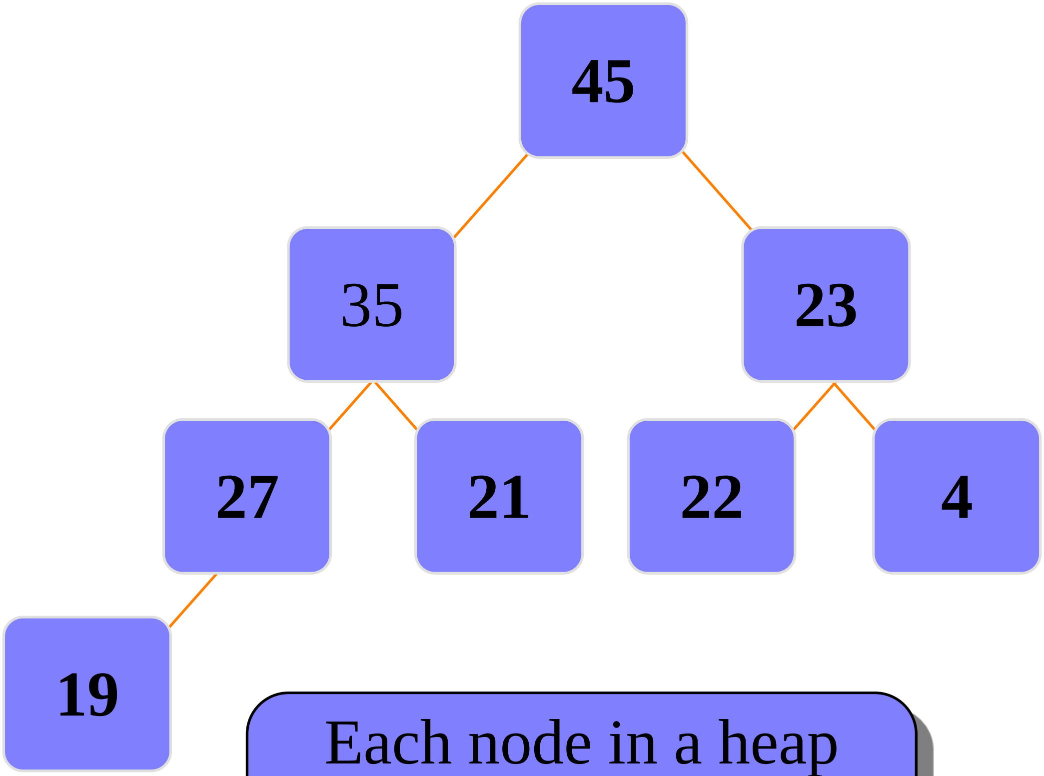
Generally, Heaps can be of two types:

- **Max-Heap**: In a Max-Heap the key present at the root node must be greatest among the keys present at all of it's children. The same property must be recursively true for all sub-trees in that Binary Tree.
- **Min-Heap**: In a Min-Heap the key present at the root node must be minimum among the keys present at all of it's children. The same property must be recursively true for all sub-trees in that Binary Tree.



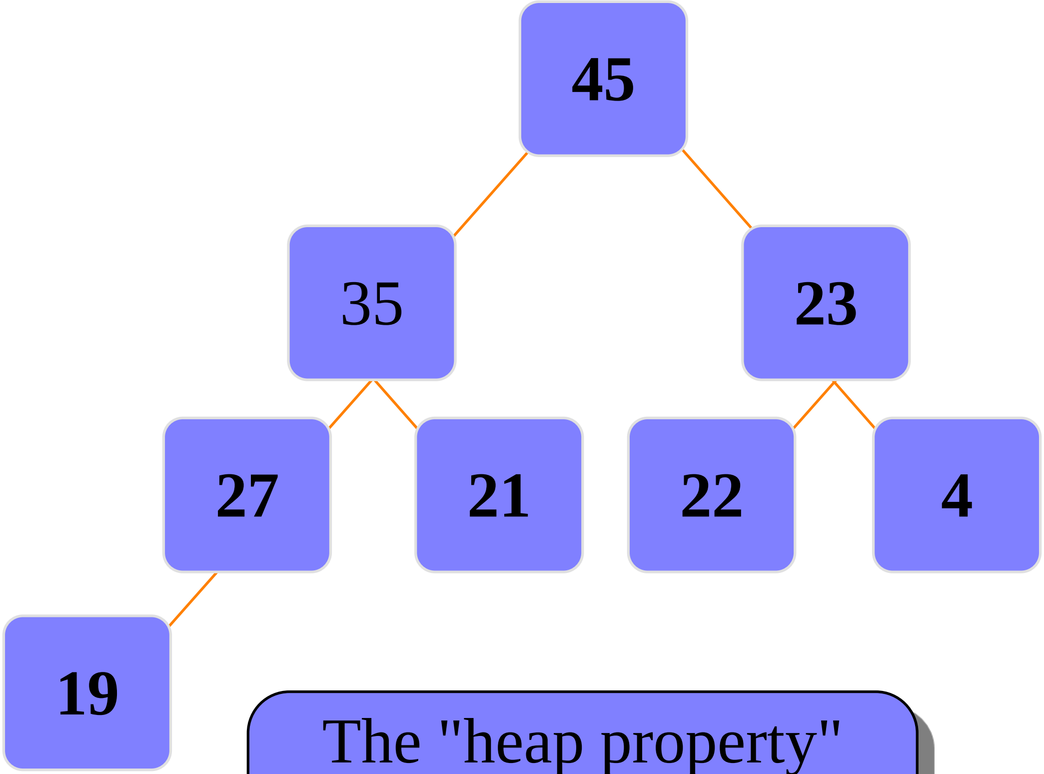


Heaps



Each node in a heap contains a key that can be compared to other nodes' keys.

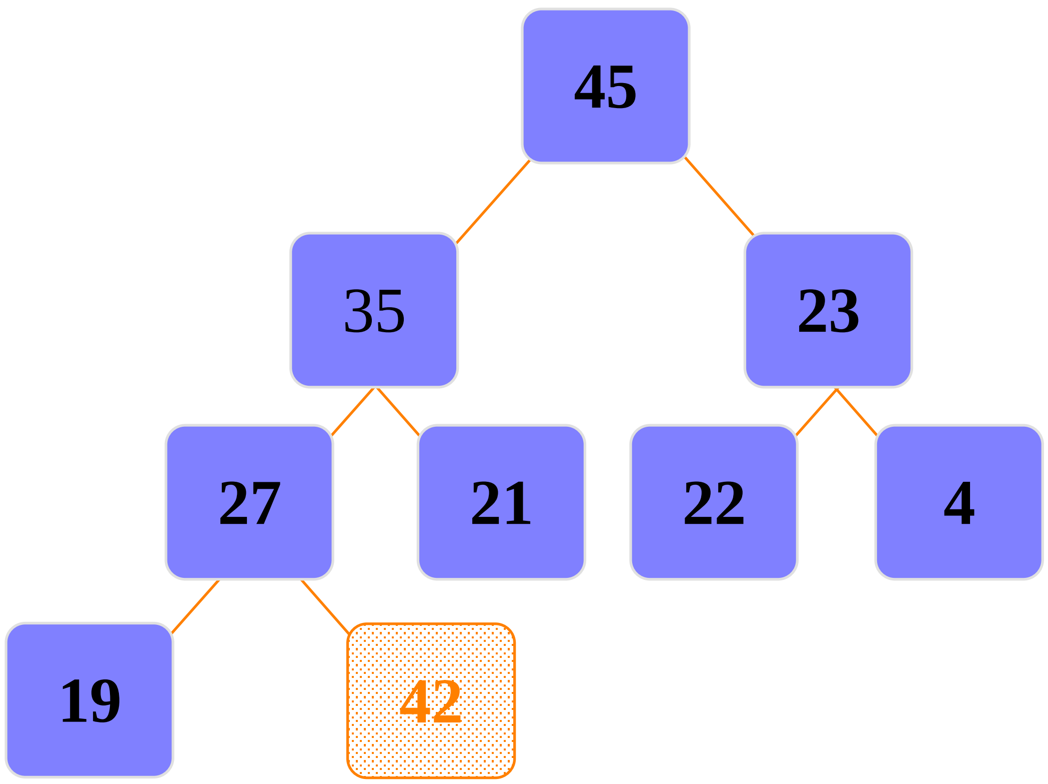
Heaps



The "heap property" requires that each node's key is >= the keys of its children

Adding a Node to a Heap

- Put the new node in the next available spot.
- Push the new node upward, swapping with its parent until the new node reaches an acceptable location.



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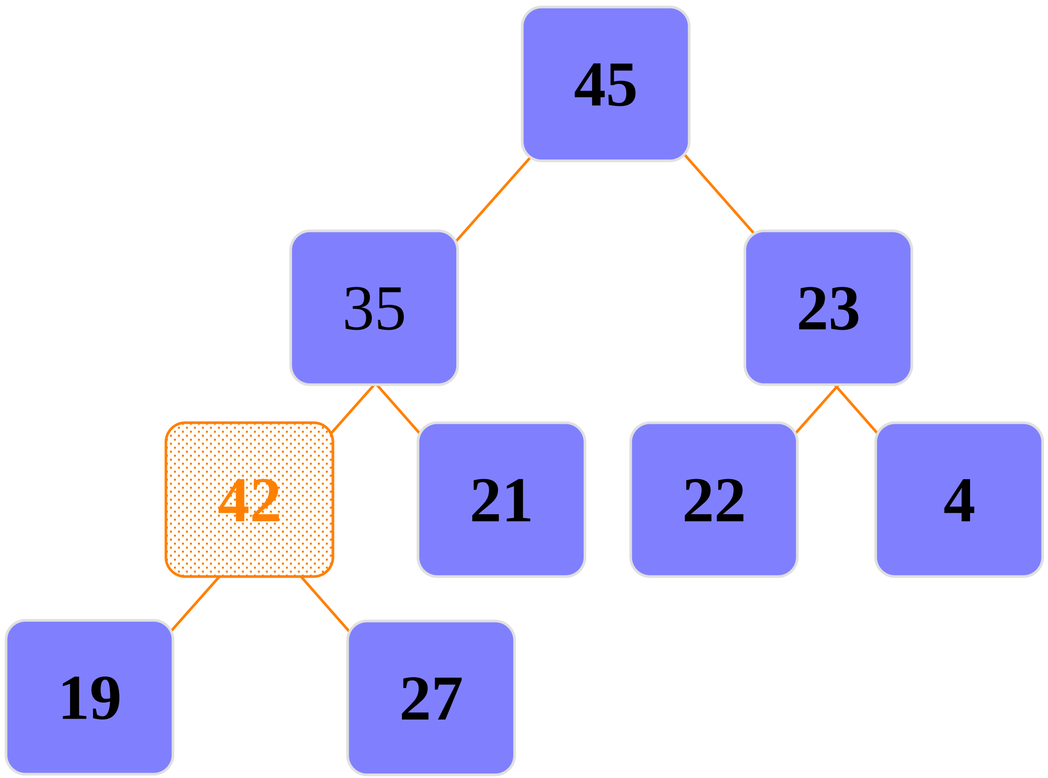

Adding a Node to a Heap

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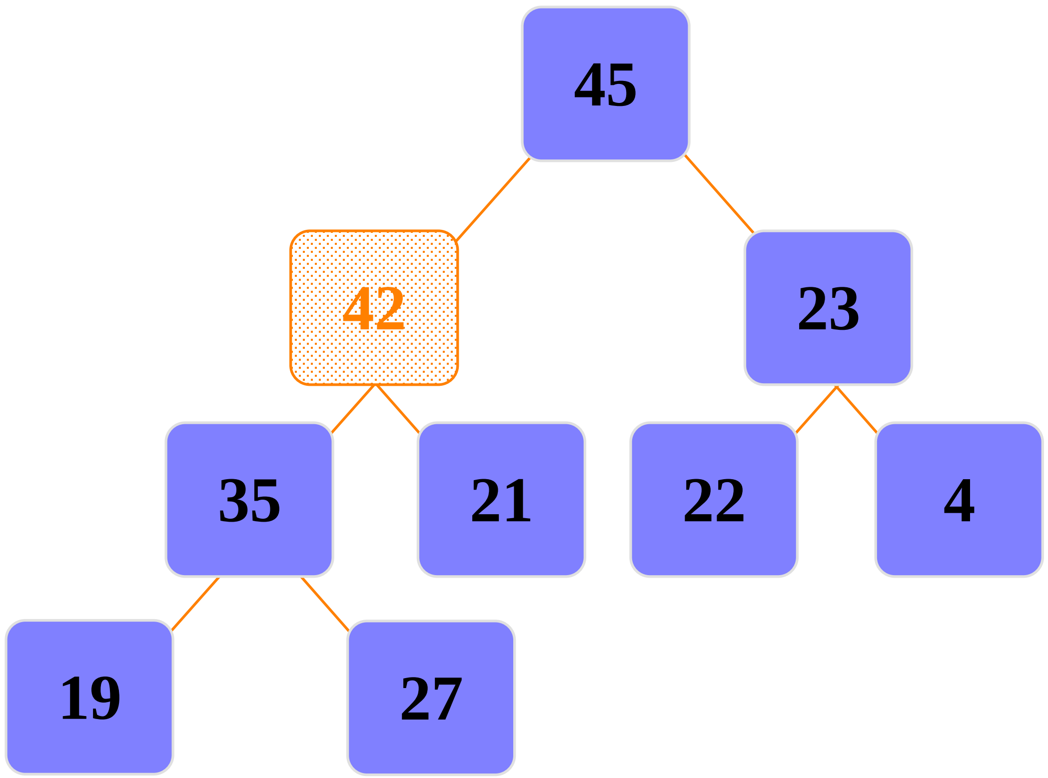
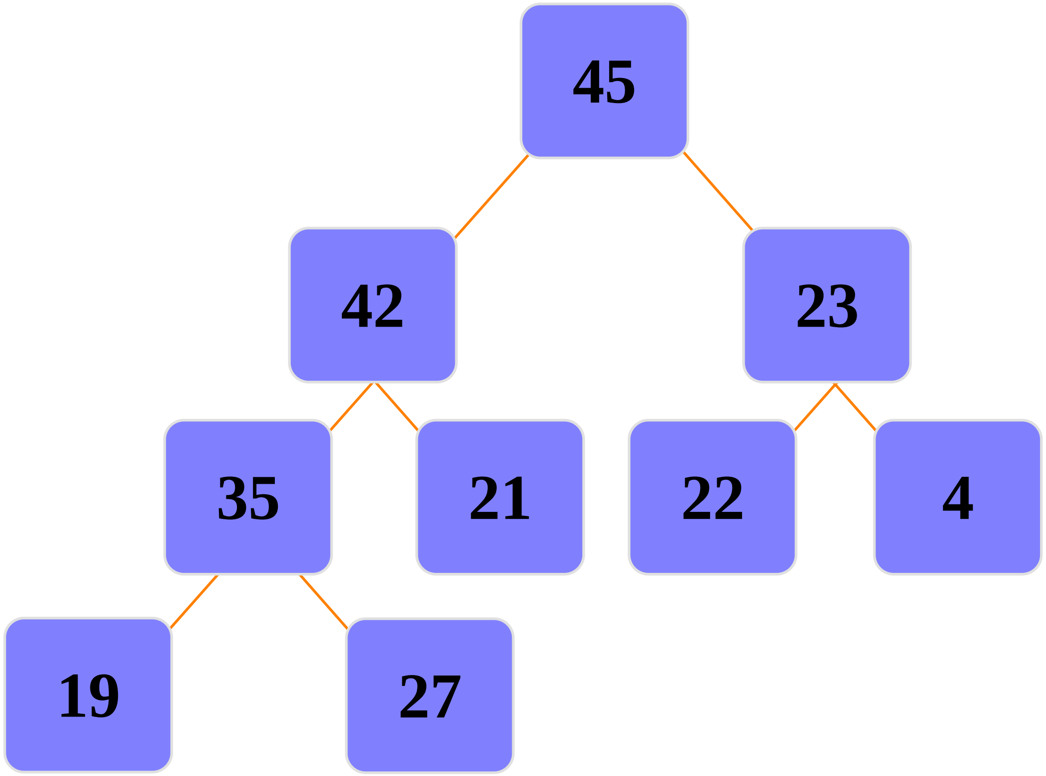


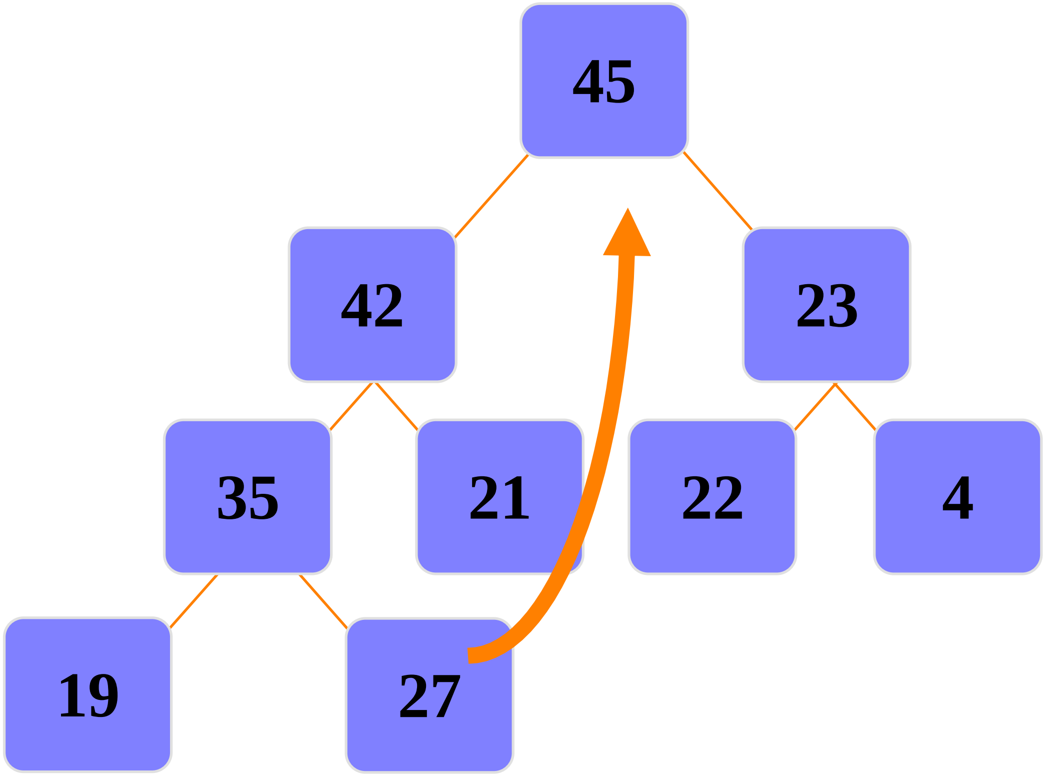
Adding a Node to a Heap

- The parent has a key that is >= new node, or
- The node reaches the root.
- The process of pushing the new node upward is called **reheapification upward**.



Removing the Top of a Heap

- ❑extractMin() or extractMax()
- ❑Move the last node onto the root.



Removing the Top of a Heap

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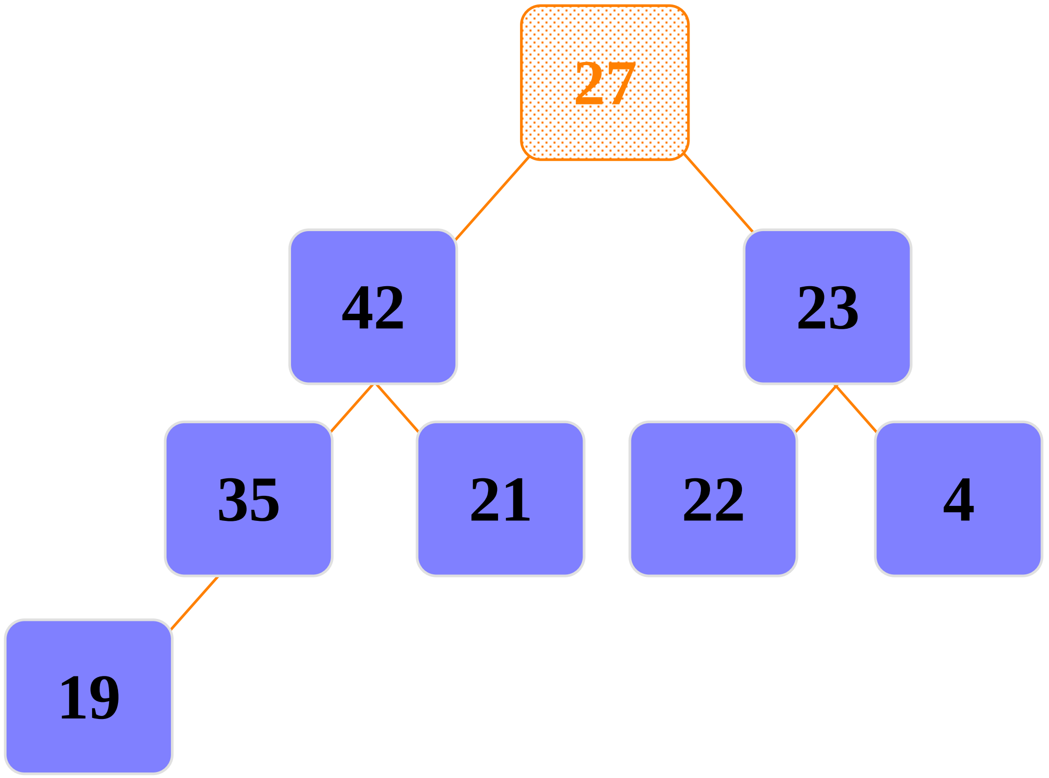
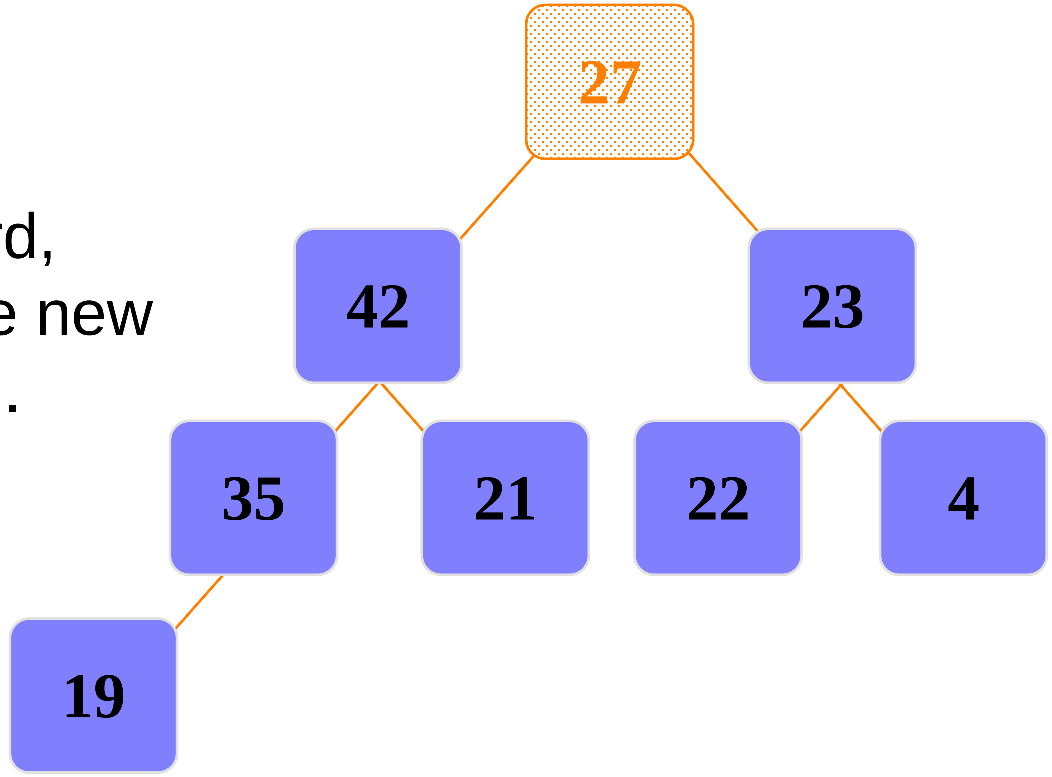


Removing the Top of a Heap

- ❑ Move the last node onto the root.
- ❑ Push the out-of-place node downward, swapping with its larger child until the new node reaches an acceptable location.



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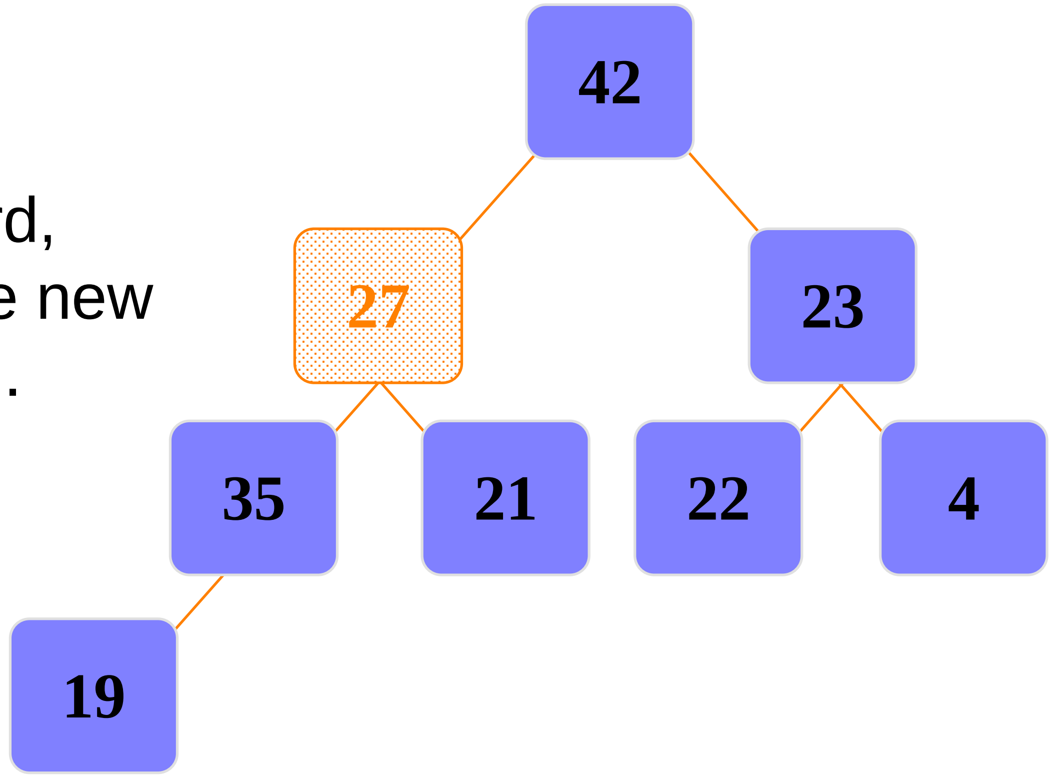

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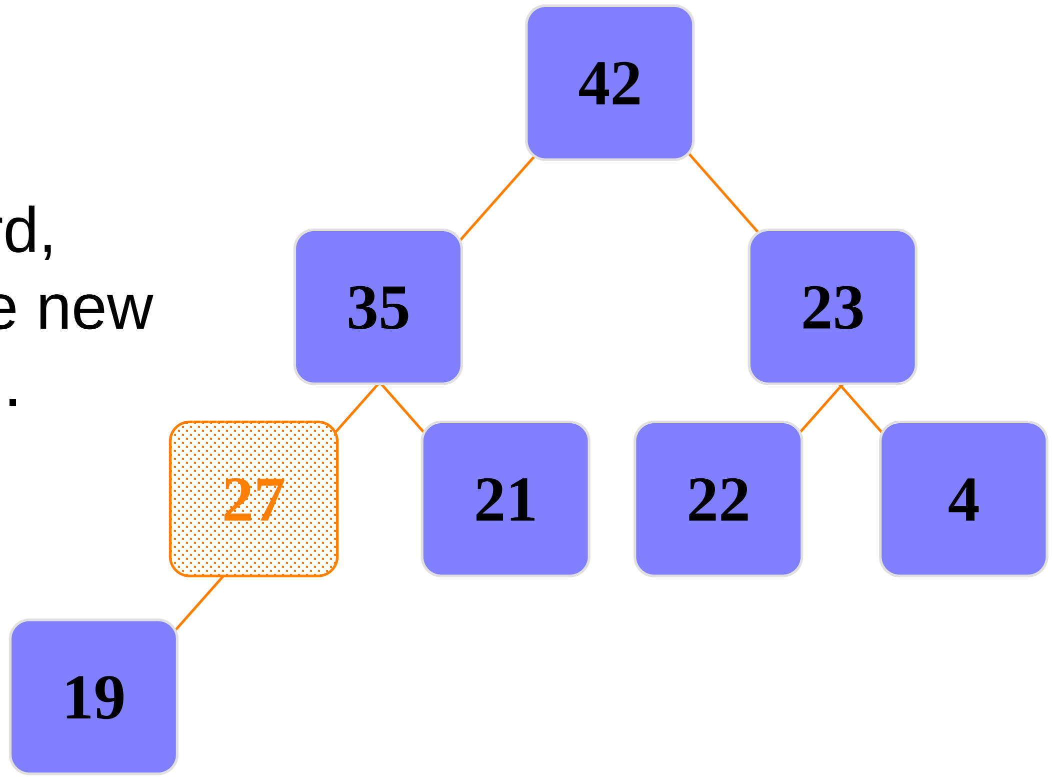
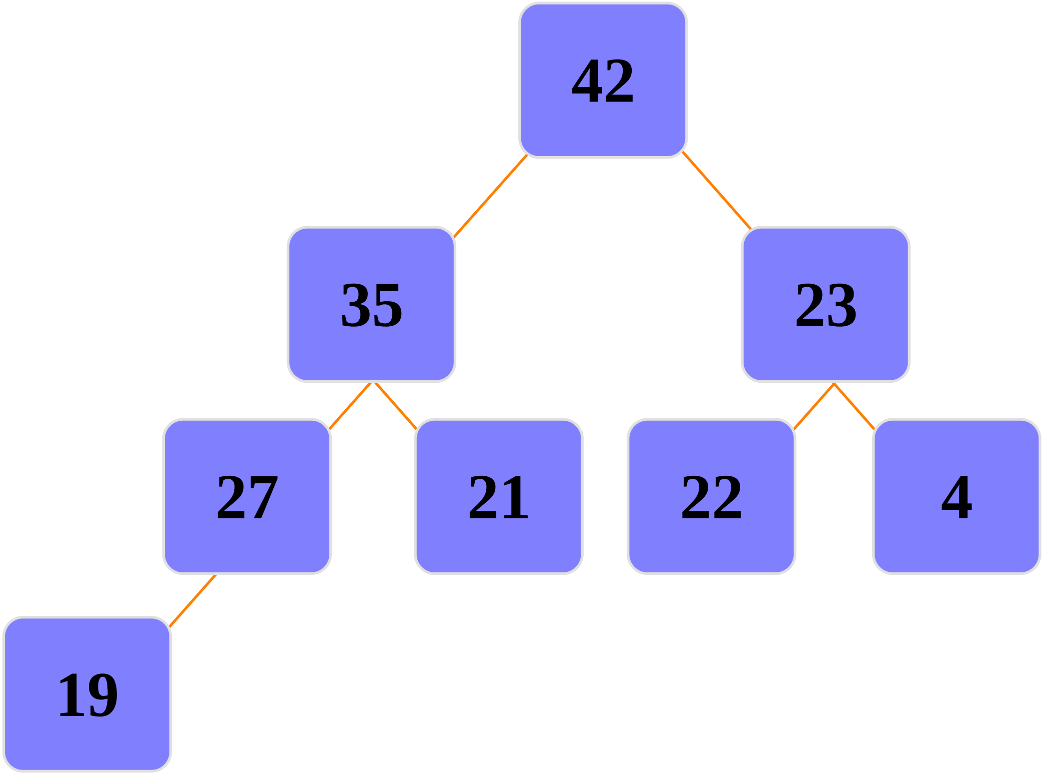


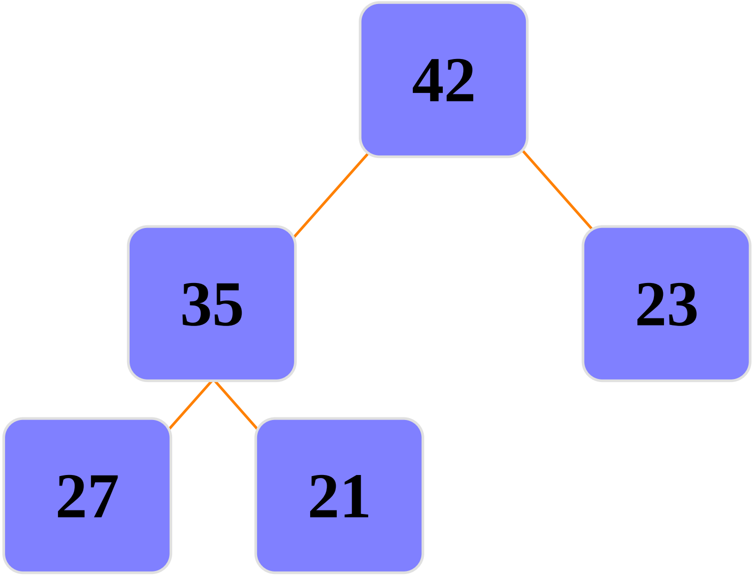
Removing the Top of a Heap

- The children all have keys <= the out-of-place node, or
- The node reaches the leaf.
- The process of pushing the new node downward is called **reheapification downward**.

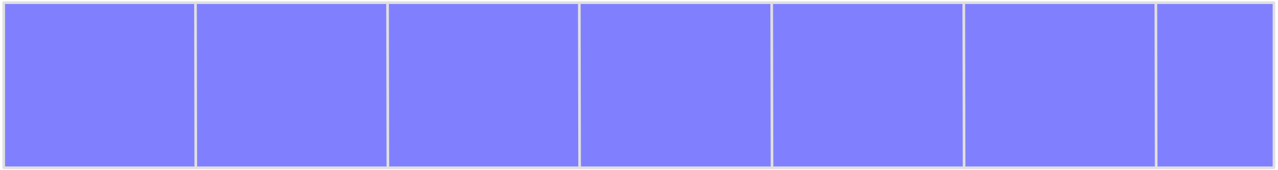


Implementing a Heap

- We will store the data from the nodes in a partially-filled array.

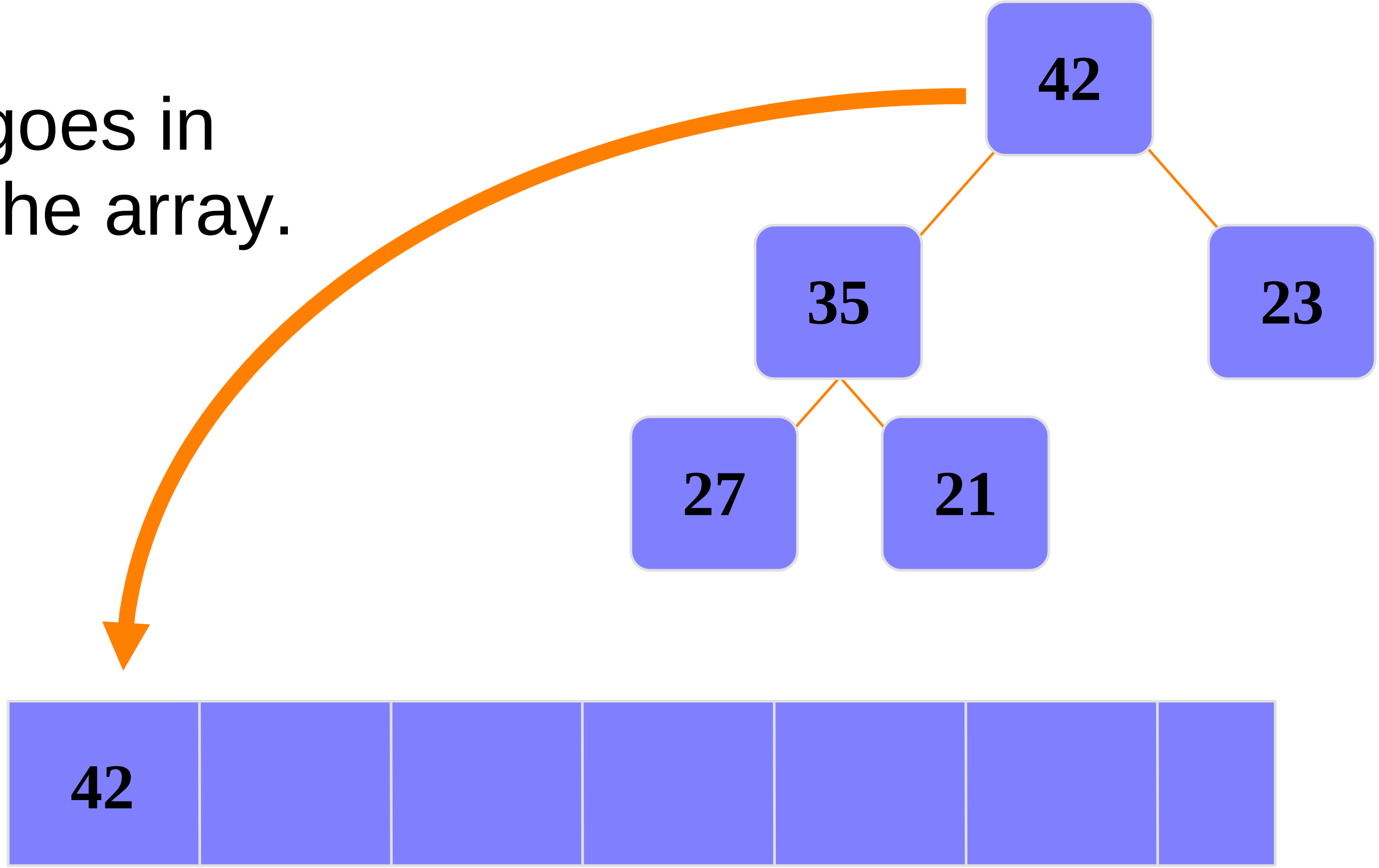


An array of data



Implementing a Heap

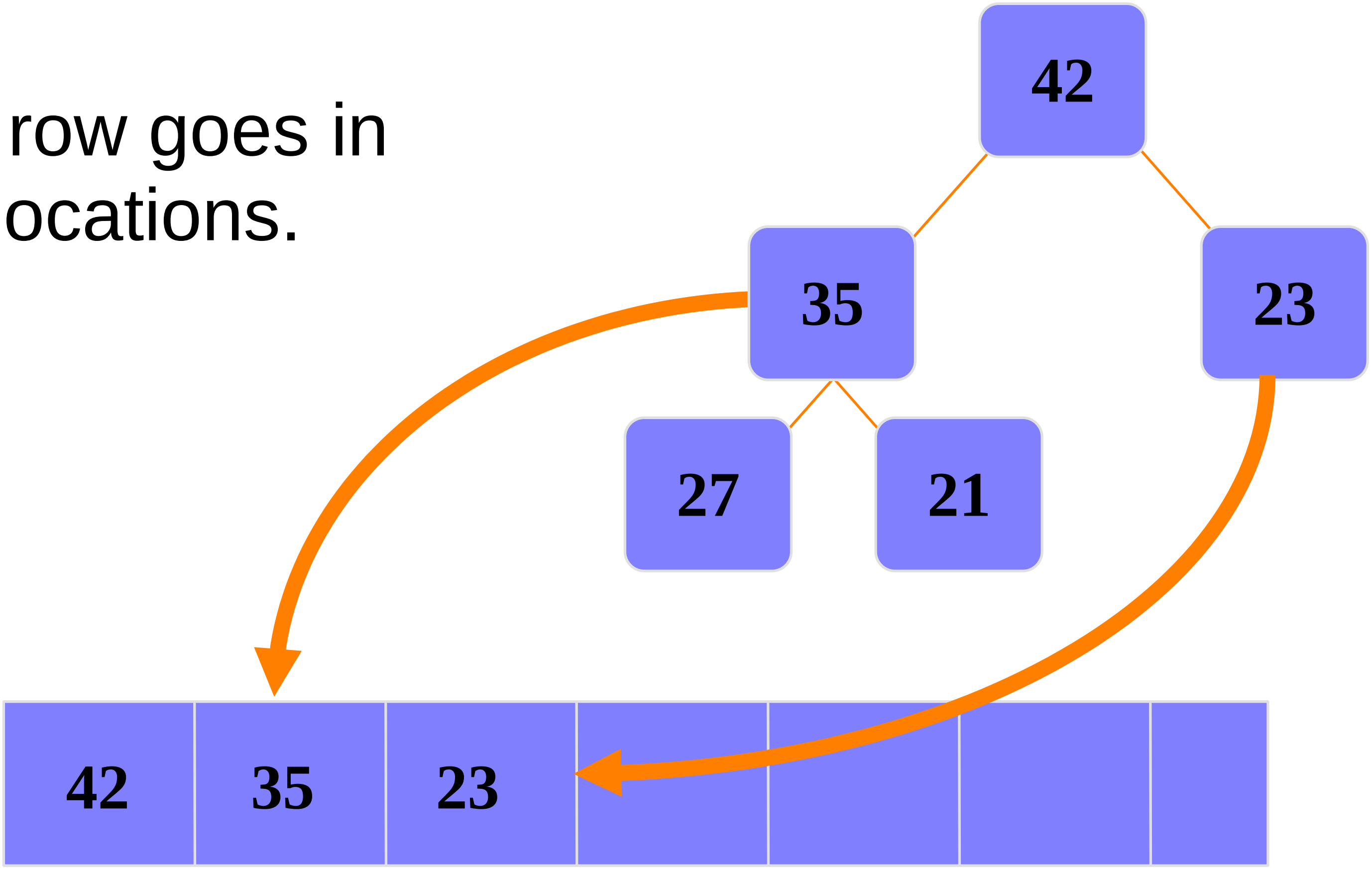
Data from the root goes in the first location of the array.



An array of data

Implementing a Heap

Data from the next row goes in the next two array locations.



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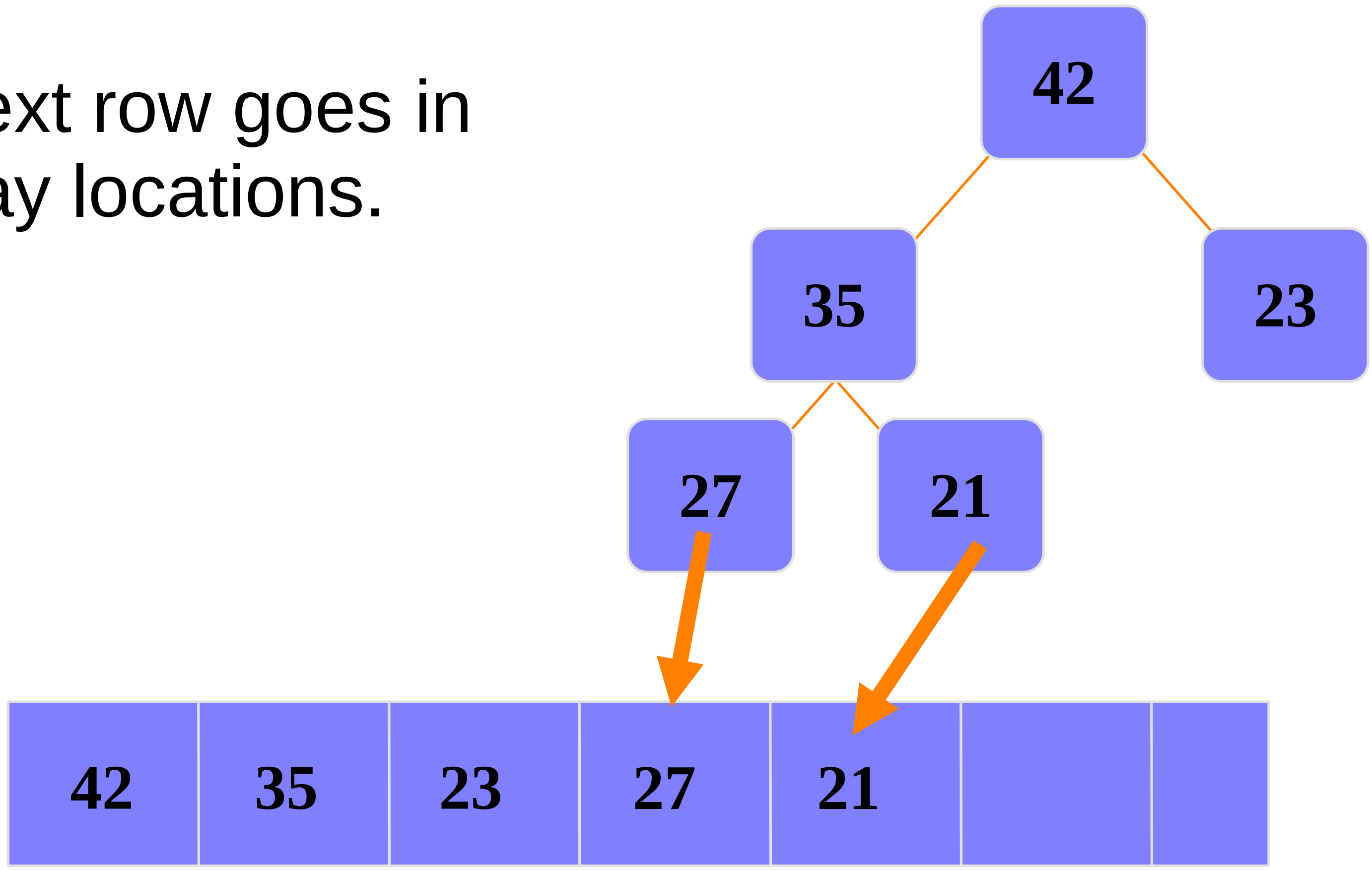

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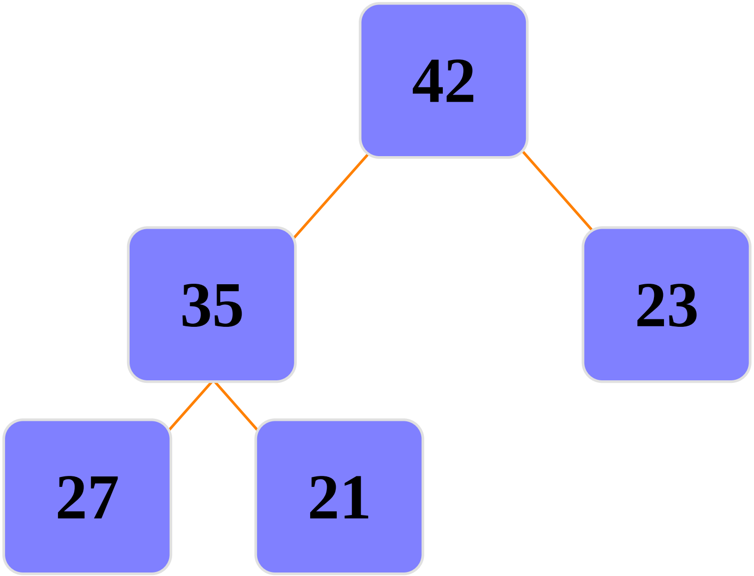


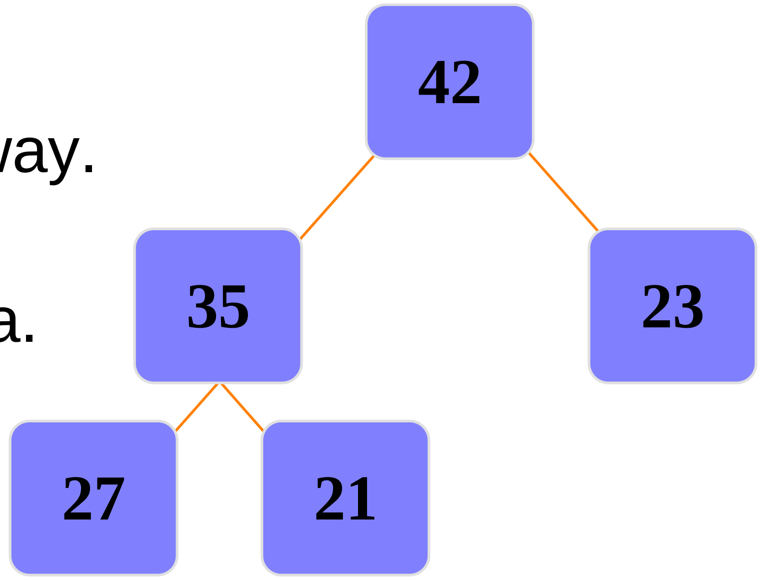
42	35	23	27	21		

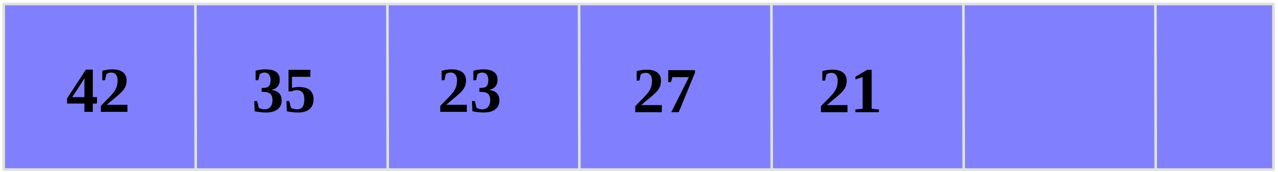
An array of data

We don't care what's in this part of the array.

Important Points about the Implementation

- The links between the tree's nodes are not actually stored as pointers, or in any other way.
- The only way we "know" that "the array is a tree" is from the way we manipulate the data.

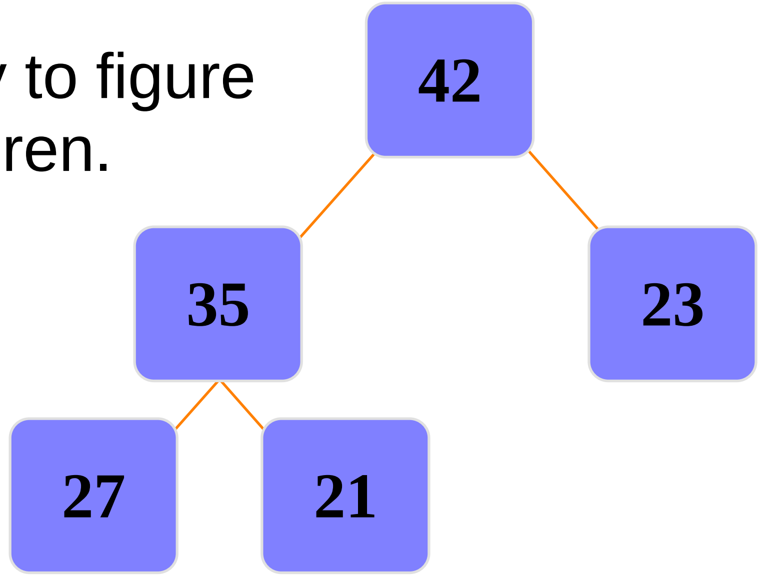


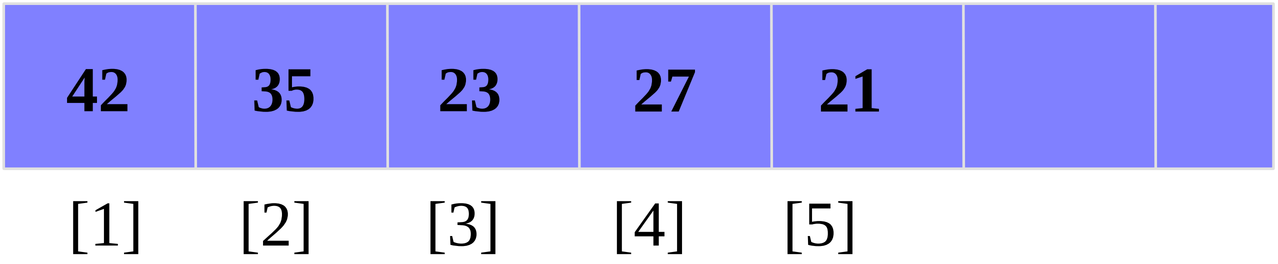


An array of data

Important Points about the Implementation

If you know the index of a node, then it is easy to figure out the indexes of that node's parent and children.





Heapify

```
MAX-HEAPIFY(A, i)
l = LEFT(i)
r = RIGHT(i)
if l ≤ A.heap-size and A[l] > A[i]
    largest = l
else largest = i
if r ≤ A.heap-size and A[r] > A[largest]
    largest = r
if largest ≠ i
    exchange A[i] with A[largest]
    MAX-HEAPIFY(A, largest)
```

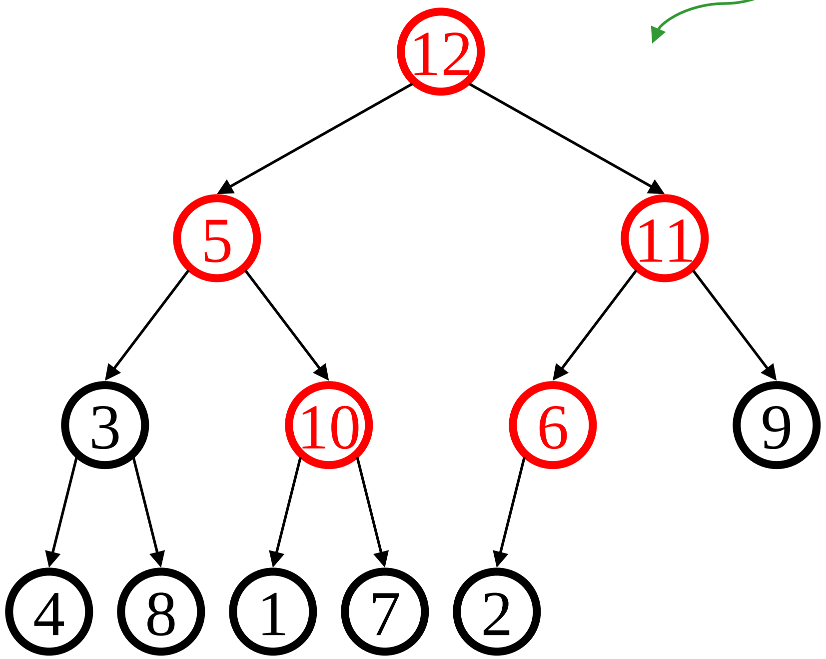
```
PARENT(i)
return ⌊i/2⌋

LEFT(i)
return 2i

RIGHT(i)
return 2i + 1
```

```
BUILD-MAX-HEAP(A)
A.heap-size = A.length
for i = ⌊A.length/2⌋ downto 1
    MAX-HEAPIFY(A, i)
```

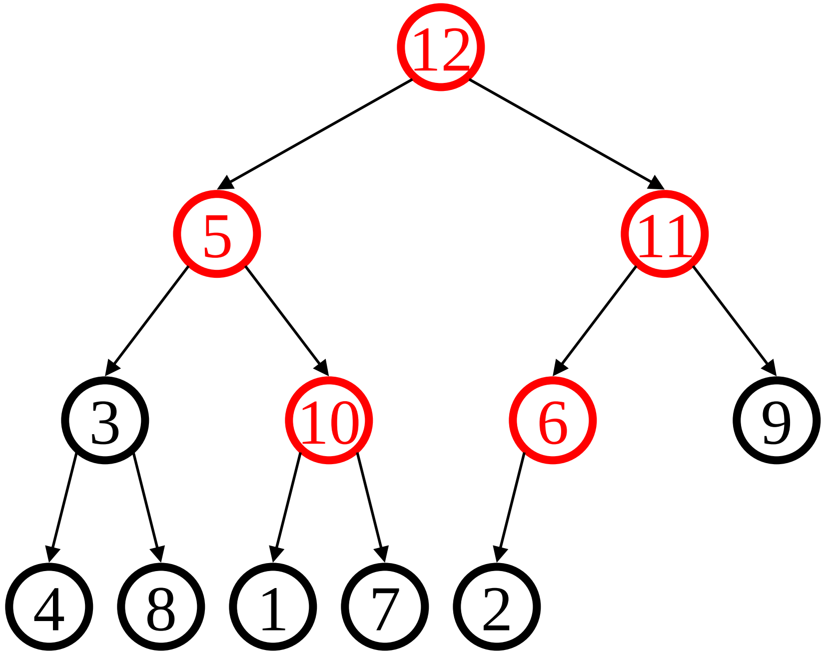
BuildHeap

12	5	11	3	10	6	9	4	8	1	7	2

Add elements arbitrarily to form a complete tree.
Pretend it's a heap and fix the heap-order property!



BuildHeap



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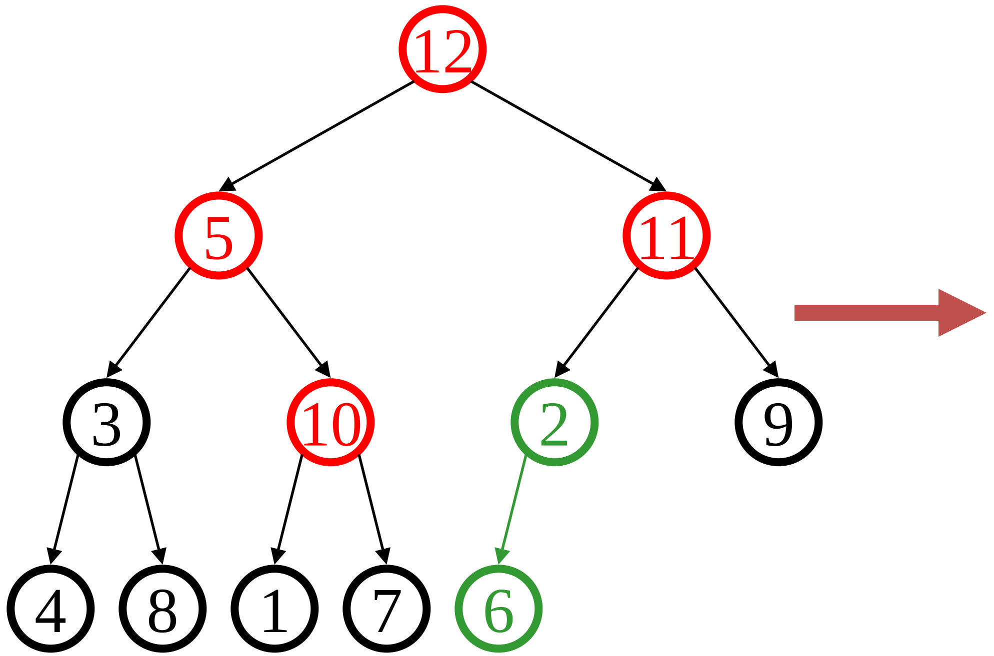

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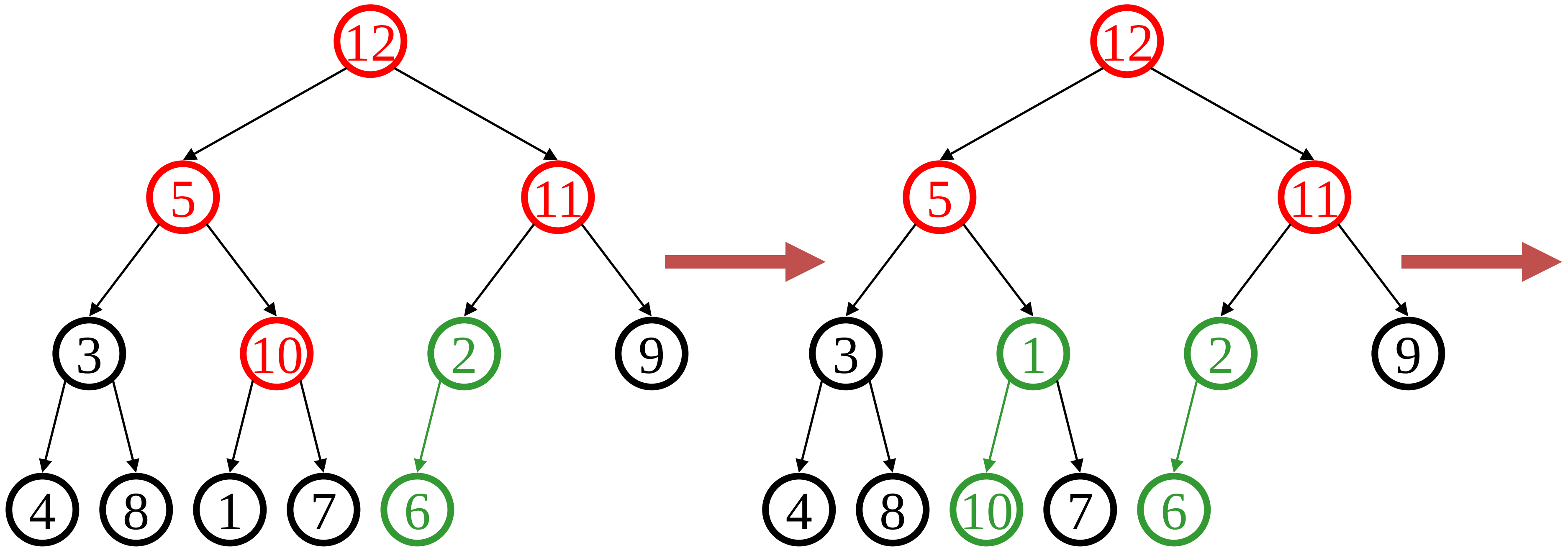

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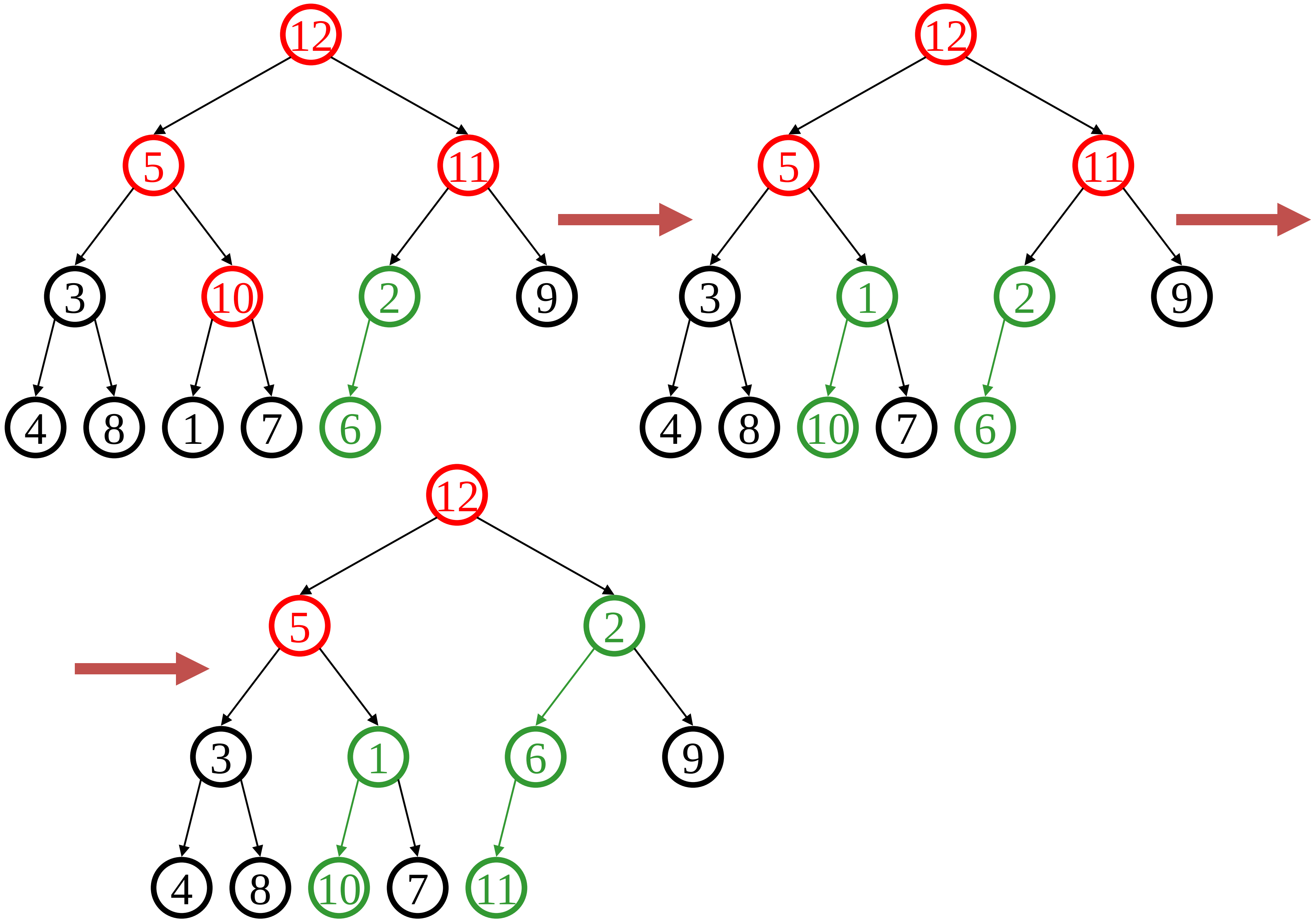

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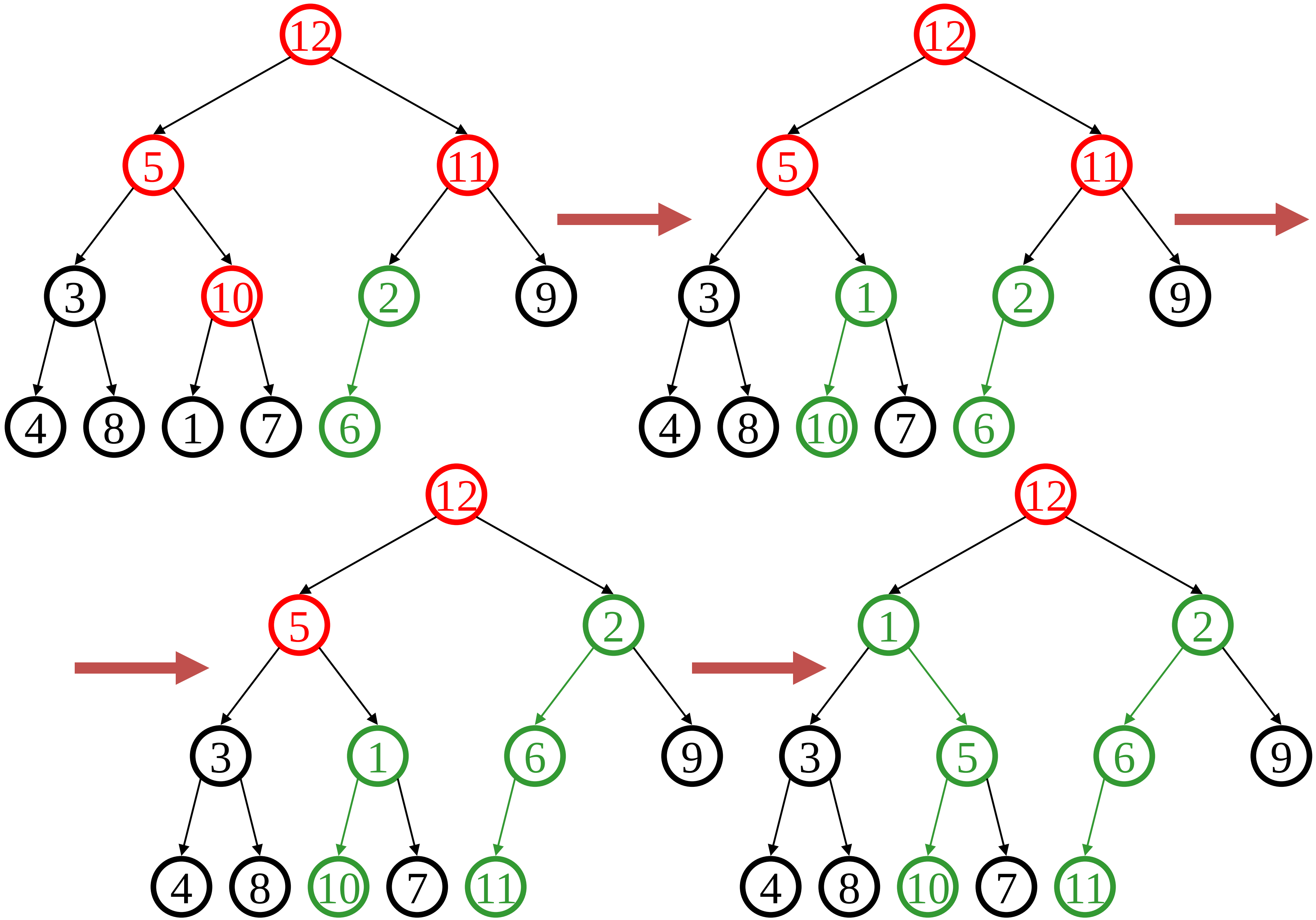
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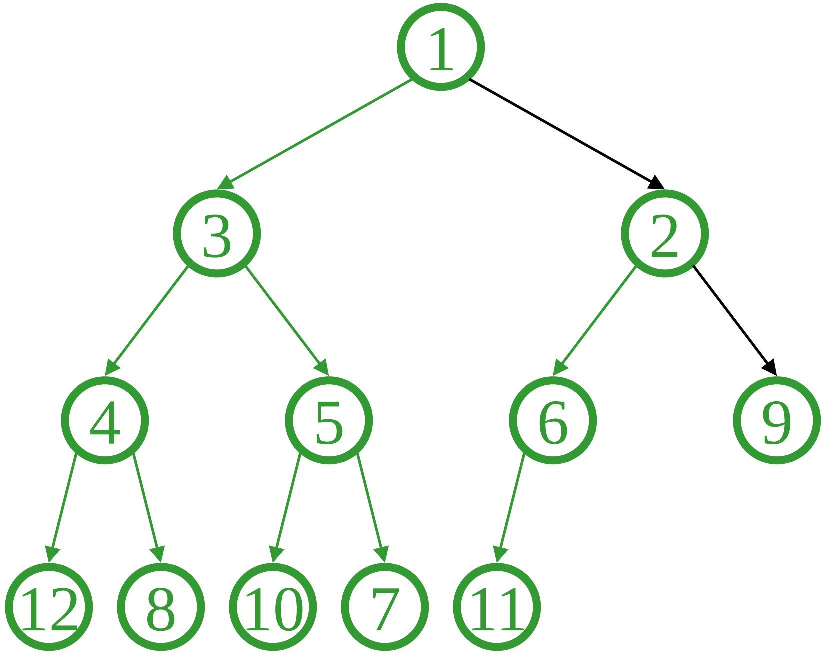
BuildHeap

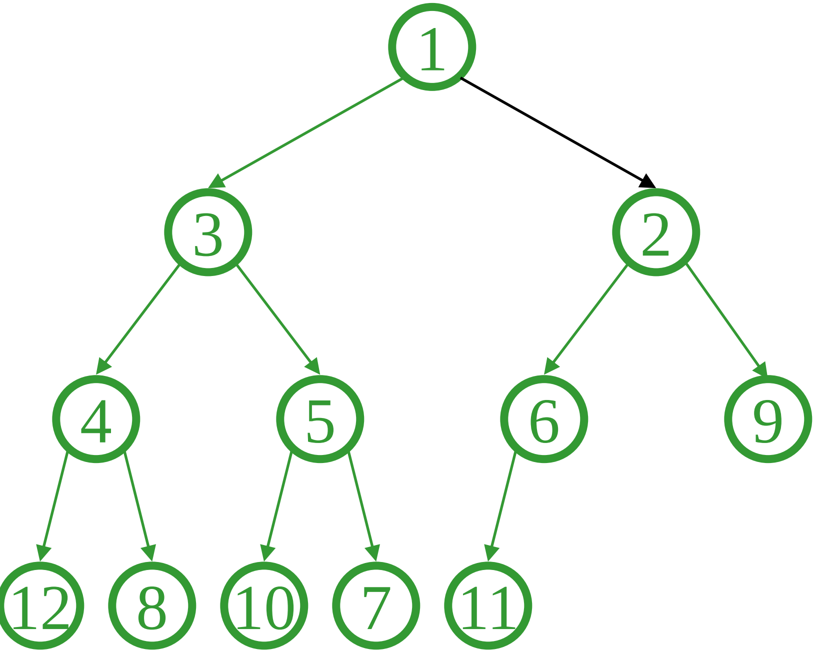


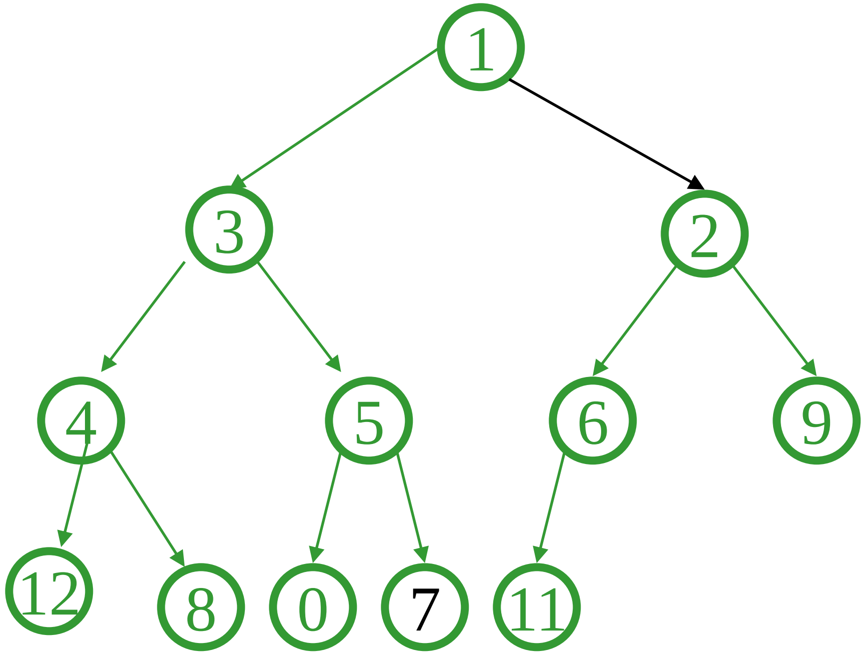
Decrease key

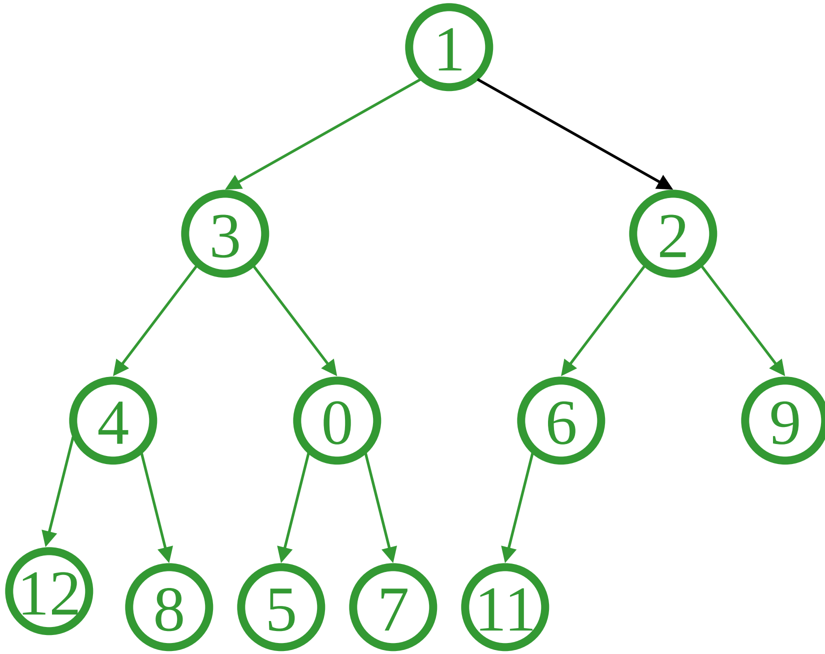
- Decrease key is used for some application to increase the priority of the key.
    - First decrease the key value
    - Perform reheapification upward
  - If it is max heap, to increase the priority, it will increase key but the steps are the same
    - First increase the key value
    - Perform reheapification upward

Example

Decrease Key 10 to 0







Example