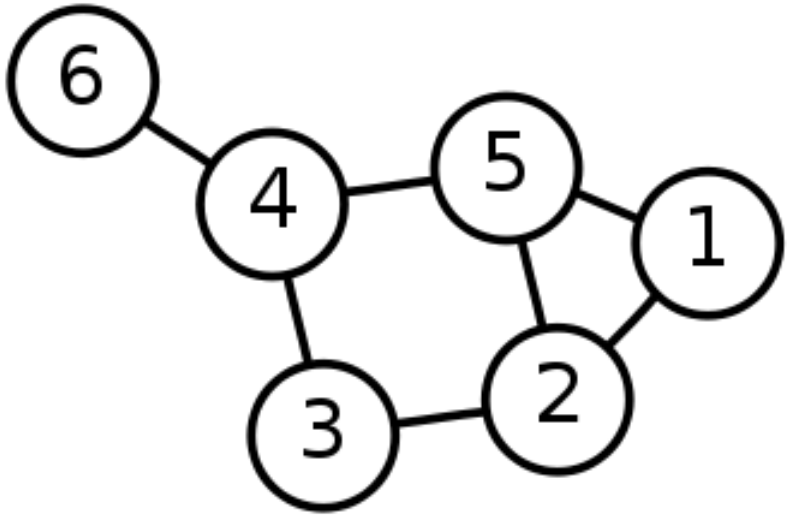
Graphs

CS223: Data Structures

Single-Source Shortest Path Problem

- The problem of finding **shortest paths** from a **source vertex v** to all other vertices in the graph.
- Weighted graph G = (E,V)
- Source vertex s ∈ V to all vertices v ∈ V



Single-Source Shortest Path Problem

- Common algorithms:
  - Dijkstra's algorithm (Discussed in the previous lecture)
  - **Bellman-Ford algorithm**

Bellman-Ford algorithm

Bellman-Ford algorithm - is a solution to the single-source shortest path problem in graph theory.

Works on both directed and undirected graphs.

Approach: Dynamic Programming

Input: Weighted graph G={E,V} and source vertex $v \in V$.

Output: Lengths of shortest paths (or the shortest paths themselves) from a given source vertex $v \in V$ to all other vertices.

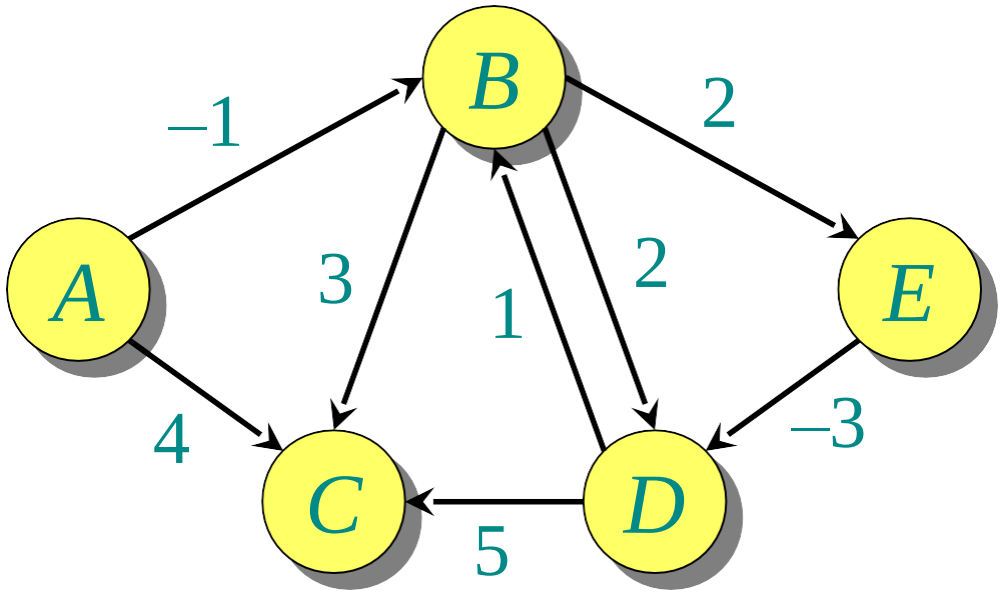
Bellman-Ford algorithm - Pseudocode

```
function bellmanFord(G, S)
        for each vertex V in G
                        distance[V] <- infinite
                        previous[V] <- NULL
        distance[S] <- 0
        for each vertex V in G
                for each edge (U,V) in G
                        tempDistance <- distance[U] + edge_weight(U, V)
                        if tempDistance < distance[V]
                           distance[V] <- tempDistance
                           previous[V] <- U

        for each edge (U,V) in G
                If distance[U] + edge_weight(U, V) < distance[V}
                        Error: Negative Cycle Exists

        return distance[], previous[]
```

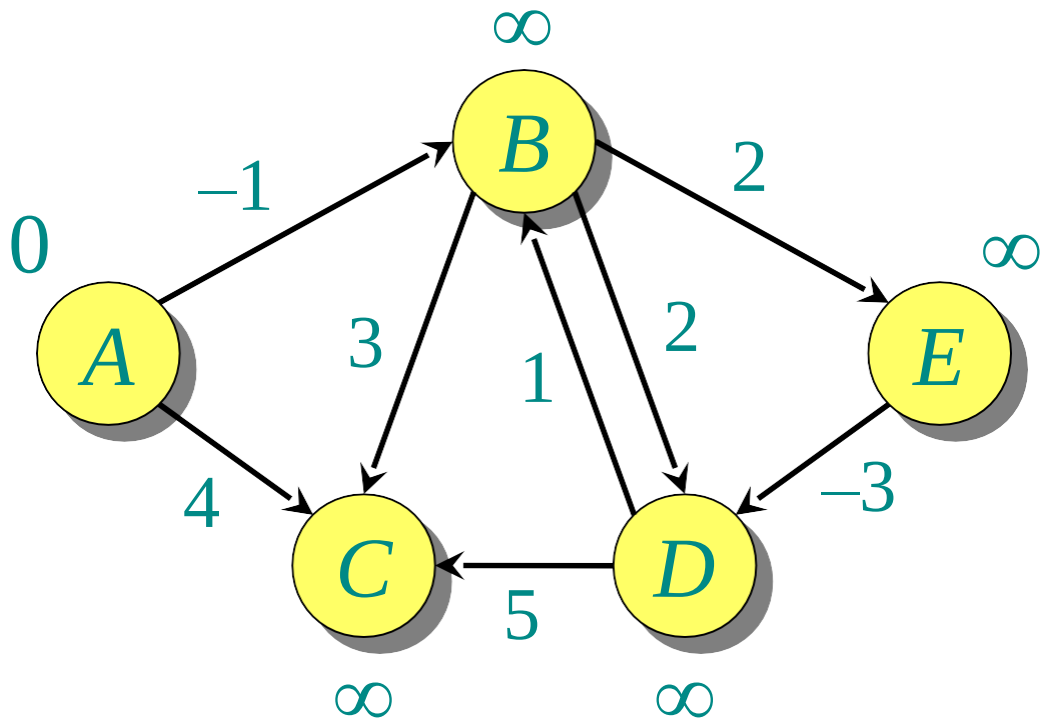
initialization

relaxation step

Example of Bellman-Ford

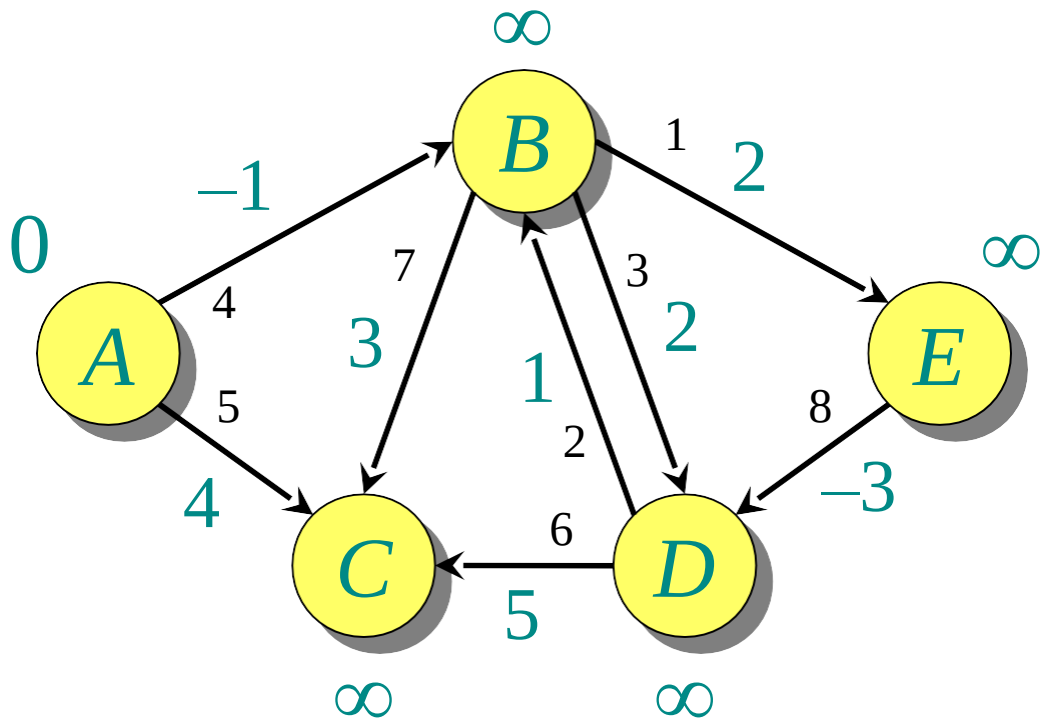


Example of Bellman-Ford



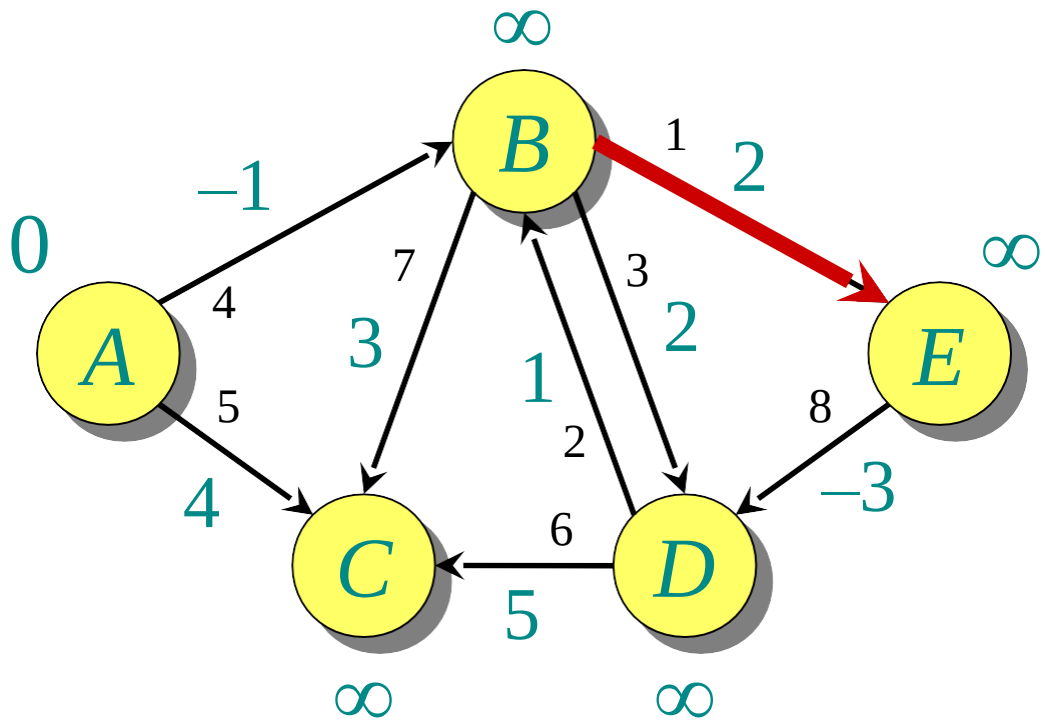
Initialization.

Example of Bellman-Ford

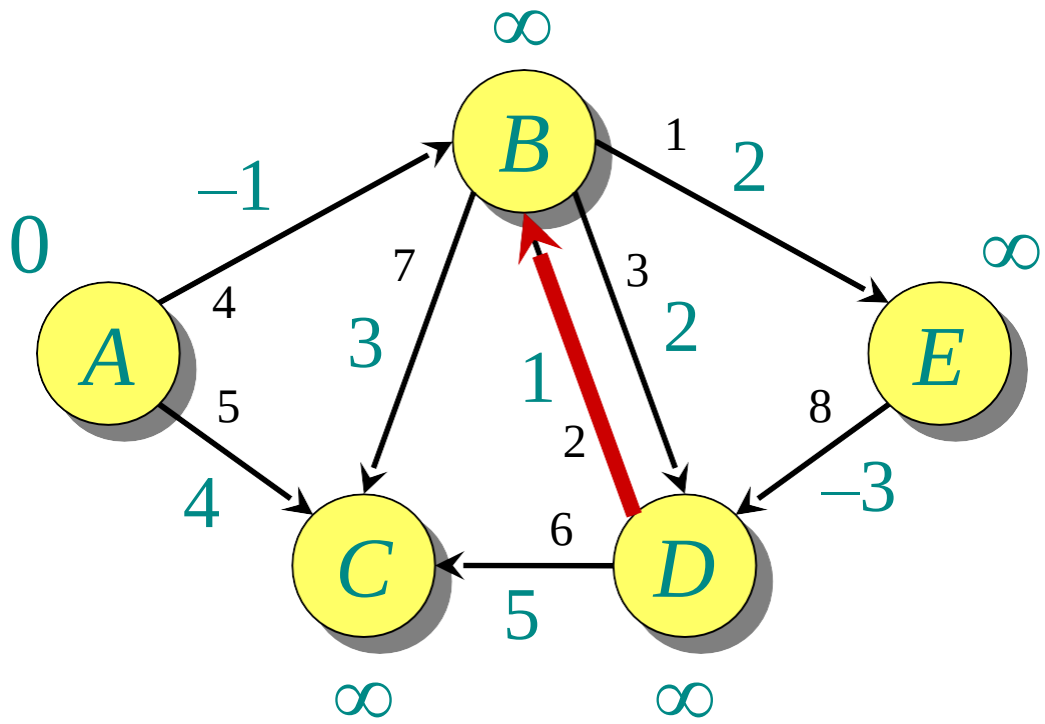


Order of edge relaxation.

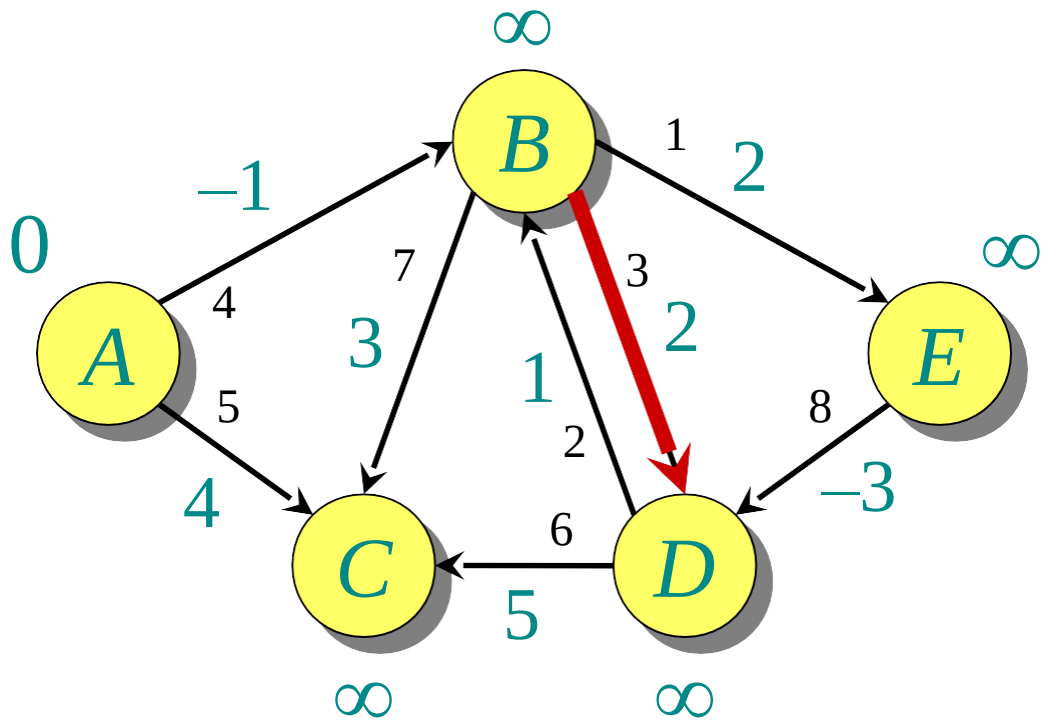
Example of Bellman-Ford



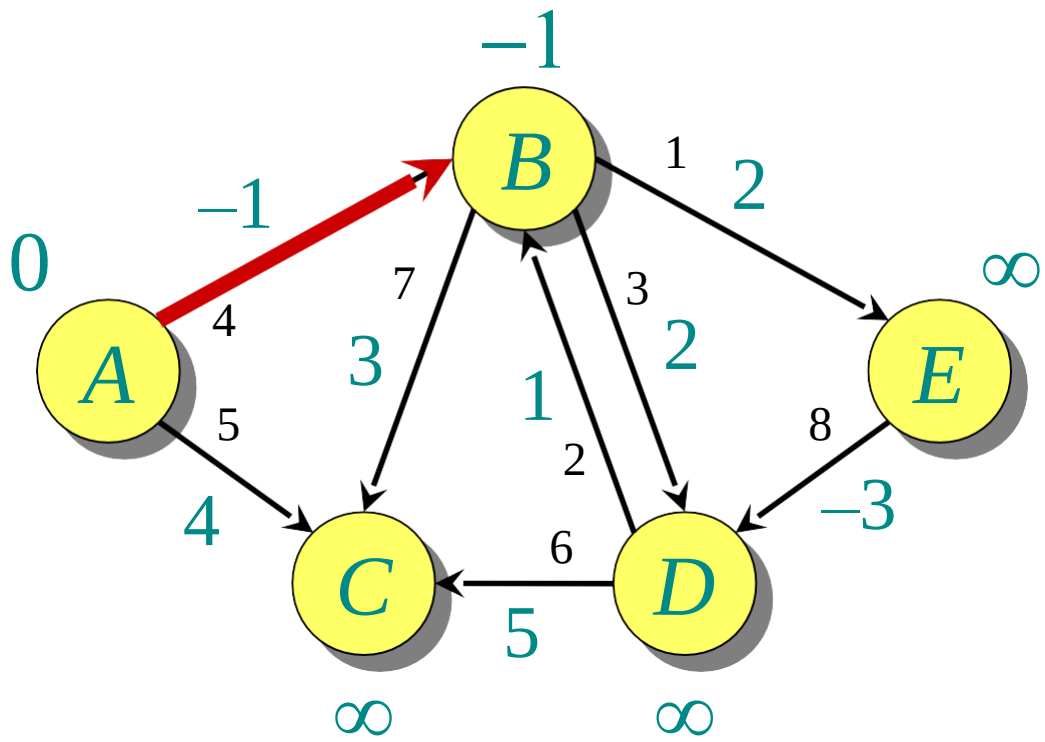
Example of Bellman-Ford



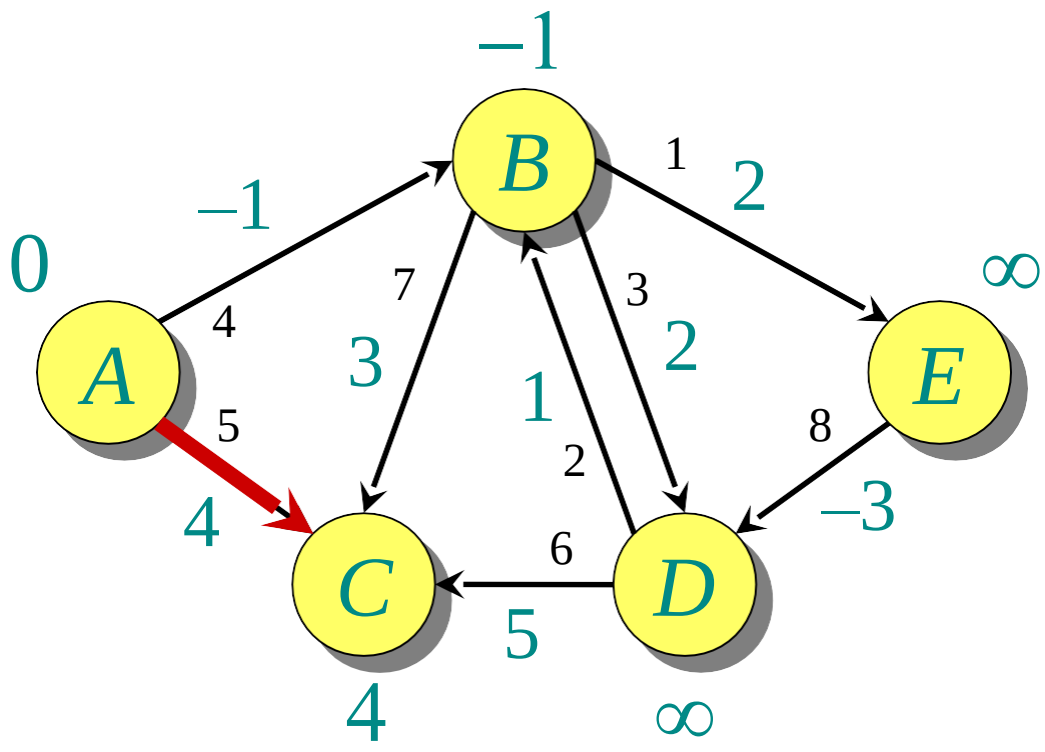
Example of Bellman-Ford



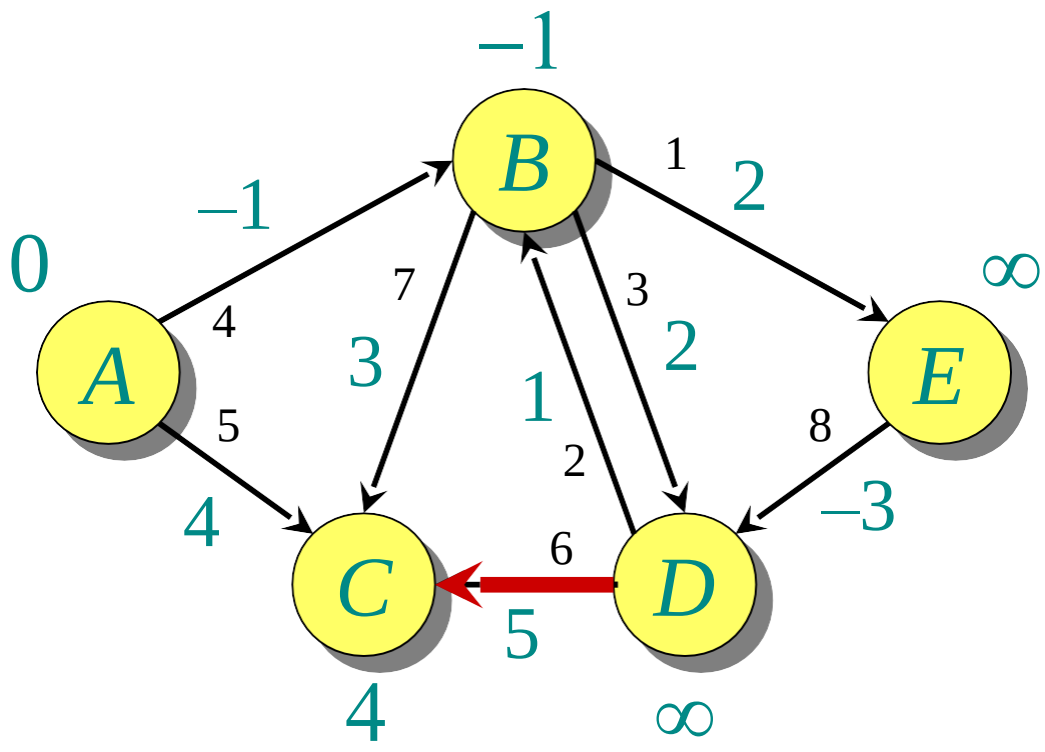
Example of Bellman-Ford



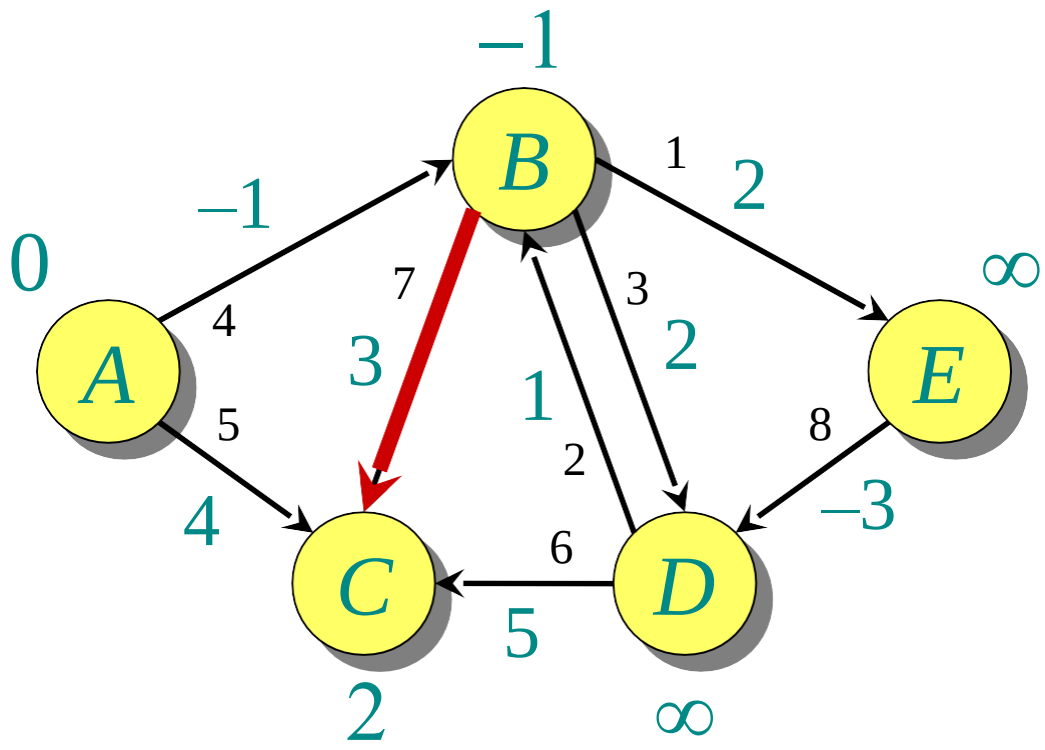
Example of Bellman-Ford



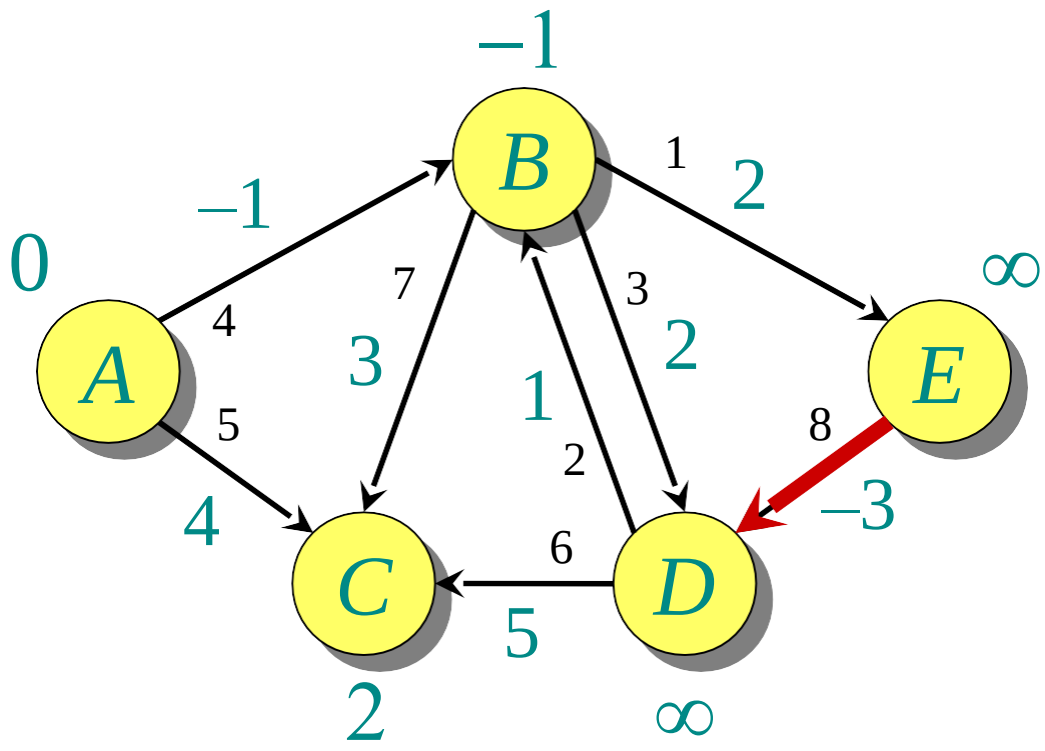
Example of Bellman-Ford



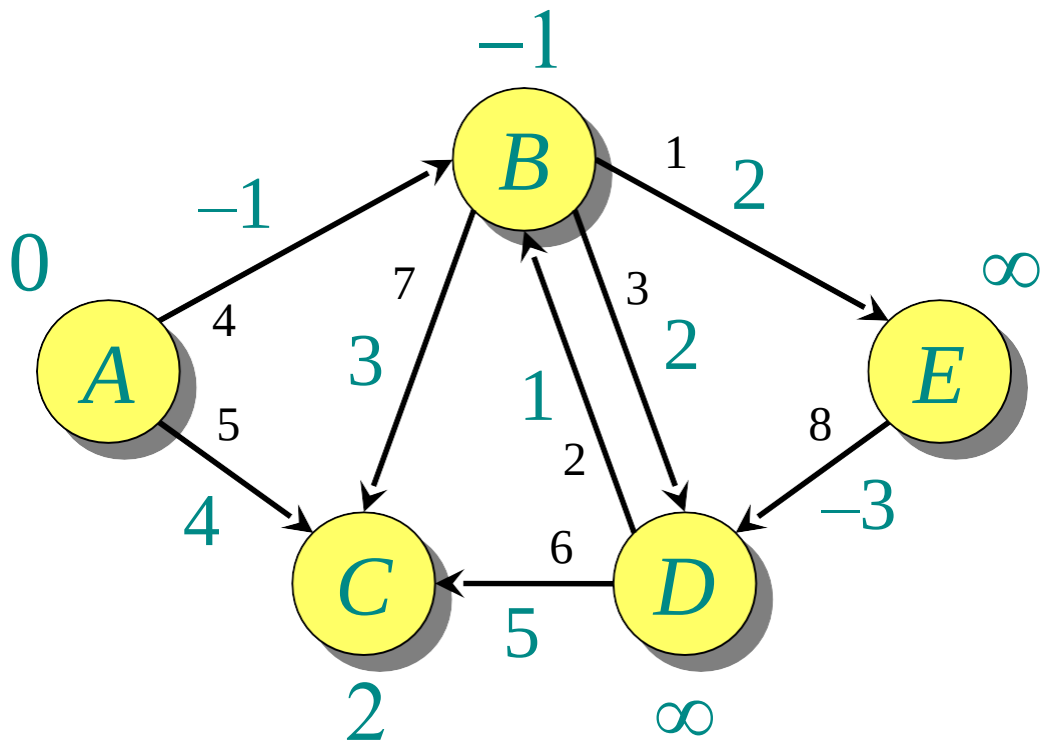
Example of Bellman-Ford



Example of Bellman-Ford

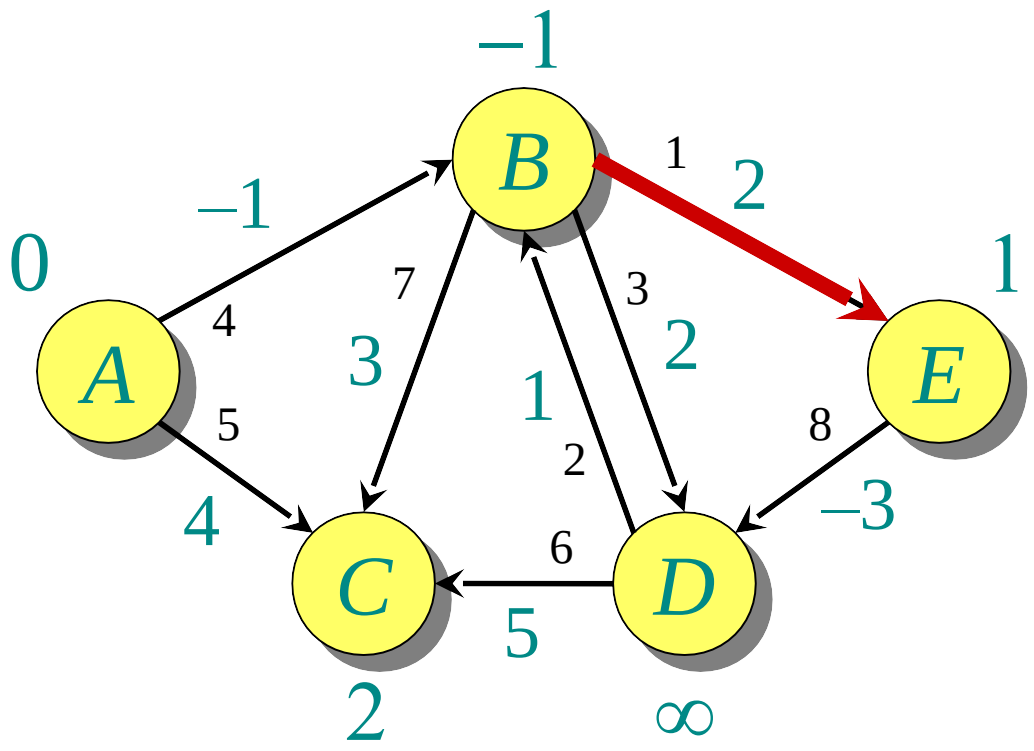


Example of Bellman-Ford

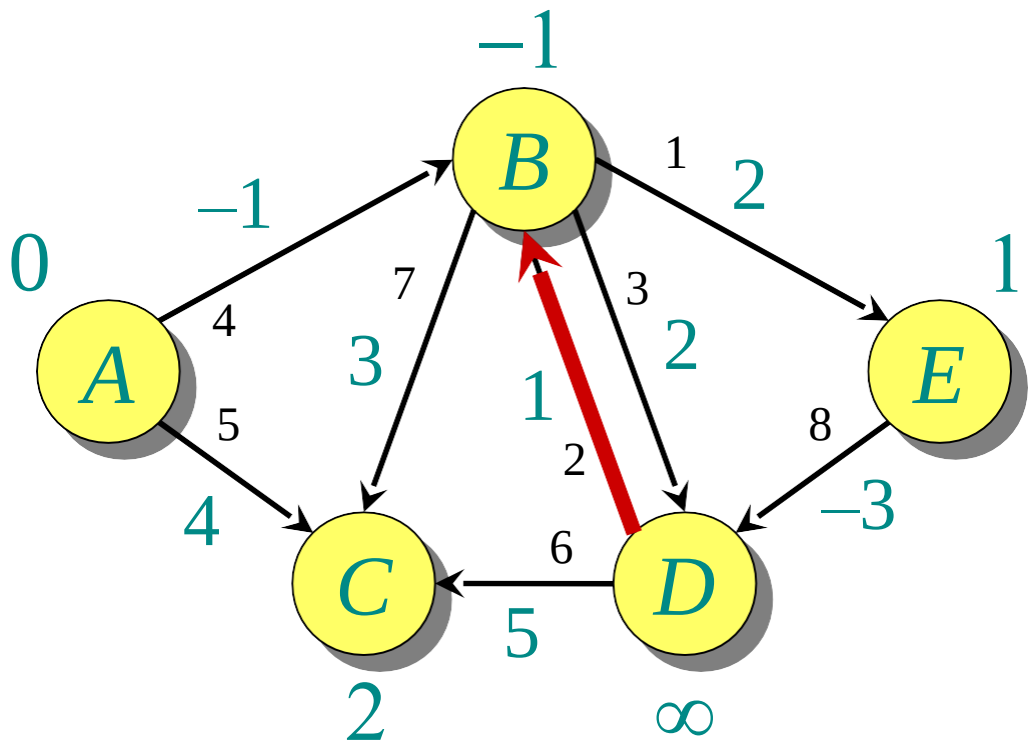


End of pass 1.

Example of Bellman-Ford



Example of Bellman-Ford



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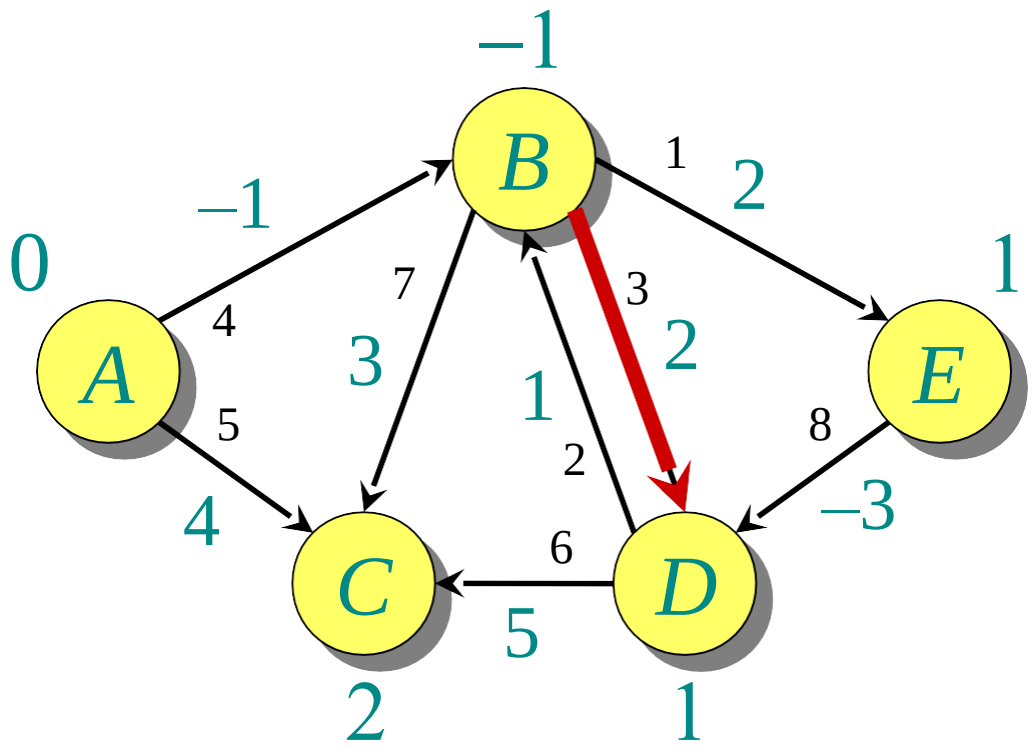
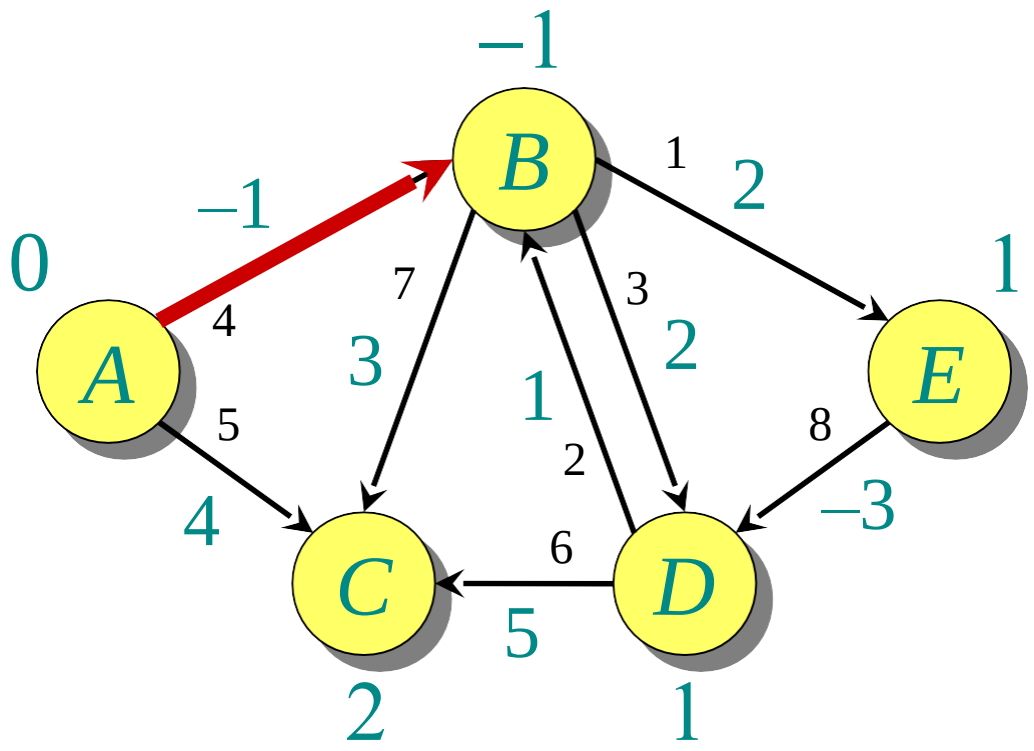
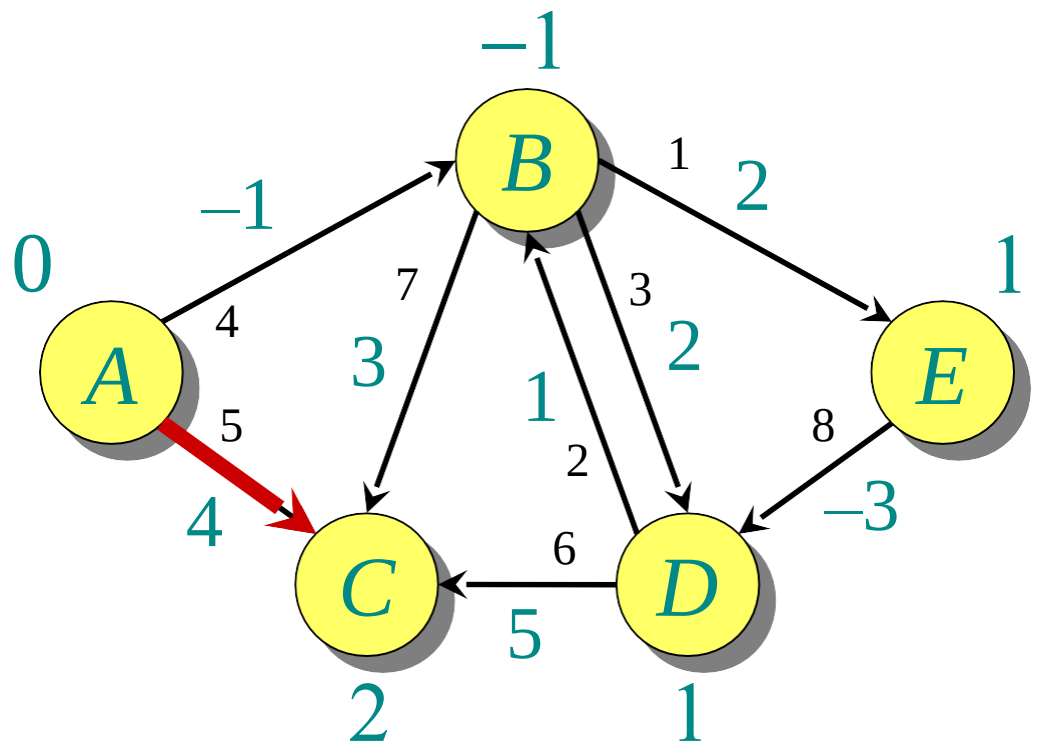


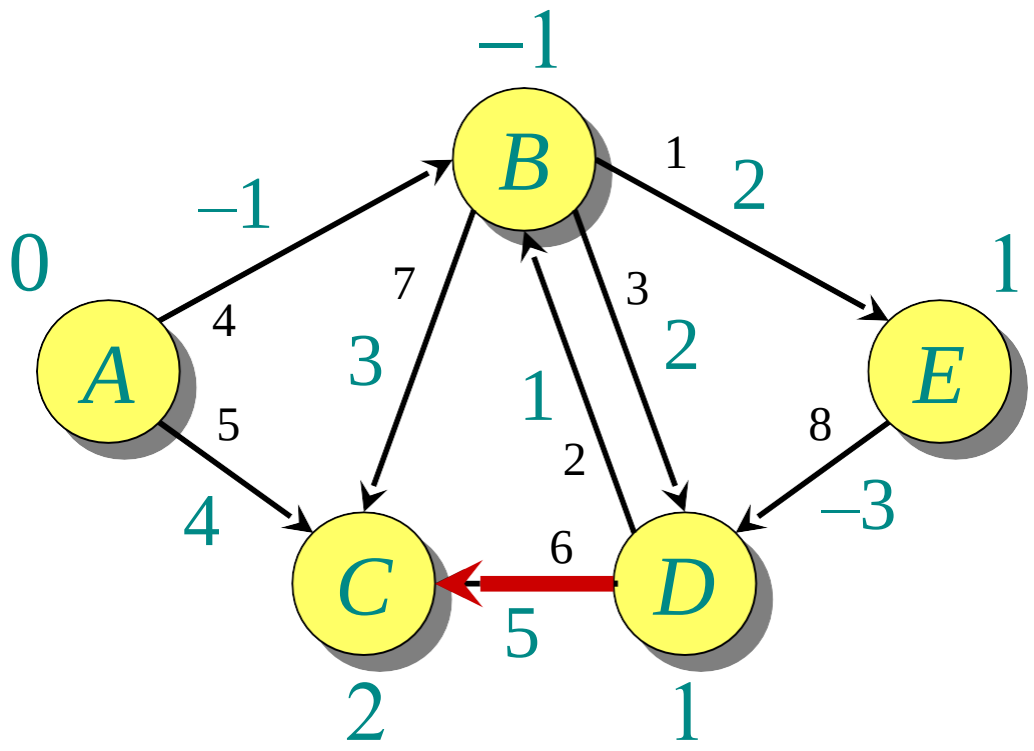
Example of Bellman-Ford



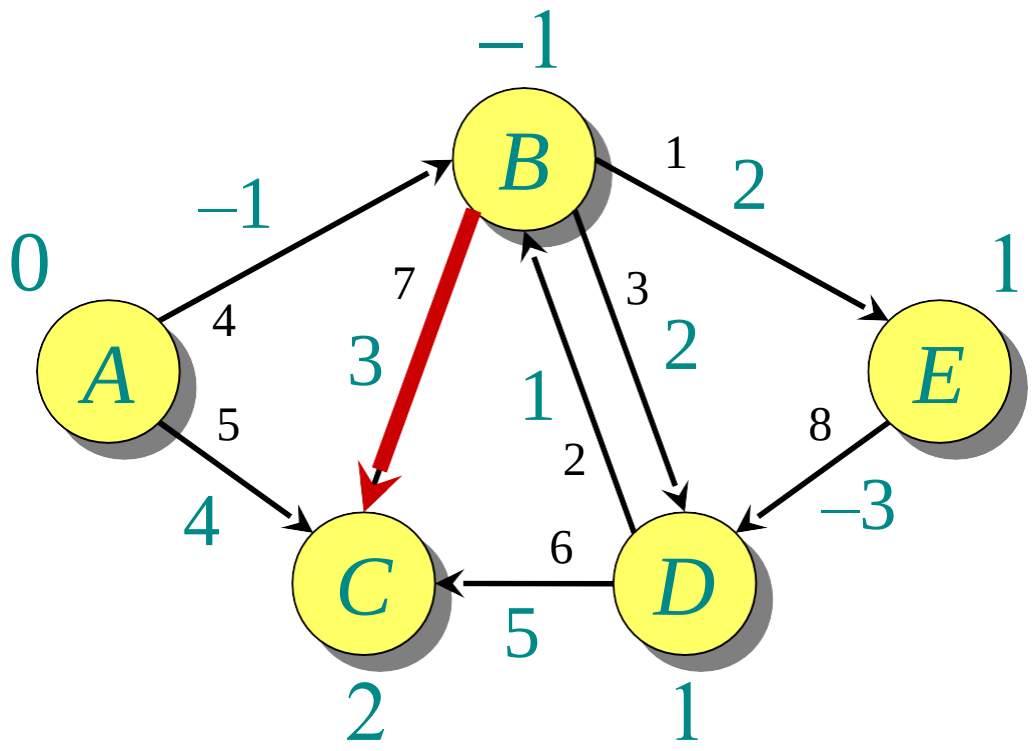
Example of Bellman-Ford



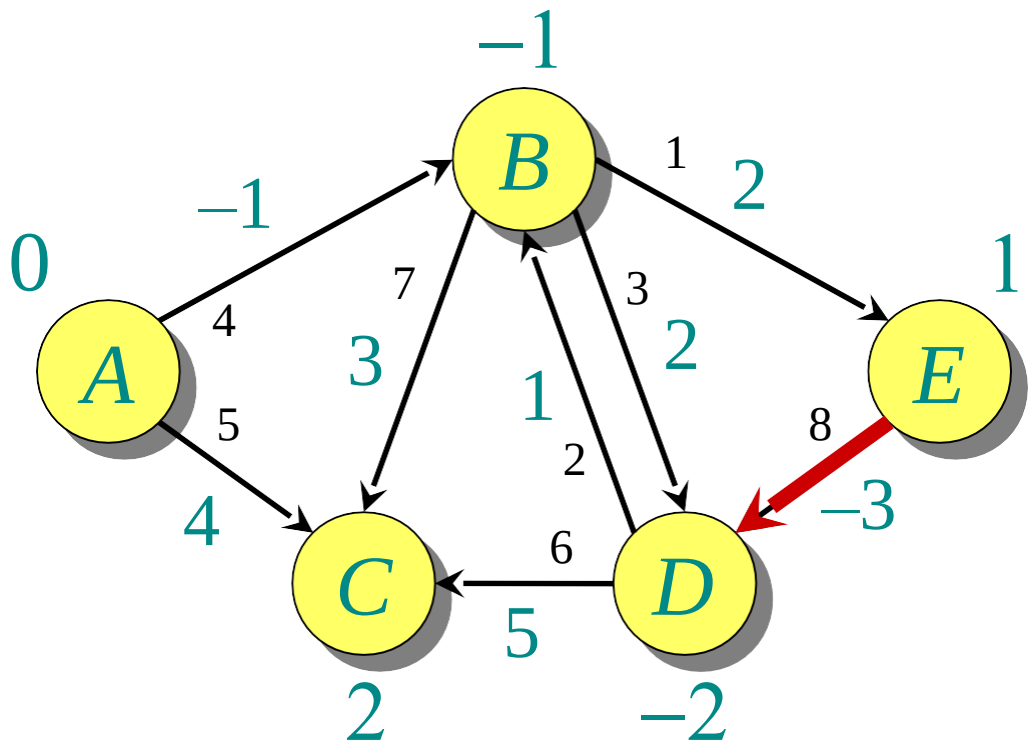
Example of Bellman-Ford



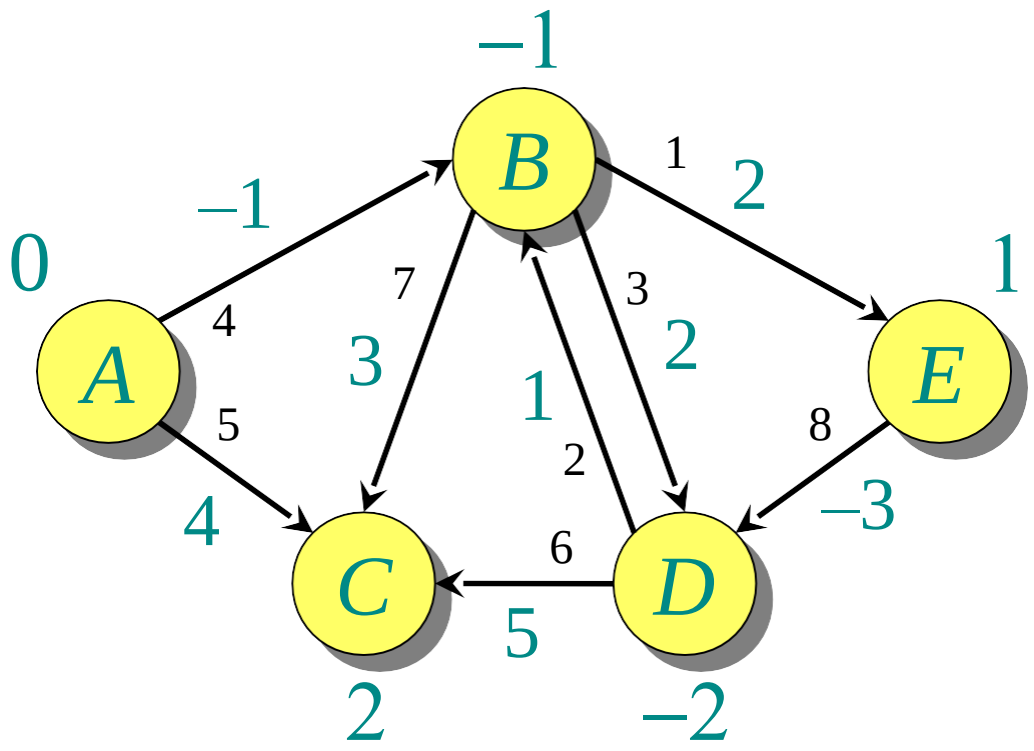
Example of Bellman-Ford



Example of Bellman-Ford

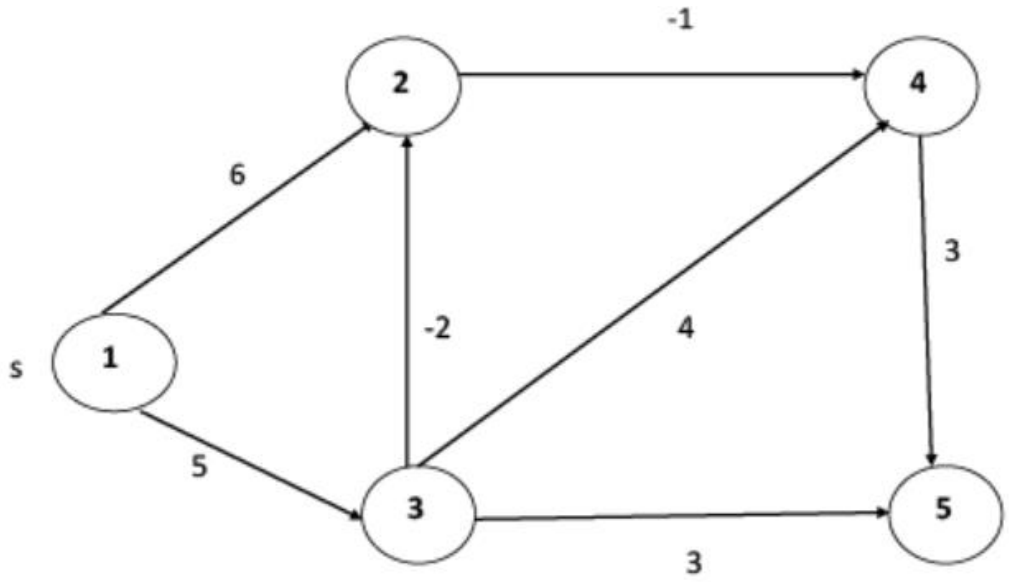


Example of Bellman-Ford

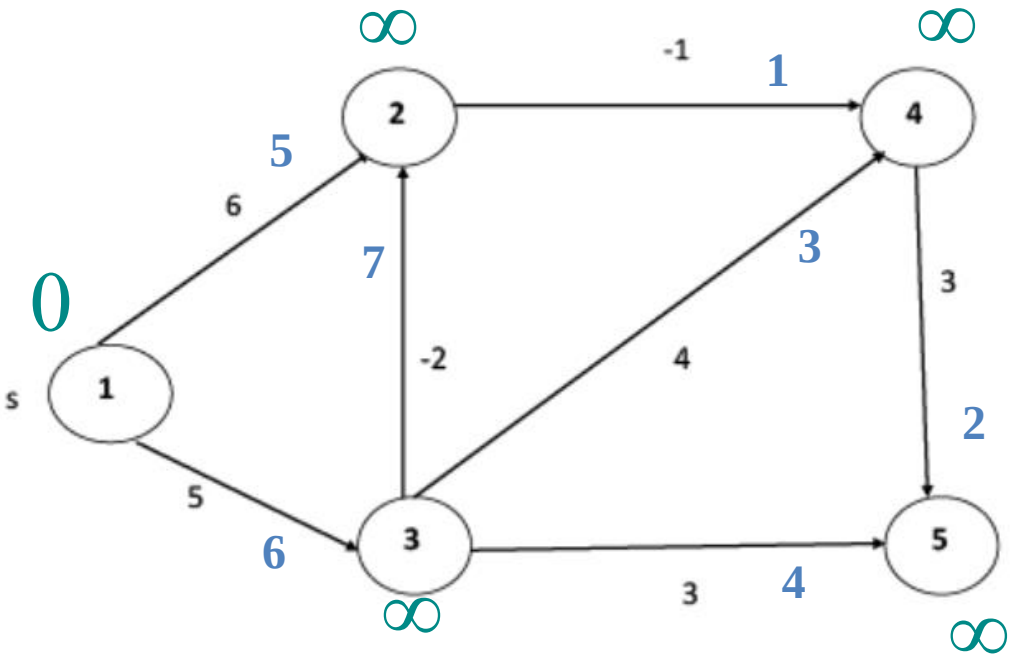


End of pass 2 (and 3 and 4).

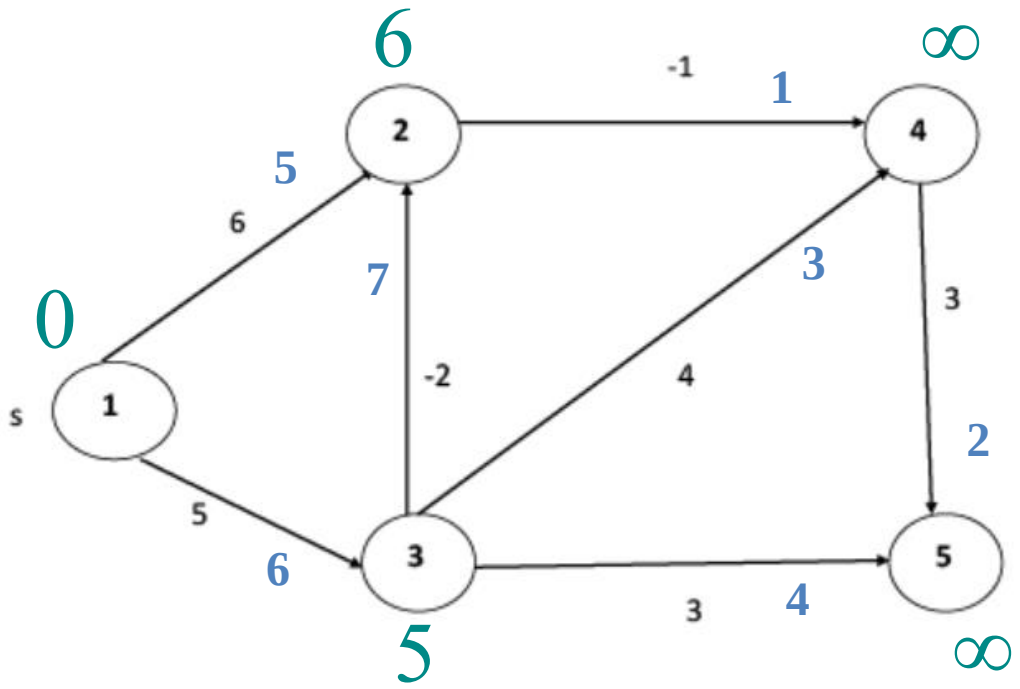
Bellman-Ford Algorithm – Example 2



Bellman-Ford Algorithm – Example 2

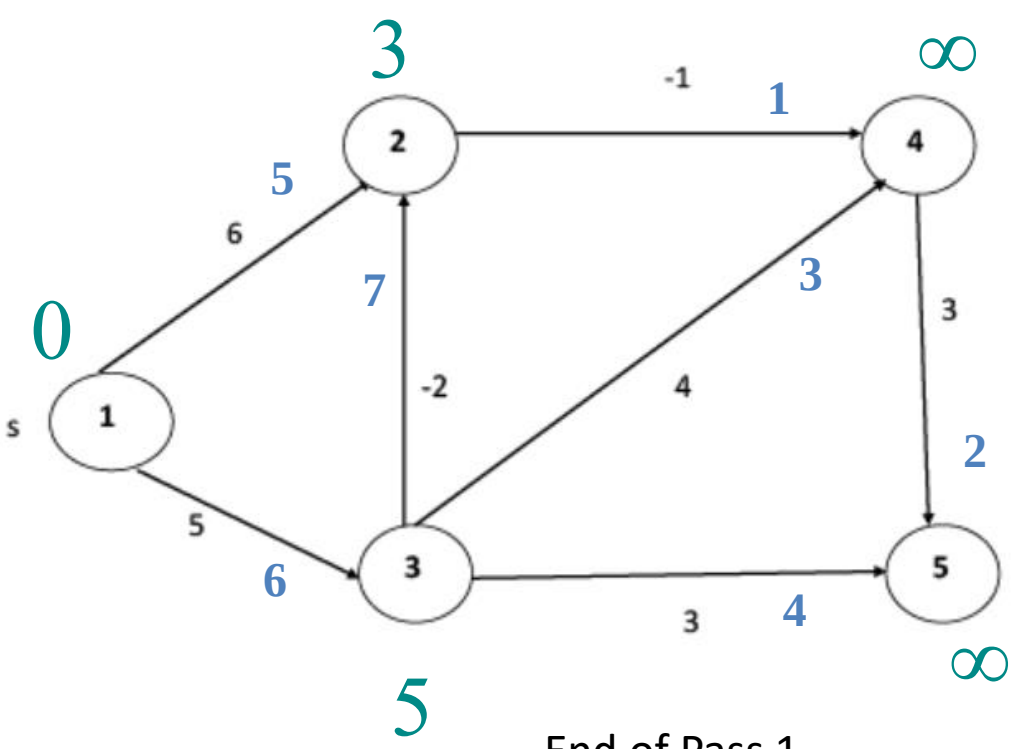


Initialization

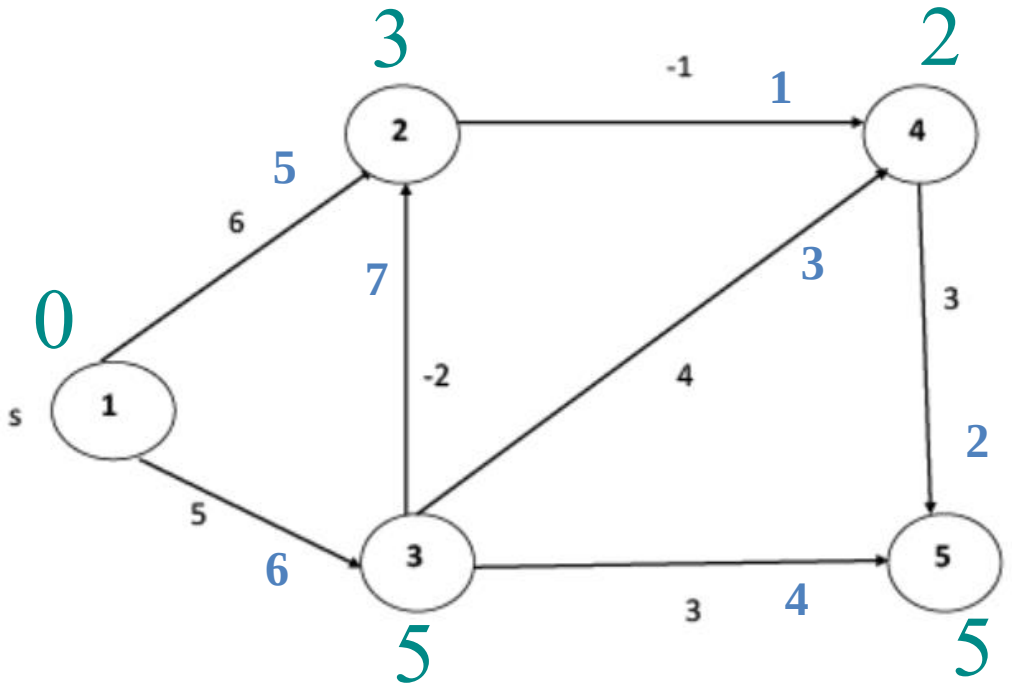


Pass 1 – edge 6

Bellman-Ford Algorithm – Example 2

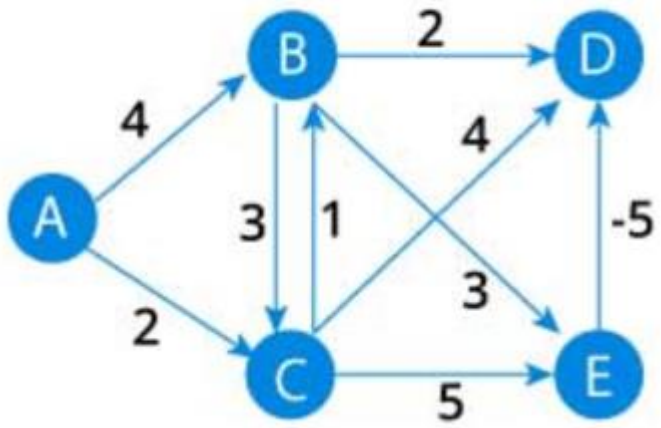


End of Pass 1

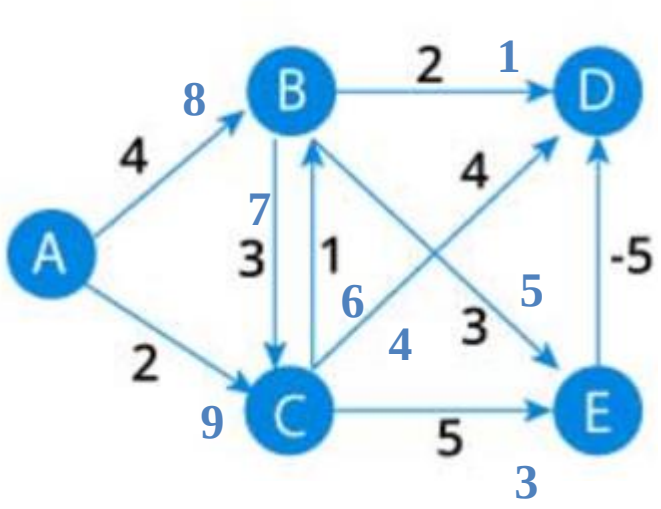


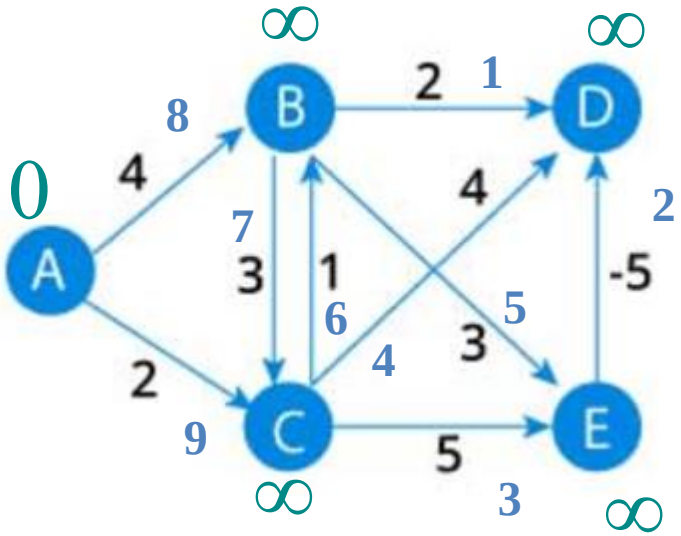
End of Pass 2, 3, and 4

Bellman-Ford Algorithm – Example 3

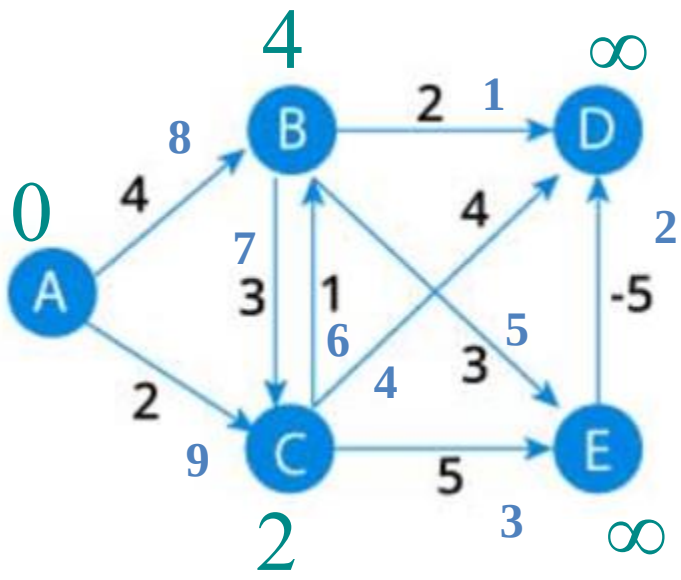


Bellman-Ford Algorithm – Example 3



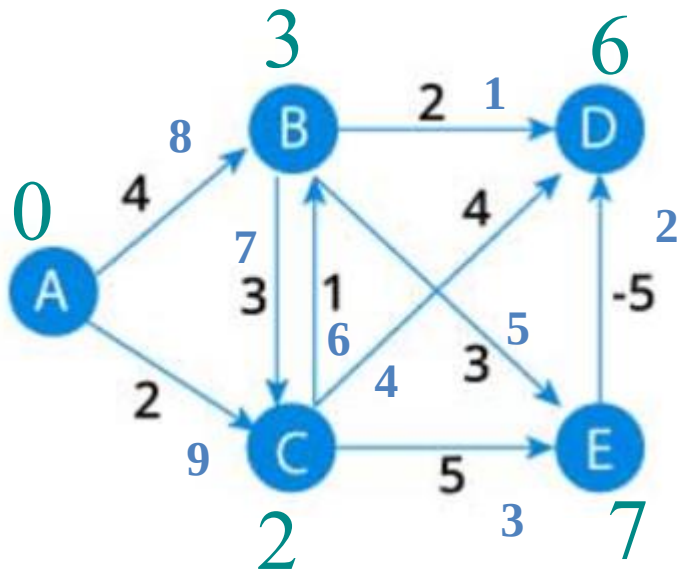


Initialization

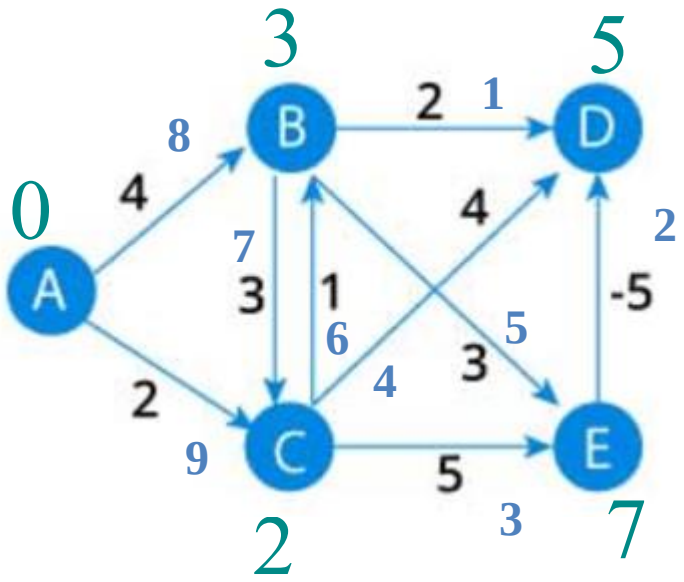


End of Pass 1

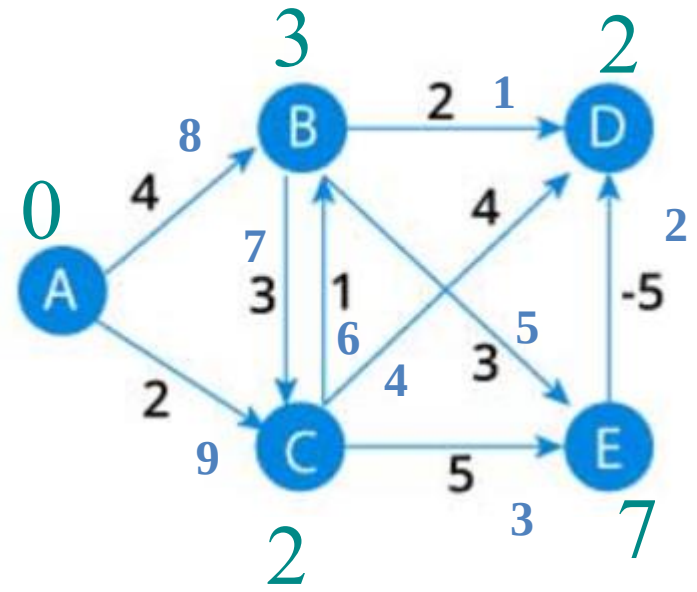
Bellman-Ford Algorithm – Example 3



End of Pass 2

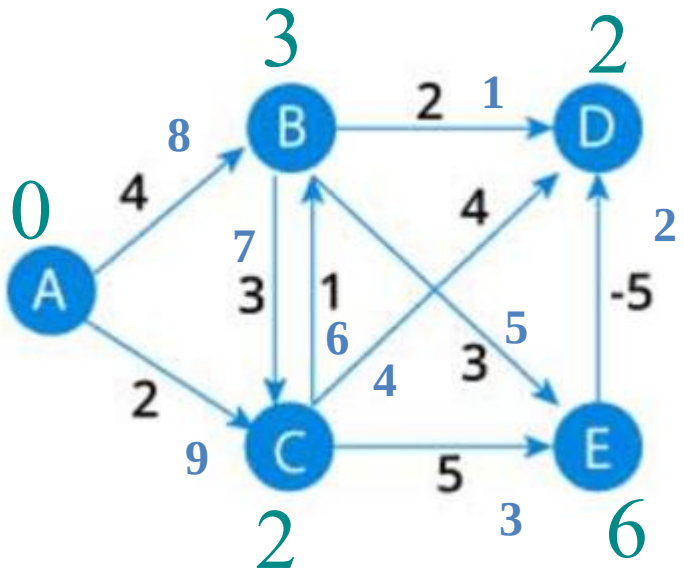


Pass 3 – edge 1

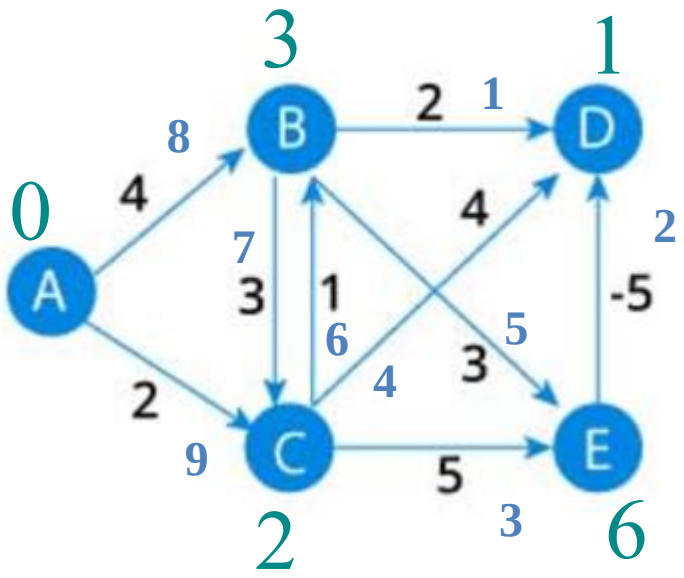


Pass 3 – edge 4

Bellman-Ford Algorithm – Example 3



End of Pass 3



End of Pass 4

(D, B) -> (C, D) -> (A, C) -> (A, B) -> (B, C)

Bellman-Ford Algorithm – Example 4