Matboard Bridge
Design Calculations

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Contents

1 Design 0

1.1 Hand Calculations

1.1.0 Cross Section

$$\begin{aligned}
\bar{y} &= \frac{\sum A_i y_i}{\sum A_i} = \frac{\sum b_i h_i y_i}{\sum b_i h_i} \\
&= \frac{(80)(1.27)(0.635) + (100)(1.27)(75.63) + 2(1.27)(73.73)(38.135) + 2(5)(1.27)(74.365)}{(80)(1.27) + (100)(1.27) + 2(1.27)(73.73) + 2(5)(1.27)} \\
&= 41.4296\,\text{mm} = \boxed{41.4\,\text{mm}}
\end{aligned}$$

$$\begin{aligned}
I = \sum I_i + A_i {d_i}^2 = \frac{b_i {h_i}^3}{12} + b_i h_i \left(\bar{y} - y_i\right)^2
&= \frac{(80)(1.27)^3}{12} + (80)(1.27)[(41.4296) - (0.635)]^2 \\
&+ \frac{(100)(1.27)^3}{12} + (100)(1.27)[(41.4296) - (75.63)]^2 \\
&+ 2\left(\frac{(1.27)(73.73)^3}{12} + (1.27)(73.73)[(41.4296) - (38.135)]^2\right) \\
&+ 2\left(\frac{(5)(1.27)^3}{12} + (5)(1.27)[(41.4296) - (74.365)]^2\right) \\
&= 418.309 \times 10^3\,\text{mm}^4 = \boxed{418 \times 10^3\,\text{mm}^4}
\end{aligned}$$

1.1.1 Flexural Tension Failure

$$M_{\text{max}} = 69\,445.3\,\text{N}\,\text{mm} \quad \text{at} \quad x_{\text{train}} \in \{557\,\text{mm}, 644\,\text{mm}\}$$

$$y_{bot} = \bar{y} - 0 = 41.4296\,\text{mm}$$

$$\sigma_{\text{demand}} = \frac{M_{max} y_{bot}}{I} = \frac{(69445.3)(41.4296)}{(418.309 \times 10^3)} = 6.877\,91\,\text{MPa}$$

$$\sigma_{\text{tension}} = 30\,\text{MPa}$$

$$\text{FOS}_{\text{tension}} = \frac{\sigma_{\text{tension}}}{\sigma_{\text{demand}}} = \frac{(30)}{(6.87791)} = 4.3618 = \boxed{4.36}$$

1.1.2 Flexural Compression Failure

$$M_{\text{max}} = 69\,445.3\,\text{N}\,\text{mm} \quad \text{at} \quad x_{\text{train}} \in \{557\,\text{mm}, 644\,\text{mm}\}$$

$$y_{top} = \bar{y} - h = (41.4296) - (76.27) = -34.8404\,\text{mm}$$

$$\sigma_{\text{demand}} = \frac{M_{max} y_{top}}{I} = \frac{(69445.3)(-34.8404)}{(418.309 \times 10^3)} = -5.784\,01\,\text{MPa}$$

$$\sigma_{\text{comp}} = -6\,\text{MPa}$$

$$\text{FOS}_{\text{comp}} = \frac{\sigma_{\text{comp}}}{\sigma_{\text{demand}}} = \frac{(-6)}{(-5.78401)} = 1.03734 = \boxed{1.037}$$

1.1.3 Material Shear Stress Failure

$$V_{\max} = 240\,\text{N} \quad \text{at} \quad x_{\text{train}} = 1\,\text{mm}$$

$$\begin{aligned} Q(\bar{y}) &= \sum A_i d_i \\ &= 2(1.27)(41.4296 - 1.27)\left(\frac{41.4296 - 1.27}{2}\right) + (80)(1.27)\left(41.4296 - \frac{1.27}{2}\right) \\ &= 6192.98\,\text{MPa} \end{aligned}$$

$$b(\bar{y}) = 2(1.27) = 2.54\,\text{mm}$$

$$\tau_{demand} = \frac{VQ}{Ib} = \frac{(240)(6192.98)}{(418.309 \times 10^3)(2.54)} = 1.398\,88\,\text{MPa}$$

$$\tau_{max} = 4\,\text{MPa}$$

$$\text{FOS}_{\text{shear}} = \frac{\tau_{max}}{\tau_{demand}} = \frac{(4)}{(1.39888)} = 2.85943 = \boxed{2.86}$$

1.1.4 Glue Shear Stress Failure

$$V_{\max} = 240\,\text{N} \quad \text{at} \quad x_{\text{train}} = 1\,\text{mm}$$

$$Q(75) = \sum A_i d_i = (100)(1.27)\left(34.8404 - \frac{1.27}{2}\right) = 4344.09\,\text{MPa}$$

$$b(75) = 2(5 + 1.27) = 12.54\,\text{mm}$$

$$\tau_{demand} = \frac{VQ}{Ib} = \frac{(240)(4344.09)}{(418.309 \times 10^3)(12.54)} = 198.754 \times 10^{-3}\,\text{MPa}$$

$$\tau_{glue} = 2\,\text{MPa}$$

$$\text{FOS}_{\text{glue}} = \frac{\tau_{max}}{\tau_{demand}} = \frac{(2)}{(198.754 \times 10^{-3})} = 10.0627 = \boxed{10.06}$$

1.1.5 Case 1 Flexural Buckling Failure

$$M_{\max} = 69\,445.3\,\text{N}\,\text{mm} \quad \text{at} \quad x_{\text{train}} \in \{557\,\text{mm}, 644\,\text{mm}\}$$

$$\sigma_{\text{demand}} = \frac{M_{\max} y_{\text{top}}}{I} = \frac{(69445.3)(34.8404)}{(418.309 \times 10^3)} = 5.784\,01\,\text{MPa}$$

$$b_{\text{crit},1} = 80 - 1.27 = 78.73\,\text{mm}$$

$$\sigma_{\text{crit},1} = \frac{K\pi^2 E}{12\left(1 - \mu^2\right)}\left(\frac{t}{b_{\text{crit},1}}\right)^2 = \frac{(4)\,\pi^2\,(4000)}{12\left[1 - (0.2)^2\right]}\left(\frac{1.27}{78.73}\right)^2 = 3.566\,93\,\text{MPa}$$

$$\text{FOS}_{\text{buck},1} = \frac{\sigma_{\text{crit},1}}{\sigma_{\text{demand}}} = \frac{(3.56693)}{(5.78401)} = 0.616688 = \boxed{617 \times 10^{-3}}$$

1.1.6 Case 2 Flexural Buckling Failure

$$M_{\text{max}} = 69\,445.3\,\text{N mm} \quad \text{at} \quad x_{\text{train}} \in \{557\,\text{mm}, 644\,\text{mm}\}$$

$$\sigma_{\text{demand}} = \frac{M_{\text{max}} y_{\text{top}}}{I} = \frac{(69445.3)(34.8404)}{(418.309 \times 10^3)} = 5.784\,01\,\text{MPa}$$

$$b_{\text{crit},2} = \frac{100 - 80}{2} + \frac{1.27}{2} = 10.635\,\text{mm}$$

$$\sigma_{\text{crit},2} = \frac{K\pi^2 E}{12\left(1-\mu^2\right)} \left(\frac{t}{b_{\text{crit},2}}\right)^2 = \frac{(0.425)\,\pi^2\,(4000)}{12\left[1-(0.2)^2\right]} \left(\frac{1.27}{10.635}\right)^2 = 20.7696\,\text{MPa}$$

$$\text{FOS}_{\text{buck},2} = \frac{\sigma_{\text{crit},2}}{\sigma_{\text{demand}}} = \frac{(20.7696)}{(5.78401)} = 3.59087 = \boxed{3.59}$$

1.1.7 Case 3 Flexural Buckling Failure

$$M_{\text{max}} = 69\,445.3\,\text{N mm} \quad \text{at} \quad x_{\text{train}} \in \{557\,\text{mm}, 644\,\text{mm}\}$$

$$y_{\text{crit},3} = y_{top} - 1.27 = 34.8404 - 2.54 = 33.5704\,\text{mm}$$

$$\sigma_{\text{demand}} = \frac{M_{\text{max}} y_{\text{crit},3}}{I} = \frac{(69445.3)(33.5704)}{(418.309 \times 10^3)} = 5.573\,17\,\text{MPa}$$

$$b_{\text{crit},3} = y_{top} - \frac{3(1.27)}{2} = 34.8404 - 1.905 = 32.9354\,\text{mm}$$

$$\sigma_{\text{crit},3} = \frac{K\pi^2 E}{12\left(1-\mu^2\right)} \left(\frac{t}{b_{\text{crit},3}}\right)^2 = \frac{(6)\,\pi^2\,(4000)}{12\left[1-(0.2)^2\right]} \left(\frac{1.27}{32.9354}\right)^2 = 30.5731\,\text{MPa}$$

$$\text{FOS}_{\text{buck},3} = \frac{\sigma_{\text{crit},3}}{\sigma_{\text{demand}}} = \frac{(30.5731)}{(5.57317)} = 5.48576 = \boxed{5.49}$$

1.1.8 Shear Buckling Failure

$$\tau_{demand} = \frac{VQ(\bar{y})}{Ib(\bar{y})} = \frac{(240)(6192.98)}{(418.309 \times 10^3)(2.54)} = 1.398\,88\,\text{MPa}$$

$$b_{\text{crit},4} = 75 - 1.25 = 73.73\,\text{mm}$$

$$a_{\text{crit},4} = 1200/3 = 400\,\text{mm}$$

$$\tau_{\text{crit}} = \frac{K\pi^2 E}{12\left(1-\mu^2\right)} \left[\left(\frac{t}{b_{\text{crit},4}}\right)^2 + \left(\frac{t}{a_{\text{crit},4}}\right)^2\right] = \frac{(5)\,\pi^2\,(4000)}{12\left[1-(0.2)^2\right]} \left[\left(\frac{1.27}{73.73}\right)^2 + \left(\frac{1.27}{400}\right)^2\right] = 5.256\,62\,\text{MPa}$$

$$\text{FOS}_{\text{buck},4} = \frac{\tau_{\text{crit}}}{\tau_{\text{demand}}} = \frac{(5.25662)}{(1.39888)} = 3.75773 = \boxed{3.76}$$

1.1.9 FOS and P_{fail}

$$\begin{aligned}\text{FOS} &= \min\{\text{FOS}_{\text{tension}}, \text{FOS}_{\text{comp}}, \text{FOS}_{\text{shear}}, \text{FOS}_{\text{glue}}, \text{FOS}_{\text{buck,1}}, \text{FOS}_{\text{buck,2}}, \text{FOS}_{\text{buck,3}}, \text{FOS}_{\text{buck,4}}\} \\ &= \text{FOS}_{\text{buck,1}} \\ &= 0.616688 = \boxed{617 \times 10^{-3}}\end{aligned}$$

$$P_{\text{fail}} = \text{FOS} \times P_{demand} = (0.616688)(400) = 246.675\,\text{N} = \boxed{247\,\text{N}}$$

1.2 Intermediate Code Calculations

```
ybar = 41.431094351923186
I = 4.183522089994237e+05
max(M) = 6.944533333333337e+04
ybot = 41.431094351923186
S_tens_app = 6.877449421183897
ytop = -34.838905648076810
S_comp_app = -5.783163955284811
Q_max = 6.193283330576414e+03
b_centroid = 2.540000000000000
T_max_app = 1.398802523651935
max(V) = 2.400000000000000e+02
Q_glue = 4.343896017305791e+03
b_glue = 12.539999999999999
T_glue_app = 0.198724338232346
S_buck1_app = 5.783163017719362
S_buck1_ult = 3.566926726812471
S_buck2_app = 5.783163017719362
S_buck2_ult = 20.769624320288855
S_buck3_app = 5.572346458284259
S_buck3_ult = 30.573884061715709
T_buck_app = 1.398802523651935
T_buck_ult = 5.256619850378164
```

All intermediate hand calculations perfectly match their respective computer-generated values to slide rule precision, excluding `T_glue_app` which is has a deviation of

$$\varepsilon = \left|0.198724338232346 - 198.754 \times 10^{-3}\right| < 30.0 \times 10^{-6}$$

which is considered consistent with zero.

1.3 Code Output

1.3.1 FOS and P_{fail}

```
FOS_tens = 4.36
FOS_comp = 1.037
FOS_shear = 2.86
FOS_glue = 10.06
FOS_buck1 = 0.617
FOS_buck2 = 3.59
FOS_buck3 = 5.49
FOS_buckV = 3.76

minFOS = 0.617
P_fail = 247 N
```

All hand calculated FOS and failure load values perfectly match their respective computer-generated values to slide rule precision.

1.3.2 Shear Force and Bending Moment Capacities Diagram

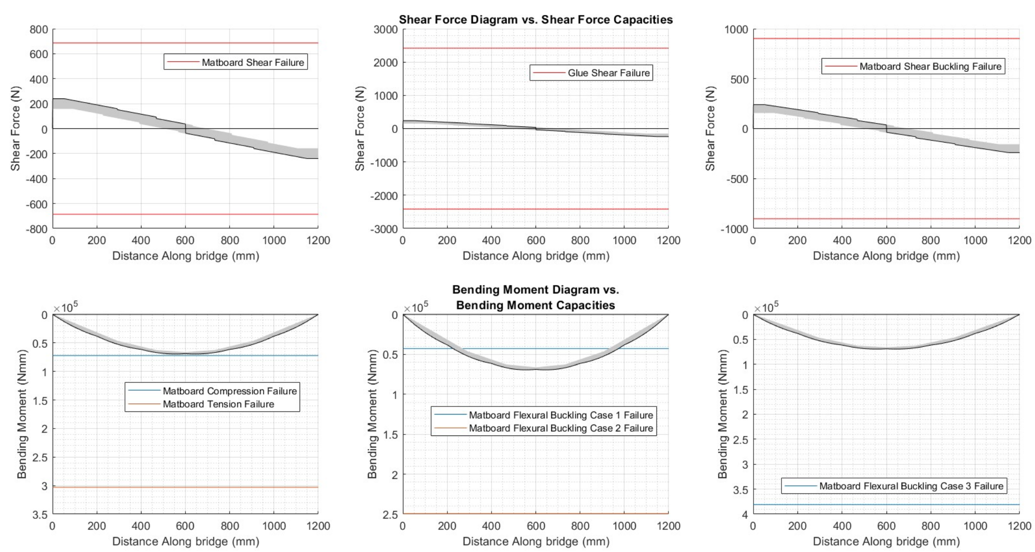

Figure 1. Subplot depicting the shear force and bending moment capacities of design 0 vs. shear force and bending moment envelope as a function of distance along the bridge.

2 Final Design

2.1 Code Output

2.1.1 FOS and P_{fail}

```
FOS_tens = 6.65
FOS_comp = 3.13
FOS_shear = 3.62
FOS_glue = 3.45
FOS_buck1 = 19.13
FOS_buck2 = 45.2
FOS_buck3 = 30.9
FOS_buckV = 3.05

minFOS = 3.05
P_fail = 1220 N
```

2.1.2 Shear Force and Bending Moment Capacities Diagram

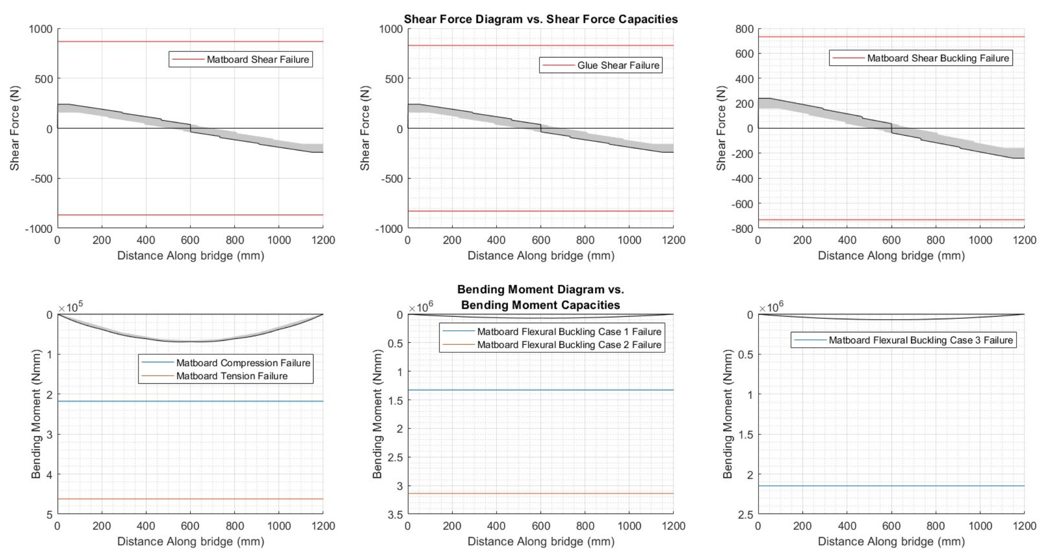

Figure 2. Subplot depicting the shear force and bending moment capacities of the final design vs. shear force and bending moment envelope as a function of distance along the bridge.