

10-07-19

DATE:

$$1) \quad A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \quad C = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$$

$$1. 2A - B$$

$$2A = 2 \times \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$2A - B = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 2-4 \\ 4-5 \\ 6-6 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$$

$$2. \quad A = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \\ = 1\hat{i} + 2\hat{j} + 3\hat{k}$$

$\hat{i}, \hat{j}, \hat{k}$ are unit vectors
along x, y and z axes
respectively

$$|A| = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{1^2 + 2^2 + 3^2} \\ = \sqrt{1+4+9} \\ = \sqrt{14}$$

distance of
vector A from origin
(0,0,0)

$$3. \quad \hat{A} = \frac{A}{|A|} = \frac{1\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{14}} \\ = \frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}$$

$$\Rightarrow \hat{A} \Rightarrow \begin{bmatrix} 1/\sqrt{14} \\ 2/\sqrt{14} \\ 3/\sqrt{14} \end{bmatrix}$$

unit vector \Rightarrow vector of distance
1 from origin (0,0,0)

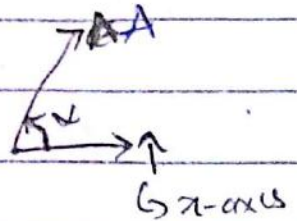
4. Let α, β, γ be the angles
A make with x, y and
z axes.

$$\cos(\alpha)$$

$$\cos(\beta)$$

$$\cos(\gamma)$$

$\left. \begin{matrix} \cos(\alpha) \\ \cos(\beta) \\ \cos(\gamma) \end{matrix} \right\} \rightarrow \text{Direction Cosines}$



Now, angle b/w

two vectors A and B

$$\cos(\theta) = \frac{A \cdot B}{|A||B|}$$

So, angle b/w A and \hat{i}

$$\cos(\alpha) = \frac{A \cdot \hat{i}}{|A||\hat{i}|} = \frac{(1\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (\hat{i})}{\sqrt{14} \cdot 1}$$

$$= \frac{1}{\sqrt{14}}$$

$$\cos(\beta) = \frac{A \cdot \hat{j}}{|A||\hat{j}|} = \frac{2}{\sqrt{14}}$$

$$\cos(\gamma) = \frac{3}{\sqrt{14}}$$

$$\begin{aligned} \hat{i} \cdot \hat{i} &= 1 & \hat{i} \cdot \hat{j} &= 0 & \hat{i} \cdot \hat{k} &= 0 \\ \hat{j} \cdot \hat{j} &= 1 & \hat{j} \cdot \hat{i} &= 0 & \hat{j} \cdot \hat{k} &= 0 \\ \hat{k} \cdot \hat{k} &= 1 & \hat{k} \cdot \hat{i} &= 0 & \hat{k} \cdot \hat{j} &= 0 \end{aligned}$$

$$\begin{aligned} \hat{i} \times \hat{i} &= 0 \\ \hat{j} \times \hat{j} &= 0 \\ \hat{k} \times \hat{k} &= 0 \end{aligned}$$

$a \times b = |a||b|\sin\theta$
 $\theta \rightarrow$ angle b/w a & b
 $\hat{n} \rightarrow$ unit vector \perp to a & b

Scalar $\leftarrow a \cdot b = |a||b|\cos\theta$

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Vector

5. $A \cdot B = (\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (4\hat{i} + 5\hat{j} + 6\hat{k})$
 $= 4 + 10 + 18 = 32$

$B \cdot A = 32$

6. Angle b/w A and $B \rightarrow \theta$

$$\cos\theta = \frac{A \cdot B}{|A||B|} = \frac{32}{\sqrt{14}\sqrt{77}} \quad \begin{matrix} 16 \\ 28 \\ 36 \end{matrix}$$

$$\theta = \cos^{-1}\left(\frac{32}{\sqrt{14}\sqrt{77}}\right) \quad 77$$

7. Let d be perpendicular to A

$$d = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ or } d = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\Rightarrow A \cdot d = 0 \Rightarrow$$

$$(\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (x\hat{i} + y\hat{j} + z\hat{k}) = 0$$

$$x + 2y + 3z = 0$$

\hookrightarrow Infinite no. of solution

Let $y = 1$ $z = 1$

$$\Rightarrow x + 2(1) + 3(1) = 0$$

$$x = -5$$

$$d = \begin{bmatrix} -5 \\ 1 \\ 1 \end{bmatrix} \text{ is } \perp \text{ to } A$$

8. $A \times B = (\hat{i} + 2\hat{j} + 3\hat{k}) \times (4\hat{i} + 5\hat{j} + 6\hat{k})$

$$\begin{aligned} &= \hat{i} \times 4\hat{i} + \hat{i} \times 5\hat{j} + \hat{i} \times 6\hat{k} + \\ &\quad 2\hat{j} \times 4\hat{i} + 2\hat{j} \times 5\hat{j} + 2\hat{j} \times 6\hat{k} + \\ &\quad 3\hat{k} \times 4\hat{i} + 3\hat{k} \times 5\hat{j} + 3\hat{k} \times 6\hat{k} \end{aligned}$$

$$= 0 + 5\hat{k} - 6\hat{j} + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0$$

$$\begin{aligned} &= 0 + 5\hat{k} - 6\hat{j} + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 \\ &= 0 + 5\hat{k} - 6\hat{j} + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 \\ &= 0 + 5\hat{k} - 6\hat{j} + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 \end{aligned}$$

$$= (12\hat{i} - 15\hat{j}) + (-6\hat{j} + 12\hat{i}) + (-5\hat{k} - 8\hat{k})$$

$$= -3\hat{i} + 6\hat{j} - 3\hat{k}$$

$$A \times B = \begin{bmatrix} -3 \\ 6 \\ -3 \end{bmatrix} \quad \text{Similarly, } B \times A = \begin{bmatrix} 3 \\ -6 \\ 3 \end{bmatrix}$$

9. $A \times B$ is \perp to both A & B

10. $x A + y B + z C = 0$ x, y, z are test numbers

$$x \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + z \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} x \\ 2x \\ 3x \end{bmatrix} + \begin{bmatrix} 4y \\ 5y \\ 6y \end{bmatrix} + \begin{bmatrix} -z \\ z \\ 3z \end{bmatrix} = 0$$

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$$10) \begin{bmatrix} x+4y-z \\ 2x+5y+z \\ 3x+6y+3z \end{bmatrix} = 0$$

Three linear equations:

$$x+4y-z=0 \quad \text{--- (1)}$$

$$2x+5y+z=0 \quad \text{--- (2)}$$

$$3x+6y+3z=0 \quad \text{--- (3)}$$

$$\textcircled{1} + \textcircled{2} \Rightarrow 3x+9y+0z=0$$

$$3x+9y=0$$

$$\textcircled{1} \quad x+4y-z=0$$

$$+ \quad 2x+5y+z=0$$

$$\textcircled{2} \quad 3x+9y=0$$

$$3x = -9y \Rightarrow x = -3y \quad \text{--- (4)}$$

$$3x+12y-3z=0$$

$$3x+6y+3z=0$$

$$6z=0$$

$$2x+5y-2z=0$$

$$2x+5y+z=0$$

$$3y-3z=0$$

$$\Rightarrow y=z \quad \text{--- (5)}$$

$$\textcircled{4} \Rightarrow \text{---} \quad \textcircled{5} \Rightarrow y=z$$

$$-\frac{x}{3} = y = z$$

$$\text{Let } y=1 \Rightarrow z=1$$

$$x = -3 \times 1 = -3$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$$

$$-3A+B+C=0$$

$$1) \quad A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

$$A^T B = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = 1 \times 4 + 2 \times 5 + 3 \times 6 = 32$$

A \rightarrow Vector or 3×1 matrix

$$A^T \rightarrow 1 \times 3 \quad B \rightarrow 1 \times 3 \quad 3 \times 1$$

$$B \rightarrow 3 \times 1$$

\downarrow
1x1

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⊗ AB^T $B^T = \begin{bmatrix} 4 & 5 & 6 \end{bmatrix}$

$A \rightarrow 3 \times 1$ $AB^T \rightarrow 3 \times 1 \cdot 1 \times 3$
 $B \rightarrow 1 \times 3$
 \downarrow
 3×3

$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 1 \times 4 & 1 \times 5 & 1 \times 6 \\ 2 \times 4 & 2 \times 5 & 2 \times 6 \\ 3 \times 4 & 3 \times 5 & 3 \times 6 \end{bmatrix}$
 $AB^T = \begin{bmatrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 12 & 15 & 18 \end{bmatrix}$

$B_3 \cdot B_2 = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix} = 3 \times 2 - 2 \times 1 + 1 \times -4$
 $= 6 - 2 - 4 = 0$

$B_1 \cdot B_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix} = 1 \times 2 + 2 \times 1 + 1 \times -4$
 $= 2 + 2 - 4 = 0$

For any vector \perp
~~orthogonal~~

$\Rightarrow B_1, B_2, B_3$ are row vectors of B
 are \perp

4. $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}$

B)

5. Orthogonal set \Rightarrow
 all vectors in the set are
 orthogonal (perpendicular $\Rightarrow a \cdot b = 0$)
 to each other

$B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -3 & 1 \end{bmatrix}$

~~$B_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$~~ $B_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ $B_2 = \begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix}$
 $B_2 \cdot B_3 = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$

$B_1 \cdot B_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} = 1 \times 3 + 2 \times -2 + 1 \times 1$
 $= 3 - 4 + 1 = 0$

$\begin{array}{ccc} \begin{array}{c|cc} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{array} & \begin{array}{c|cc} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{array} & \begin{array}{c|cc} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{array} \\ \downarrow & \downarrow & \downarrow \\ 2(4 \times 1 - 3 \times 0) & 2(4 \times 1 - 3 \times 0) & 3(4 \times 5 - 2 \times 0) \\ 2(-4 - 0) & 2(-4 - 0) & 3(20 - 0) \\ 1(-2 \times -1 - 5 \times 3) & 1(-2 \times -1 - 5 \times 3) & \\ = 2 - 15 & = 2 - 15 & \\ = -13 & = -13 & \\ \downarrow & \downarrow & \\ -13 & -13 & 60 \\ \downarrow & \downarrow & \\ -13 + -(-8) + 60 & = 55 & \end{array}$