10-07-19 [redacted]

DATE:

A) $A = \begin{bmatrix}1\\2\\3\end{bmatrix}$ $B = \begin{bmatrix}4\\5\\6\end{bmatrix}$ $C = \begin{bmatrix}7\\1\\3\end{bmatrix}$

1. $2A - B$

$$2A = 2 * \begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{bmatrix}2\\4\\6\end{bmatrix}$$

$$2A - B = \begin{bmatrix}2\\4\\6\end{bmatrix} - \begin{bmatrix}4\\5\\6\end{bmatrix} = \begin{bmatrix}2-4\\4-5\\6-6\end{bmatrix} = \begin{bmatrix}-2\\-1\\0\end{bmatrix}$$

2. $A = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$= 1\hat{i} + 2\hat{j} + 3\hat{k}$

$\hat{i}, \hat{j}, \hat{k}$ are unit vectors along x, y and z axes respectively

$$|A| = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{1^2+2^2+3^2}$$
$$= \sqrt{1+4+9}$$
$$= \sqrt{14}$$

distance of vector A from origin (0, 0, 0)

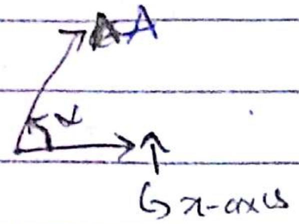
3. $$\hat{A} = \frac{A}{|A|} = \frac{\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{14}}$$
$$= \frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}$$

$$\Rightarrow \hat{A} \rightarrow \begin{bmatrix}1/\sqrt{14}\\2/\sqrt{14}\\3/\sqrt{14}\end{bmatrix}$$

unit vector ⇒ vector of distance 1 from origin (0, 0, 0)

4. Let α, β, γ be the angles A make with x, y and z axes.

$$\left.\begin{matrix}\cos(\alpha)\\ \cos(\beta)\\ \cos(\gamma)\end{matrix}\right\} \rightarrow \text{Direction Cosines}$$



Now, angle b/w two vectors A and B is $\cos(\theta) = \frac{A \cdot B}{|A||B|}$

So, angle b/w A and $\hat{i}$

$$\cos(\alpha) = \frac{A \cdot \hat{i}}{|A||\hat{i}|} = \frac{(\hat{i}+2\hat{j}+3\hat{k})\cdot(\hat{i})}{\sqrt{14}\sqrt{1}}$$
$$= \frac{1}{\sqrt{14}}$$

$$\cos(\beta) = \frac{A \cdot \hat{j}}{|A||\hat{j}|} = \frac{2}{\sqrt{14}}$$

$$\cos(\gamma) = \frac{3}{\sqrt{14}}$$

$\hat{i}\cdot\hat{i} = 1$ $\quad \hat{i}\cdot\hat{j}=0 \quad \hat{i}\cdot\hat{k}=0$ $\quad \hat{i}\times\hat{i}=0$
$\hat{j}\cdot\hat{j} = 1$ $\quad \hat{j}\cdot\hat{i}=0 \quad \hat{j}\cdot\hat{k}=0$ $\quad \hat{j}\times\hat{j}=0$
$\hat{k}\cdot\hat{k} = 1$ $\quad \hat{k}\cdot\hat{i}=0 \quad \hat{k}\cdot\hat{j}=0$ $\quad \hat{k}\times\hat{k}=0$

$\Rightarrow a\times b = |a||b|\sin\theta\,\hat{n}$
$\theta \to$ angle b/w a & b
$\hat{n} \to$ unit vector $\perp$ to a & b

scalar $\leftarrow$ $a\cdot b = |a||b|\cos\theta$ $\quad$ DATE: $\quad$ Vector

5. $A\cdot B = (1\hat{i}+2\hat{j}+3\hat{k})\cdot(4\hat{i}+5\hat{j}+6\hat{k})$
$= 4+10+18 = \underline{32}$

$B\cdot A = \underline{32}$

6. Angle b/w A and B $\to \theta$

$\cos\theta = \dfrac{A\cdot B}{|A||B|} = \dfrac{32}{\sqrt{14}\sqrt{77}}$ $\qquad$ 16 + 25 + 36 = 77

$\theta = \cos^{-1}\left(\dfrac{32}{\sqrt{14}\sqrt{77}}\right)$

7. Let d be perpendicular to A

$d = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ or $d = x\hat{i}+y\hat{j}+z\hat{k}$

$\Rightarrow A\cdot d = 0 \Rightarrow$

$(\hat{i}+2\hat{j}+3\hat{k})\cdot(x\hat{i}+y\hat{j}+z\hat{k}) = 0$

$x + 2y + 3z = 0$

$\hookrightarrow$ Infinite no. of solution

Let $y=1 \quad z=1$

$\Rightarrow x + 2*1 + 3*1 = 0$

$x = -5$

$d = \begin{bmatrix} -5 \\ 1 \\ 1 \end{bmatrix}$ is $\perp$ to A

8. $A\times B = (\hat{i}+2\hat{j}+3\hat{k})\times(4\hat{i}+5\hat{j}+6\hat{k})$
$= \hat{i}\times4\hat{i} + \hat{i}\times5\hat{j} + \hat{i}\times6\hat{k} +$
$2\hat{j}\times4\hat{i} + 2\hat{j}\times5\hat{j} + 2\hat{j}\times6\hat{k} +$
$3\hat{k}\times4\hat{i} + 3\hat{k}\times5\hat{j} + 3\hat{k}\times6\hat{k}$

$=$ ~~0+ ...~~
$4(0) + 5(\hat{k}) + 6(-\hat{j}) +$
$8(-\hat{k}) + 10(0) + 12(\hat{i}) +$
$12(\hat{j}) + 15(-\hat{i}) + 18(0)$
$= 0 + 5\hat{k} - 6\hat{j} - 8\hat{k} + 0 + 12\hat{i}$
$+ 12\hat{j} - 15\hat{i} + 0$
$= (12\hat{i} - 15\hat{i}) + (-6\hat{j} + 12\hat{j}) +$
$(5\hat{k} - 8\hat{k})$
$= -3\hat{i} + 6\hat{j} - 3\hat{k}$

$A\times B = \begin{bmatrix} -3 \\ 6 \\ -3 \end{bmatrix}$ $\quad$ Similarly, $B\times A = \begin{bmatrix} 3 \\ -6 \\ 3 \end{bmatrix}$

9. $A\times B$ is $\perp$ to both A & B

10. $xA + yB + zC = 0$ $\quad$ x, y, z are real numbers

$x\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + z\begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix} = 0$

$\begin{bmatrix} x \\ 2x \\ 3x \end{bmatrix} + \begin{bmatrix} 4y \\ 5y \\ 6y \end{bmatrix} + \begin{bmatrix} -z \\ z \\ 3z \end{bmatrix} = 0$

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10) $\begin{bmatrix} x+4y-z \\ 2x+5y+z \\ 3x+6y+3z \end{bmatrix} = 0$

Three linear equations:

$x + 4y - z = 0$ — ①

$2x + 5y + z = 0$ — ②

$3x + 6y + 3z = 0$ — ③

~~① + ② ⇒ 3x + 9y + 0z = 0~~

$3 \times$ ① $-$ ③ ⇒

$$\begin{array}{rl} ① & x + 4y - z = 0 \\ + & \\ ② & 2x + 5y + z = 0 \\ \hline & 3x + 9y = 0 \end{array}$$

$3x = -9y \Rightarrow x = -3y$ — ④

$$\begin{array}{rl} 3\times① & 3x + 12y - 3z = 0 \\ + & \cancel{3x + 6y + 3z = 0} \\ ③ & \cancel{6x} \end{array}$$

$$\begin{array}{rl} 2\times① & 2x + 8y - 2z = 0 \\ - & 2x + 5y + z = 0 \\ ② & \hline \\ & 3y - 3z = 0 \end{array}$$

$\Rightarrow y = z$ — ⑤

④ ⇒ [illegible] ⑤ ⇒ $y = z$

$-\frac{x}{3} = y = z$

Let $y = 1 \Rightarrow z = 1$

$x = -3 \times 1 = -3$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$$

$-3A + B + C = 0$

11) $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$

$A^T B = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = 1 \times 4 + 2 \times 5 + 3 \times 6$

$= 32$

$A \rightarrow$ Vector or 3×1 matrix

$A^T \rightarrow 1 \times 3$

$B \rightarrow 3 \times 1$

$A^T B = 1 \times 3 \quad 3 \times 1$

$\downarrow$

1×1

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AB^T $\quad B^T = [4 \quad 5 \quad 6]$

$A \to 3\times1$
$B \to 1\times3$

$AB^T \quad 3\times1 \quad 1\times3$
$\Downarrow$
3×3

$$\begin{bmatrix}1\\2\\3\end{bmatrix}[4 \quad 5 \quad 6] = \begin{bmatrix}1*4 & 1*5 & 1*6\\2*4 & 2*5 & 2*6\\3*4 & 3*5 & 3*6\end{bmatrix}$$

$$AB^T = \begin{bmatrix}4 & 5 & 6\\8 & 10 & 12\\12 & 15 & 18\end{bmatrix}$$

B)

~~5.~~ orthogonal set $\Rightarrow$
all vectors in the set are orthogonal (perpendicular $\Rightarrow a\cdot b = 0$) to each other

$$B = \begin{bmatrix}1 & 2 & 1\\2 & 1 & -4\\3 & -2 & 1\end{bmatrix}$$

~~$B_1 = [1 \quad 2 \quad 1]$~~ $\quad B_1 = \begin{bmatrix}1\\2\\1\end{bmatrix} \quad B_2 = \begin{bmatrix}2\\1\\-4\end{bmatrix}$

~~B_2~~ $B_3 = \begin{bmatrix}3\\-2\\1\end{bmatrix}$

$$B_1 \cdot B_3 = \begin{bmatrix}1\\2\\1\end{bmatrix}\cdot\begin{bmatrix}3\\-2\\1\end{bmatrix} = 1*3 + 2*-2 + 1*1$$
$$= 3 - 4 + 1 = 0$$

$$B_3 \cdot B_2 = \begin{bmatrix}3\\-2\\1\end{bmatrix}\begin{bmatrix}2\\1\\-4\end{bmatrix} = 3*2 - 2*1 + 1*-4$$
$$= 6 - 2 - 4 = 0$$

$$B_1 \cdot B_2 = \begin{bmatrix}1\\2\\1\end{bmatrix}\cdot\begin{bmatrix}2\\1\\-4\end{bmatrix} = 1*2 + 2*1 + 1*-4$$
$$= 2 + 2 - 4 = 0$$

~~for any vector ... ...~~

$\Rightarrow B_1, B_2, B_3$ i.e. row vectors of B are $\perp$

4. $$A = \begin{bmatrix}1 & 2 & 3\\4 & -2 & 3\\0 & 5 & -1\end{bmatrix}$$

$$\begin{array}{c|cc}1 & 2 & 3\\\hline 4 & -2 & 3\\0 & 5 & -1\end{array} \qquad \begin{array}{c|c|c}1 & 2 & 3\\\hline 4 & -2 & 3\\0 & 5 & -1\end{array} \qquad \begin{array}{cc|c}1 & 2 & 3\\\hline 4 & -2 & 3\\0 & 5 & -1\end{array}$$

1 $(-2*-1 - 5*3) =$
$= 2 - 15$
$= -13$

$2(4*-1 - 3*0)$
$2(-4 - 0)$
-8

$3(4*5 - -2*0)$
$3(20 - 0)$
60

$-13 + -(-8) + 60 = \underline{\underline{55}}$