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C. $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$

1) ~~If~~

$AV = \lambda V$

V → eigenvector, λ → eigenvalue

λI : λ is a scalar

$I \Rightarrow$ Identity matrix

$(A - \lambda I) V = 0$

Let eigen vector V, be non zero

$\Rightarrow (A - \lambda I) = 0 \Rightarrow |A - \lambda I| = 0$

$\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$

$\begin{bmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{bmatrix} = 0$

$(1-\lambda)(2-\lambda) - 2 * 3 = 0$

$2 - \lambda - 2\lambda + \lambda^2 - 6 = 0$

$\lambda^2 - 3\lambda - 4 = 0$

$\lambda^2 - 4\lambda + \lambda - 4 = 0$

$\lambda(\lambda - 4) + 1(\lambda - 4) = 0$

$(\lambda - 4)(\lambda + 1) = 0 \Rightarrow \lambda = 4, -1$

Now, let $\lambda = 4$ $V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

$(A - \lambda I) V = 0$

$\begin{bmatrix} 1-4 & 2 \\ 3 & 2-4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = 0 \Rightarrow \begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = 0$

$\begin{bmatrix} -3V_1 + 2V_2 \\ 3V_1 - 2V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow 3V_1 - 2V_2 = 0$

$3V_1 = 2V_2$

$\Rightarrow$ any V_1, V_2 satisfying this eqn is an eigen vector of A $\Rightarrow V_1 = 2 \Rightarrow V_2 = 3$

$V_{\lambda=4} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

let $\lambda = -1$

$\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = 0 \Rightarrow V_1 = V_2$

$V_{\lambda=-1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

2) $V = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}$ $C(V) = \begin{bmatrix} 1 & -3 \\ -1 & 2 \end{bmatrix}$

$\text{adj}(V) = C(V)^T = \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix}$

$|V| = 2 - 3 = -1$

$V^{-1} = \frac{1}{|V|} \text{adj}(A) = -1 * \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix}$

$V^{-1} = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}$

~~$AV = A \begin{bmatrix} V^1 & V^2 \end{bmatrix} = \begin{bmatrix} AV_1 & AV_2 \end{bmatrix}$~~

~~$AV = A \begin{bmatrix} V_{\lambda=4} & V_{\lambda=-1} \end{bmatrix} = \begin{bmatrix} AV_{\lambda=4} & AV_{\lambda=-1} \end{bmatrix}$~~

$AV = \begin{bmatrix} \lambda^1 V^1 & \lambda^2 V^2 \end{bmatrix} = \begin{bmatrix} 4 \begin{bmatrix} 2 \\ 3 \end{bmatrix} & -1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{bmatrix}$

$= \begin{bmatrix} 8 & -1 \\ 12 & -1 \end{bmatrix}$

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$\vec{V}^{-1}AV = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 8 & -1 \\ 12 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 0 & -5 \end{bmatrix}$

3. $V_A^1 \cdot V_A^2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 5$

A ⇒

4. Eigen vectors of B, $\begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$

$BV = \lambda V$

$|B - \lambda I| = 0 \Rightarrow \begin{vmatrix} 2-\lambda & -2 \\ -2 & 5-\lambda \end{vmatrix} = 0$

$(2-\lambda)(5-\lambda) - 4 = 0$

$10 - 2\lambda - 5\lambda + \lambda^2 - 4 = 0 \Rightarrow \lambda^2 - 7\lambda + 6 = 0$

$\lambda^2 - 6\lambda - \lambda + 6 = 0$

$(\lambda - 1)(\lambda - 6) = 0 \Rightarrow \lambda = 1, 6$

$\lambda = 1$

$\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$

$v_1 - 2v_2 = 0$

$v_1 = 2v_2$

$v_B^1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\lambda = 6$

$\begin{bmatrix} -4 & -2 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$

$-4v_1 - 2v_2 = 0$

$v_2 = -2v_1$

$v_B^2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$v_B^1 \cdot V_B^2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix} = 0$

5. Eigen vectors of B are perpendicular to each other ⇒ orthogonal

∵ B is ~~an~~ symmetric matrix

Proof: $V = [V^1 \; V^2]$

~~B and V^1 and V^2 are different eigenvectors~~ $\lambda = [\lambda^1 \; \lambda^2]$

$\lambda^1 \neq \lambda^2$

Now,

$V^1 = \begin{bmatrix} v_1^1 \\ v_2^1 \end{bmatrix}_{2\times 1}$ $V^2 = \begin{bmatrix} v_1^2 \\ v_2^2 \end{bmatrix}_{2\times 1}$

we know $BV = \lambda V$

$(AB)^T = B^T A^T$

$B = B^T$

$$\lambda^1 V^1 \cdot V^2 = (\lambda^1 V^1)^T V^2$$
$$= (BV^1)^T V^2$$
$$= V^1 B^T V^2$$
$$\boxed{= V^1 B V^2 \\ = V^1(BV^2)} \rightarrow \text{(symmetric}$$
$$= V^1(\lambda^2 V_2^2)$$

$\lambda^1 V^1 \cdot V^2 = \lambda^2 \; V^1 \cdot V_B^2$

$(\lambda^1 - \lambda^2)\, V_1^1 \cdot V^2 = 0$ As $\lambda^1 \neq \lambda^2$

$V_B^1 \cdot V^2 = 0$

⇒ V^1, V^2 are orthogonal.

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$$\frac{d}{dx}(x^n) = n x^{n-1} \qquad \frac{d}{dx}(x) = 1 \cdot x^{1-1} = 1$$

$$\frac{d}{dx}(x^2) = 2x^{2-1} = 2x$$

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D) $f(x) = x^2+3 \qquad g(x,y) = x^2+y^2$

1. $f'(x) = \frac{d}{dx}(f(x)) = \frac{d}{dx}(x^2+3)$

$$= \frac{d}{dx}(x^2) + \frac{d}{dx}(3)$$

$$= 2x + 0 = 2x$$

$$f''(x) = \frac{d}{dx}(f'(x)) = \frac{d}{dx}(2x)$$

$$= 2\frac{d}{dx}(x) = 2*1 = 2$$

2. $\frac{\partial g}{\partial x} = \frac{\partial}{\partial x}(x^2+y^2) = \frac{\partial}{\partial x}(x^2) + \frac{\partial}{\partial x}(y^2)$

$$= 2x + 0$$

$$= 2x$$

W.r.t x ⇒ all other variables are considered as constants

$$\frac{\partial g}{\partial y} = \frac{\partial}{\partial y}(x^2+y^2) = \frac{\partial}{\partial y}(x^2) + \frac{\partial}{\partial y}(y^2)$$

$$= 0 + 2y = 2y$$

3. $\nabla g(x,y) = \begin{bmatrix} \frac{\partial g}{\partial x} \\ \frac{\partial g}{\partial y} \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$

gradient

4. $p(x;\mu,\sigma) = \cancel{\frac{1}{\sqrt{2\pi}\sigma}} \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right]$