Day: Monday

Date: March 10, 25

: Mathematical Expectation :

Basic Furmulae :

1. $E(X+Y) = E(X) + E(Y)$
2. $E(X_1 + X_2 + \cdots + X_n) = E(X_1) + \cdots E(X_n)$
3. $E(XY) = E(X)\,E(Y)$
4. $E[ax] = a\,E(X)$

For any Random variable $x > 0$, $E(X) > 0$
if R. Var X & Y are $Y \geq x$;
$E(Y) \geq E(X)$

Questions : Two independent RV with parameters :

$E(X) = 4$; $V(X) = 3$
$E(Y) = 9$; $V(Y) = 6$

i) $E(X+Y+2)$
$= E(X) + E(Y) + 2$
$= 4 + 9 + 2$
$= 15$

ii) $E[3x + 2y - 5]$
$= 3E(X) + 2E(Y) - 5 = 3(4) + 2(9) - 5$
$= 25$

iii) $V(3X+2)$
$= 3^2 V(X) = 3($
$= 3^2 V(X) = 3^2(3)$
$= 27$

$V(a) = 0$
$\because$ a is constant

$V[ax+b]$ $\quad$ a & b $\rightarrow$ const.
$= a^2 V(x)$

iv) $V[2(X+Y+1)]$
$= 2^2 V(X) + 2^2 (Y)$
$= 4(3) + 4(6)$
$= 36$

$V(a_1 X_1 + a_2 X_2 + \cdots + a_n X_n)$
$= a^2 V(X_1) \cdots a_n^2 V(X_n)$

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Mean of a Random Variable: (u_x / u)

Assume a fair coin was tossed thrice, the sample space of our experiment is:

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

'x' is the random variable

↳ in our case: how many heads coming ....

favorable / total..

x	$f(x)$	$xf(x)$	$x^2f(x)$	$(x-u)^2f(x)$
0	1/8	0	0	$(0-1.5)^2 \times 1/8$
1	3/8	3/8	3/8	
2	3/8	6/8	12/8	
3	1/8	3/8	9/8	
	$\Sigma = 1$	$= 12/8$	$= 24/8 = 3$	

$$\boxed{u = E(x) = \Sigma x f(x)} = \frac{12}{8} = 1.5$$

$$\approx 2.$$

$$V(x) = x^2 f(x) - (u)^2$$
$$= 3 - (1.5)^2 = 0.75$$

Day: Date:

Question :

In A shipment of 8 similar micro-computers to a retail outlet contains 3 hour defective and 5 hour non-defective.

If a school make random purchase of two of these computers Find the expected number of defective computers purchased.

$X \sim$ no. of defective comp. purchased.

$X = x : 0, 1, 2$ (no. of defective computers)

x	$f(x)$	$xf(x)$
0	5/14	0
1	15/28	15/28
2	3/28	6/28
	1	$\Sigma = 21/28$

$\cancel{x^2 f(x)}$ $\cancel{(x-u)^2 f x}$

$n(S) = \text{comb.}$

$= \binom{8}{2} = {}^8C_2$

$$f(x) = \frac{\binom{3}{x}\binom{5}{2-x}}{\binom{8}{2}}$$

$$E(X) = \frac{21}{28}$$

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Question:

A box containing 7 components is sampled by a quality inspector, the box contain 4 good components, 3 defective comp. A sample of 3 is taken by the inspector. Find the expected value of no. of good component.

X = no. of good comp

$X : x = 0 \quad 1 \quad 2 \quad 3$

x	$f(x)$	$x(f(x))$
0	1/35	0
1	12/35	12/35
2	18/35	36/35
3	4/35	12/35
	$\Sigma = 1$	$\Sigma = 12/7 = 1.7$

$$f(x) = \frac{\binom{4}{x}\binom{3}{3-x}}{\binom{7}{3}}$$

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Let x be random variable with prob distribution $f(x)$; expected value of random var $g(X)$ is :

$$\mu_{g(x)} = E[g(X)] = \sum_x g(x) f(x)$$

$\longrightarrow$ $X \rightarrow$ is discrete.

$$\mu_{g(x)} = E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x)\, dx$$

$\rightarrow$ X is contineous

For Discrete:

$$\mu_{g(x,y)} = E[g(X,Y)] = \sum_x \sum_y g(x,y) f(x,y)$$

For Cont. :

$$\mu_{g(x,y)} = E[g(X,Y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) f(x,y)\, dx\, dy$$

Variance :

for cont :

$$V(X) = \sigma_X^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$$

×(A box containing 7 components is sampled by a quality ;)×

1) Let 'x' be a discrete random variable with the following probability distribution

x	$f(x)$
0	10/28
1	15/28
2	3/28

$(x-1)^2 f(x)$

Find expected value of $E[g(x)]$

$\because \quad g(x) = (x-1)^2$

M1 $E[g(x)] = \sum g(x)f(x) = \sum (x-1)^2 f(x)$

M2 $E[(x-1)^2] = E[X^2 + 1 - 2X]$

$= E(X^2) + 1 - 2E(X)$

$= \sum x^2 f(x) + 1 - 2\sum x f(x)$

x	0	1	2
$f(x)$	10/28	15/28	3/28
$(x-1)^2 f(x)$	10/28	0	3/8

$\sum = 13/28 = 0.463$

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2) Suppose that the no. of car 'x' that pass through a car wash between 4 and 5 pm on any sunny Friday has the following probability distribution

x	4	5	6	7	8	9
$P(X=x)$	$1/12$	$1/12$	$1/4$	$1/4$	$1/6$	$1/6$

Let $g(x) = 2x - 1$ represent the amount of money in Dollars; paid to the attendent by the manger. Find the attendents expected money for the given time interval.

M_1: $\Sigma(2X-1)f(x)$

$7(1/12) +$
$9(1/12) +$
$11(1/4) +$
$13(1/4) +$
$15(1/6) +$
$17(1/6) =$

M_2: $2E(X) - 1 = 2[\Sigma x f(x)] - 1$

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Let x and y be the random variables with the joint probability distribution

$f(x,y)$	x = 0	1	2	$f(y)$
0	3/28	9/28	3/28	15/28
1	3/14	3/14	0	3/7
2	1/28	0	0	1/28
	5/14	15/28	3/28	

Find the expected value of

$$g(x,y) = xy$$

$$E(XY) = \sum_x \sum_y g(x,y) f(x,y)$$

$$= \sum\sum xy\, f(x,y)$$

$$= \left(0 \times 0 \times \frac{3}{28}\right) + \left(0 \times 1 \times \frac{3}{14}\right) + \cdots + (2 \times 2 \times 0)$$

$$= \frac{3}{14}$$

$$\mu_x = E(X) = \sum x f(x) = 0.75 = \frac{3}{4}$$

$$\mu_y = E(Y) = \sum y f(y) = 1/2 = 0.5$$

$$S_x = \sqrt{Var(X)}$$
$$= \sqrt{\sum (X - \mu_x)^2 f(x)}$$

or $\sqrt{\sum x^2 f(x) - (\mu x)^2} = 0.6338$

$$S_y = \sqrt{Var(Y)} = \sqrt{\sum (y - \mu_y)^2 f(y)}$$
$$\sqrt{9/28} = 0.5669$$

$E(XY) = E(X)E(Y)$
X \$ Y are independent

$$\frac{3}{14} \neq \left(\frac{3}{4}\right)\left(\frac{1}{2}\right)$$

X \$ Y are not independent.

Pearson's coef of correlation $\quad -1 \leq x \leq +1$

strong $\quad$ moderate $\quad$ weak $\quad$ moderate $\quad$ strong

$-1 \leftarrow \quad \longrightarrow \quad | \quad \longleftarrow \quad \rightarrow +1$

$\qquad\qquad\qquad 0$

perfect inverse

sample $\qquad\qquad\qquad$ population

$r_{xy} = r_{yx} = r \;;\; e = e_{yx} = e_{xy}$

$$\frac{Cov(x, y)}{S.dx . S.dy}$$

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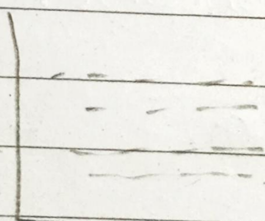
$$r = \frac{S_{xy}}{S_x S_y} \qquad \rho = \frac{S_{xy}}{S_x S_y}$$

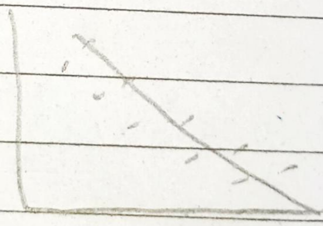
$$\rho = \frac{-9/56}{\sqrt{(45/112)(9/28)}} = -0.4472$$

A weak linear relation b/w x and y.

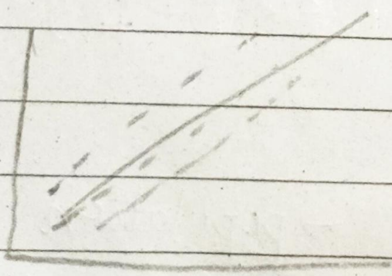
* $$\text{Cov}(x, y) = E(xY) - \mu_x \mu_y$$
$$= E(XY) - E(X)E(Y)$$
$$= 3/14 - (3/4)(1/2) = -9/56$$



correlation = 0



negative -1



+1

Day: Wednesday Date: 19March25

$\int_0^1 2(1-x) = 1$

1. In a machine learning model, time taken (in hours) to process a data batch follows a contineous R.V. The pdf is given as:

$$f(x) = \begin{cases} K(1-x) & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

a) Find K
b) $E[(g(x)]$; $f(x) = x^2$
c) Find cdf.

2. Two ball points pens are selected at random from a box that contains three blue pens, two red pens and three green pens.
If x is the number of blue pens selected and 'y' is the number of red pen selected (a) Find the joint probability function and joint probability distribution.

b) $P[(X,Y) \in A]$ A is the region $\{(x,y) \mid x+y \le 1\}$

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c) Find the conditional distribution of X, given that Y = 1 and use it to answer P(X = 0 | Y = 1)

(a) joint prob. function & joint prob distribution

$$f(x\,y) = \frac{\binom{3}{x}\binom{2}{y}\binom{3}{2-x-y}}{\binom{8}{2}}$$

y \ x	0	1	2
0	3/28	9/28	3/28
1	6/28	6/28	0
2	1/28	0	0

b)

$x + y \leq 1$ ← where A is the region

$$= \frac{3}{28} + \frac{6}{28} + \frac{9}{28} = \frac{9}{14}$$

(c) $P(X=0 \mid Y=1) = \dfrac{P(X=x \cap Y=1)}{P(X=1)}$

$\because P(X=0 \mid Y=1) = \dfrac{3/14}{\frac{3}{14} + 3/14} = 1/2$

$\because P(X=1 \mid Y=1) = \dfrac{6/28}{3/14 + 3/14} = \cancel{0}\ 1/2$

x	0	1	2
$y=1$	1/2	1/2	0

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Ques 1 Solution:

a) Find k:

$$\int_0^1 k(1-x)\,dx = 1$$

$$k\int_0^1 (1-x)\,dx = 1$$

$$k\left[x - \frac{x^2}{2}\right]\Big|_0^1 = 1$$

$$k\left[1-\frac{1}{2}\right] - k\cancel{\left[0-\frac{0}{2}\right]} = 1$$

0

$$\frac{k}{2} = 1$$

$$\boxed{k = 2}$$

(b) $E[g(x)]$ $\qquad \because f(x) = x^2$

$$= \int_{-\infty}^{\infty} g(x) f(x)\,dx$$

$$= \int_0^1 x^2 [2(1-x)]\,dx$$

$$= 2\int_0^1 (x^2 - x^3)\,dx$$

$$= 2\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$$

$$= 2\left[\frac{1}{3} - \frac{1}{4}\right] - 2[0]$$

$$= 2\left[\frac{4-3}{12}\right] = \frac{2}{12} = \boxed{\frac{1}{6}}$$ Ans

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c) Find CDF (commulative distribution function)

→ For $x<0$

$$\int_{-\infty}^{0} dx = 0$$

→ For $0 \leq x \leq 1$

$$= \int_0^x 2(1-u)\,du$$

$$= 2\int_0^x (1-u)\,du = 2\left[u-\frac{u^2}{2}\right]_0^x$$

$$= 2\left[x-\frac{x^2}{2}\right]$$

→ For $x>1$

$$= \int_{-\infty}^{0} du + \int_0^1 2(1-u)\,du + \int_1^x du$$

$$= 0 + 2\left(u-\frac{u^2}{2}\right) + 0$$

$$= 0 + 1 + 0 = 1 \quad \text{(always 1)}$$

So:

$$f(x) = \begin{cases} 0 & x<0 \\ 2\left[x-\frac{x^2}{2}\right] & 0\leq x\leq 1 \\ 1 & x>1 \end{cases}$$