

# Lecture #05

Date: Sept 2nd, 25

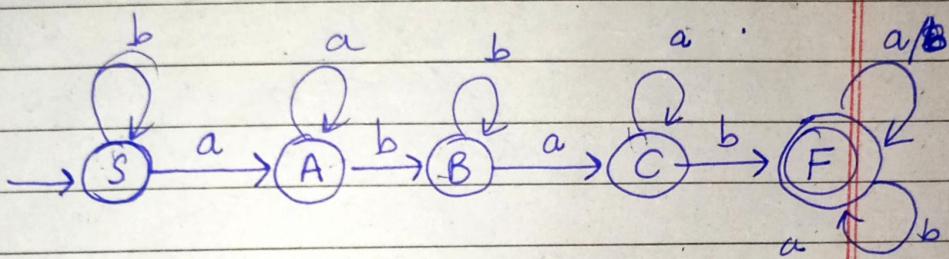
Day: Tuesday

Construct a DFA:

$L = \{ x \mid x \in \Sigma^* ; x \text{ contains at least two occurrences of } 'ab' \}$

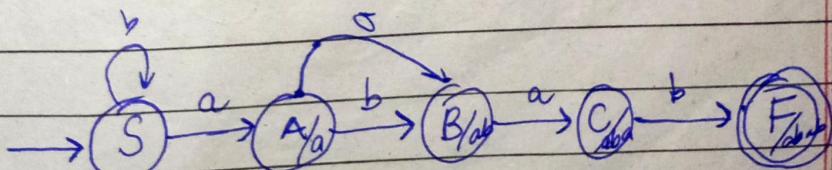
$$\Sigma = \{a, b\}$$

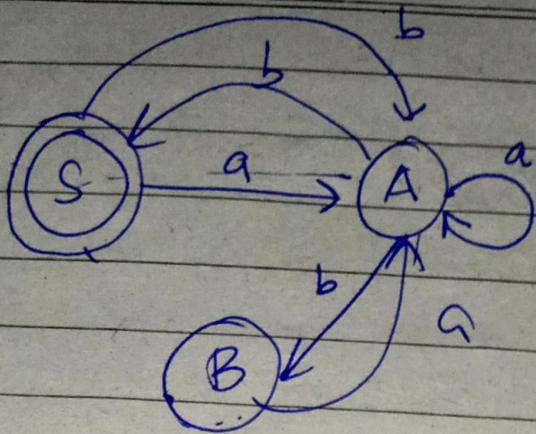
$L = \{ abab, aabab, ababa, aba\cancel{bab} \\ \cancel{ababa}, ababb, ba\cancel{abbab}, \cancel{abaab} \\ aababa, aaababab \}$



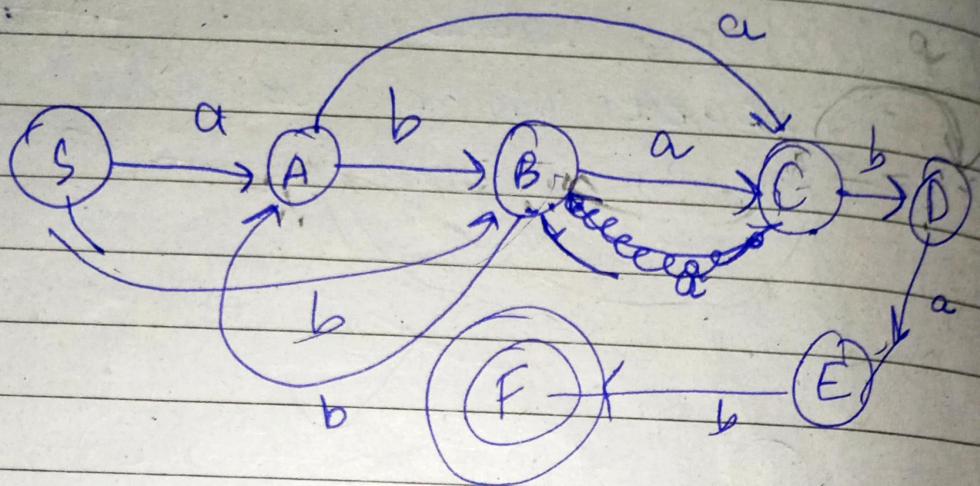
$L = \{ x \mid x \in \Sigma^* ; x \text{ contains at least 2 occurrences of } 'ab' \text{ and } |x| \leq 6 \}$

$L = \{ abab, aabab, babab, abaab, abbab \\ ababa, ababb, aaabab, ababab, bbabab \\ ababaa, ababbb \}$

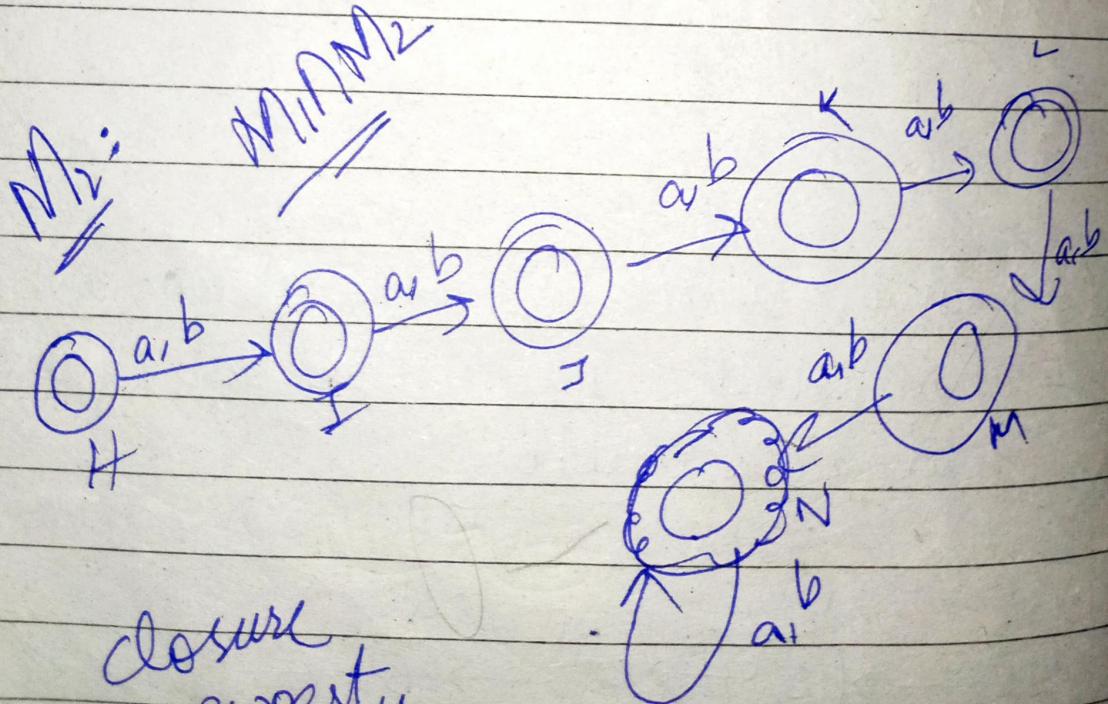




~~Agabab~~



Dead State  $\leftarrow$



Closure  
property

$$\mathcal{L}_4 = \{ abab \}$$

$$\mathcal{L}_5 = \{ aabab, babab, \dots \}$$

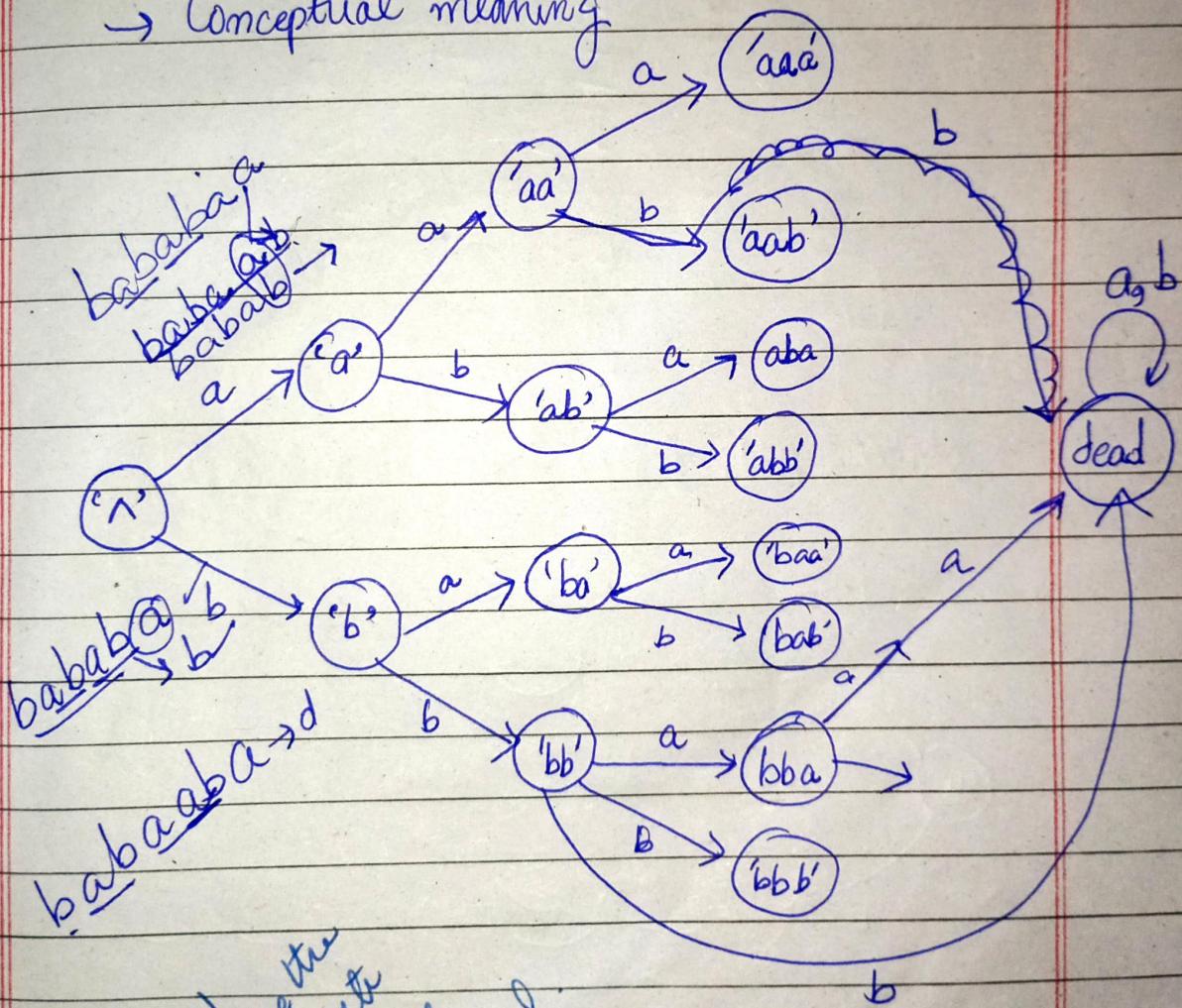
$$\mathcal{L}_6 = \{ aaabab, bbabab, ababab, baabab, \dots \}$$

$\Sigma^2 abab$       abaaabab, abbbab, ababab, abbaab

$ab\Sigma^2 ab$

$abab\Sigma^2$

→ Conceptual meaning



$N-1$   
will be the  
final state  
only one  
is nonfinal.

Date: Sept 4, 25

Day: Thurs.

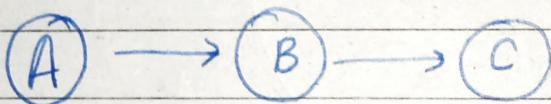
# Topics Finite Automata

DFA
NFA

For NFA:

Transition function is diff.

$$\delta_{NFA} = Q \times \Sigma \Rightarrow P(Q) / 2^0$$



$$Q = \{A, B, C\}$$

Transition can be skipped (mapped to dead state)

$$P(Q) = \{\emptyset, \{A\}, \{B\}, \{C\}, \{A, B\}, \{A, C\}, \{B, C\}, \{A, B, C\}\}$$

\* DFA is a subset of NFA.

can be transitioned to multiple states.

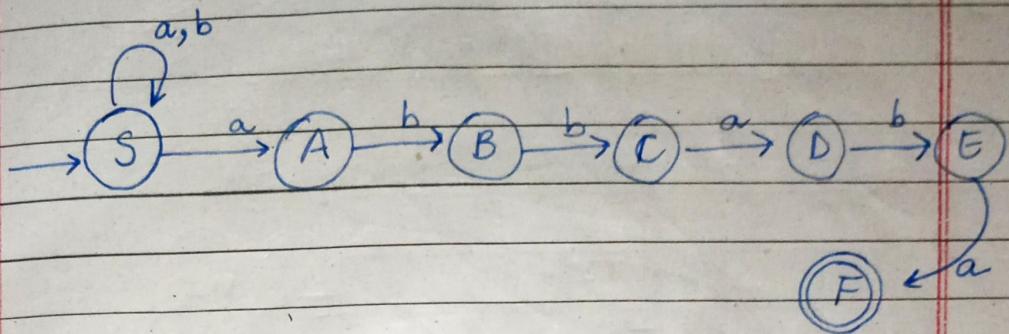
ex accept  $\epsilon$ ,  $S \xrightarrow{a,b} a,b$

Date: \_\_\_\_\_

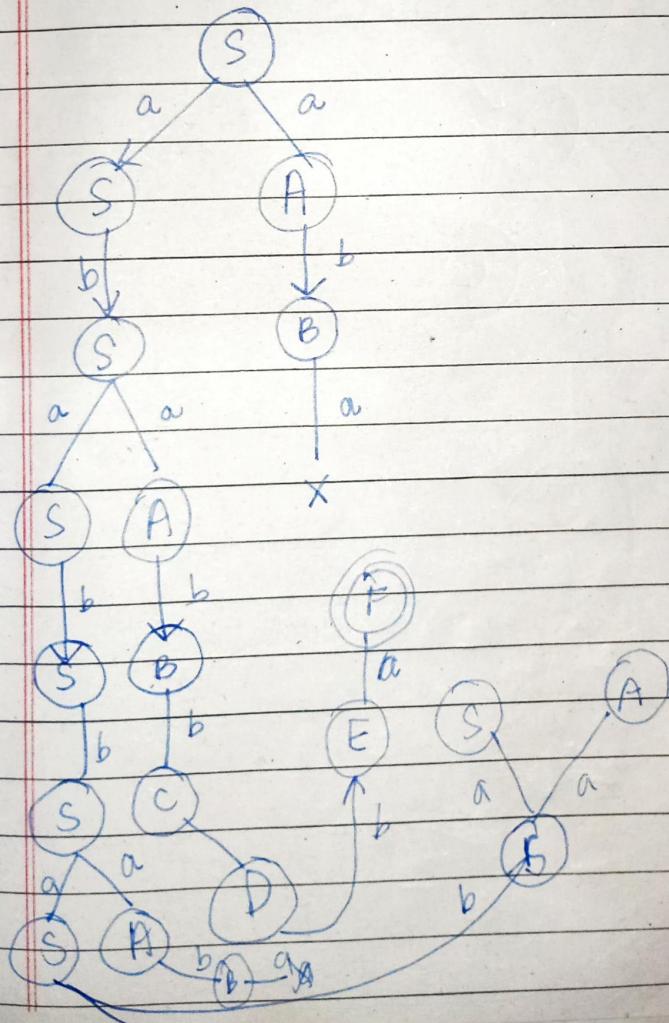
Day: \_\_\_\_\_

strings ends with 'abbaba'

$L = \{ abbaba, aabbaba, babbaba, \dots \}$



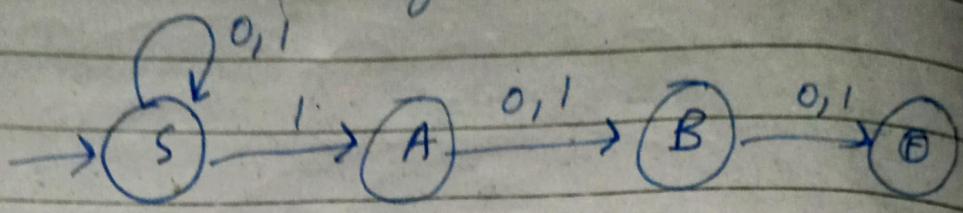
TraceTree



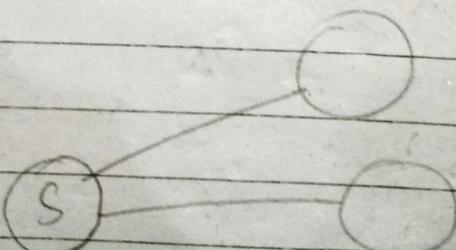
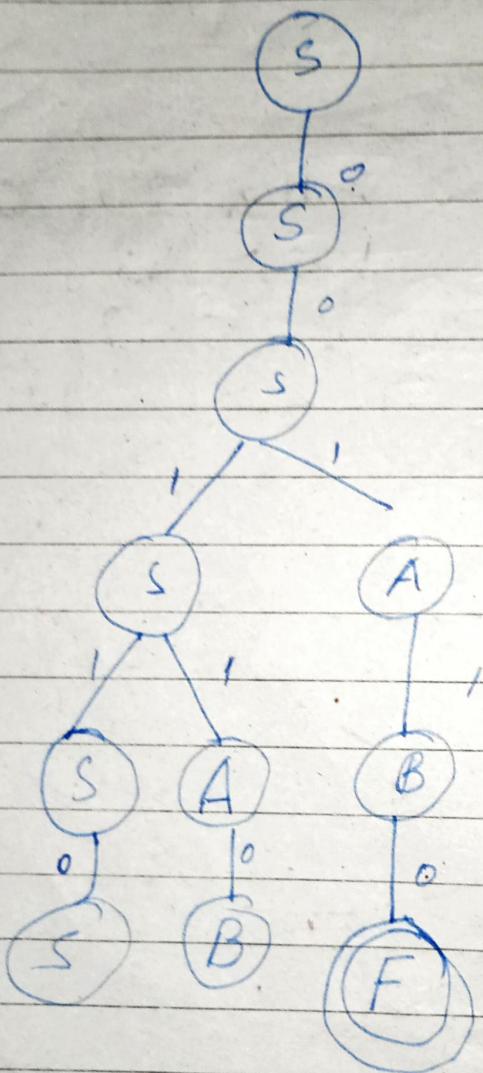
Date: \_\_\_\_\_

Day: \_\_\_\_\_

Third last digit is 1



00110



# Lecture # 07

Date: Sept 9<sup>th</sup>, 25

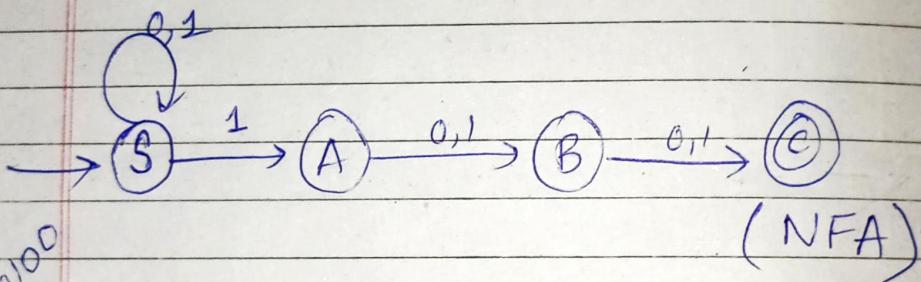
" ToA "

Tuesday

$d = \{ x | x \in \Sigma^* ; 3^{\text{rd}} \text{ last digit in } 'x' \text{ is '1'} \}$

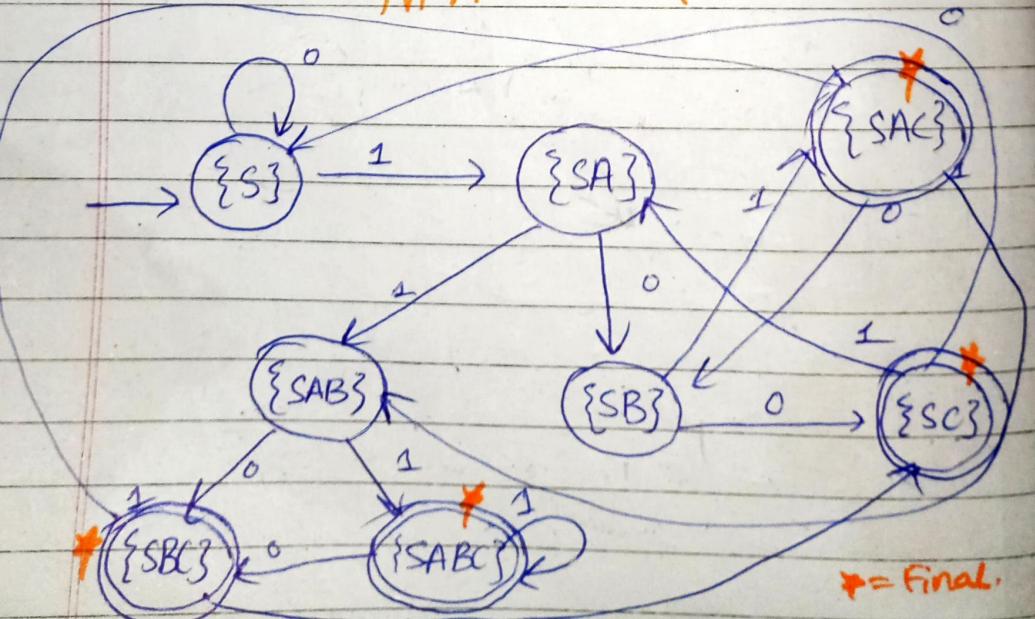
$d = \{ 100, 101, 110, 111, 0100, 0101, 0110, 0111, 1100, 1101, 1110, 1111, \dots \}$

$$= \Sigma^* \{ 100 \mid 101 \mid 110 \\ \mid 111 \}$$



Subset Construction Method :

NFA  $\rightarrow$  DFA



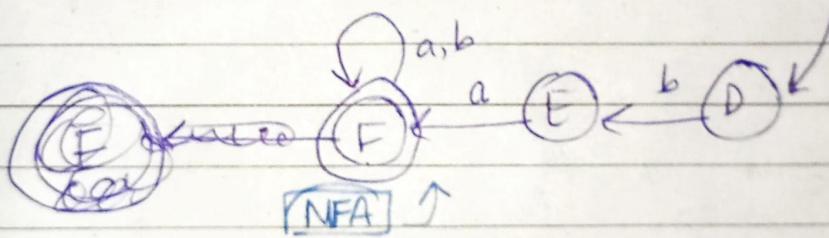
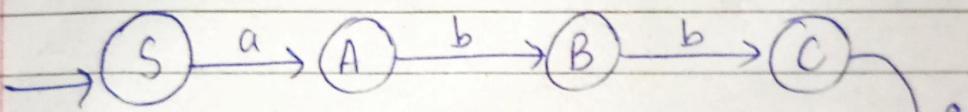
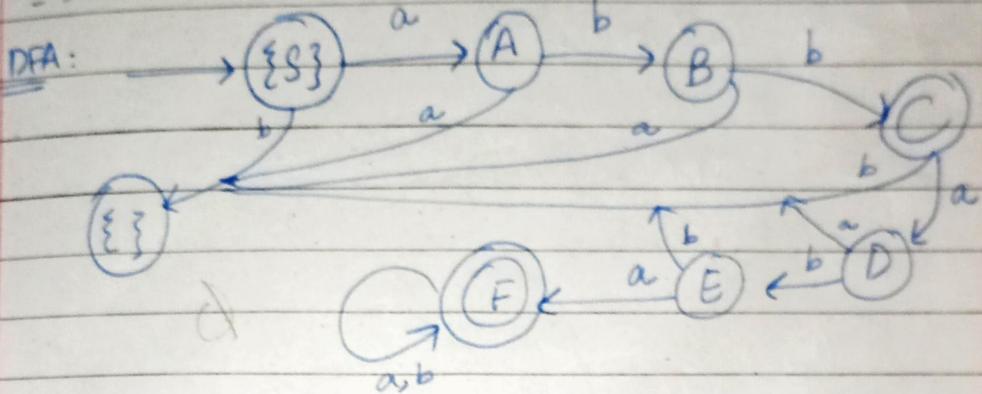
{a, b}

Day \_\_\_\_\_

Date \_\_\_\_\_

Starts with 'abbaba'  
 $\text{abbaba} \in ^*$

$L = \{ \text{abbaba}, \text{abbabaa}, \text{abbabab}, \dots \}$



we cannot extract pattern for negation ones... only  
so we will first make NFA for the patterned  
one  $\hookrightarrow$  make DFA and then apply  
NEGATION.

Date: \_\_\_\_\_

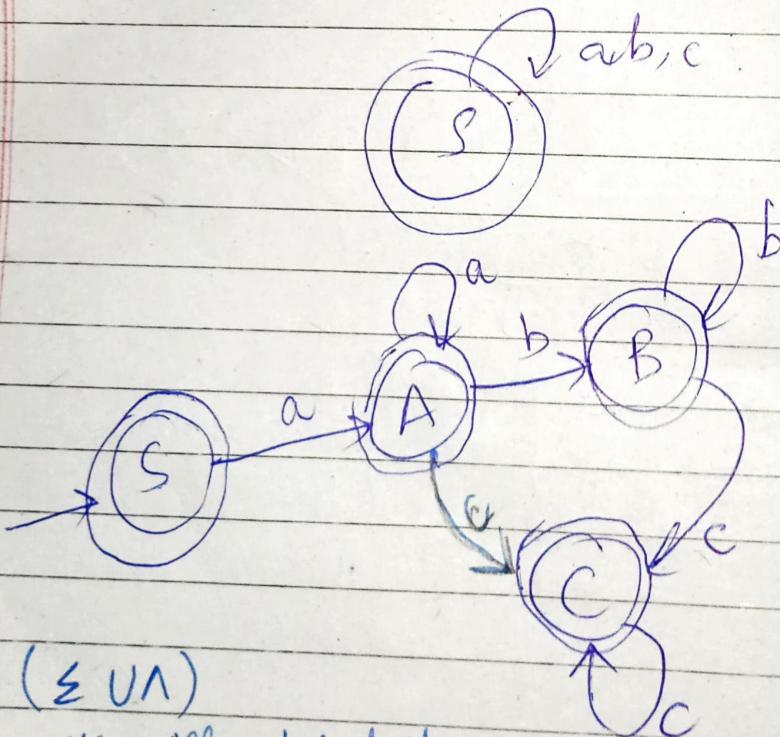
Day: \_\_\_\_\_

# NFA- $\Lambda$ ; NFA-NULL

$$S_{\text{NFA-}\Lambda} : Q * (\Sigma \cup \Lambda) \rightarrow P(Q)$$

$$\begin{aligned} L &= \{ a^* b^* c^* \} & a^* b^* c^* & \xrightarrow{\text{aa, bb, cc}} \\ L &= \{ \Lambda, a, b, c, ab, ac, bc, abc, aabc, abbc, abcc, \dots \} & a^0 b^0 c^0 & \end{aligned}$$

$\Lambda, a, b, c, ab, bc, ac, da, bb, cc,$   
 $abc, aabc, abbc, abcc,$



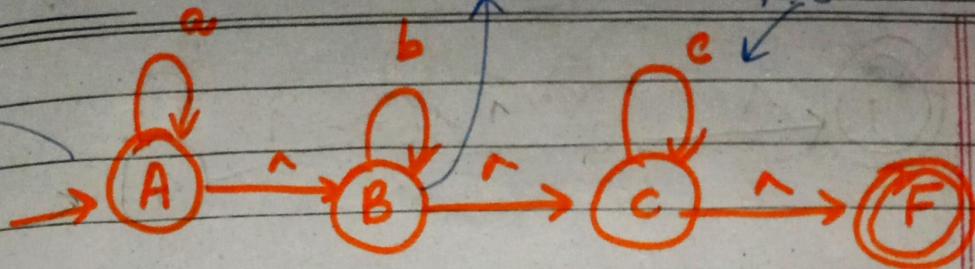
$(\Sigma \cup \Lambda)$

We will not include  
 $'\Lambda'$  in  $\Sigma$  as ' $\Sigma$ ' it is  
 the set of alphabets having

not real character

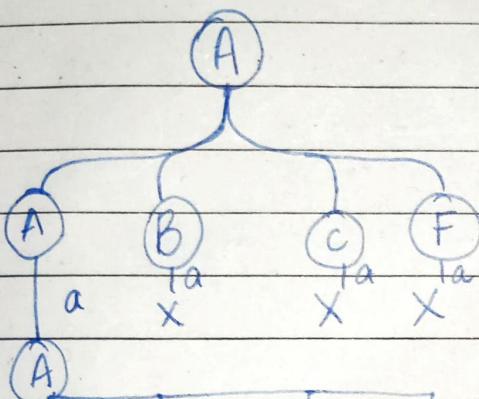
Date:

Final\*



Here  $\lambda$ : without reading any character we can move to other/next state.

## Trace Tree / Computational Tree

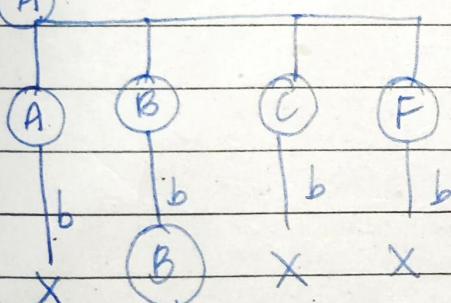


$\lambda$ -closure then

$\in$  then

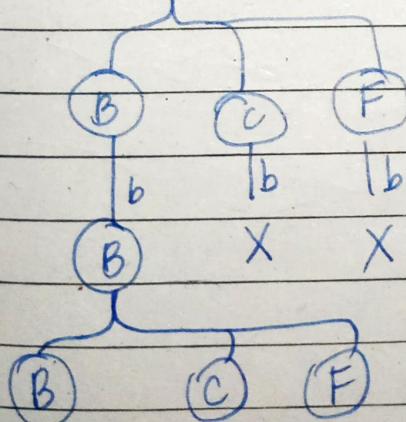
$\lambda$ -closure

string 'abb'



First take

$\lambda$ -closure then  
check alphabet then  
again closure and  
so-on... If the  
leaf of the tree has  
any state belongs to fn  
its accepted.....



F belongs to fn

← leaf

# \* Extended Transition Function

$$\delta^*(A, abb) = ?$$

$$\delta^*(A, \lambda) = \Delta(A) = \{A, B, C, F\}$$

$$\delta^*(A, a) = \Delta \left( \bigcup_{k \in \{A, B, C, F\}} \delta^*(k, a) \right)$$

$$= \Delta \left( \delta(A, a) \cup \delta(B, a) \cup \delta(C, a) \cup \delta(F, a) \right)$$

$$= \Delta(\{A\} \cup \{B\} \cup \{C\} \cup \{F\})$$

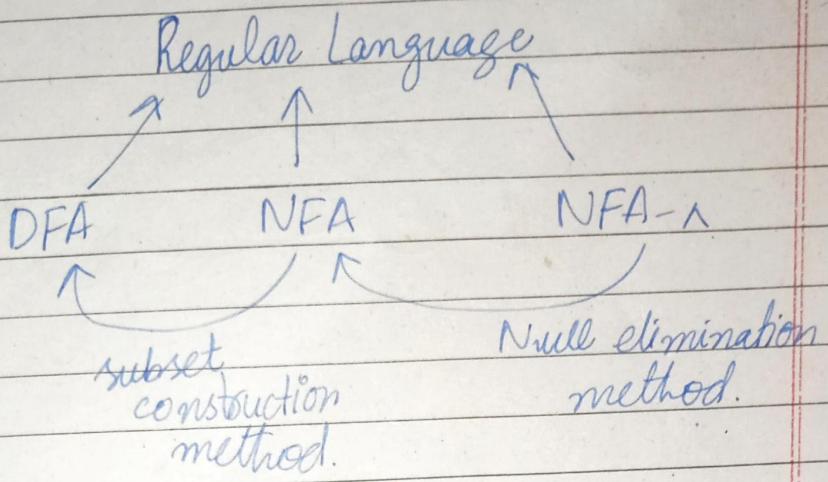
$$= \Delta(\{A\})$$

$$\delta(a, b) = \{A, B, C, F\}$$

# Lecture # 08

Date: \_\_\_\_\_ Day: \_\_\_\_\_

Topic : Null Elimination Method  
NFA- $\lambda$  to NFA conversion



No. of state doesn't change ....

	a	b	$\lambda$	null transition
- S	{S}	{B}	{A}{A}	
A	{F}	{-}	{B}	
B	{}	{A}	{G}	
G	{B}	{}	{}	
+ F	{F, G}	{}	{}	

$$\Lambda(S) = \{S, A, B, G\}$$

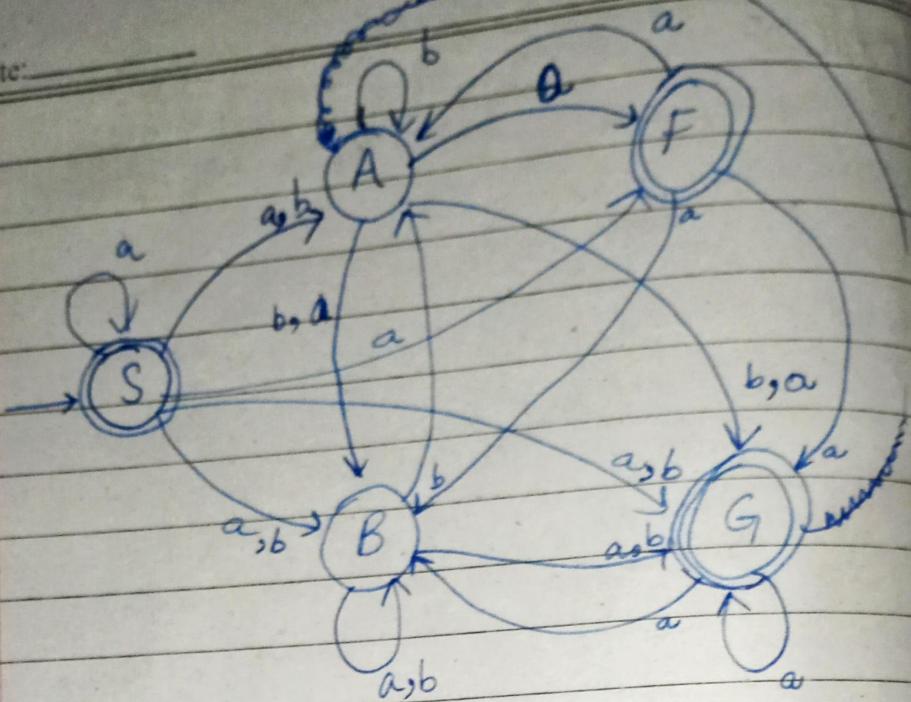
$$\Lambda(A) = \{A, B, G\}$$

$$\Lambda(B) = \{B, G\}$$

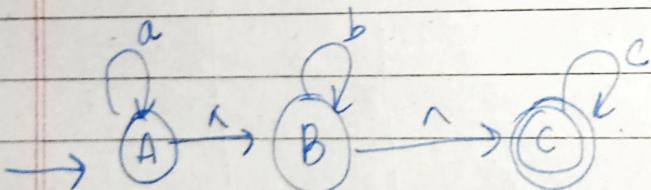
$$\Lambda(F) = \{F\}$$

Date: \_\_\_\_\_

Day: \_\_\_\_\_



if we can back track to 'S'  
 from any of the final state  
 using 'λ' then start  
 will also be final.



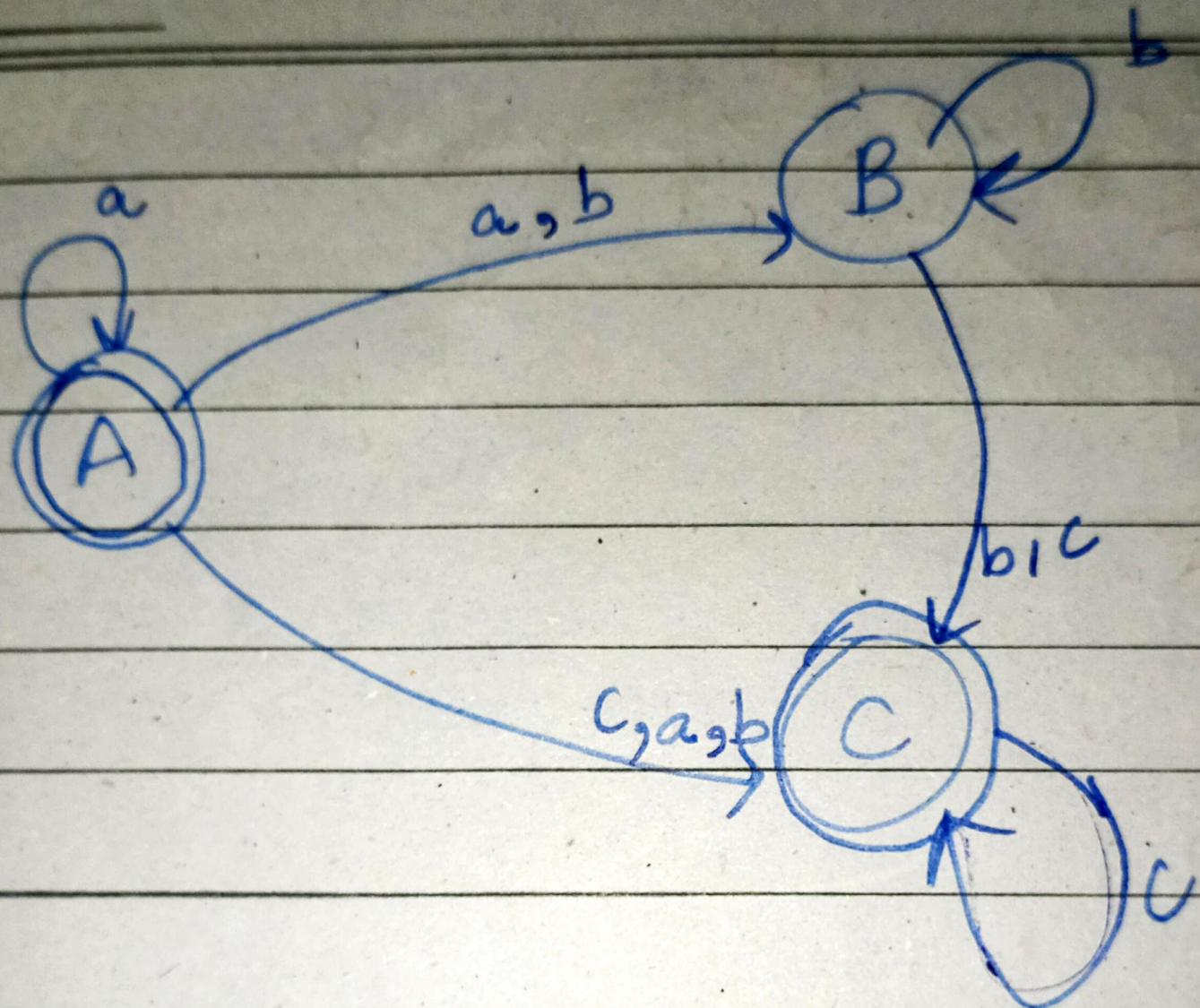
	a	b	c	λ
A	{A}	{B}	{C}	{B, C}
B	{ }	{B}	{C}	{C}
C	{ }	{ }	{ }	{ }

$$\lambda(A) = \{A, B, C\}$$

$$\lambda(B) = \{B, C\}$$

$$\lambda(C) = \{C\}$$

Day: \_\_\_\_\_



# Lecture # 09

Date: 11 Sept 2025

Day: Thursday

## Topic: Regular Expressions

Regular operants  
 $\Sigma$

Regular operators  
Union (+), Concatenation  
Kleene Star (\*), Positive Star (+)  
Paranthesis

$$\Sigma = \{a, b\}$$

$$\text{Union} = a + b$$

$$\text{Concatenation} = a \cdot b$$

$$\text{Kleene star} = \Sigma^*$$

$$\text{Positive star} = \Sigma^+$$

$$\text{Paranthesis} = (a + b)$$

$\min = 0$   
 $\min = 1$

Smallest regular expression  
= minimal

$$L_1 = \{a\} \Rightarrow R.E = a$$

$$L_2 = \{b\} \Rightarrow R.E = b$$

$$L = L_1 \cup L_2 \Rightarrow a + b$$

$$L = L_1 \cdot L_2 \Rightarrow ab$$

$$L = L_1^* \Rightarrow a^*$$

$$L = L_2^* L_1 \Rightarrow b^* a$$

$$L = L_1 \cdot L_2^+ + L_1$$

Precedence : 1. Stars  
2. Concatenation  
3. Union

Date: \_\_\_\_\_

Day: \_\_\_\_\_

$$\begin{aligned} L &= ((L_1(L_2^* + L_1) \\ L &= ab^* + a \end{aligned}$$

$$L = \{a, ab, abb, abbb, abbbb, \dots \}$$

$$L = \{aa, ab, ba, bb\}$$

$$\begin{aligned} R \cdot E &= b(a+b) \cdot (a+b) \\ &= (a+b)^2 \end{aligned}$$

$$\begin{aligned} L &= \{a, b, aa, ab, ba, bb\} \\ &= (a+b)(a+b+\lambda) \end{aligned}$$

$$\begin{aligned} L &= \{\lambda, a, b, aa, ab, ba, bb\} \\ &= (a+b+\lambda)(a+b+\lambda) \end{aligned}$$

$$a\lambda = a$$

$$L = \{x \mid x \in \Sigma^* ; a \text{ is followed by exactly } 3 b's\}$$

$$\begin{aligned} L &= \{abbb, abbb\cancel{ab}, abbb\cancel{bbb}, abbb\cancel{bbbb}\} \\ L &= \{\lambda, b, bb, bbb, abbb, bbbb, \dots\} \\ &= b^* \cdot (abb)^* \end{aligned}$$

Date:

Day

$$d = \{x \mid x \in \Sigma^* \text{ and } x \text{ is not divisible by 2}\} \quad \Sigma = \{0, 1\}$$

## State Elimination method

DFA  $\rightarrow$  RFA

$\rightarrow$  Aöden Theorem

$\rightarrow$  Elimination method.

$$d = \{1, 01, 11, 001, 011, \\ 00, 0111, \dots\}$$

