Date Sep 30, 2025   Day Tuesday

Programming

$$n = 3 \quad aaabbb$$
$$n = 1 \quad ab$$
$$n = 5 \quad aaaaabbbbb$$

```
void S(n){

    if(n < 1)
        return;

    cout << "a";
    S(n--);
    cout << "b";

}
```

Content Free Language:

CFG
(content free grammar)

PDA (push down automata)
↓
FA + Stack

To represent CFG
  4 Tuples :

$\Sigma \rightarrow$ Terminals
$V \rightarrow$ Set of variables
$S_V \rightarrow$ Start variable
$P \rightarrow \$ \alpha$ (rule)

$\Sigma = \{a, b\}$
$V = \{S\}$
$S_V = S$
$P \rightarrow \alpha \in (\Sigma \cup V)^*$
$\rightarrow \in V$

bakers
$:=$ biggest
moyer
form

1. $S \rightarrow ab$ (base case)
   $S \rightarrow aSb$

$$ P \xrightarrow{\quad} \overset{1}{\alpha_1} \,|\, \overset{2}{\alpha_2} \,|\, \overset{3}{\alpha_3} \,|\, \cdots \; \overset{n}{\alpha_n} $$
$P \rightarrow \alpha_1$ Pipe operator.
$P \rightarrow \alpha_2$
$P \rightarrow \alpha_3$
$\vdots$
$P \rightarrow \alpha_n$

Palindrome $\Sigma = \{a\,b\}$

$L = \{\lambda a, b, aa, bb, aba, aaa, bbb,$
$\quad bab, aaaa, abba,$

P:

$$S \to a + b \mid \wedge \mid aSa \mid$$
$$bSb$$

$\Sigma = \{a, b\}$
$V = \{S\}$
$S_V = S$

Trace Tree :

$$a\,b\,a\,b\,b\,a\,b\,a$$

Not Palindrome:
$L = \{ab, ba, aab, baa, aaab, abaa,$
$\quad aaba,$

~~$S \to ab \mid ba \mid bSa \mid aSb$~~
$S = aSa \mid bSb \mid aPb \mid bPa$
$P \to \wedge \mid aP \mid bP$

Write CFG:

$*\ L = \{a^n b^m ;\ n > m ;\ m \geq 1\}$

$L = \{aab,\ aaab,\ aaabb,\ aaaab,\ aaaabb$
$\qquad \xi 3,\quad \xi 4,\quad \xi 5$

$S \to aSb \mid aS \mid aab \longrightarrow$ base case.
$\qquad\quad 1 \qquad 2 \quad 3 \qquad P \to \alpha \to \varepsilon (\xi \cup V)^*$
$\qquad\qquad\qquad\qquad\qquad\quad G \in V$

‖ String Processing:
$\qquad \to$ Derivation Tree (LM, RM)
$\qquad \to$ Substitution Method (LM, RM)

aaaab
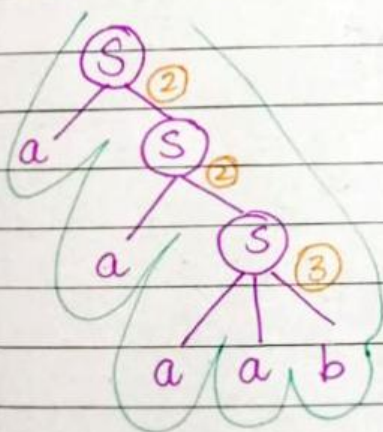
no variable at leaf
Leaf should belong
to ξ or $\wedge$

Substitution Method:

$L(G) = \{x \in \xi^* \mid S_v \overset{*}{\underset{G}{\Longrightarrow}} x\} \quad \to$ step

$S \overset{2}{\Rightarrow} aS \overset{2}{\Rightarrow} aaS \overset{3}{\Rightarrow} aaaab$

should not
contain
variable

as/bs

Without writing a Language

$$a^n b^m \qquad n > m \,;\, m \geq 1$$

$$a^p \leftarrow \overbrace{a^+ \quad a^m b^m}^{} \qquad \text{(closure property)}$$

Lang: A $\qquad$ B

$$S \to AB \quad \text{①}$$
$$A \to a \,\text{②}\,|\, aA \,\text{③}$$
$$B \to aBb \,\text{④}\,|\, \underline{ab} \,\text{⑤}$$

base case.

> if $m > 1$
> Rule #5:
> $aabb$
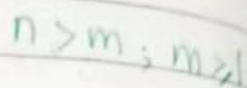
aaaabb

LM ε
Rule: 1, 2, 2, 4, 5

Left Most :
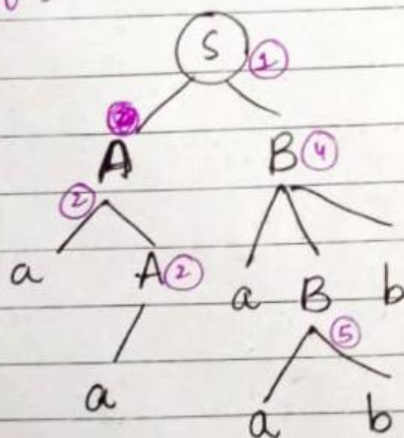
// Both trees
LM & RM are
same.

Right Most :

Rules : 1, 4, 5, 2, 3
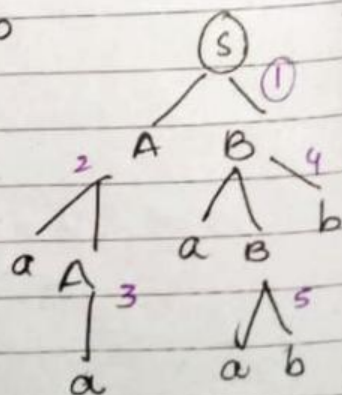
Substitution Method:-

$$S \overset{LM}{\underset{1}{\Rightarrow}} AB \overset{2}{\Rightarrow} aAB \overset{3}{\Rightarrow} aaB \overset{4}{\Rightarrow} aaaBb \overset{5}{\Rightarrow} aaaabb$$

$$S \overset{RM}{\underset{1}{\Rightarrow}} AB \overset{4}{\Rightarrow} AaBb \overset{5}{\Rightarrow} Aaabb \overset{2}{\Rightarrow} aAaabb$$
$$\overset{3}{\Rightarrow} aaaabb$$

if for a single string : multiple trees
can be formed $\longrightarrow$ Ambigous Grammer ?

$$\begin{cases} n > m \to L \\ \text{or} \\ n = m \to L_2 \end{cases}$$

$\bigstar \; \mathcal{L} = \{ a^n b^m \; ; \; n \geqslant m \; ; \; m \geqslant 1 \}$

$\mathcal{L} = \{ ab, aab, aaab, aabb, \dots \}$

$S \to aSb \,|\, aS \,|\, ab$ [Method 1]

$$\boxed{\begin{array}{l} L = L_1 \,|\, L_2 \\[4pt] L_1 \to A \cdot L_2 \\[4pt] L_2 \to aL_2B \,|\, ab \\[4pt] A \to aA \,|\, a \end{array}}$$

$n \geqslant 1; m \geqslant 0$

$$\mathcal{L} = \{a^n b^m c^p; \quad p = n+m; \quad n, m \geqslant 0\}$$

$$\mathcal{L} = \{\wedge, ac, bc, abcc, aabcc, abbcc, \ldots\}$$

$$a^n b^m c^{n+m}$$
$$a^n b^m c^n c^m$$
$$a^n b^m c^m c^n$$

$$S \to aSc^{①} \mid A^{②}$$

$$A \to bAc^{③} \mid \wedge^{④}$$

✳ $$\mathcal{L} = \{(a^i b^i)^k (c^j d^j)^k; \quad i, j, k \geqslant 0\}$$

$$S \to ASB \mid \wedge$$

$$A \to aAb \mid \wedge$$

$$B \to \{cBd \mid \wedge$$

* $d = \{a^n b^m c^p; n > 1; m \geqslant 0\}$

$$S \to aSc \mid aaAcc$$
$$A \to bAc \mid \wedge$$

* $d = \{a^{2n} b^{3m} c^p \quad p = n+m; n > 1; m \geqslant 0$

$$S \to aaScc \mid aaaaAcc$$
$$A \to bbbAc \mid \wedge$$

* $d = \{a^{2n} b^{3m} c^p \quad p = 2n + 3m; n > 1; m \geqslant 0$

$$S \to aaScc \mid aaaaAcccc$$
$$A \to bbbAccc \mid \wedge$$

* $d = \{x_a(n) = x_b(n)\}$

$$d = \{\wedge, ab, ba, aabb, bbaa,$$
$$baab, abab, baba, \cdots \}$$

$$S \to aB \mid bA \mid \wedge$$
$$B \to bS \mid aBB$$
$$A \to aS \mid bAA \qquad OR$$
$$S \to aSb \mid bSa \mid \wedge \mid SS$$