# Assignment 4 Math

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### 1. Using matrix operations, describe the solutions for the following family of equations:

$$x + 2y - 3z = 5$$

$$2x + y - 3z = 13$$

$$-x + y = -8$$

To solve the equation I put them in matrix form:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & -3 \\ -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \\ -8 \end{bmatrix}$$

By hand I used Gauss-Jordan elimination:

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 2 & 1 & -3 & 13 \\ -1 & 1 & 0 & -8 \end{array}\right] \xrightarrow{R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 1 & -1 & 0 & 8 \\ -1 & 1 & 0 & -8 \end{array}\right] \xrightarrow{R_1 - R_2} \left[\begin{array}{ccc|c} 0 & 3 & -3 & -3 \\ 1 & -1 & 0 & 8 \\ -1 & 1 & 0 & -8 \end{array}\right]$$

$$\xrightarrow{\frac{R_1}{3}} \left[\begin{array}{ccc|c} 0 & 1 & -1 & -1 \\ 1 & -1 & 0 & 8 \\ -1 & 1 & 0 & -8 \end{array}\right] \xrightarrow{R_3 + R_2} \left[\begin{array}{ccc|c} 0 & 1 & -1 & -1 \\ 1 & -1 & 0 & 8 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{R_2 + R_1} \left[\begin{array}{ccc|c} 0 & 1 & -1 & -1 \\ 1 & 0 & -1 & 7 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \leftrightarrow R_1} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 7 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x - z = 7; \quad y - z = -1; \quad z \text{ is a free variable}$$

### 2. Provide a solution for #1, using R functions of your choice.

By using the `pracma` package:

```
library(pracma)
A <- matrix(data=c(1, 2, -3, 2, 1, -3, -1, 1, 0), nrow = 3, ncol = 3, byrow =
TRUE)
```

```
B <- matrix(data=c(5, 13, -8), ncol = 1, nrow = 3)
K <- cbind(A, B) # To create a matrix in the form so we can use Gauss Jordan elimination method.
K

##      [,1] [,2] [,3] [,4]
## [1,]    1    2   -3    5
## [2,]    2    1   -3   13
## [3,]   -1    1    0   -8

rref(K) #Produces the reduced row echelon form with the use of Gauss Jordan elimination with partial pivoting.

##      [,1] [,2] [,3] [,4]
## [1,]    1    0   -1    7
## [2,]    0    1   -1   -1
## [3,]    0    0    0    0
```

**3. Solve for AB by hand:**

$$A = \begin{bmatrix} 4 & -3 \\ -3 & 5 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 4 \\ 3 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -3 \\ -3 & 5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} x1 & y1 \\ x2 & y2 \\ x3 & y3 \end{bmatrix}$$

$$\begin{bmatrix} x1 & y1 \\ x2 & y2 \\ x3 & y3 \end{bmatrix}$$

$$\rightarrow x1 = (4 * 1 + \ -3 * 3) = \ -5, \qquad y1 = (4 * 4 + \ -3 * -2) = 22$$

$$x2 = (-3 * 1 + \ 5 * 3) = 12, \qquad y2 = (-3 * 4 + \ 5 * -2) = -22$$

$$x3 = (0 * 1 + 1 * 3) = 3, \qquad y3 = (0 * 4 + \ 1 * -2) = -2$$

$$\therefore = \ AB = \begin{bmatrix} -5 & 22 \\ 12 & -22 \\ 3 & -2 \end{bmatrix}$$

**4. Solve AB from #3 using R functions of your choice.**

```r
D <- matrix(data=c(4,-3,-3,5,0,1), nrow=3, ncol=2, byrow=TRUE)
E <- matrix(data=c(1,4,3,-2), nrow=2, ncol=2, byrow=TRUE)
D %*% E #This multiplies the two matrices.
```

```
##      [,1] [,2]
## [1,]   -5   22
## [2,]   12  -22
## [3,]    3   -2
```