McGill University

Assignment 3

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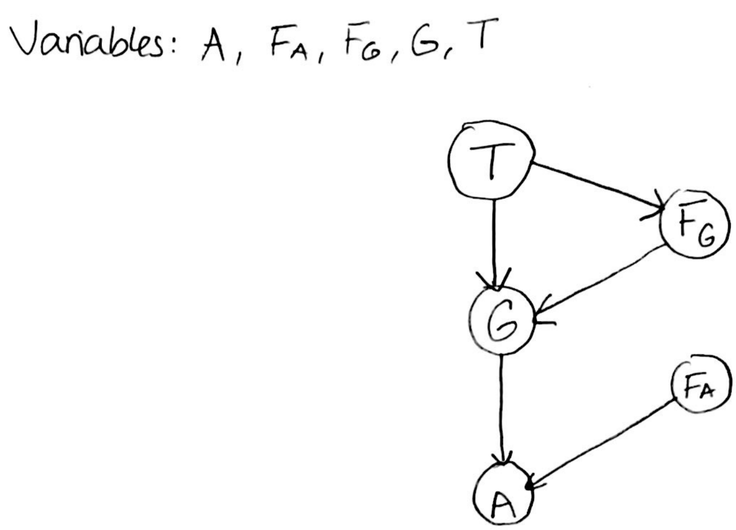
COMP 424

1 Designing a Bayesian Network

1.1 Drawing the network



1.2 Is the network polytree?

No. We know that the gauge reading G depends on temperature T and faultiness F_G. However, F_G also depends on T as the gauge is more likely to fail when the temperature gets too high. Therefore, replacing the directed edges with undirected ones produces a cycle, and so the network is not a polytree.

1.3 Conditional probability table associated with G

G=normal=0
G=high=1
T=normal=0

T=high=1
F_G=working=0
F_G=faulty=1

	G=0	G=1
T=0, F_G=0	x	1 - x
T=0, F_G=1	y	1 - y
T=1, F_G=0	1 - x	x
T=1, F_G=1	1 - y	y

1.4 Conditional probability table associated with A

A=no sound=0
A=sounds=1
G=normal=0
G=high=1
F_A=working=0
F_A=faulty=1

	A=0	A=1
G=0, F_A=0	1	0
G=0, F_A=1	1	0
G=1, F_A=0	0	1
G=1, F_A=1	1	0

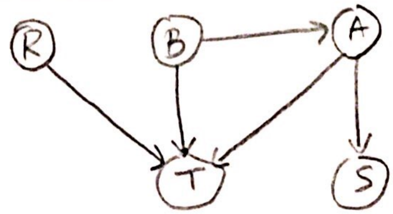
1.5 Calculating expression for the probability that the temperature of the core is too high

$$P(T=1 \mid \underbrace{A=1, F_A=0}_{\text{implies } G=1}, F_G=0)$$

$$P(T=1 \mid G=1, F_G=0) = \frac{P(T=1, G=1, F_G=0)}{P(G=1, F_G=0)}$$

$$= \frac{P(T=1, G=1, F_G=0)}{P(T=0, G=1, F_G=0) + P(T=1, G=1, F_G=0)}$$

$$= \frac{P(G=1 \mid F_G=0, T=1)\ P(F_G=0 \mid T=1)\ P(T=1)}{P(G=1 \mid F_G=0, T=0)\ P(F_G=0 \mid T=0)\ P(T=0) + P(G=1 \mid F_G=0, T=1)\ P(F_G=0 \mid T=1)\ P(T=1)}$$

Q2) 

a) $$P(s,r) = \sum_{a,b,t} P(\underline{R=r}, B=b, A=a, T=t, \underline{S=s})$$

$$= \sum_{a,b,t} \underbrace{P(r)}_{\substack{\text{constant} \\ 0.15}} P(b)\, P(a|b)\, P(t|r,b,a)\, P(s|a)$$

$$= 0.15 \sum_{a,b,t} P(b) P(a|b) P(t|r,b,a) P(s|a)$$

$$= 0.15 \Big[\underbrace{0.2(0.6)(0.95)(0.8)}_{b,a,t} + \underbrace{0.2(0.6)(0.05)(0.8)}_{b,a,\bar{t}}$$

$$+ \underbrace{0.2(0.4)(0.90)(0.2)}_{b,\bar{a},t} + \underbrace{0.2(0.4)(0.10)(0.2)}_{b,\bar{a},\bar{t}}$$

$$+ \underbrace{0.8(0.4)(0.92)(0.8)}_{\bar{b},a,t} + \underbrace{0.8(0.4)(0.08)(0.8)}_{\bar{b},a,\bar{t}}$$

$$+ \underbrace{0.8(0.6)(0.85)(0.2)}_{\bar{b},\bar{a},t} + \underbrace{0.8(0.6)(0.15)(0.2)}_{\bar{b},\bar{a},\bar{t}} \Big]$$

$$= 0.15 [0.0912 + 0.0048 + 0.0144 + 0.0016 + 0.23552 + 0.02048 + 0.0816 + 0.0144]$$

$$= \underline{\underline{0.0696}}$$

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b) $P(a, \neg t) = \sum_{r,b,s} P(R=r, B=b, A=a, T=\neg t, S=s)$

$$= \sum_{r,b,s} P(r)\, P(b)\, P(a|b)\, P(\neg t|r,b,a)\, P(s|a)$$

$$= \underbrace{0.15(0.2)(0.6)(0.05)(0.8)}_{r,b,s} + \underbrace{0.15(0.2)(0.6)(0.05)(0.2)}_{r,b,\bar{s}}$$

$$+ \underbrace{0.15(0.8)(0.4)(0.08)(0.8)}_{r,\bar{b},s} + \underbrace{0.15(0.8)(0.4)(0.08)(0.2)}_{r,\bar{b},\bar{s}}$$

$$+ \underbrace{0.85(0.2)(0.6)(0.65)(0.8)}_{\bar{r},b,s} + \underbrace{0.85(0.2)(0.6)(0.65)(0.2)}_{\bar{r},b,\bar{s}}$$

$$+ \underbrace{0.85(0.8)(0.4)(0.4)(0.8)}_{\bar{r},\bar{b},s} + \underbrace{0.85(0.8)(0.4)(0.4)(0.2)}_{\bar{r},\bar{b},\bar{s}}$$

$$= 0.0009 + 0.00384 + 0.0663 + 0.1088$$

$$\underline{= 0.17984}$$

c) $P(t|s) = \dfrac{P(t \wedge s)}{P(s)} = \dfrac{\sum_{r,b,a} P(R=r, B=b, A=a, T=t, S=s)}{\sum_{r,b,a,s} P(R=r, B=b, A=a, T=t, S=s)}$

<u>numerator</u>

$\sum_{r,b,a} P(r)\, P(b)\, P(a|b)\, P(t|r,b,a)\, P(s|a) = P(t \wedge s)$

$$= \underbrace{0.15(0.2)(0.6)(0.95)(0.8)}_{rba} + \underbrace{0.15(0.2)(0.4)(0.9)(0.2)}_{rb\bar{a}}$$

$$+ \underbrace{0.15(0.8)(0.4)(0.92)(0.8)}_{r\bar{b}a} + \underbrace{0.15(0.8)(0.6)(0.85)(0.2)}_{r\bar{b}\bar{a}}$$

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$$+\underbrace{0.85(0.2)(0.6)(0.35)(0.8)}_{\bar{r}ba} + \underbrace{0.85(0.2)(0.4)(0.4)(0.2)}_{\bar{r}b\bar{a}}$$

$$+\underbrace{0.85(0.8)(0.4)(0.6)(0.8)}_{\bar{r}\bar{b}a} + \underbrace{0.85(0.8)(0.6)(0.05)(0.2)}_{\bar{r}\bar{b}\bar{a}}$$

$$= 0.01368 + 0.00216 + 0.035328 + 0.01224$$

$$+ 0.02856 + 0.00544 + 0.13056 + 0.00408$$

$$= 0.232048$$

Denom

Marginalize r, b, a, t

for $t=1$ for $t=0$

$$\Rightarrow \underline{0.232048} + [0.00072 + 0.00024 + 0.003072$$

$$+ 0.00216 + 0.05304 + 0.00816 + 0.08704 + 0.07752]$$

$$= 0.464$$

$$\frac{0.232048}{0.464} \approx \underline{\underline{0.5001}}$$

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R, S, B, A, T

Q3

$P(T|a)$

R/active: $\cancel{P(R)}$, $\cancel{P(T|R,B,A)}$, $P(B)$, $P(A|B)$, $P(S|A)$, $\delta(A,a)$

		$M_R(T,B,A)$
t,b,a	$0.15(0.95) + 0.85(0.35)$	0.44
$t,b,\bar{a}$	$0.15(0.9) + 0.85(0.4)$	0.475
$t,\bar{b},a$	$0.15(0.92) + 0.85(0.6)$	0.648
$t,\bar{b},\bar{a}$	$0.15(0.85) + 0.85(0.05)$	0.17
$\bar{t},b,a$	$0.15(0.05) + 0.85(0.65)$	0.56
$\bar{t},b,\bar{a}$	$0.15(0.1) + 0.85(0.6)$	0.525
$\bar{t},\bar{b},a$	$0.15(0.08) + 0.85(0.4)$	0.352
$\bar{t},\bar{b},\bar{a}$	$0.15(0.15) + 0.85(0.95)$	0.83

S/active: $P(B)$, $P(A|B)$ $\cancel{P(S|A)}$ $\delta(A,a)$ $M_R(T,B,A)$

$M_S(A) = \sum_S P(S|A)$

a	$0.8 + 0.2$	1
$\bar{a}$	$0.2 + 0.8$	1

B/active: $\cancel{P(B)}$ $\cancel{P(A|B)}$ $\delta(A,a)$ $\cancel{M_R(T,B,A)}$, $M_S(A)$

$M_B(A,T) = \sum_B P(B)\, P(A|B) M_R(T,B,A)$

		$M_B(A,T)$
a,t	$0.2(0.6)(0.44) + 0.8(0.4)(0.648)$	0.26016
$a,\bar{t}$	$0.2(0.6)(0.56)$ $0.8(0.4)(0.352)$	0.17584
$\bar{a},t$	$0.2(0.4)(0.475)$ $0.8(0.6)(0.17)$	0.1196
$\bar{a},\bar{t}$	$0.2(0.4)(0.525)$ $0.8(0.6)(0.83)$	0.4404

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A/

active: $\cancel{\delta(A,a)}$, $\cancel{M_S(A)}$, $\cancel{M_B(A,T)}$

$$M_A(T) = \sum_A \delta(A,a)\, M_S(A)\, M_B(A,T)$$

	$M_A(t)$
t	0.26016
$\bar{t}$	0.17984

$$\boxed{\begin{array}{l} P(t \mid a) = 0.26016 \\ P(\bar{t} \mid a) = 0.17984 \end{array}}$$

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Q4) a)

i)	ii) max likelihood	iii) Laplace smoothing
$\theta_A = P(A)$	$49/146$	$50/148$
$\theta_{B1} = P(B \mid A)$	$43/49$	$44/51$
$\theta_{B2} = P(B \mid \neg A)$	$68/97$	$69/99$
$\theta_{C1} = P(C \mid A)$	$19/49$	$20/51$
$\theta_{C2} = P(C \mid \neg A)$	$56/97$	$57/99$
$\theta_{D1} = P(D \mid B, C)$	$0/46$	$1/48$
$\theta_{D2} = P(D \mid B, \neg C)$	$21/65$	$22/67$
$\theta_{D3} = P(D \mid \neg B, C)$	$8/29$	$9/31$
$\theta_{D4} = P(D \mid \neg B, \neg C)$	$4/6$	$5/8$

b i) 1st E step

$$w_{D_1=0} = P(D_1=0 \mid B_1=0, C_1=1) = \frac{21}{29}$$

$$w_{D=1} = P(D=1 \mid B_1=0, C_1=1) = \frac{8}{29}$$

$$w_{C_2=0} = P(C_2=0 \mid A_2=1) = \frac{30}{49}$$

$$w_{C_2=1} = P(C_2=1 \mid A_2=1) = \frac{19}{49}$$

ii)

$$\theta_A = \frac{49+2}{146+2} = \frac{51}{148}$$

$$\theta_{B1} = \frac{43+1}{49+2} = \frac{44}{51}$$

$$\theta_{B2} = \frac{68+0}{97+0} = \frac{68}{97}$$

$$\theta_{C1} = \frac{19+1+19/49}{49+2} = \frac{20.39}{51}$$

$$\theta_{C2} = \frac{56+0}{97+0} = \frac{56}{97}$$

$$\theta_{D1} = \frac{0+0}{46+30/49} = \frac{0}{46.61}$$

$$\theta_{D2} = \frac{21+8/29}{65+30/49} = \frac{21.28}{65.61}$$

$$\theta_{D3} = \frac{8+8/29}{29+1} = \frac{8.27}{30}$$

$$\theta_{D4} = \frac{4+0}{6+0} = \frac{4}{6}$$

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iii) 2nd E step

$$\left.\begin{array}{l} W_{D_1=0} = P(D_1=0 \mid B_1=0, C_1=1) = \dfrac{21.73}{30} \\[2ex] W_{D_1=1} = P(D_1=1 \mid B_1=0, C_1=1) = \dfrac{8.27}{30} \end{array}\right\} = 1$$

$$\left.\begin{array}{l} W_{C_2=0} = P(C_2=0 \mid A_2=1) = \dfrac{30.61}{51} \\[2ex] W_{C_2=1} = P(C_2=1 \mid A_2=1) = \dfrac{20.39}{51} \end{array}\right\} = 1$$

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