

MTH3035 Mathematics group project

You need to be interested in person for this module

~~Thursday 10am~~ office hour

choice by today

5% group project plan (written)

5% (2) short group presentations in class

10% final (60 mins) group presentation

75% final written report

5% individual self assessment (January)

Adjustment for low or high contribution

Week 6 reading week

Week	date	Monday 10:30	Thursday 1:30
2	27 Sept	Project plan lecture	no class
3	4 Oct	presentation lecture	no class
4	11 Oct	group presentation 1	group presentation 1

Project Management Skills, teamwork, communications

include MTH3035 in 2 subject line

Team's office hour Meeting Tuesday 10-11

Doesn't decrease marks for doing bad or being nervous
be understanding of different people & their cultures

already have project plan title

Critical path analysis

risk analysis: how can things go wrong? how likely?

getting covid was an impact more than 1 (nurse hospital)

room being double book is low prob but high impact

need to interview people involved in this industry like
who makes these predictions professionally

can mitigate for laptop battery while bringing charger

Strategy in Making Money? what skills? experience or personality?
are you set up to fail?
blaming others makes you look bad
not responding to communications
making up excuses looks bad
how will you get your team to force yourself to contribute
is someone is not like your team, then advisor - then mark
upload by 12:00 Friday 8th October etc or email
each group only hand in once

new slides use 16:9 aspect ratio
use transitions sparingly so use it once but not everytime
don't use cliparts

make sure light doesn't wash it out
aim, audience, medium, content, style

keep it short & simple

how confident are you while?

assume less knowledge in the audience

unreadable, invisible, too much material, unclear message

poor explanation, run out of time or finish too early

mental block (embarrassing silence), poor response to questions

about 1-2 mins per slide

what is key message? can you simplify it?

start sentence & or non facts with "in my opinion"

Cite sources

don't have to present in order which it happened

use visuals

choose most important

avoid irrelevant material

attention grabber (who wants

to be rich)

don't use Comic-Sans

use the entire slide - but don't be ridiculous

1) not ok to right but a complete script

can leave optional extra material to aster
make sure not double booked

Project voice but don't shout

Speak to the audience

Drink water before & after speaking

Keep an eye on the time - leave enough time for conclusion

explain everything carefully

repeat & even rephrase questions & answer the audience

can paraphrase question to make them open

don't have to answer immediately ask for clarification

conference posters?

be prepared for everything - make a plan

Follow up by emails

tuesday 8A 3rd last Max 4 mins

1) don't produce graphs that produce odd data info that
doesn't exist ("chart junk")

pie charts have a place not necessarily in Maths

www.vischeck.com to check if it works best for everyone

maybe our bar chart should be a standard bar chart or even maybe a

pie chart (probably a bad idea)

to publish you need to add labels maybe even labels on the graph instead
of a key

1) can do a key of green least danger & red most
no rainbow keys

Soton villa 2-1

united city 2-0

Brentford Norwich 3-0

Chelsea v. Reading 0-1

Crystal Palace 0-2

Brighton vs Newcastle 1-1

Arsenal Walsley 2-0

Leeds Leicester 3-2

Everton Tottenham 0-0

Brighton Newcastle

West Ham Wolves 1-0

MTM Mathematics group project - Statistical Forecasting

- Target - who will win
- Lead time - predict one day ahead, one week, whole season before it starts, how long? whole season? all next month lead/horizon

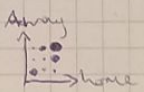
• type of forecasts - exact single value like 23°C is a point forecast

- range/interval forecast for like first goal within 30 to 40 mins

probability/probabilistic forecast - probs for different results happen - like prob of rain is 35%.

probabilities add up to 100%. So win & not win or can do win/lose/draw

represent as probs for normal distribution a continuous dist

time teams can affect each other  discrete 2-dimensional

Poisson is best for number of goals for one team is 1D
 $\text{poi}(2.1)$

home advantage - league position, current form
keyplayer missing - history of results

convergence and correlation, but better to just assume home and away goals are independent

strong team does not affect weak team but

being away on a new pitch does affect performance

is Exeter $\sim \text{poi}(1.5)$ Walsall (2.1) \therefore

$P(\text{Exeter score } \geq 2) = 0.22$ $P(\text{Walsall score } 1) = 0.18$

is independence $S(x, y) = S(x)S(y)$ is true for all numbers
then independence

Dissertation, the source

how many players are still playing - the longer the window - how relevant, so the further back is less relevant and introduces bias

but too small data introduces noise outliers even if data is only the most relevant

when do you throw away data - do you grow your window or shift it along

measure performance - how to tell which prediction is better?

what do other people do to forecast - professional like betting companies

keep things really simple at the start

use whole season to accurate for whole years seasons good at fitting data but not good at predicting

scoring rules on hypothesis tests to tests its fitting & prediction

need more than chi-squared

data should come with estimated standard error like a confidence interval

aim said predict most likely outcome

we need to be reading & what other people have done

adding form - how well they've done in the last 5 games

linear and cubic statistical model of football results

just because it fits well doesn't mean it predicts real \therefore don't over fit

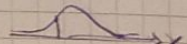
\therefore use cross-validation

simplest

proper scoring rules \rightarrow Brier score \rightarrow logarithmic \rightarrow ranked probab score

Brier score - Binary do $(y-x)^2$ for each forecast then find average

logarithmic score: $-\log(p(x))$



\rightarrow we need to do more than one

ranked prob to score & logarithmic score for football use logarithmic
use consistent state like 2 teams but not 2 years

can compare true ranking joined by hypothesis test t-test

can check for covariance with Diaboy & Morine (1995)

decision table on reading list

want to be fighting about 20 sources

first line of raw position code use away averages

very final part of SP code weights average and sum into 50-50
weighting but we can change the weighting to 30-70 then do a
logarithmic or Brier score to compare which is best

Southampton only has 2 game away in recent form but arsenal has 3

in weighting calculation it means home games do exactly Models expected
away average

would be give 50% weighting for away-Models expected for 3 home
& only 2 away games

for Fulham home vs Man City Away

Model 4 predicts: Most probable Score 0-1 \therefore away win most probable
with prob 55% win Draw home win Draw Away win:

H: 0.1012704 D: 0.2014161 A: 0.6972599

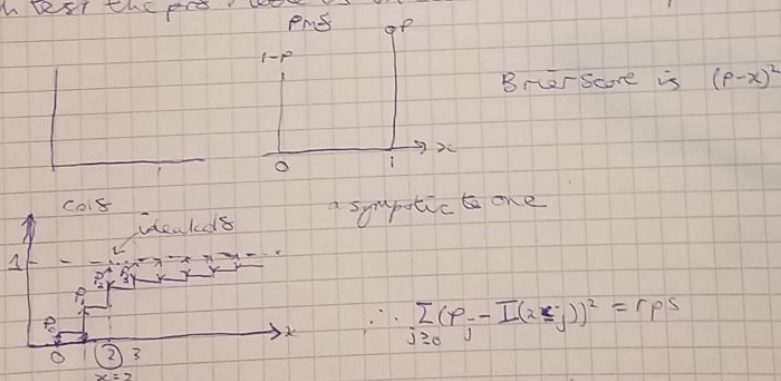
True result: Away win \therefore True Results: (H: 0, D: 0, A: 1)

$$\therefore -(0 \times \log_2(0.1012704) + 0 \times \log_2(0.2014161) + 1 \times \log_2(0.6972599)) = \\ -\log_2(0.6972599) = 0.5202 \approx 0.52 \text{ is Log Scoring}$$

\therefore do that for every game and add up all the log scoring values
for that model to get the Logarithmic Scoring Results
then just find out which one has the lowest one

Southampton	SOU
Arsenal	ARS
Aston Villa	AST
Brighton	BHA
Burnley	BUR
Chelsea	CHE
Crystal Palace	CRY
Everton	EVE
Leeds	LEE
Leicester	LEI
Liverpool	LIV
Man City	MCI
Man United	MUN
Newcastle	NEW
Tottenham	TOT
West Brom	WBA
West Ham	WHU
Wolves	WOL
Fulham	FUL
Sheffield United	SHU
Norwich	NOR
Brentford	BRE

2 good models are kept very secretive so cross validation is hard
with test the ~~pro~~ ^{pro} model is on our data to compare to our models

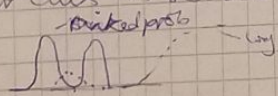


$x=2$
Still works i.e. home-draw-away can be ordered

sensitive to difference

	H	D	A
F_1	0.4	0.2	0.4
F_2	0.4	0.3	0.3

So by score is not sensitive to dissonance so would weight equal but this is sensitive to dissonance so say S_{F2} way better



$HT \sim Poi(\lambda)$
 $AT \sim Poi(\rho)$
 $S \sim \text{multinomial}(A, \text{Form})$
 $C \sim \text{multinomial}(A, \text{Form})$

home W.	Draw	Away W.	H?	D?	A?
α	β	γ	r	s	t

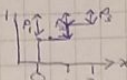
$$\alpha, \beta, \gamma \in \mathbb{R}_{\geq 0}, \alpha + \beta + \gamma = 1 \quad 0 \leq \alpha, \beta, \gamma, \leq 1$$

only one $r, s, t = (0 \text{ or } 1)$ only one of r, s, t can be 1 at once.

$$\text{if } r=1 \Rightarrow s, t=0$$

$$\text{if } s=1 \Rightarrow r, t=0$$

$$\text{if } t=1 \Rightarrow r, s=0$$

eg if $r=1$: $rps = (R\alpha)^2$  $\therefore \sum_{j=0}^{\infty} (F_j - I(x \leq j))^2 = rps$

$$\therefore r=0.254 \quad \alpha=0.54 \quad \beta=0.24 \quad \gamma=0.21$$

$$\therefore (1-\alpha)=0.46 \quad (1-\beta)=0.76 \quad (1-\gamma)=0.79$$

$$\alpha + \beta = 0.78 \quad (1-0.78)=0.22$$

$$\alpha + \beta + \gamma = 1 \quad 1-(\alpha + \beta + \gamma) = 0$$

$$I(x \leq j) \text{ is } 0 \text{ or } 1 \quad \therefore \text{for } r=1 \quad \therefore x=0 \quad \therefore$$

$$I(0 \leq 0)=1 \text{ and } I(0 \leq 1)=1, I(0 \leq 2)=1 \quad \therefore \text{do}$$

$$(0.46-1)^2 + (0.22-1)^2 + (0-1)^2 = rps = 1.9 \text{ which } rps \text{ to be as low as possible}$$

$$\text{if } \alpha=0.11, \beta=0.22, \gamma=0.65, t=1 \quad \therefore x=2$$

$$1-1=0 \quad \therefore I(x \leq 0)=0 \quad I(x \leq 1)=0 \quad I(x \leq 2)=1$$

$$(0.11-0)^2 + (0.22-0)^2 + (0-1)^2 = rps = 1.0605$$

$$\text{if } \alpha=0.35, \beta=0.28, \gamma=0.35, t=1 \quad \therefore x=2$$

$$\therefore 1-1=0 \quad I(x \leq 0)=0 \quad I(x \leq 1)=0 \quad I(x \leq 2)=1$$

$$(0.35-0)^2 + (0.28-0)^2 + (0-1)^2 = rps = 1.063$$

$$(0.35-0)^2 + (0.63-0)^2 + (0-1)^2 = rps = 1.5194$$

$$\text{if } \alpha=0.11, \beta=0.22, \gamma=0.65, t=1 \quad \therefore x=2$$

$$I(x \leq 0)=0, I(x \leq 1)=0, I(x \leq 2)=1, 1-1=0, \alpha + \beta = 0.33$$

$$(0.11-0)^2 + (0.33-0)^2 + (0-1)^2 = rps = 1.121$$

1.121 < 1.5194 which is expected for a better model

$$\text{if } \alpha = 0.40, \beta = 0.26, \gamma = 0.35, S = 1 \therefore x = 1$$

$$I(x \leq 0) = 0, I(x \leq 1) = 1, I(x \leq 2) = 1 \therefore 1 - 1 = 0$$

$$\alpha + \beta = 0.40 + 0.26 = 0.66 \quad 1 - 0.66 = 0.34$$

$$(0.40 - 0)^2 + (0.34 - 1)^2 + (0 - 1)^2 = rps = 1.5756 \text{ is high as expected}$$

$$\text{Equivalently: } (0.40 - 0)^2 + (0.66)^2 + 1 = 1.5756$$

$$\text{Equivalently: } (0.40 - I(x \leq 0))^2 + (x + \beta)^2 + 1 \quad \text{for } S = 1 \cdot (x - H)^2 + (x + \beta)^2 + 1$$

$$\text{for } r = 1: \text{ if } \alpha = 0.54, \beta = 0.24, \gamma = 0.21, r = 1 \therefore x = 0$$

$$I(x \leq 0) = 1, I(x \leq 1) = 1, I(x \leq 2) = 1 \therefore 1 - 1 = 0$$

$$1 - \alpha = 0.46, \alpha + \beta = 0.54 + 0.24 = 0.78 \therefore 1 - (\alpha + \beta) = 1 - 0.78 = 0.22$$

$$(0.46 - 1)^2 + (1 - (\alpha + \beta) - 1)^2 + (0 - 1)^2 = 1.90$$

$$(0.46 - 1)^2 + (0.78)^2 + 1 = 1.9$$

$$(\alpha)^2 + (\alpha + \beta)^2 + 1 = 1.9$$

$$\text{for } S = 1: H = 0 \therefore (x)^2 + (x + \beta)^2 + 1 = rps$$

$$\text{for } r = 1 \quad (x)^2 + (\alpha + \beta)^2 + 1 = rps$$

$$\text{for } t = 1: \alpha = 0.11, \beta = 0.22, \gamma = 0.65 \therefore t = 1 \therefore x = 2$$

$$I(x \leq 0) = 0, I(x \leq 1) = 0, I(x \leq 2) = 1 \therefore 1 - 1 = 0 \therefore \alpha + \beta = 0.33$$

$$(\alpha - I(x \leq 0))^2 + ((\alpha + \beta) - I(x \leq 1))^2 + (0 - 1)^2 =$$

$$(\alpha - 0)^2 + ((\alpha + \beta) - 0)^2 + 1 =$$

$$\alpha^2 + (\alpha + \beta)^2 + 1 = 0.11^2 + (0.33)^2 + 1 = 1.121$$

$$\text{if } r = 1 \quad \alpha = 0.6, \beta = 0.24, \gamma = 0.16$$

$$\alpha^2 + (\alpha + \beta)^2 + 1 = (0.6)^2 + (0.6 + 0.24)^2 + 1 = rps = 2.0656$$

$$rps = [Pr(\text{Home } W)]^2 + [Pr(\text{Home } W) + Pr(\text{Draw})]^2 + 1$$

$$rps = [Pr(\text{Home } W)]^2 + [Pr(\text{Home } W) + Pr(\text{Draw})]^2 + [Pr(\text{Home } W) + Pr(\text{Draw}) + Pr(\text{Away } W)]^2$$

So $\Pr(\text{Home Win}) = 0.4 = P_1$, $\Pr(\text{Draw}) = 0.26 = P_2$, $\Pr(\text{Away Win}) = 0.34 = P_3$
true result Draw.

$\therefore H=0, D=1, A=0 \therefore 0.4 + 0.26 = 0.66$, $0.4 + 0.26 + 0.34 = 1$

$$\begin{aligned} \therefore \frac{1}{2} [(P_1 - H)^2 + (P_1 - H + P_2 - D)^2 + (P_1 - H + P_2 - D + P_3 - A)^2] &= \\ \frac{1}{2} [(0.4 - 0)^2 + (0.4 - 0 + 0.26 - 1)^2 + (0.4 - 0 + 0.26 - 1 + 0.34 - 0)^2] &= \\ \frac{1}{2} [(0.4 - 0)^2 + (0.66 - 1)^2 + (1 - 1)^2] &= \\ \frac{1}{2} [(0.4 - 0)^2 + (0.66 - 1)^2] &= 0.1378 \end{aligned}$$

$\therefore D=1, \Pr(\text{Home W}) = 0.3 = P_1, \Pr(\text{Draw}) = 0.25 = P_2, \Pr(\text{Away W}) = 0.45 = P_3$
 $H=0, D=1, A=0 \therefore 0.3 + 0.25 = 0.55$, $0.3 + 0.25 + 0.45 = 1$

$$\begin{aligned} \frac{1}{2} [(P_1 - H)^2 + (P_1 - H + P_2 - D)^2 + (P_1 - H + P_2 - D + P_3 - A)^2] &= \\ \frac{1}{2} [(0.3 - 0)^2 + (0.3 - 0 + 0.25 - 1)^2 + (0.3 - 0 + 0.25 - 1 + 0.45 - 0)^2] &= \\ \frac{1}{2} [(0.3 - 0)^2 + (0.55 - 1)^2 + (1 - 1)^2] &= \\ \frac{1}{2} [(0.3 - 0)^2 + (0.55 - 1)^2] &= 0.1625 \end{aligned}$$

~~So~~ $H=1, \Pr(\text{Home W}) = 0.54 = P_1, \Pr(\text{Draw}) = 0.24 = P_2, \Pr(\text{Away W}) = 0.22 = P_3$
 $\therefore H=1, D=0, A=0 \therefore 0.54 + 0.24 = 0.78$, $0.54 + 0.24 + 0.22 = 1$

$$\begin{aligned} \frac{1}{2} [(P_1 - H)^2 + (P_1 - H + P_2 - D)^2 + (P_1 - H + P_2 - D + P_3 - A)^2] &= \\ \frac{1}{2} [(0.54 - 1)^2 + (0.54 - 1 + 0.24 - 0)^2 + (0.54 - 1 + 0.24 - 0 + 0.22 - 0)^2] &= \\ \frac{1}{2} [(0.54 - 1)^2 + (0.78 - 1)^2 + (1 - 1)^2] &= \\ \frac{1}{2} [(0.54 - 1)^2 + (0.78 - 1)^2] &= 0.13 \quad \left\{ \text{pretty good since H with prob } 0.54 \right\} \end{aligned}$$

~~So~~ $A=1, \Pr(\text{Home W}) = 0.11 = P_1, \Pr(\text{Draw}) = 0.23 = P_2, \Pr(\text{Away W}) = 0.66 = P_3$
 $H=0, D=0, A=1 \therefore 0.11 + 0.23 = 0.34$, $0.11 + 0.23 + 0.66 = 1$

$$\begin{aligned} \frac{1}{2} [(P_1 - H)^2 + (P_1 - H + P_2 - D)^2 + (P_1 - H + P_2 - D + P_3 - A)^2] &= \\ \frac{1}{2} [(0.11 - 0)^2 + (0.11 - 0 + 0.23 - 0)^2 + (0.11 - 0 + 0.23 - 0 + 0.66 - 1)^2] &= \\ \frac{1}{2} [(0.11 - 0)^2 + (0.34 - 0)^2 + (1 - 1)^2] &= \\ \frac{1}{2} [(0.11 - 0)^2 + (0.34 - 0)^2] &= 0.06385 \quad \left\{ \text{very good! A with prob } 0.66 \right\} \end{aligned}$$

\therefore Formula: $\frac{1}{2} [(P_1 - H)^2 + (P_1 - H + P_2 - D)^2]$

We tried to improve Z's first model by factoring in recent form, that is weighting Z average goals in Z previous 5 games more heavily than less recent results. We calculated recent form as a team's average goals from Z last 5 games, ignoring whether they were played at home or away. We tried weighting form at 25, 50 and 75% however all 3 attempts had worse results than Z's original model (where $w=0$). We believe this is due to Z's fact that ignoring whether Z team was playing home or away when calculating recent form was adding more noise than accuracy to Z predictions.

We next tried calculating recent form separately for home & away games, using a similar formula as before, but considering only home or away games. We again tested for 25, 50 and 75% weighting of recent form. We found that 25% gave Z best predictions, & was an improvement on Z's first model. Calculating form separately for home & away Z noise induced when we calculated it as a single variable.

We tried again to improve our model further by including conceded goals by Z opponent to our third model. We attempted this by adding goals conceded by Z away team into Z home team's expected goal number, & vice versa. This would then mean our model took into account Z opponent Z teams were facing. We added two factors for conceded goals, (1) conceded average goals, & (2) conceded form, both relative to home & away matches.

As previously stated, we added Z conceded goals factor to our previous model, leading to our expected goal numbers calculated like so. We weighted all 4 factors equally, & also 12.5% weightings on form's, as that worked well with model 3. Z results showed that although both weightings were good, adding this factor didn't change things too much.

We tried to improve Z's first model by factoring in recent form, that is weighting Z average goals in Z previous 5 games more heavily than less recent results. We calculated recent form as a team's average goals from Z last 5 games, ignoring whether they were played at home or away. We tried weighting form at 25, 50 & 75% however all 3 attempts had worse results than Z's original model (where $w=0$). We believe this is due to Z's fact that ignoring whether Z team was playing home or away when calculating recent form was adding more noise than accuracy to Z predictions.

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We tried again to improve our model further by including conceded goals by Z opponent to our 3rd model. We attempted this by adding goals conceded by Z away team into Z home team's expected goal number & vice versa. This would then mean Z opponent Z teams were would matter & be taken into account by Z model. We added in two factors for conceded goals, conceded average goals, & conceded form, both relative to home & away matches.

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- we tried to improve Model 3 by including conceded goals by the opponent
- do this by adding goal conceded by away team to home team's expected and vice versa
- this means our Model takes into account the opponents
- conceded goals was made as two factors: conceded away goals, & conceded home
- both relative to home and away

~~how we improved~~ → how we tried to improve →
 how we implemented this improvement →
 why this is an improvement →
 specifics of the improvement
 extra specifics

we tried to improve our Model further by adding conceded goals. that is adding the conceded goals of the away team to the home team's expected goals and vice versa. this means the Model takes into account who the opponent is. we did this relative to home and away

as previously stated we added the conceded goals factors to our previous model, leading to our expected goal numbers calculated like so, we weighted each goal equally, & also for 12.5% weighting on home's, as that worked well with Model 3. 2 results showed that although both weighting were good, adding this factor didn't change things too much