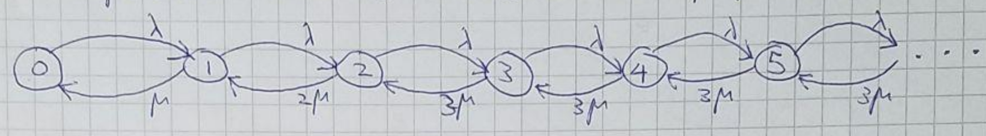
(Q1a)

Mean treatment time of $\frac{1}{\mu}$ hours $\therefore$

independent rate of treatment is μ patients/hour $\therefore$



is an M/M/3 queue with infinite capacity with $\lambda, \mu > 0$

(Q1b)

$\rho = \frac{\lambda}{3\mu} \quad \therefore \quad \frac{\lambda}{2\mu} = \frac{3}{2}\frac{\lambda}{3\mu} = \frac{3}{2}\rho, \quad \frac{\lambda}{\mu} = 3\frac{\lambda}{3\mu} = 3\rho \quad \therefore$

$P_0 + P_1 + P_2 + P_3 + \dots = 1 \quad \therefore$

$\frac{dP_0}{dt} = -\lambda P_0 + \mu P_1 \quad , \quad \frac{dP_1}{dt} = \lambda P_0 - \mu P_1 - \lambda P_1 + 2\mu P_2 \quad ,$

$\frac{dP_2}{dt} = \lambda P_1 - 2\mu P_2 - \lambda P_2 + 3\mu P_3 \quad , \quad \frac{dP_3}{dt} = \lambda P_2 - 3\mu P_3 - \lambda P_3 + 3\mu P_4 \quad ,$

$\frac{dP_4}{dt} = \lambda P_3 - 3\mu P_4 - \lambda P_4 + 3\mu P_5 \quad \therefore$

$\frac{dP_n}{dt} = \lambda P_{n-1} - 3\mu P_n - \lambda P_n - 3\mu P_{n+1}$ for $n \geq 3 \quad \therefore$

at steady state : $\frac{dP_0}{dt} = 0, \quad \frac{dP_1}{dt} = 0, \quad \frac{dP_2}{dt} = 0, \quad \frac{dP_3}{dt} = 0, \dots$

$\therefore \frac{dP_n}{dt} = 0 \quad \forall n \in \mathbb{Z}_{\geq 0} \quad \therefore$

$0 = -\lambda P_0 + \mu P_1 \quad , \quad \lambda P_0 - \mu P_1 - \lambda P_1 + 2\mu P_2 = 0,$

$\lambda P_1 - 2\mu P_2 - \lambda P_2 + 3\mu P_3 = 0 \quad , \quad \lambda P_2 - 3\mu P_3 - \lambda P_3 - 3\mu P_4 = 0 \quad ,$

$\lambda P_{n-1} - 3\mu P_n - \lambda P_n - 3\mu P_{n+1} = 0$ for $n \geq 3 \quad \therefore$

$\lambda P_0 = \mu P_1 \quad \therefore \quad -\lambda P_1 + 2\mu P_2 = 0 \quad \therefore$

$-\lambda P_2 + 3\mu P_3 = 0 \quad \therefore \quad -\lambda P_3 - 3\mu P_4 = 0 \quad \therefore$

$-\lambda P_n - 3\mu P_{n+1} = 0$ for $n \geq 3 \quad \therefore$

$P_1 = \frac{\lambda}{\mu} P_0 = 3\rho P_0 \quad , \quad 2\mu P_2 = \lambda P_1 \quad \therefore \quad P_2 = \frac{\lambda}{2\mu} P_1 = \frac{3}{2}\rho P_1 \quad , \quad \therefore$

$3\mu P_3 = \lambda P_2 \quad \therefore \quad P_3 = \frac{\lambda}{3\mu} P_2 = \rho P_2 \quad , \quad \therefore \quad 3\mu P_4 = \lambda P_3 \quad \therefore \quad P_4 = \frac{\lambda}{3\mu} P_3 = \rho P_3 \quad \therefore$

$P_1 = 3\rho P_0 \quad , \quad P_2 = \frac{3}{2}\rho P_1 \quad , \quad P_3 = \rho P_2 \quad , \quad P_{n+1} = \rho P_n$ for $n \geq 3 \quad \therefore$

$P_2 = \frac{3}{2}\rho(3\rho P_0) = \frac{1}{2}3^2\rho^2 P_0 = \frac{9}{2}\rho^2 P_0 = P_2 \quad \therefore$

$P_3 = \rho\frac{9}{2}\rho^2 P_0 = \frac{9}{2}\rho^3 P_0 \quad \therefore \quad P_n = \frac{9}{2}\rho^n P_0$ for $n \geq 3 \quad \therefore$

$P_0 + P_1 + P_2 + P_3 + \dots = 1 = P_0 + 3\rho P_0 + \frac{9}{2}\rho^2 P_0 + \sum_{n=3}^{\infty} \frac{9}{2}\rho^n P_0$

$= P_0 + 3\rho P_0 + \frac{9}{2} P_0 \left[\sum_{n=2}^{\infty} \rho^n\right] =$

$P_0 + 3\rho P_0 + \frac{9}{2} P_0 \left[-\rho^1 + \sum_{n=1}^{\infty} \rho^n\right] =$

$P_0 + 3\rho P_0 + \frac{9}{2} P_0 \left[-\rho + \frac{\rho}{1-\rho}\right] = 1 \qquad$ (for $|\rho| < 1$) $\therefore$

(Q1b continued)

$$p_0 + 3\rho p_0 + \frac{9}{2} p_0 \left[-\rho\right] + \frac{9}{2}\frac{\rho}{1-\rho} p_0 = 1$$

$$\cancel{p_0\left[3\rho - \frac{9}{2}\rho\right.} = p_0\left[1 + 3\rho - \frac{9}{2}\rho + \frac{9}{2}\frac{\rho}{1-\rho}\right] =$$

$$\cancel{p_0\left[1 - \frac{9}{2}\rho\right.} \quad p_0\left[1 - \frac{3}{2}\rho + \frac{9}{2}\left(\frac{\rho}{1-\rho}\right)\right] =$$

$$p_0\left[\frac{1-\rho}{1-\rho} - \frac{3}{2}\,\frac{\rho-\rho^2}{1-\rho} + \frac{9}{2}\,\frac{\rho}{1-\rho}\right] =$$

$$p_0\left[\frac{1-\rho}{1-\rho} + \frac{-\frac{3}{2}\rho + \frac{3}{2}\rho^2}{1-\rho} + \frac{\frac{9}{2}\rho}{1-\rho}\right] =$$

$$p_0\left[\frac{1-\rho-\frac{3}{2}\rho+\frac{3}{2}\rho^2+\frac{9}{2}\rho}{1-\rho}\right] = \cancel{p_0\left[\frac{9}{2}\rho\right.} \quad p_0\left[\frac{\frac{3}{2}\rho^2+2\rho+1}{1-\rho}\right] = 1 \;\therefore$$

$$p_0 = \frac{1-\rho}{\frac{3}{2}\rho^2+2\rho+1} \;\therefore$$

$$p_1 = 3\rho p_0 = 3\rho\,\frac{1-\rho}{\frac{3}{2}\rho^2+2\rho+1} = \frac{3\rho-3\rho^2}{\frac{3}{2}\rho^2+2\rho+1} \;\therefore$$

$$p_2 = \frac{9}{2}\rho^2 p_0 = \frac{9}{2}\rho^2\,\frac{1-\rho}{\frac{3}{2}\rho^2+2\rho+1} = \frac{\frac{9}{2}\rho^2-\frac{9}{2}\rho^3}{\frac{3}{2}\rho^2+2\rho+1},$$

$$p_3 = \frac{9}{2}\rho^3 p_0 = \frac{9}{2}\rho^3\,\frac{1-\rho}{\frac{3}{2}\rho^2+2\rho+1} = \frac{\frac{9}{2}\rho^3-\frac{9}{2}\rho^4}{\frac{3}{2}\rho^2+2\rho+1} \;\therefore$$

$$p_n = \frac{9}{2}\rho^n p_0 = \frac{9}{2}\rho^n\,\frac{1-\rho}{\frac{3}{2}\rho^2+2\rho+1} = \frac{\frac{9}{2}\rho^n-\frac{9}{2}\rho^{n+1}}{\frac{3}{2}\rho^2+2\rho+1}, \text{ for } n \geq 2$$

(Q1c:)

Let X be the random variable denoting the number of individuals in the system at steady state. $\therefore$

The expected number ~~in the~~ of patients in the system is: $E(X)=G_X'(1)$

$$\therefore G_X(\theta)=\sum_{n=0}^{\infty}P_n\theta^n = P_0\theta^0+P_1\theta^1+\sum_{n=2}^{\infty}P_n\theta^n = P_0+P_1\theta+\sum_{n=2}^{\infty}P_n\theta^n =$$

$$\frac{1-\rho}{\frac{3}{2}\rho^2+2\rho+1}+\frac{3\rho-3\rho^2}{\frac{3}{2}\rho^2+2\rho+1}\theta+\sum_{n=2}^{\infty}\frac{9}{2}\frac{1-\rho}{(\frac{3}{2})\rho^2+2\rho+1}\rho^n\theta^n =$$

$$\frac{1-\rho}{\frac{3}{2}\rho^2+2\rho+1}\left[1+3\rho\theta+\frac{9}{2}\sum_{n=2}^{\infty}\rho^n\theta^n\right]=$$

$$\frac{1-\rho}{\frac{3}{2}\rho^2+2\rho+1}\left[1+3\rho\theta-\frac{9}{2}\rho^1\theta^1+\frac{9}{2}\sum_{n=1}^{\infty}(\rho\theta)^n\right]=$$

$$\frac{1-\rho}{\frac{3}{2}\rho^2+2\rho+1}\left[1-\frac{3}{2}\rho\theta+\frac{9}{2}\frac{\rho\theta}{1-\rho\theta}\right]=G_X(\theta) \quad (\text{for } |\rho\theta|<1)\ \therefore$$

$$G_X'(\theta)=\frac{d}{d\theta}\left(G_X(\theta)\right)=\frac{d}{d\theta}\left(\frac{1-\rho}{\frac{3}{2}\rho^2+2\rho+1}\left[1-\frac{3}{2}\rho\theta+\frac{9}{2}\frac{\rho\theta}{1-\rho\theta}\right]\right)=$$

$$\frac{1-\rho}{\frac{3}{2}\rho^2+2\rho+1}\left[\frac{d}{d\theta}(1)-\frac{3}{2}\rho\frac{d}{d\theta}(\theta)+\frac{9}{2}\rho\frac{d}{d\theta}\left(\frac{\theta}{1-\rho\theta}\right)\right]=$$

$$\frac{1-\rho}{\frac{3}{2}\rho^2+2\rho+1}\left[-\frac{3}{2}\rho+\frac{9}{2}\rho\frac{1-\rho\theta-\theta(-\rho)}{(1-\rho\theta)^2}\right]=$$

$$\frac{1-\rho}{\frac{3}{2}\rho^2+2\rho+1}\left[-\frac{3}{2}\rho+\frac{9}{2}\rho\frac{1}{(1-\rho\theta)^2}\right]=G_X'(\theta)\quad\therefore$$

$$E(X)=G_X'(1)=G_X'(\theta)\Big|_{\theta=1}=\frac{1-\rho}{\frac{3}{2}\rho^2+2\rho+1}\left[-\frac{3}{2}\rho+\frac{9}{2}\rho\frac{1}{(1-\rho\theta)^2}\right]\Bigg|_{\theta=1}=$$

$$\frac{1-\rho}{\frac{3}{2}\rho^2+2\rho+1}\left[-\frac{3}{2}\rho+\frac{9}{2}\rho\frac{1}{(1-\rho)^2}\right]=\frac{1}{\frac{3}{2}\rho^2+2\rho+1}\left[-\frac{3}{2}\rho(1-\rho)+\frac{9}{2}\rho\frac{1}{(1-\rho)}\right]=$$

$$\frac{1}{\frac{3}{2}\rho^2+2\rho+1}\left[\frac{(-\frac{3}{2}\rho+\frac{3}{2}\rho^2)(1-\rho)+\frac{9}{2}\rho}{(1-\rho)}\right]=\frac{-\frac{3}{2}\rho+\frac{3}{2}\rho^2+\frac{3}{2}\rho^2-\frac{3}{2}\rho^3+\frac{9}{2}\rho}{(\frac{3}{2}\rho^2+2\rho+1)(1-\rho)}=$$

$$\frac{-\frac{3}{2}\rho^3+3\rho^2+3\rho}{(\frac{3}{2}\rho^2+2\rho+1)(1-\rho)}=L_s\equiv \text{expected number of patients in the system}$$

(Q1 d:)

Let W_s be the expected waiting time ~~in~~ of the system. $\therefore$

$$W_s = \frac{L_s}{\lambda_{eff}} \text{ by littles theorem,}$$

where λ_{eff} is the effective arrival rate.

$$\therefore \lambda_{eff} = \sum_{n=0}^{\infty} \lambda_n P_n = \sum_{n=0}^{\infty} 3P_n = 3\sum_{n=0}^{\infty} P_n = 3(1) = 3 \quad , \because \lambda \text{ is constant with time,}$$

$$\rho = \frac{\lambda}{3\mu} = \frac{3}{3(2)} = \frac{1}{2} \Rightarrow \rho = 0.5, \therefore$$

$$L_s = \frac{-\frac{3}{2}\rho^3 + 3\rho^2 + 3\rho}{(\frac{3}{2}\rho^2 + 2\rho + 1)(1-\rho)} = \frac{-\frac{3}{2}(0.5)^3 + 3(0.5)^2 + 3(0.5)}{(\frac{3}{2}(0.5)^2 + 2(0.5) + 1)(1-0.5)} = \cancel{\frac{(-39/16)}{(\frac{19}{8})(\frac{1}{2})}} =$$

$$\cancel{\frac{(-39/16)}{(19/16)}} \cancel{=} \frac{(33/16)}{(\frac{19}{8})(0.5)} = \frac{(33/16)}{(19/16)} = \frac{33}{19} \approx 1.74 \text{ (3S.F.)}$$

$$\therefore L_s = \frac{33}{19} \therefore$$

$$W_s = \frac{(33/19)}{3} = \frac{11}{19} \text{ hours} \approx 0.579 \text{ hours (3S.F.)}$$

$$W_s = \frac{11}{19} \times 60 \text{ minutes} = \cancel{34.7m} \frac{660}{19} \text{ minutes} \approx 34.7 \text{ minutes (3S.F.)}$$

is the expected waiting time of the system.

(Q1e:)

Treatment took on average 50 minutes $= \frac{5}{6}$ hours $\therefore$

$$\frac{1}{(5/6)} = \frac{6}{5} \text{ patients per hour} = \mu \quad \therefore$$

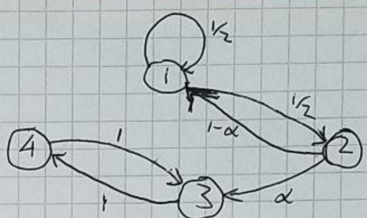
with $\lambda = 4$:

$$\rho = \frac{\lambda}{3\mu} = \frac{4}{3 \times \left(\frac{6}{5}\right)} = \frac{10}{9} > 1 \quad \text{and}$$

$$L_s = \frac{-\frac{3}{2}\rho^3 + 3\rho^2 + 3\rho}{\left(\frac{3}{2}\rho^2 + 2\rho + 1\right)(1-\rho)} = \frac{-1210}{137} < 0$$

$\therefore$ Because $\rho > 1$ and the queue has infinite capacity: the number of patients in the queue will ~~only~~ increase, and with enough time; explode to infinity. Because it is an M/M/3 queue with infinite capacity, with $\rho > 1$.

(2 a) transition diagram:



$\therefore$ States $\{1,2\}$ form a irreducible subchain

states $\{3,4\}$ form a irreducible subchain

Let $f_2^{(n)} = \Pr(\text{first return to state 2 at time } n) \therefore$

$f_2^{(1)} = 0$, $f_2^{(2)} = (1-\alpha)\frac{1}{2}$, $f_2^{(3)} = (1-\alpha)\frac{1}{2} \times \frac{1}{2}$, $f_2^{(4)} = (1-\alpha)\left(\frac{1}{2}\right)^2 \frac{1}{2}$ $\therefore$

$f_2^{(n)} = (1-\alpha)\left(\frac{1}{2}\right)^{n-2}\frac{1}{2}$ for $n \geq 3$ $\therefore$

$$f_2 = \sum_{n=1}^{\infty} f_2^{(n)} = \cancel{0 + \sum_{n=2}} \; 0 + (1-\alpha)\frac{1}{2} + \sum_{n=3}^{\infty}(1-\alpha)\left(\frac{1}{2}\right)^{n-2}\frac{1}{2} =$$

$$(1-\alpha)\frac{1}{2} + \sum_{n=1}^{\infty}(1-\alpha)\left(\frac{1}{2}\right)^n \frac{1}{2} = (1-\alpha)\frac{1}{2} + (1-\alpha)\frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n = \qquad \left(\left|\frac{1}{2}\right| < 1 \therefore \right)$$

$$(1-\alpha)\frac{1}{2} + (1-\alpha)\frac{1}{2}\left(\frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)}\right) = (1-\alpha)\frac{1}{2} + (1-\alpha)\frac{1}{2}\times 1 = (1-\alpha)\frac{1}{2} + (1-\alpha)\frac{1}{2} = 1-\alpha$$

$\therefore \because \alpha \in (0,1) \therefore 0 < \alpha < 1 \therefore 1-\alpha < 1 \therefore f_2 < 1 \therefore$

state 2 is not recurrent $\therefore$ State 2 is transient

Let $f_3^{(n)} = \Pr(\text{first return to state 3 at step } n) \therefore$

$f_3^{(1)} = 0$, $f_3^{(2)} = 1 \times 1 = 1$, $f_3^{(3)} = 0$, $f_3^{(4)} = 0$, $f_3^{(5)} = 0$ $\therefore$

$f_3^{(n)} = 0$ for $n \geq 3$ $\therefore$

$$f_3 = \sum_{n=1}^{\infty} f_3^{(n)} = f_3^{(1)} + f_3^{(2)} + \sum_{n=3}^{\infty} f_3^{(n)} = 0 + 1 + \sum_{n=3}^{\infty} 0 = 0 + 1 + 0 = 1 = f_3 \therefore$$

State 3 is recurrent

(Q2b)

states $\{1,2\}$ form a irreducible subchain and state 2 is transient $\therefore$ state 1 is also transient

$f_2^{(1)} = 0$, $f_2^{(2)} = (1-\alpha)\frac{1}{2}$, $f_2^{(3)} = (1-\alpha)(\frac{1}{2})\frac{1}{2}$ $\therefore$

2 and 3 are both prime $\therefore$

state 2 is aperiodic $\therefore$

subchain $\{1,2\}$ is aperiodic, transient, $\therefore$ not ergodic

states $\{3,4\}$ form a irreducible subchain

state 3 has a period of 2 $\therefore$

subchain $\{3,4\}$ has a period of 2

state 3 is recurrent $\therefore$

subchain $\{3,4\}$ is recurrent $\therefore$

subchain $\{3,4\}$ has a period of 2, is recurrent $\therefore$ is not ergodic

$\therefore$ only ergodic if subchain is aperiodic and positively recurrent.

(Q2C)

Let steady state vector $\tilde{P} = (\tilde{P}_1, \tilde{P}_2, \tilde{P}_3, \tilde{P}_4) = (\tilde{P}_1 \ \tilde{P}_2 \ \tilde{P}_3 \ \tilde{P}_4)$

Such that $\tilde{P} = \tilde{P}\, T = (\tilde{P}_1 \ \tilde{P}_2 \ \tilde{P}_3 \ \tilde{P}_4) \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 1-\alpha & 0 & \alpha & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = (\tilde{P}_1 \ \tilde{P}_2 \ \tilde{P}_3 \ \tilde{P}_4)$

$= \left(\tilde{P}_1 \tfrac{1}{2} + \tilde{P}_2(1-\alpha) + 0\tilde{P}_3 + 0\tilde{P}_4 \ , \tfrac{1}{2}\tilde{P}_1 \ , \alpha\tilde{P}_2 + 1\tilde{P}_4 \ , 1\tilde{P}_3\right) =$

$\left(\tfrac{1}{2}\tilde{P}_1 + (1-\alpha)\tilde{P}_2 \ , \tfrac{1}{2}\tilde{P}_1 \ , \alpha\tilde{P}_2 + \tilde{P}_4 \ , \tilde{P}_3\right) =$

$\left(\tfrac{1}{2}\tilde{P}_1 + (1-\alpha)\tilde{P}_2 \quad \tfrac{1}{2}\tilde{P}_1 \quad \alpha\tilde{P}_2 + \tilde{P}_4 \quad \tilde{P}_3\right) = \cancel{(\tilde{P}_1 \ \tilde{P}_2} \ (\tilde{P}_1 \quad \tilde{P}_2 \quad \tilde{P}_3 \quad \tilde{P}_4) \ \therefore$

$\tfrac{1}{2}\tilde{P}_1 + (1-\alpha)\tilde{P}_2 = \tilde{P}_1 \ , \ \tfrac{1}{2}\tilde{P}_1 = \tilde{P}_2 \ , \ \alpha\tilde{P}_2 + \tilde{P}_4 = \tilde{P}_3 \ , \ \tilde{P}_3 = \tilde{P}_4 \ \therefore$

$\tfrac{1}{2}\tilde{P}_1 + (1-\alpha)\tfrac{1}{2}\tilde{P}_1 = \tilde{P}_1 \quad \therefore (1-\alpha)\tfrac{1}{2}\tilde{P}_1 = \tilde{P}_1 - \tfrac{1}{2}\tilde{P}_1 = \tfrac{1}{2}\tilde{P}_1 \ \therefore$

$\cancel{\Rightarrow (1-\alpha)\tfrac{1}{2}\tilde{P}_1} \quad (1-\alpha)\tilde{P}_1 = 1\tilde{P}_1$ and $\alpha \in (0,1) \ \therefore \ 0 < \alpha < 1 \ \therefore$

$1-\alpha < 1 \ \therefore \ \tilde{P}_1 = 0 \ \therefore$

$\tfrac{1}{2}\tilde{P}_1 = \tilde{P}_2 = 0 \ \therefore$

$\alpha\tilde{P}_2 + \tilde{P}_4 = \alpha(0) + \tilde{P}_4 = \tilde{P}_4 = \tilde{P}_3$

and $\tilde{P}_1 + \tilde{P}_2 + \tilde{P}_3 + \tilde{P}_4 = 1 = \tilde{P}_3 + \tilde{P}_4 \ \therefore$

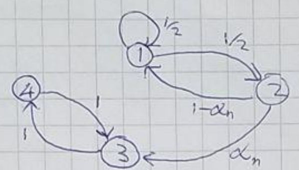
$1 = \tilde{P}_4 + \tilde{P}_4 = 2\tilde{P}_4 \ \therefore \ \tfrac{1}{2} = \tilde{P}_4 \ \therefore$

$\tilde{P}_3 = \tfrac{1}{2} \ \therefore$

$\tilde{P} = (0 \quad 0 \quad \tfrac{1}{2} \quad \tfrac{1}{2})$

(Q 2d)

$\therefore \quad TP(n) = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 1-\alpha_n & 0 & \alpha_n & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad \alpha_n \in (0,1) \quad \therefore$



$\therefore$

Let $f_1^{(n)} = Pr($first return to state 1 at time $n) \therefore$

$f_1^{(1)} = \frac{1}{2}$, $f_1^{(2)} = \frac{1}{2}(1-\alpha_2)$, $f_1^{(3)} = 0$, $f_1^{(4)} = 0$, $f_1^{(5)} = 0$, ...

$\therefore f_1^{(n)} = 0$ for $n \geq 3 \quad \therefore$

$f_1 = \sum_{n=1}^{\infty} f_1^{(n)} = f_1^{(1)} + f_1^{(2)} + \sum_{n=3}^{\infty} f_1^{(n)} = \frac{1}{2} + \frac{1}{2}(1-\alpha_2) + \sum_{n=3}^{\infty} 0 =$

$\frac{1}{2} + \frac{1}{2}(1-\alpha_2) + 0 = \frac{1}{2} + \frac{1}{2} - \frac{1}{2}\alpha_2 = 1 - \frac{1}{2}\alpha_2 = f_1 \quad \therefore$

for state 1 to be recurrent: $f_1 = 1 \quad \therefore$

$f_1 = 1 - \frac{1}{2}\alpha_2 = 1 \quad \therefore \qquad \frac{1}{2}\alpha_2 = 0 \quad \therefore$

$\alpha_2 = 0 \quad \therefore$

$\alpha_2 = 0$ for state 1 to be recurrent

$\mu_1 = \sum_{n=1}^{\infty} n f_1^{(n)} = 1 f_1^{(1)} + 2 f_1^{(2)} + \sum_{n=3}^{\infty} n f_1^{(n)} =$

$\frac{1}{2} + (2)\frac{1}{2}(1-\alpha_2) + \sum_{n=3}^{\infty} n(0) = \frac{1}{2} + (1-0) + \sum_{n=3}^{\infty} 0 = \frac{1}{2} + 1 + 0 =$

$\frac{3}{2} = \mu_1 < \infty \quad \therefore$

state 1 is positively recurrent