Laboratory 5

Moodle quiz: 5/5/25 – 5/12/25

Goal

Experiment with resonant circuits.

1 Series RLC Circuit

Consider the series RLC circuit in Fig. 1.

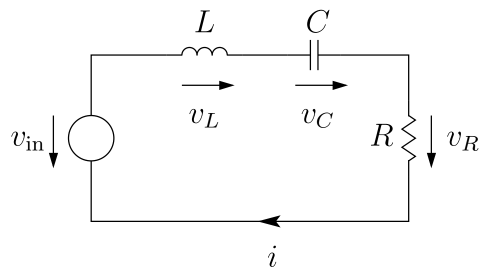

Figure 1: Series RLC circuit.

The circuit is easy to analyze in the Laplace domain. The device equations are:

$$V_R \;=\; RI\,, \qquad V_L = LsI\,, \qquad I = CsV_C\,.$$

Apply KVL to the circuit loop:

$$V_{\text{in}} \;=\; V_L + V_C + V_R\,.$$

We are interested in output voltage V_R, and deduce the corresponding transfer function:

$$V_{\text{in}} \;=\; LsI + \frac{I}{Cs} + V_R \;=\; \Big(Ls + \frac{1}{Cs}\Big)\frac{V_R}{R} + V_R$$

$$\Rightarrow \qquad H(s) \;=\; \frac{V_R}{V_{\text{in}}} \;=\; \frac{\frac{R}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}\,.$$

Plot the frequency response using the transfer-function model:

```
R = 100, L = 1e-3, C = 10e-9;        % see Lab 2

num = [ R/L, 0 ];
den = [ 1, R/L, 1/(L*C) ];
sys = tf(num, den);
bode(sys);                           % plot frequency response
```

Matlab/Octave enable us to compare the theoretical model of the frequency response graphically with experimental data. First, get the model data from function `bode`:

```
[mag, pha, w] = bode(sys);
```

Assuming we have recorded our experimental data in a file using the comma-separated value (csv) format,[1] read the experimental data of the network analysis into Octave:

```
x = csvread('rlcss_scope.csv');  % columns: freq, magnitude (linear), phase
```

In Matlab you may have to adapt to the dimension of `x`. Plot the magnitude response:

```
figure(2);
subplot(2,1,1), semilogx(w, 20*log10(mag)), grid;
xlabel('Frequency [rad/s]'), ylabel('Magnitude Response [dB]');
hold on
plot(2*pi*x(:,1), 20*log10(x(:,2)))
legend({'model', 'data'})
```

and the phase response, see Fig. 2:

```
subplot(2,1,2), semilogx(w, pha), grid;
xlabel('Frequency [rad/s]'), ylabel('Phase Response [deg]');
hold on
plot(2*pi*x(:,1), x(:,3))
```

By visual comparison, model and reality match reasonably well. In particular, the predicted and measured resonant frequencies agree at $f_0 \approx 310/2\pi\,\mathrm{kHz} \approx 50\,\mathrm{kHz}$.

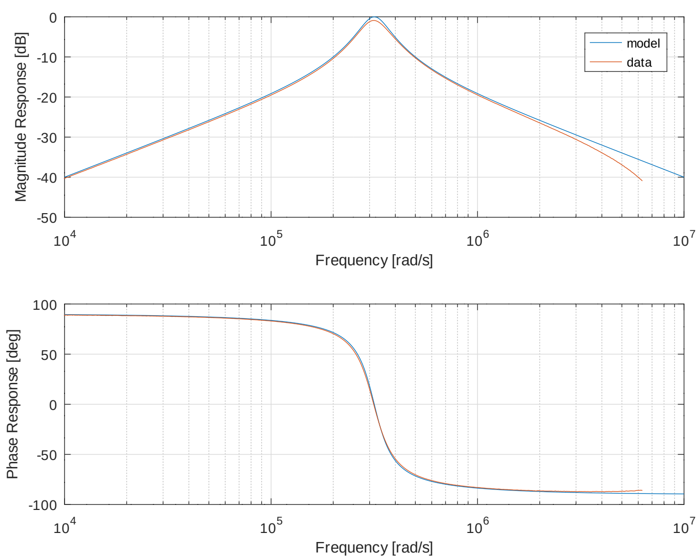

Figure 2: Frequency response of RLC circuit: comparison of model and experimental data.

[1] In Waveforms select the desired unit (gain or dB) in the Magnitude section, and unit Degree in the gear tool of the Phase section. Then, select Export in the gear tool of the channel (1 or 2) you wish to export.

We confirm the value of resonant frequency f_0 by recalling that the inductive and capacitive impedances should cancel each other at f_0:

$$Z_L + Z_C \;=\; j2\pi f_0 L + \frac{1}{j2\pi f_0 C} \;=\; 0$$

$$\Rightarrow \qquad f_0 \;=\; \frac{1}{2\pi\sqrt{LC}} \;=\; \frac{1}{2\pi\sqrt{10^{-3}10^{-8}}} \;=\; 50.33\,\text{kHz}\,.$$

At resonant frequency f_0 the circuit is purely resistive, and the phase shift between V_R and V_{in} should be $0°$, as seen in Fig. 2.

The frequency response shows that the RLC circuit is a frequency-selective bandpass filter. It passes sinusoidal input voltages with frequencies about the resonant frequency and attenuates significantly sinusoids with frequencies much smaller and much larger than the resonant frequency. We can simulate this behavior using function `lsim`, see Fig. 3:

```
f0 = 1/(2*pi*sqrt(L*C));      % resonant frequency
t = [0:0.01/f0:10/f0];        % time array
x = sin(2*pi*f0*t);           % sinusoid
lsim(sys, x, t);              % plot system response for input x
```

Change the frequency and plot the system response. For example, Fig. 3 shows the system response for $f = f_0/10$ on the right. The output voltage is significantly attenuated compared to the resonant frequency. The resonance phenomenon also occurs in mechanical systems, as demonstrated in this video: `www.youtube.com/watch?v=rvwwQAfdBWs`.

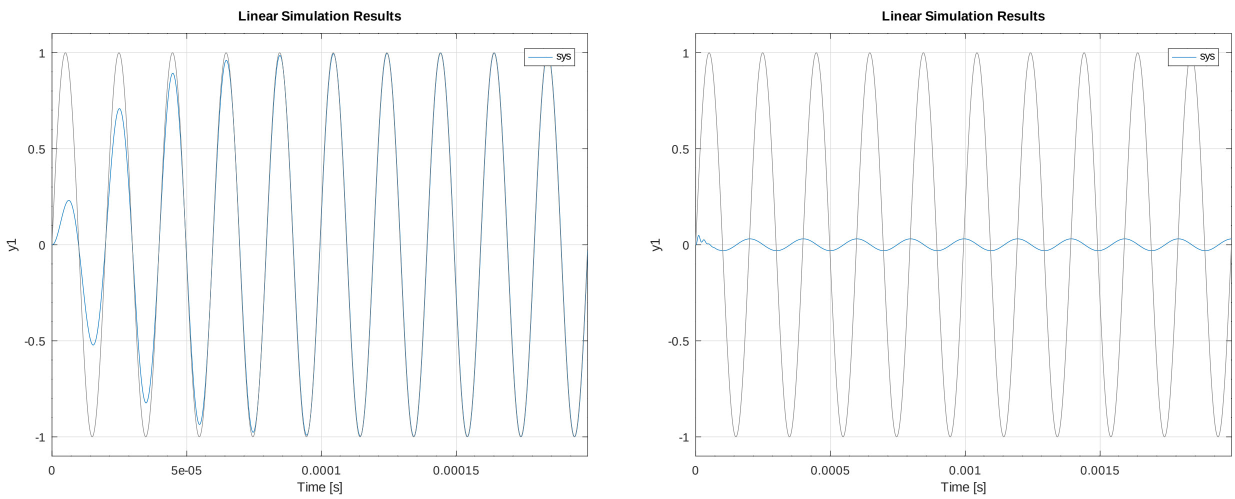

Figure 3: System response (blue) of RLC circuit: input sinusoid (black) with $f = f_0$ (left) and $f = f_0/10$ (right).

Exercises

1. Derive a state-space model (see Lab 2) of the RLC circuit in Fig. 1 for output voltage $v_{\text{out}}(t) = v_L(t) + v_C(t)$.

2. Plot the frequency response of the RLC circuit with output voltage $v_{\text{out}}(t)$.

3. Generate the pole-zero plot using function `pzmap`.

4. Plot the step response of the RLC circuit with output voltage $v_{\mathrm{out}}(t)$.

2 PreLab Problems

Use Matlab/Octave to analyze the following circuits.

1. Derive a state-space model of the circuit in Fig. 4 and plot the frequency response for output $v_R(t)$. Determine the resonant frequency of the circuit.

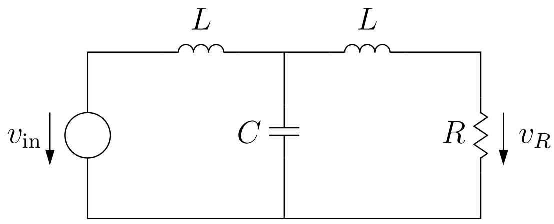

Figure 4: RLC circuit with two inductors each with inductance L.

2. Derive a state-space model of the circuit in Fig. 5 and plot the frequency response for output $v_R(t)$. Determine the resonant frequency of the circuit.

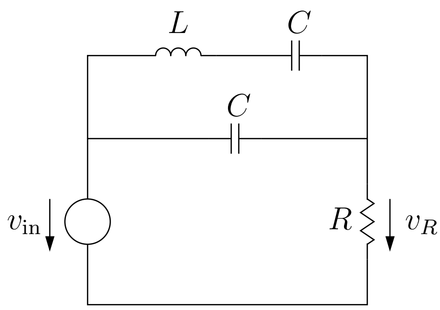

Figure 5: RLC circuit with two capacitors each with capacitance C.

3 Lab Problems

We build resonant circuits and compare experimental measurements with the theoretical models of Sec. 1 and Sec. 2. Use the *Network Analyzer* of the Analog Discovery 2 to measure the frequency response of the RLC circuit.

Plot the measured frequency response together with your simulated frequency response to perform a meaningful comparison of theory and experiment for these circuits:

1. series RLC circuit in Fig. 1 with output voltage $v_{\mathrm{out}}(t) = v_L(t) + v_C(t)$,

2. the RLC circuit in Fig. 4 with output voltage $v_R(t)$, and

3. the RLC circuit in Fig. 5 with output voltage $v_R(t)$.

Explain potential deviations between model and experiment.