Laboratory 7

Moodle quiz: 5/19/25 – 5/26/25

Goal

Experiment with ladder filters.

1 Ladder Filters

We design improved filter circuits that come closer to their frequency-selective ideals.

1.1 *RC*-Ladder Filter

An RC low-pass filter (LPF) has a roll-off (slope of the transition region) of $-20\,\text{dB/decade}$. The steeper the roll-off the closer a circuit comes to realize an ideal LPF, which has no transition region between pass-band and stop-band. Ladder filters offer a simple design recipe to obtain a steeper transition region: increase the number of stages. Consider the series composition of two RC circuits in Fig. 1.

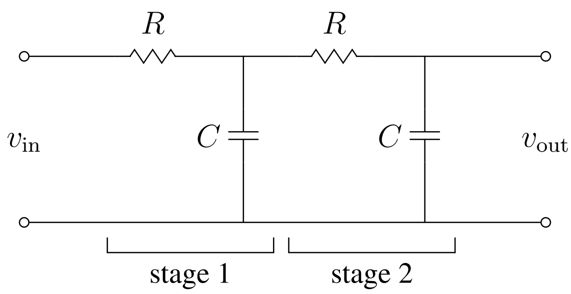

Figure 1: Two-stage ladder: series composition of two identical RC low-pass filters.

Fig. 2 below compares the frequency response of an RC (1-stage ladder) LPF with that of the 2-stage ladder in Fig. 1. The roll-off of the 2-stage ladder is $-40\,\text{dB/decade}$, i.e. twice as steep as for a single stage. However, the cut-off frequency of the 2-stage ladder is smaller than that of the 1-stage ladder, and the kink at the cut-off frequency is smoother or wider for the 2-stage ladder. The Matlab/Octave script for the frequency response in Fig. 2 is:

```
R = 100;  C = 10e-9;

% single RC stage
num = [ 1 ];
den = [ R*C, 1 ];
sys1 = tf(num, den);

% two-stage RC ladder
```

```
A = [ -2/(R*C), 1/(R*C); 1/(R*C), -1/(R*C) ];   % s = [v_C1, v_C2]^T
B = [ 1/(R*C); 0 ];
C = [ 0, 1 ];
D = 0;
sys2 = ss(A,B,C,D);

bode(sys1, sys2);       % compare 1-stage with 2-stage frequency response
```

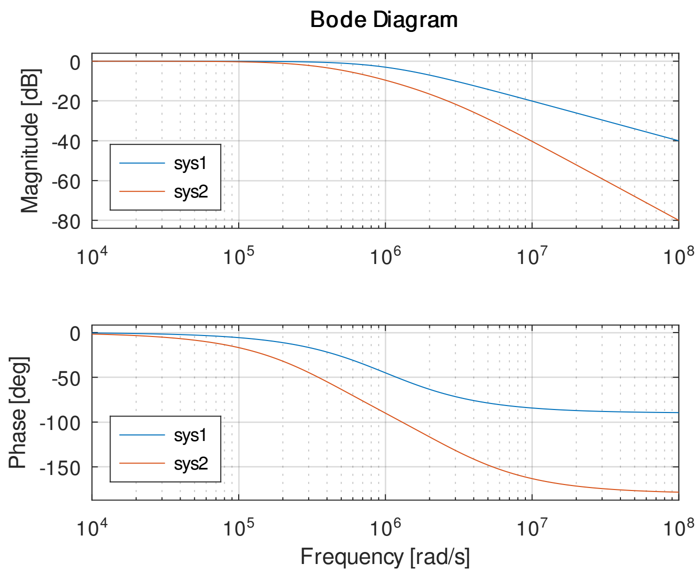

Figure 2: Frequency response of 1-stage (sys1) and 2-stage (sys2) RC ladder.

Longer ladders increase the slope of the transition region further. The longer the ladder the smaller the cut-off frequency and the smoother the kink between pass-band and transition region. The widening of the kink can be avoided by section isolation.

1.2 Isolation of Filter Sections

Isolation of ladder stages, aka filter sections, is accomplished with a voltage follower that serves as a buffer between ladder stages, see Fig. 3. Under the ideal op-amp assumptions the voltage follower's output voltage equals the voltage applied to its non-inverting input.

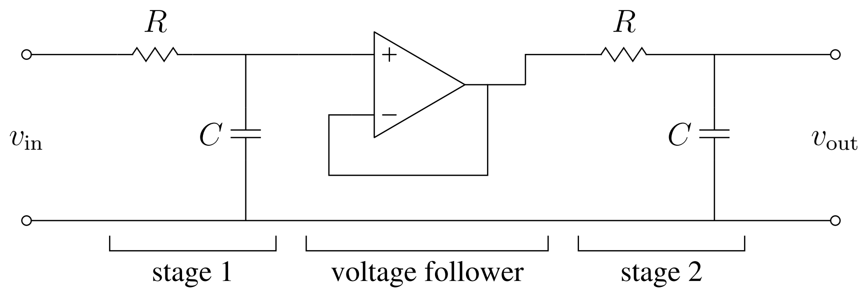

Figure 3: Two-stage ladder with section isolation using a voltage follower.

The transfer function of the voltage follower is the identity function $v_{\text{out}}/v_{\text{in}} = 1$, see Fig. 4. Since no current flows into the input of an ideal op-amp, we model the voltage follower with a dependent source, more succinctly a voltage-controlled voltage source, shown in Fig. 4 on the right. The circuit driving the input of the voltage follower does not supply any current, and the power supply of the op-amp (not shown in the schematic) supplies output voltage $v_{\text{out}} = v_{\text{in}}$ to the load, no matter how much current the load draws. This is the behavior of an ideal electrical buffer that isolates the load from the driver circuit.

Figure 4: Buffer model: under ideal op-amp assumptions, the voltage follower (left) can be modeled as a voltage-controlled voltage source (right).

Using the buffer model for the voltage follower in the 2-stage RC ladder of Fig. 3 leads to the circuit model in Fig. 5. The buffer turns the 2-stage ladder into a series composition of isolated RC stages. Each stage has transfer function

$$H(s) = \frac{V_C}{V_{\text{in}}} = \frac{1}{1 + RCs}\,.$$

Due to the isolating buffer, the transfer function of the series composition of two ladder stages is the product of their transfer functions:

$$H_2(s) = H_{\text{stage1}}(s)H_{\text{stage2}}(s) = \frac{V_C}{V_{\text{in}}}\frac{V_{\text{out}}}{V_C} = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{(1 + RCs)^2}\,.$$

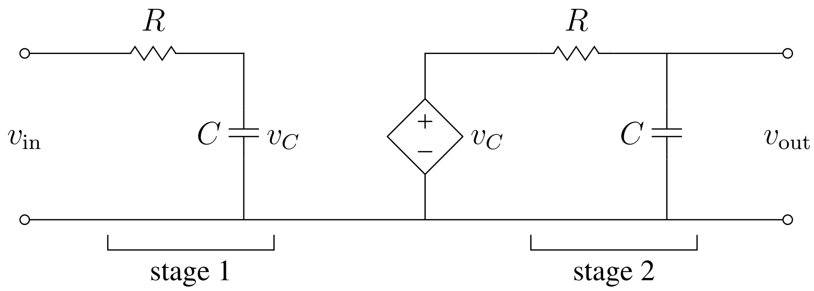

Figure 5: Two-stage ladder with ideal buffer modeled as voltage-controlled voltage source.

The product of transfer functions is supported by Matlab/Octave, which simplifies plotting the frequency response considerably:

```
R = 100;  C = 10e-9;

% single RC stage
num = [ 1 ];
```

```
den = [ R*C, 1 ];
sys1 = tf(num, den);

% two-stage RC ladder with buffer
sys2 = sys1 * sys1;      % series composition

bode(sys1, sys2);        % compare 1-stage with 2-stage frequency response
```

The comparison of the 1-stage RC circuit and the 2-stage ladder is shown in Fig. 6. Furthermore, comparing the 2-stage frequency response in Fig. 6 (sys2) with the frequency response in Fig. 2 (sys2) shows that the voltage follower sharpens the kink around the cut-off frequency, resulting in a better approximation of an ideal low-pass filter.

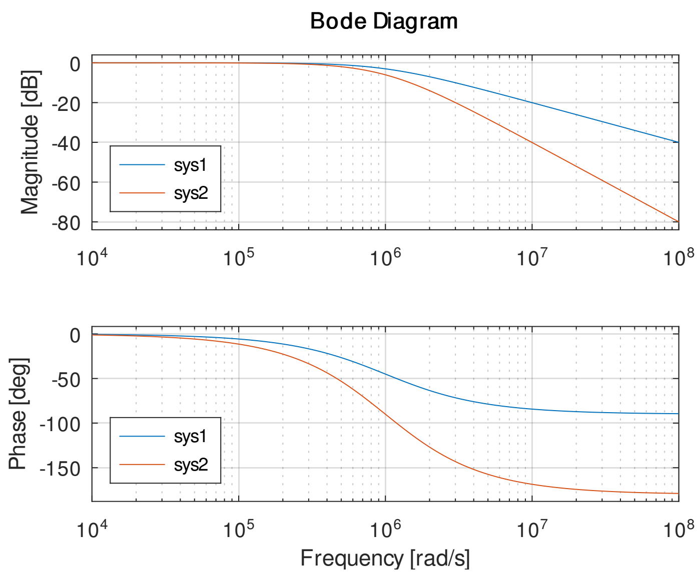

Figure 6: Frequency response of 1-stage (sys1) and buffered 2-stage (sys2) RC ladder.

Note that the isolating buffer is the key enabler for modeling the series composition of RC stages as a product of transfer functions from the systems perspective. Without the buffer, the circuit in Fig. 1 is a series composition of RC stages, where the presence of stage 2 affects the voltage across the capacitor of stage 1. From the circuit perspective, stage 2 forms a parallel composition with the capacitor of stage 1. Therefore, the circuit in Fig. 1 must be analyzed as a whole, which leads to the state-space model of the Matlab script in Sec. 1.1.

2 PreLab Problems

1. Analyze the LC ladder filter in Fig. 7 and plot the frequency response. Compare the frequency response of the two-stage ladder with that of a single stage. Which filter characteristic does the circuit exhibit?

2. Isolate the stages in Fig. 7 with a voltage follower. Analyze the circuit and plot the frequency response.

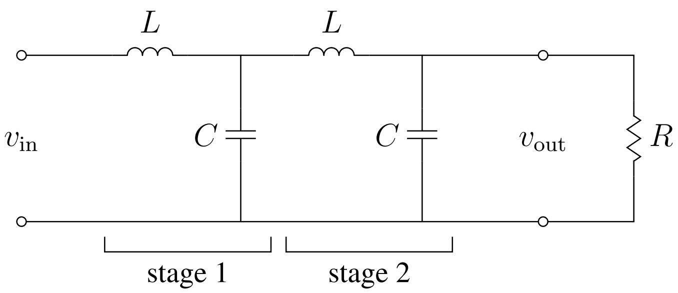

Figure 7: Two-stage ladder of identical LC circuits and load resistor R.

3. Analyze the LC ladder filter in Fig. 8 and plot the frequency response. Compare the frequency response of the two-stage ladder with that of a single stage. Which filter characteristic does the circuit exhibit?

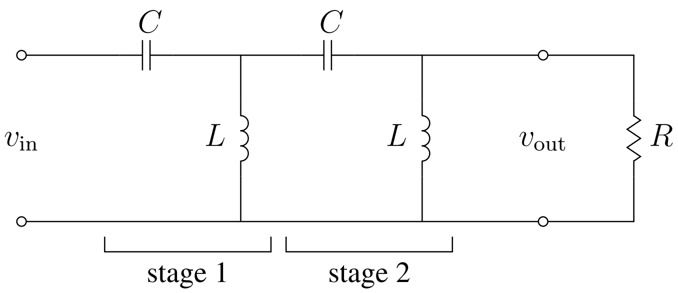

Figure 8: Two-stage ladder of identical LC circuits and load resistor R.

4. Isolate the stages in Fig. 8 with a voltage follower. Analyze the circuit and plot the frequency response.

3 Lab Problems

1. Build a voltage follower using the OP27 or one of the four op-amps in the OP484 package.

   (a) Download the datasheet for the pin configurations from:

   `wiki.analog.com/university/tools/adalp2000/parts-index`

   (b) Connect the V+ pin of the op-amp chip to the *AD2* positive power supply (red wire V+), and the V- pin of the op-amp to ground (one of the black *AD2* wires). You can switch the power on or off in the *Supplies* section of the *Waveforms* software.

   (c) Do not connect the TRIM pins of the op-amp chip.

   (d) Connect the op-amp output with a jumper wire to the inverting op-amp input.

   (e) Connect *Wavegen1* to the non-inverting op-amp input and the *Scope* to the op-amp output.

(f) Select a DC source type for *Wavegen1*, and set both amplitude and offset to 0 V initially. Then, switch on the positive power supply using a voltage of 5 V to power up the op-amp. Thereafter, run *Wavegen1* to drive the DC input voltage.

(g) Vary the DC input voltage between 0 V and 5 V, and observe the output voltage using the *Scope*.

(h) Power down the op-amp, and connect its V- pin to the negative power supply of the *AD2* (white wire V-). Set the negative power supply to −5 V. Power up the op-amp and vary the DC input voltage between −5 V and 5 V. Observe the input and output voltages with the *Scope*.

(i) Verify the functionality of the voltage follower by loading the op-amp output with various resistances, and identify its operating range.

2. Verify the *RC* ladder filters of Sec. 1 experimentally.

3. Plot the measured frequency response together with your simulated frequency response, and compare theory and experiment for the circuits in Prelab Problems 1–4.