Analog and Digital Signal Processing May 5, 2025
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Laboratory 6

Moodle quiz: 5/12/25 – 5/19/25

Goal

Experiment with passive filter circuits.

1 Passive Filters

A series RC circuit realizes either a low-pass filter (LPF) or a high-pass filter (HPF), depending on our choice of the output voltage.

1.1 RC Low-Pass Filter

We derive a transfer-function model of the RC circuit in Fig. 1.

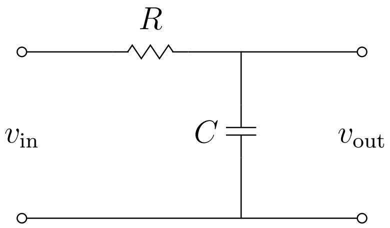

Figure 1: Series RC circuit with $v_{\text{out}} = v_C$ serves as low-pass filter.

The device equations of the resistor and the capacitor in the Laplace domain are:

$$V_R \;=\; RI\,, \qquad I \;=\; CsV_C\,.$$

Apply KVL to the circuit loop recognizing that $V_{\text{out}} = V_C$:

$$V_{\text{in}} \;=\; V_R + V_{\text{out}}\,.$$

Rearrange the equations to obtain the transfer function:

$$V_{\text{in}} \;=\; RCsV_{\text{out}} + V_{\text{out}}$$

$$\Rightarrow \qquad H(s) \;=\; \frac{V_{\text{out}}}{V_{\text{in}}} \;=\; \frac{1}{RCs+1}$$

The frequency response exhibits low-pass characteristic, see Fig. 2:

```
R = 100, C = 10e-9;                   % see Lab 5

num = [ 1 ];
den = [ R*C, 1 ];
sys = tf(num, den);
bode(sys);                            % plot frequency response
```

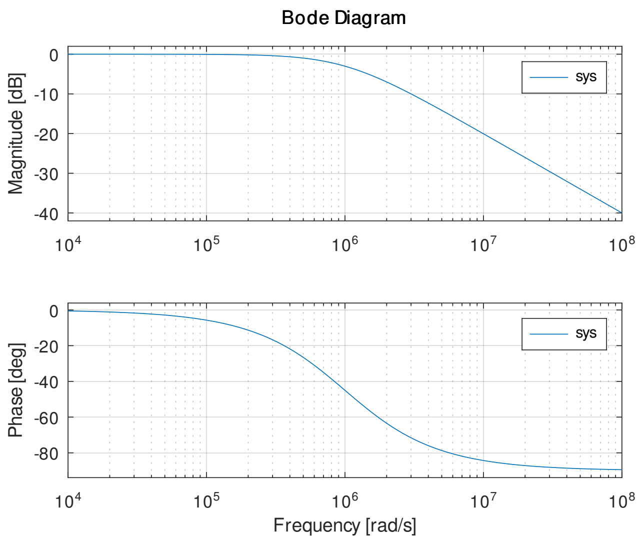

Figure 2: Frequency response of RC low-pass filter.

The cutoff or 3 dB frequency characterizes the location of the kink in the magnitude response:

$$\omega_c = \frac{1}{RC} = \frac{1}{100 \cdot 10 \times 10^{-9}} = 10^6 \mathrm{rad/s}$$

or $f_c = \omega_c / 2\pi = 159\,\mathrm{kHz}$.

1.2 RC High-Pass Filter

The RC circuit in Fig. 3 uses V_R as output voltage, after exchanging R and C of the circuit in Sec. 1.1.

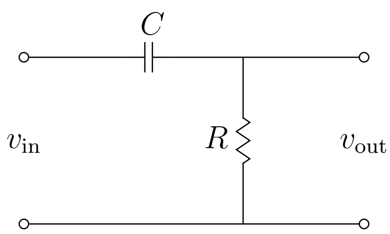

Figure 3: Series RC circuit with $v_{\text{out}} = v_R$ serves as high-pass filter.

We deduce the transfer function analogously to Sec. 1.1, using $V_{\text{out}} = V_R$:

$$V_{\text{in}} = V_R + V_C = V_{\text{out}} + V_C = V_{\text{out}} + \frac{1}{RCs} V_{\text{out}}$$

$$\Rightarrow \qquad H(s) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{1 + \frac{1}{RCs}} = \frac{RCs}{RCs + 1}$$

Compared to the LPF, the transfer function changes in the numerator, and the frequency response in Fig. 4 exhibits a high-pass characteristic:

```
R = 100, C = 10e-9;                 % see RC LPF

num = [ R*C, 0 ];
den = [ R*C, 1 ];
sys = tf(num, den);
bode(sys);                          % plot frequency response
```

Note that cutoff frequency $\omega_c = 1/RC = 10^6$ rad/s of the HPF equals that of the LPF in Sec. 1.1.

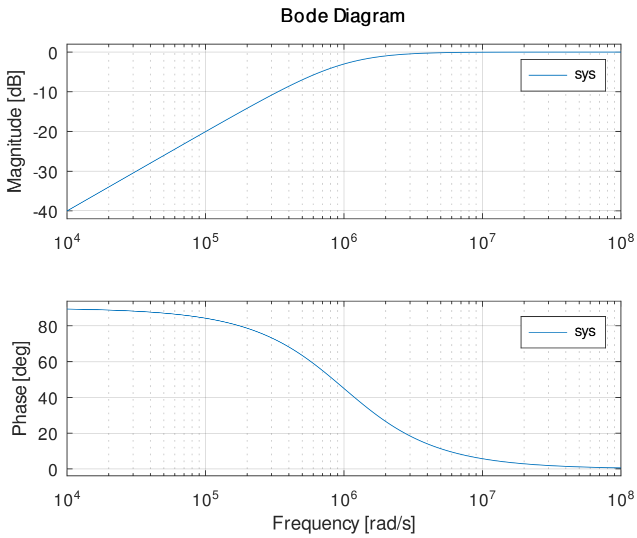

Figure 4: Frequency response of RC high-pass filter.

Exercises

1. Choose resistance R and capacitance C s.t. the cutoff frequency of the LPF and HPF is $f_c \in \{159\,\text{Hz}, 1.59\,\text{kHz}, 15.9\,\text{kHz}, 159\,\text{kHz}\}$. Plot the associated frequency response and step response.

2. Use Matlab/Octave function `lsim` (see Lab 5) and plot the system response of the LPF with cutoff frequency f_c of your choice when the input is a sinusoid with frequency $f_0 \in \{f_c/100, f_c/10, f_c, 10f_c, 100f_c\}$.

3. Use Matlab/Octave functions `lsim` (see Lab 5) and `square` (see Lab 1), and plot the system response of the LPF with cutoff frequency f_c when the input is a square wave with 50 % duty cycle and frequency $f_0 \in \{f_c/100, f_c/10, f_c, 10f_c, 100f_c\}$.

4. Redo Exercises 2 and 3 for the HPF in Sec. 1.2 instead of the LPF.

2 PreLab Problems

1. Recall the band-pass filter (BPF) in Fig. 1 of Lab 2 with v_C as output. Assume we wish to retain resistance R, how can you change the bandwidth of the BPF without changing the center frequency of its pass-band?

   Pick R, L, and C, and plot the frequency response of (a) the BPF, (b) the BPF with the same center frequency and a narrower pass-band, and (c) the BPF with the same center frequency and a wider pass-band.

2. Analyze the RLC circuit in Fig. 5 and plot the frequency response. Which filter characteristic does the circuit exhibit?

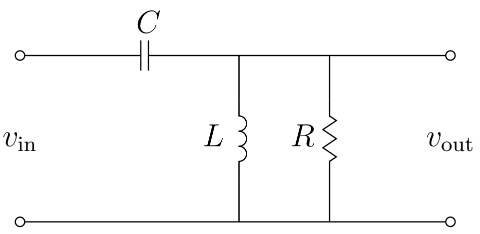

Figure 5: RLC filter.

3. Analyze the RLC circuit in Fig. 6 and plot the frequency response. Which filter characteristic does the circuit exhibit?

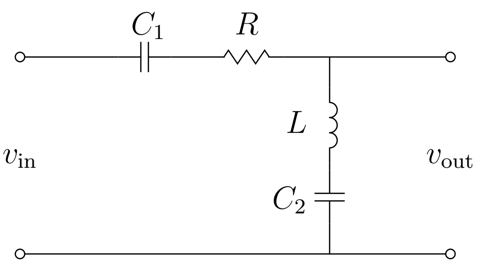

Figure 6: RLC filter with two capacitors.

3 Lab Problems

1. Verify Exercises 1–4 of Sec. 1 experimentally.
2. Verify Prelab Problem 1 experimentally.
3. Plot the measured frequency response together with your simulated frequency response for the RLC circuits in Fig. 5 and Fig. 6, respectively. Explain potential deviations between model and experiment.