Laboratory 10

Moodle quiz: 6/16/25 – 6/23/25

Goal

Explore circuits for amplitude modulation, aka mixing.

1 Mixers

Amplitude modulation implements frequency shifting by $\pm\omega_c$ when the modulating signal $x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(j\omega)$ is multiplied by a carrier sinusoid of frequency ω_c:

$$y(t) \;=\; x(t)\cos(\omega_c t) \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad Y(j\omega) \;=\; \frac{1}{2}X\big(j(\omega+\omega_c)\big) + \frac{1}{2}X\big(j(\omega-\omega_c)\big)\,.$$

Multiplication appears to be a trivial signal operation in theory. However, a multiplier is anything but obvious to implement with electrical components. In this Lab we study two passive multiplier implementations, aka mixer circuits, that are popular in communication systems.

The circuits in this Lab use nonlinear devices. Thus, they are nonlinear systems, that cannot be analyzed with the state-space and transfer-function methods we used in previous Labs. You may use a simulator such as LTspice[1] to explore nonlinear circuits in more detail, or study the circuits experimentally as we do in this Lab.

1.1 Single-Diode Mixer

Consider modulating signal $x(t) = \cos(\omega_0 t)$ and carrier sinusoid $\cos(\omega_c t)$, where $\omega_c \gg \omega_0$. Forming the product generates two new frequencies $\omega_c + \omega_0$ and $\omega_c - \omega_0$:

$$y(t) \;=\; \cos(\omega_0 t)\cos(\omega_c t) \;=\; \frac{1}{2}\cos\big((\omega_c+\omega_0)t\big) + \frac{1}{2}\cos\big((\omega_c-\omega_0)t\big)$$

The input frequency components ω_0 and ω_c disappear in the product. Multiplication by the carrier cosine is linear but not time-invariant, or ***linear time-variant (LTV)***. Unlike LTI systems, LTV systems can output frequencies other than their input frequencies. Thus there is no hope to build an LTV mixer with LTI components like R, L, or C alone. Instead, we employ a diode with nonlinear current-voltage relationship, known as the ***Shockley diode equation***:

$$i_D(t) \;=\; I_s\Big(e^{\frac{v_D(t)}{V_T}} - 1\Big)\,.$$

We treat saturation current I_s and thermal voltage V_T as constants.

Consider the circuit in Fig. 1, where $v_0(t) = x(t)$ represents the modulating sinusoid, $v_c(t)$ the carrier sinusoid, and $v_R(t) = Ri(t)$ shall be the output voltage. Current $i(t)$ is the diode current, which depends in a nonlinear fashion on diode voltage $v_D = v_c + v_0 - v_R$.

[1] `https://www.analog.com/en/resources/design-tools-and-calculators/ltspice-simulator.html`

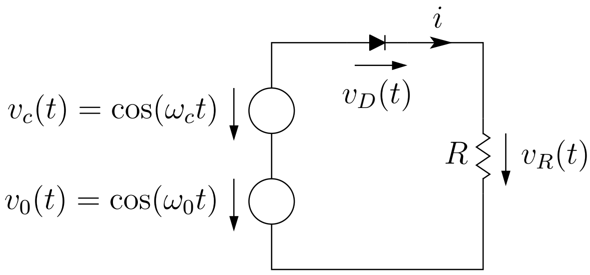

Figure 1: Diode circuit for modulation.

The spectrum of $v_R(t)$ is shown in Fig. 2 for $f_0 = \omega_0/2\pi = 1\,\text{kHz}$, $f_c = \omega_c/2\pi = 50\,\text{kHz}$, and $R = 100\,\Omega$. Observe that the output spectrum contains the desired output frequencies $f_c + f_0 = 51\,\text{kHz}$ and $f_c - f_0 = 49\,\text{kHz}$ of the modulated signal $y(t)$, and many more.

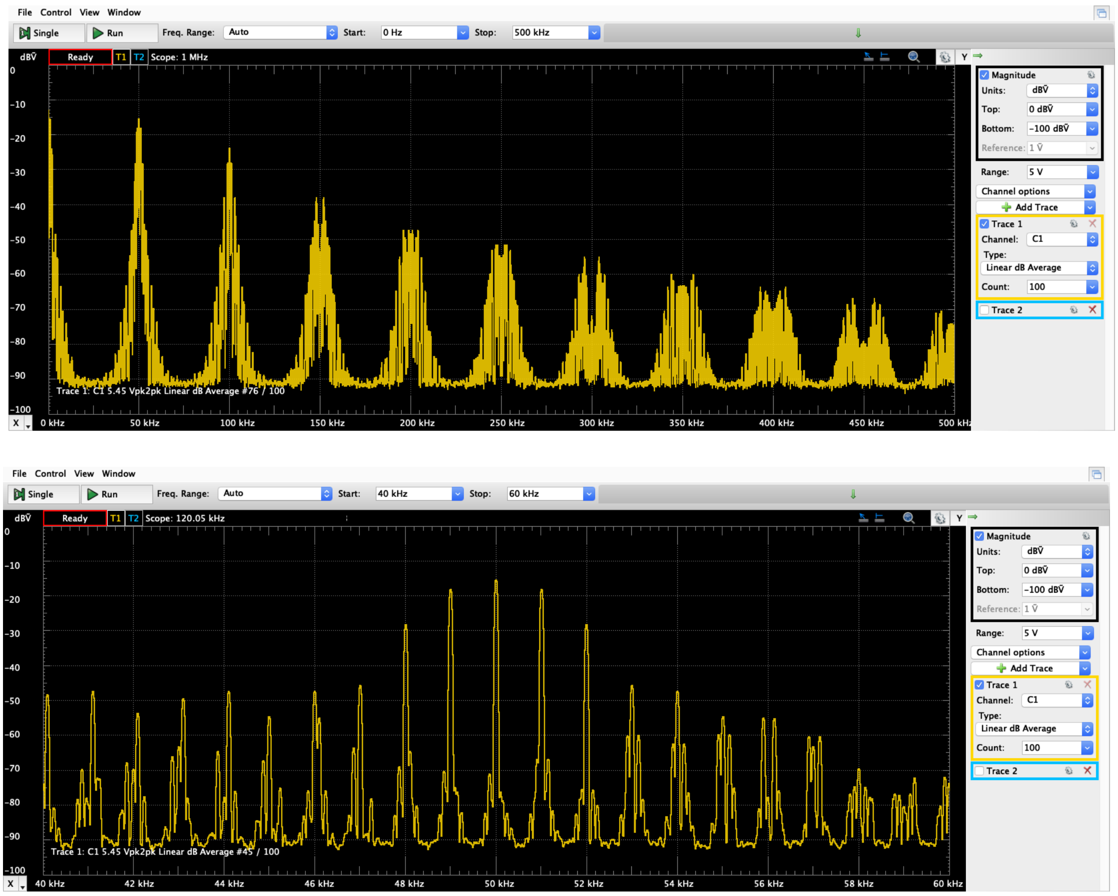

Figure 2: Magnitude spectrum of $v_R(t)$ of diode circuit; zoom into 50 kHz region (bottom).

To understand the spectrum of $v_R(t) = Ri(t)$ we inspect the Taylor series of the exponential function in Shockley's diode equation:

$$i \;=\; I_s\Big(e^{\frac{v_D}{V_T}} - 1\Big) \;=\; I_s\Big(e^{\frac{v_c + v_0 - v_R}{V_T}} - 1\Big) \;=\; I_s e^{-\frac{v_R}{V_T}} e^{\frac{v_c+v_0}{V_T}} \;-\; I_s\,.$$

The Taylor series of the exponential term with input voltages v_c and v_0 is:

$$e^{\frac{v_c+v_0}{V_T}} \;=\; 1+\frac{v_c+v_0}{V_T}+\frac{1}{2!}\Big(\frac{v_c+v_0}{V_T}\Big)^2+\frac{1}{3!}\Big(\frac{v_c+v_0}{V_T}\Big)^3+\frac{1}{4!}\Big(\frac{v_c+v_0}{V_T}\Big)^4+\cdots$$

The linear term contains frequencies ω_c and ω_0:

$$v_c+v_0 \;=\; \cos(\omega_c t)+\cos(\omega_0 t)\,.$$

The quadratic term contains the desired frequency components of modulated signal $y(t)$,

$$(v_c+v_0)^2 \;=\; v_c^2+2v_cv_0+v_0^2$$

in the mixed summand:

$$v_cv_0 \;=\; \cos(\omega_c t)\cos(\omega_0 t) \;=\; y(t)\,.$$

Term v_c^2 introduces frequency $2\omega_c$, term v_0^2 frequency $2\omega_0$, and the higher powers of the series introduce the other frequency components in Fig. 2. In general the frequency components are linear combinations of the form:

$$\forall m,n\in\mathbb{Z}\,,\quad m\omega_c+n\omega_0 \;=\; 2\pi(mf_c+nf_0)\,.$$

We conclude that the simple diode circuit in Fig. 1 generates an output signal $v_R(t)$ which contains the desired modulated signal.

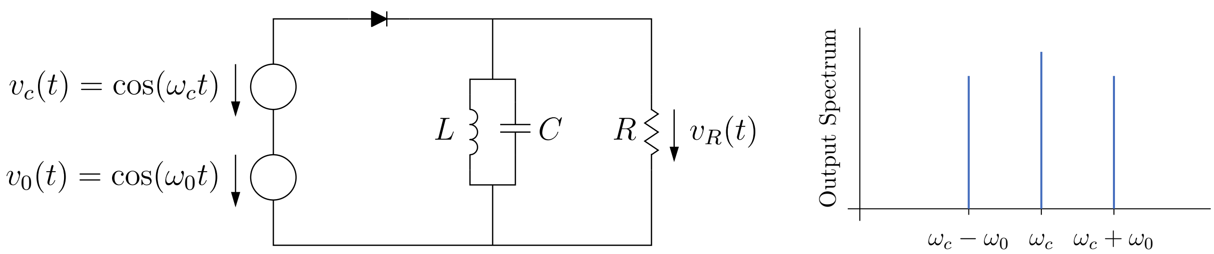

Figure 3: Single-diode mixer with BPF tuned to resonate at carrier frequency $\omega_c=1/\sqrt{LC}$.

The removal of undesired frequency components can be accomplished with a band-pass filter (BPF) centered at carrier frequency ω_c. Fig. 3 shows the resulting single-diode mixer circuit. The LC-circuit approximates an ideal BPF that passes the modulated signal $y(t)$ plus the carrier frequency component, sketched in Fig. 3 on the right. The corresponding signal is the ***amplitude modulated signal with large carrier***.

1.2 Diode-Ring Mixer

A diode-ring mixer, see Fig. 4, is used in communication systems to generate an ***amplitude modulated signal with suppressed carrier***, i.e. frequency component ω_c is significantly attenuated relative to the desired frequency components $\omega_c\pm\omega_0$ of the modulated signal $y(t)$. Output signal $v_R(t)$ contains the desired frequency components of product $y(t)$.

The waveforms in Fig. 5 illustrate the functionality of the mixer circuit. The key is to realize that the diode ring acts like two switches: if $v_c(t)>0$ then diodes D_1 and D_2 conduct current while D_3 and D_4 do not, and if $v_c(t)<0$ then D_3 and D_4 conduct. The transformers

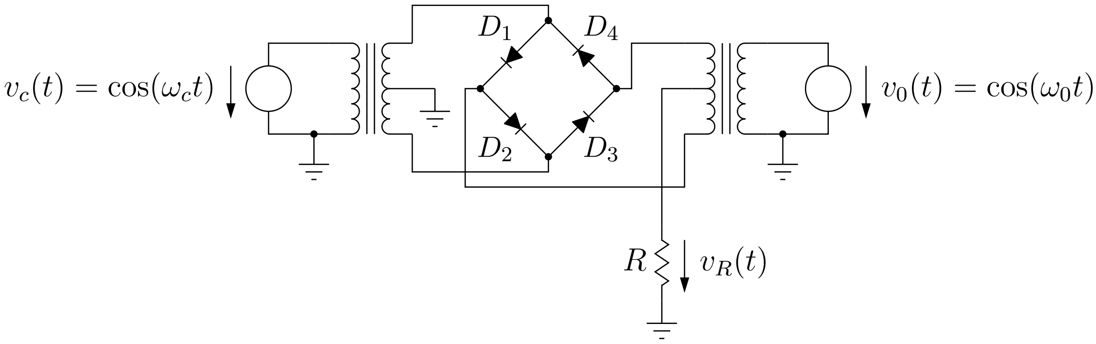

Figure 4: Diode ring for modulation with suppressed carrier.

provide distinct current paths for the positive and negative half-cycles of carrier $v_c(t)$, s.t. the mixer essentially computes output voltage

$$v_R(t) \;=\; v_0(t)\,\mathrm{sign}\big(v_c(t)\big)\,,$$

as shown in the middle of Fig. 5. Band-pass filtering the output signal with an additional LC circuit in parallel to R, analogous to Fig. 3, approximates modulated signal $y(t)$ shown in the bottom part of Fig. 5.

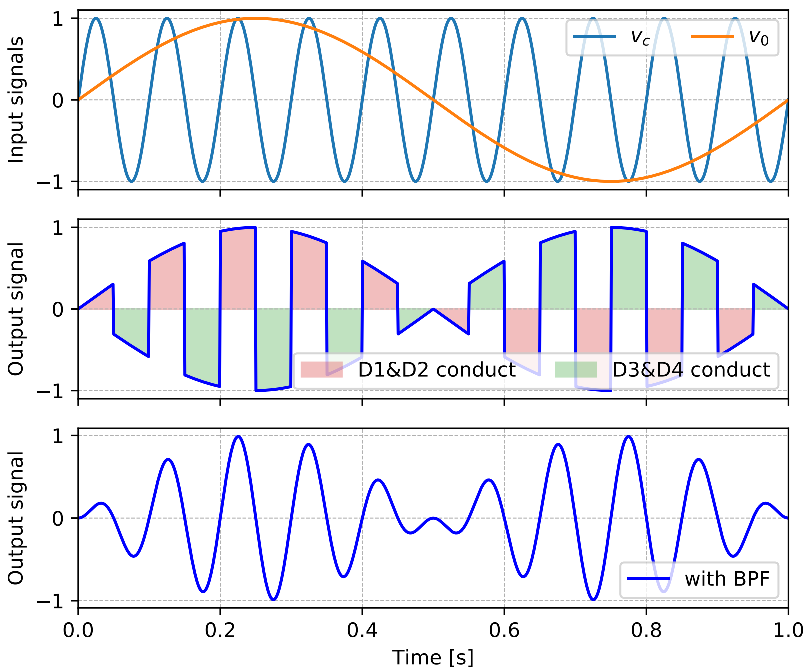

Figure 5: Illustration of diode ring functionality.

Similar to the single-diode mixer, the diode ring also generates frequency components $mf_c + nf_0$ for all $m, n \in \mathbb{Z}$, although with different magnitudes. In particular, the diode ring suppresses the carrier frequency and emphasizes the desired product frequencies $f_c \pm f_0$, as seen in the spectrum of Fig. 6 for $f_c = 50\,\mathrm{kHz}$ and $f_0 = 5\,\mathrm{kHz}$. The other frequencies contribute to the sharp edges of the product $v_0(t)\,\mathrm{sign}(v_c(t))$.

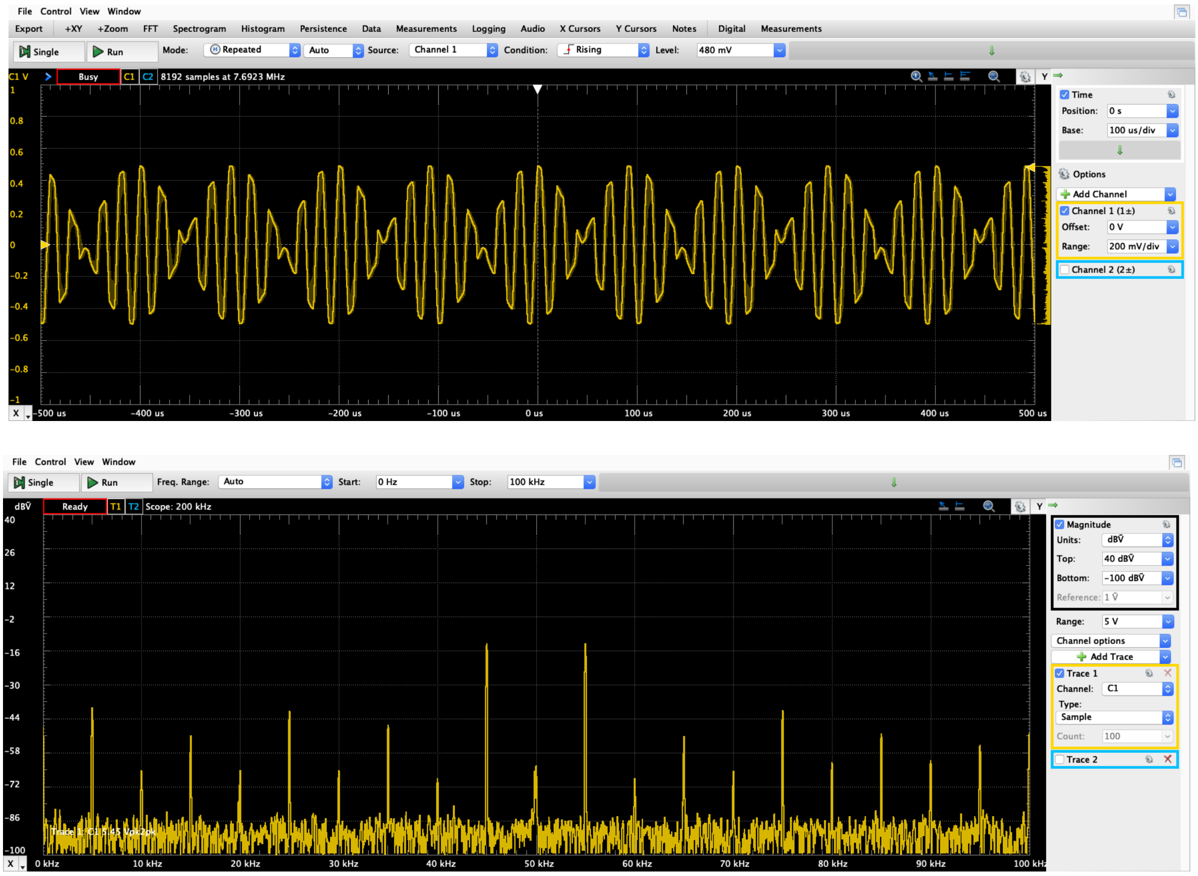

Figure 6: Output signal $v_R(t)$ and magnitude spectrum of diode ring.

Analagous to the single-diode mixer, we can attenuate all frequencies other than $f_c \pm f_0$ using a BPF with center frequency f_c that passes the desired frequency components of product $y(t)$. Augmenting the circuit in Fig. 4 with a parallel LC filter tuned to f_c outputs the amplitude modulated signal with suppressed carrier shown in Fig. 7.

2 PreLab Problems

1. Use Matlab/Octave to plot the expected output signals of the single-diode mixer and the diode-ring mixer assuming an ideal BPF.
2. Vary the frequencies f_c and f_0 relative to each other to explore the shapes of the output signals.
3. Vary the amplitudes A_c of $v_c(t) = A_c \cos(\omega_c t)$ and A_0 of $v_0(t) = A_0 \cos(\omega_0 t)$. How does the output signal change between ratios $A_c/A_0 > 1$, $A_c/A_0 = 1$, and $A_c/A_0 < 1$?

3 Lab Problems

1. Build the single-diode mixer in Fig. 3 with a 1N914 diode. Tune the BPF by determining values for L and C s.t. the desired output spectrum is approximated as good as possible. Generate one of the input signals with *WaveGen1* and the other input signal

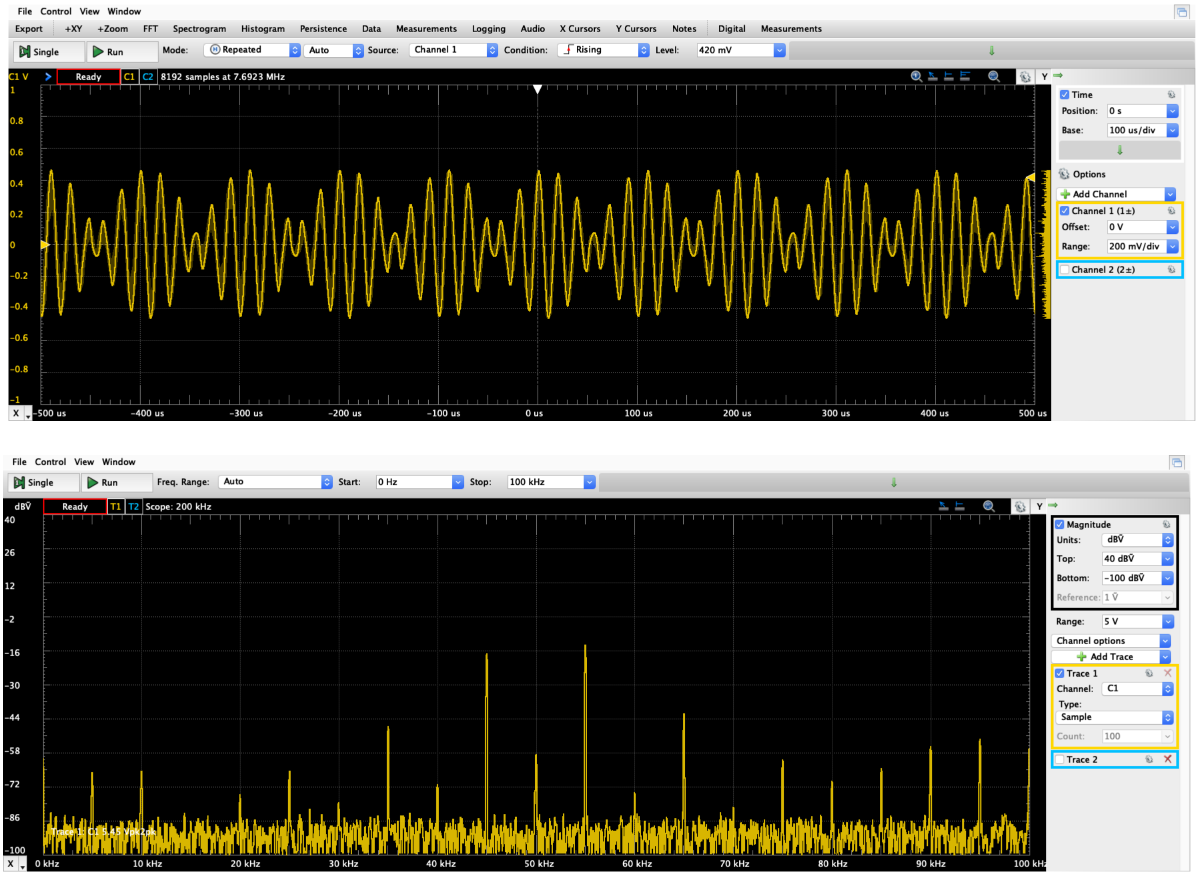

Figure 7: Output signal $v_R(t)$ and magnitude spectrum of diode ring with BPF tuned to carrier frequency $f_c = 50\,\text{kHz}$.

with *WaveGen2*. Use the *Spectrum Analyzer* of the *Waveforms* software to display the spectrum of output signal $v_R(t)$. Furthermore, use the *Scope* to verify the expected output signal $v_R(t)$ in the time domain.

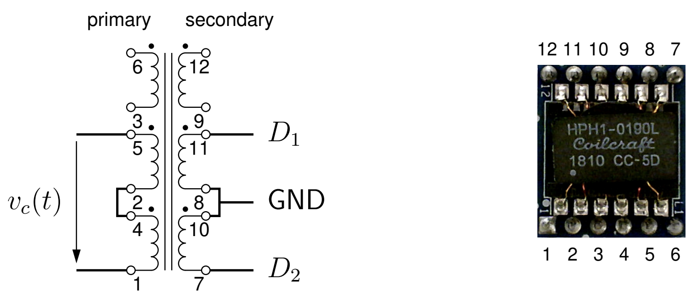

Figure 8: HPH1-0190L configuration for $v_c(t)$. Analogous for HPH1-1400L and $v_0(t)$.

2. Build the diode-ring mixer in Fig. 4 with 1N914 diodes, augmented with a tuned BPF at the output. Verify the expected output signal and its spectrum with the *AD2*. Use the Coilcraft hexa-path transformer HPH1-0190L for the carrier input $v_c(t)$, see Fig. 8, with $f_c = 50\,\text{kHz}$ and $1\,\text{V}$ amplitude. Analogously, use transformer HPH1-1400L for modulating signal input $v_0(t)$ with $f_0 = 5\,\text{kHz}$ and $1\,\text{V}$ amplitude.