Laboratory 3

Moodle Quiz: combined with Lab 4

Goal

Explore harmonically-related sinusoids.

1 Sinusoids, Sounds, and Spectra

We generate a pure tone of frequency $f_0 = 400\,\text{Hz}$, plot the signal and its line spectrum, and listen to its sound.

On a digital computer, we can generate and manipulate discrete numbers only. Math functions such as $\sin\alpha$ or $\cos\alpha$ operate on double-precision floating-point numbers with a machine epsilon of $\epsilon = 2^{-53} \approx 1.11 \times 10^{-16}$. The machine epsilon represents the smallest spacing between consecutive floating-point numbers, including the values of α and the values computed by the sin and cos functions. For all practical purposes in this Lab, we consider ϵ to be sufficiently small to qualify as *good enough* for approximating CT signals.

We generate the pure tone by sampling a CT cosine at the standard compact disk (CD) audio sampling rate of $f_s = 44.1\,\text{KHz}$:

```
fs = 44100;                 % sampling rate
t  = [0:1/fs:2];            % sample times for 2 seconds of sound
f0 = 400;                   % pure tone frequency
x  = cos(2*pi*f0*t);        % DT cosine signal
```

As discussed in Lab 1, we may listen to the tone with Matlab/Octave command:

```
sound(x, fs);
```

We zoom into the first three periods of signal $x(nT_s)$ to obtain the graph in Fig. 1:

```
n = [1 : round(3 * fs/f0)];  % index range covering first 3 periods
plot(t(n), x(n));
xlabel('Time [s]'), ylabel('cos(2 \pi f_0 t)');
```

The number of samples in the first three periods of the pure tone is the number of samples per second, i.e. f_s, multiplied by the number of seconds spanned by three periods of the pure tone, i.e. $3T_0 = 3/f_0$.

The line spectrum of a periodic signal is the graph of its frequency components. The frequency components of periodic signal $x(t)$ are the coefficients of the harmonically related frequencies $k\omega_0 = k2\pi f_0$ in the Fourier series (CTFS). Here the harmonic or trigonometric form of the CTFS is convenient:

$$x(t) \;=\; A_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \phi_k)\,.$$

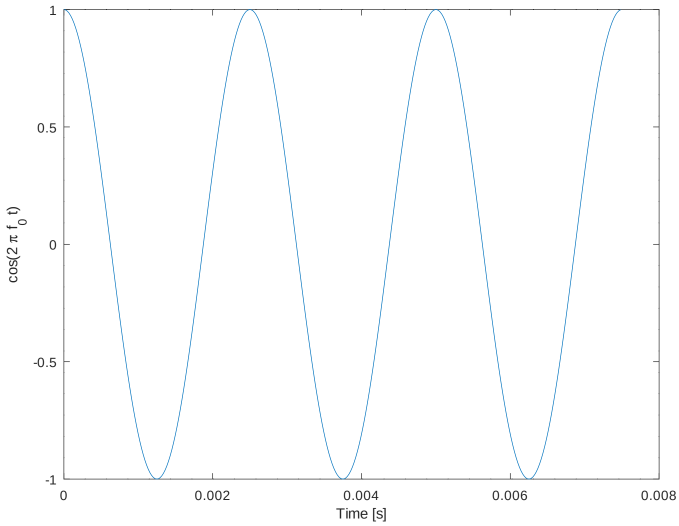

Figure 1: First three periods of cosine signal with $f_0 = 400\,\text{Hz}$.

The pure cosine constitutes the first and only harmonic of its CTFS:

$$\begin{aligned}
\cos(2\pi f_0 t) &= A_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \phi_k) \\
\Rightarrow \quad A_k &= \begin{cases} 1\,, & k = 1 \\ 0\,, & k > 1 \ \wedge \ k = 0 \end{cases}, \qquad \forall k \in \mathbb{N}\,,\ \phi_k = 0
\end{aligned}$$

Plotting the line spectrum of the pure tone is straightforward, see Fig. 2:

```
K = 20;                              % max harmonic number
k = [0:K-1];
A = zeros(1, K); A(2) = 1;           % A(1) = A_0, A(2) = A_1, ...
stem(k*f0, A, 'marker', 'none');
xlabel('f = k f_0'), ylabel('A_k');
```

Both representations the time-domain signal graph in Fig. 1 and the frequency-domain line spectrum in Fig. 2 provide the same information about the pure cosine signal.

2 Lab Problems

Use Matlab/Octave to solve these problems:

1. Generate the first ten harmonically-related sine waves with common fundamental frequency $f_0 = 200\,\text{Hz}$, all with amplitude 1. Add the ten harmonics and listen to the associated signal. Plot three periods of the compound signal and its line spectrum.

2. Using the same common frequency $f_0 = 200\,\text{Hz}$, generate the first ten odd harmonics, $k = 1, 3, 5, \ldots$ with amplitude $A_k = 1/(\pi k)$. Add the ten odd harmonics and listen to the associated signal. Plot the line spectrum and three periods of the signal.

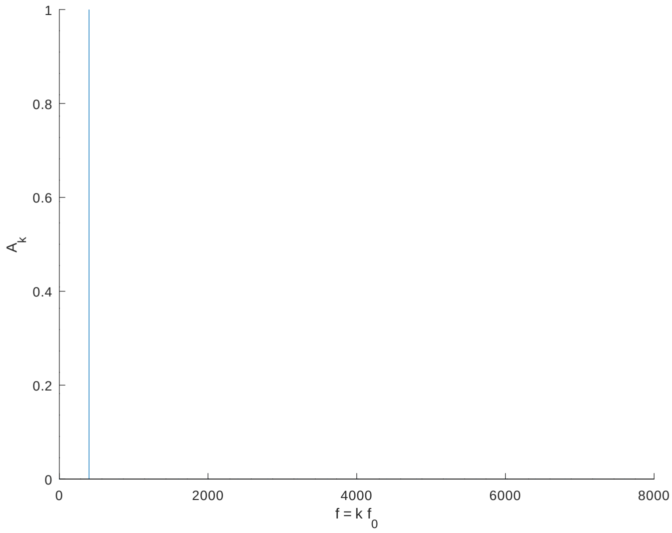

Figure 2: Line spectrum of pure cosine.

3. Given your solution to Problem 2, change the sinusoid from sine to cosine, and the amplitudes of the harmonics from $1/(\pi k)$ to $A_k = 1/(\pi k)^2$. Add the ten odd harmonics and listen to the associated wave. Plot the line spectrum and three periods of the signal.

4. Given the sounds and the signal waveforms of the problems above, characterize the key features of the three signals.