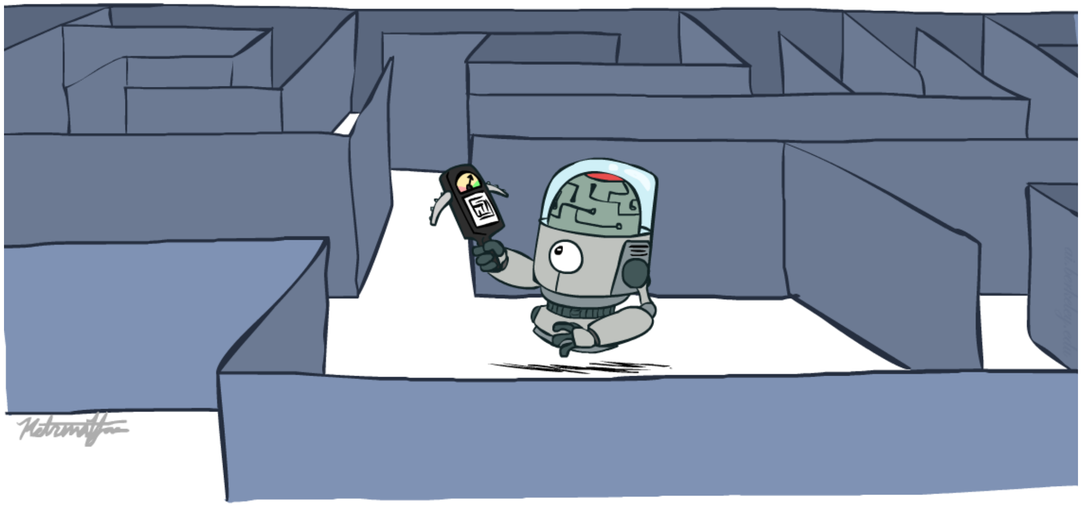
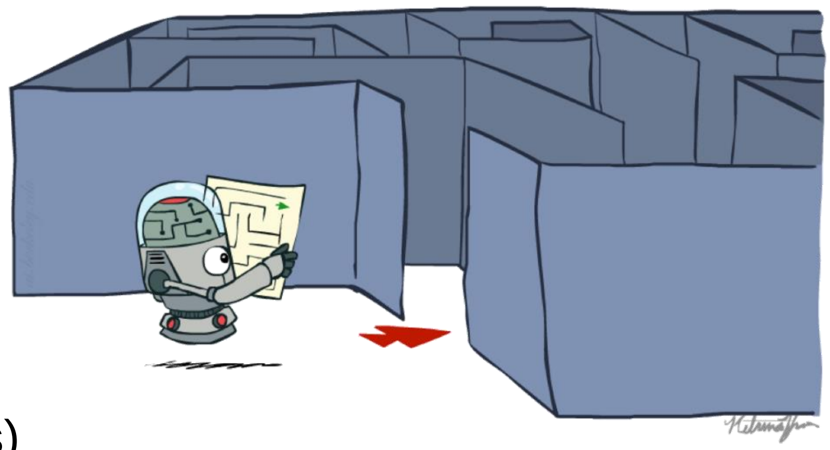
Informed Search

- Informed Search Methods
  - Heuristics
  - Greedy Search
  - A* Search
- Graph Search

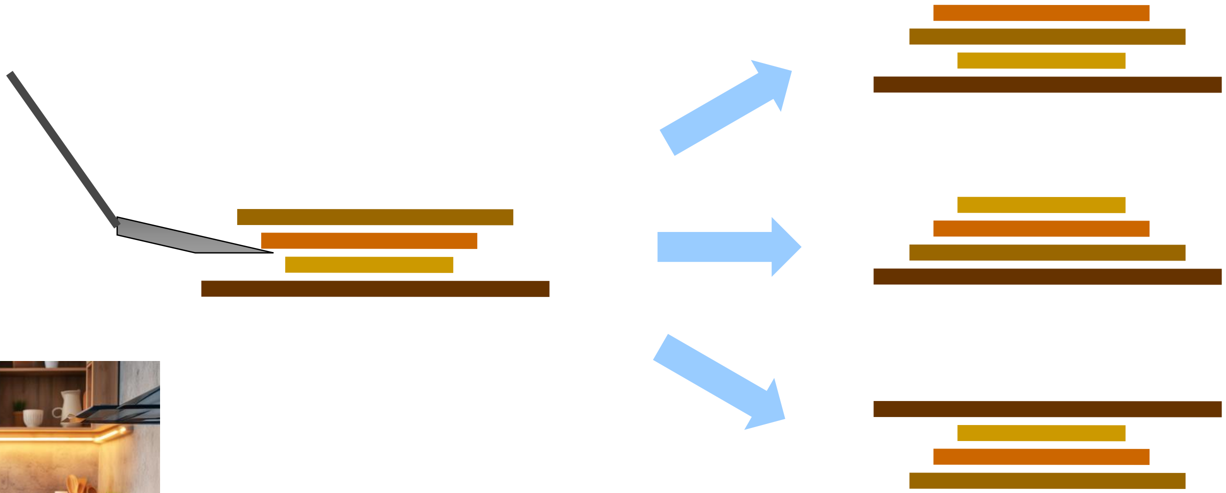


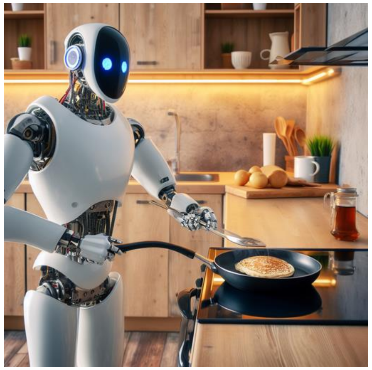
Recap: Search



- Search problem:
  - States (configurations of the world)
  - Actions and costs
  - Successor function (world dynamics)
  - Start state and goal test
- Search tree:
  - Nodes: represent plans for reaching states
  - Plans have costs (sum of action costs)
- Search algorithm:
  - Systematically builds a search tree
  - Chooses an ordering of the fringe (unexplored nodes)
  - Optimal: finds least-cost plans

Example: Pancake Problem





Cost: Number of pancakes flipped

Example: Pancake Problem

BOUNDS FOR SORTING BY PREFIX REVERSAL

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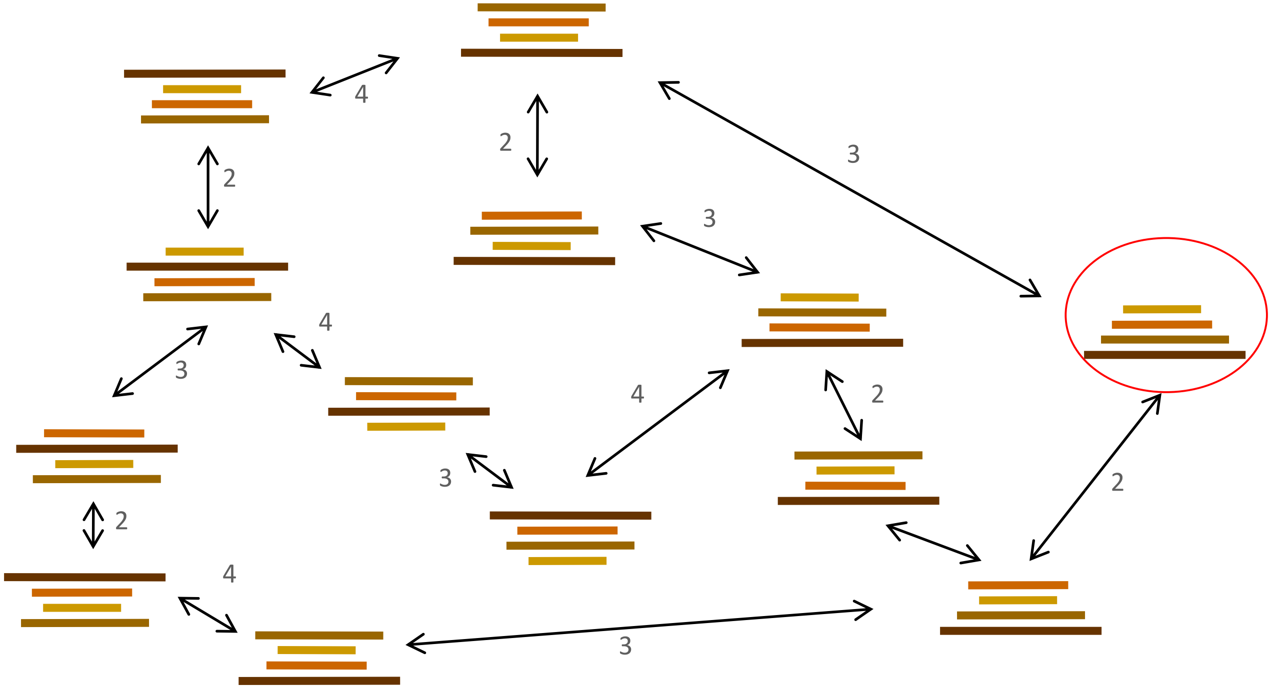
For a permutation σ of the integers from 1 to n, let $f(\sigma)$ be the smallest number of prefix reversals that will transform σ to the identity permutation, and let $f(n)$ be the largest such $f(\sigma)$ for all σ in (the symmetric group) S_n. We show that $f(n) \leqslant (5n+5)/3$, and that $f(n) \geqslant 17n/16$ for n a multiple of 16. If, furthermore, each integer is required to participate in an even number of reversed prefixes, the corresponding function $g(n)$ is shown to obey $3n/2 - 1 \leqslant g(n) \leqslant 2n+3$.

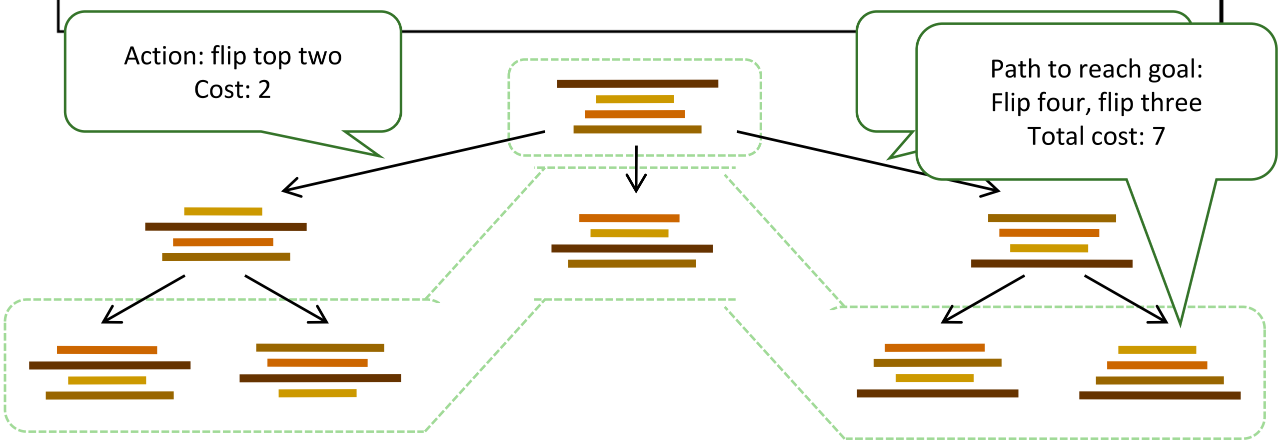
Example: Pancake Problem

State space graph with costs as weights



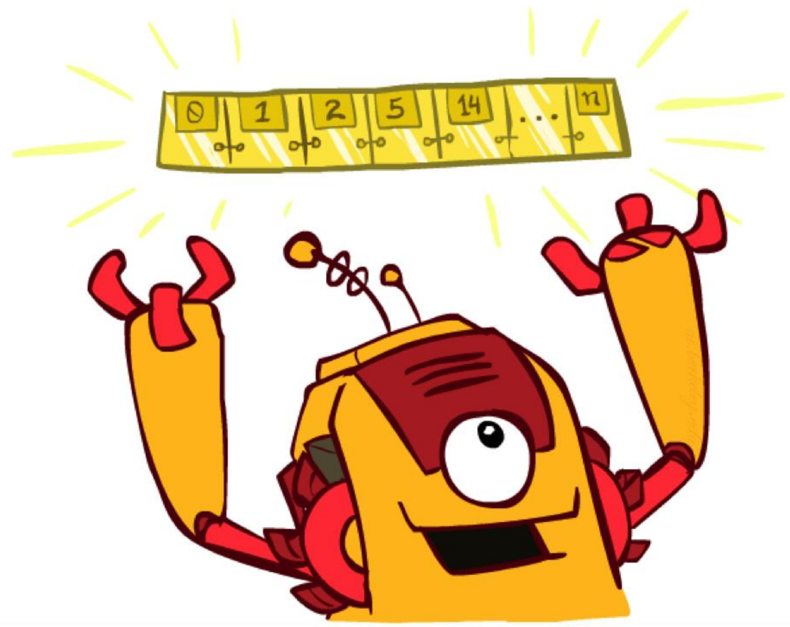
General Tree Search

function Tree-Search(*problem, strategy*) **returns** a solution, or failure
  initialize the search tree using the initial state of *problem*
  loop do
    if there are no candidates for expansion **then return** failure
    choose a leaf node for expansion according to *strategy*
    if the node contains a goal state **then return** the corresponding solution
    else expand the node and add the resulting nodes to the search tree
  end

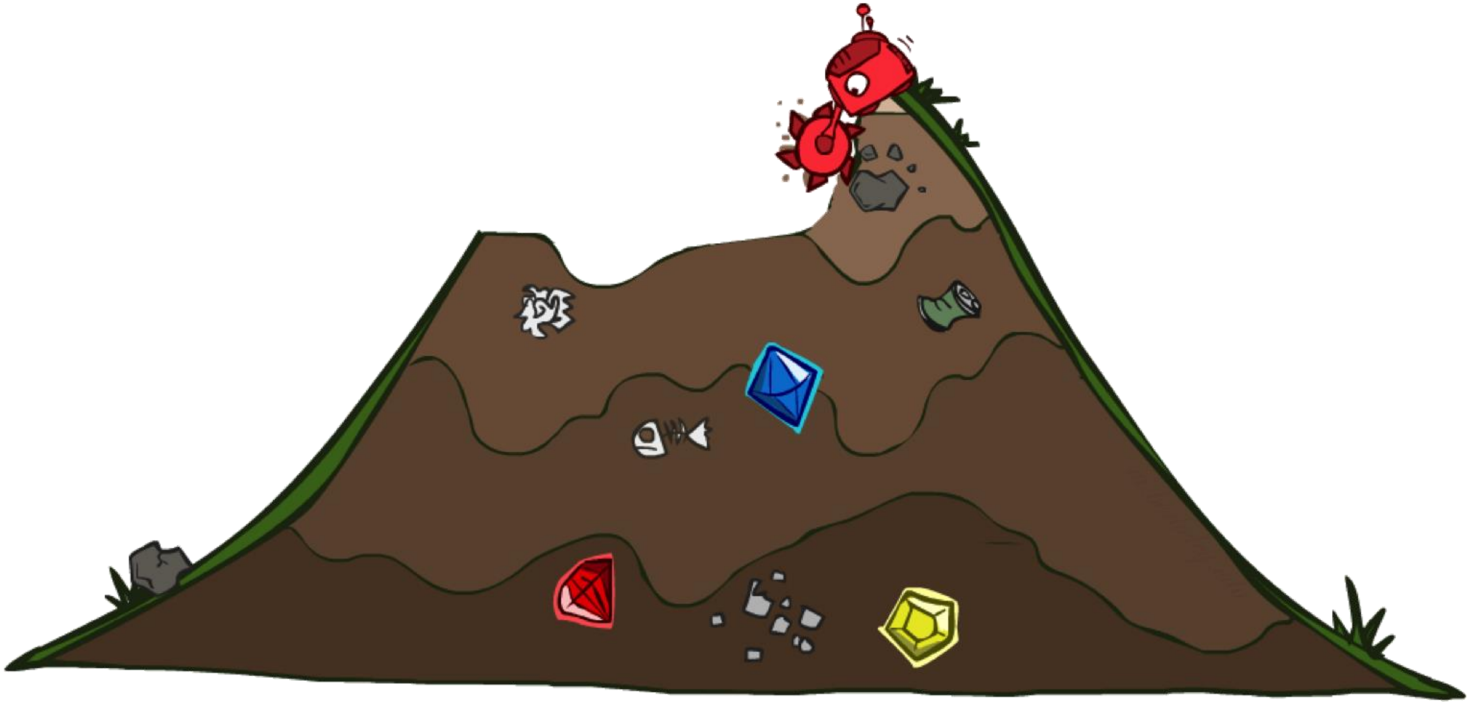


The One Queue

- All these search algorithms are the same except for fringe strategies
  - Conceptually, all fringes are priority queues (i.e. collections of nodes with attached priorities)
  - Practically, for DFS and BFS, you can avoid the log(n) overhead from an actual priority queue, by using stacks and queues
  - Can even code one implementation that takes a variable queuing object

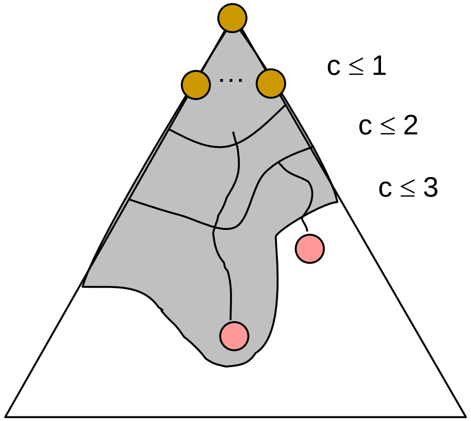


Uninformed Search



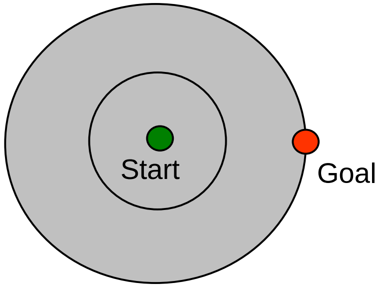
Uniform Cost Search

- Strategy: expand lowest path cost



- The good: UCS is complete and optimal!

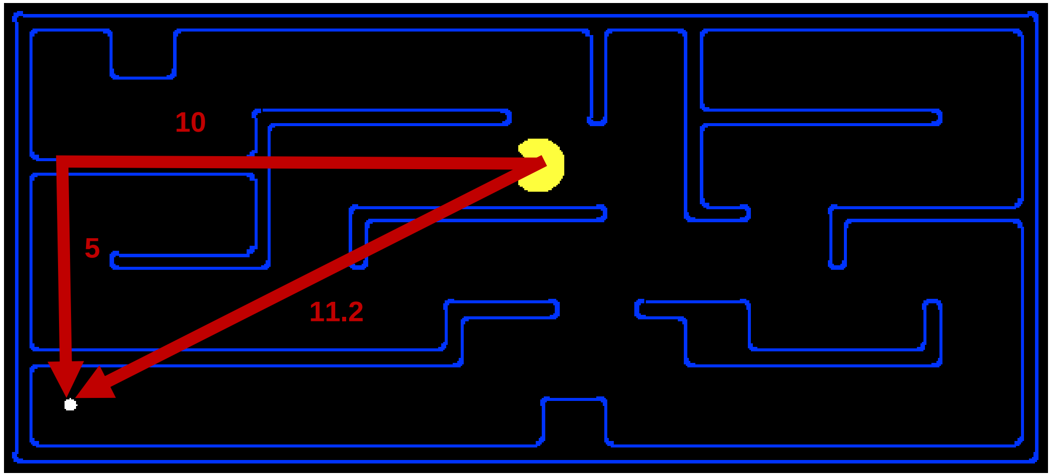
- The bad:
  - Explores options in every “direction”
  - No information about goal location

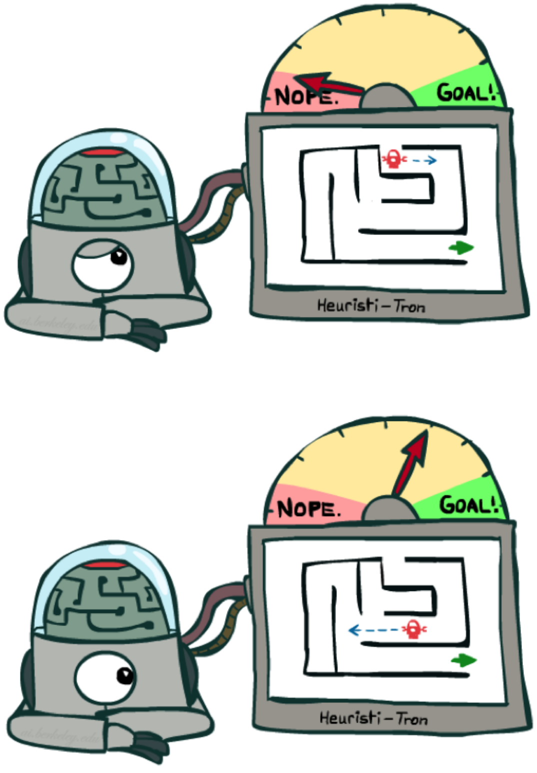


Informed Search

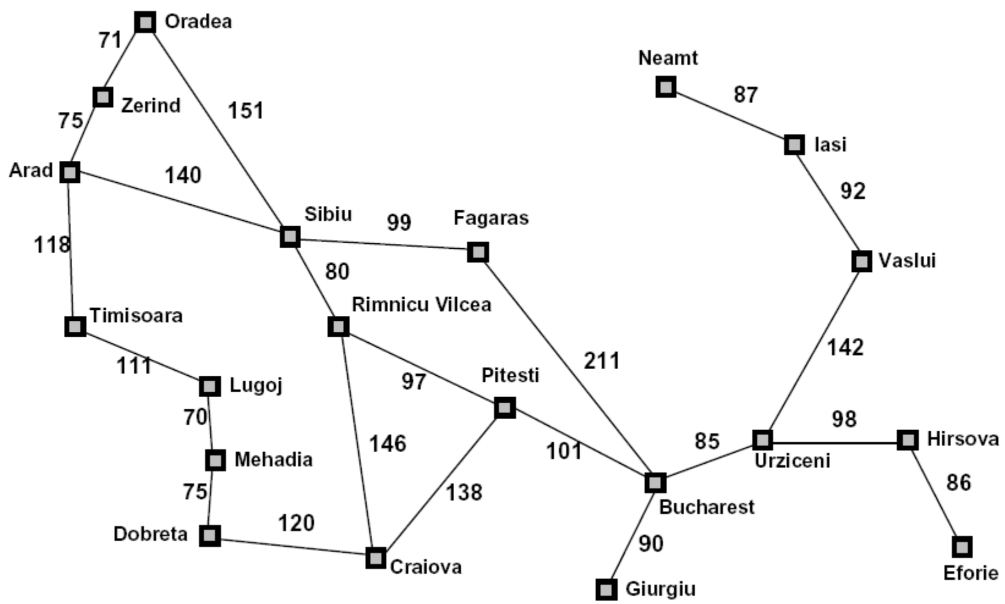
Search Heuristics

- A heuristic is:
  - A function that *estimates* how close a state is to a goal
  - Designed for a particular search problem
  - Examples: Manhattan distance, Euclidean distance for pathing





Example: Heuristic Function



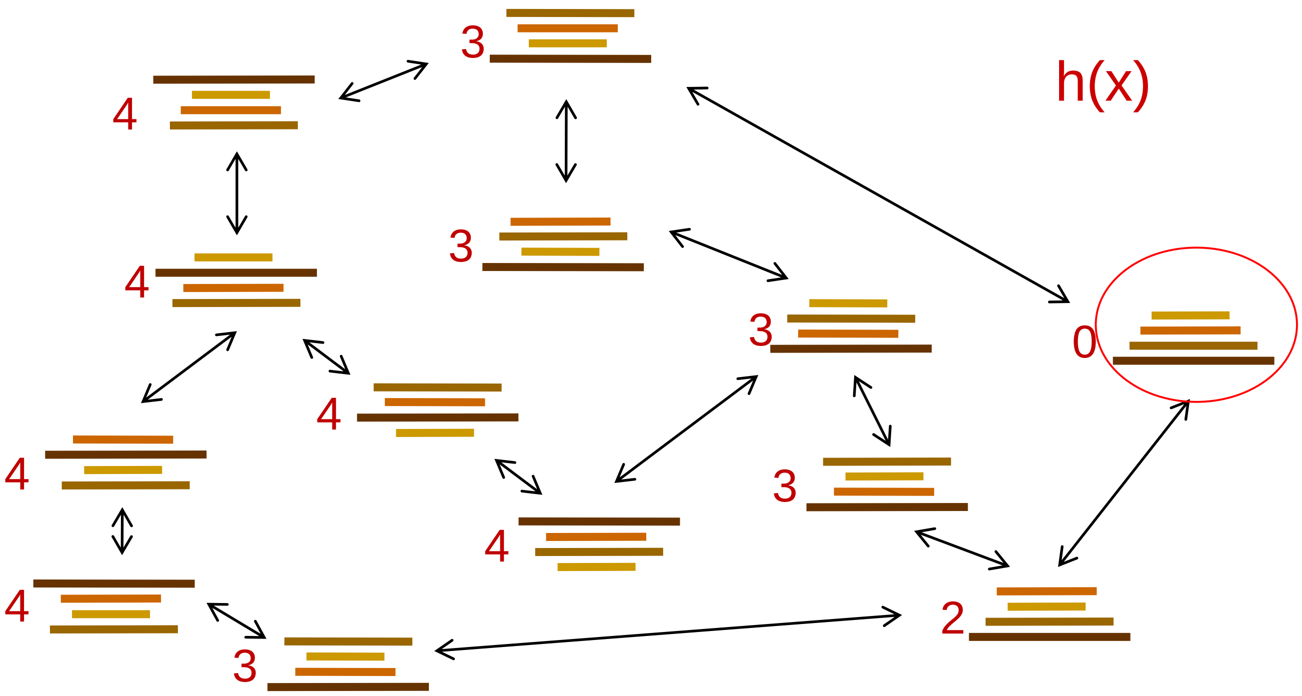
Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

h(x)

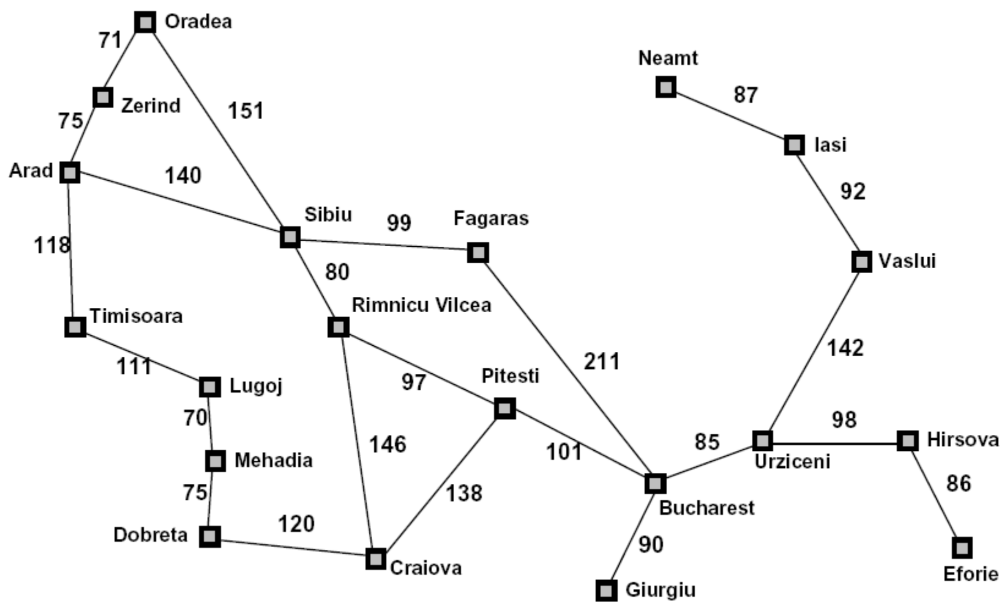
Example: Heuristic Function

Heuristic: the number of the largest pancake that is still out of place



Greedy Search

Example: Heuristic Function



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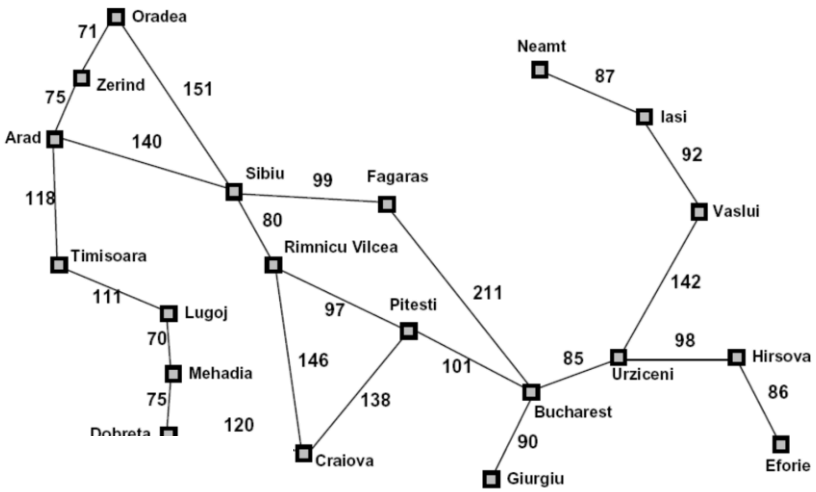
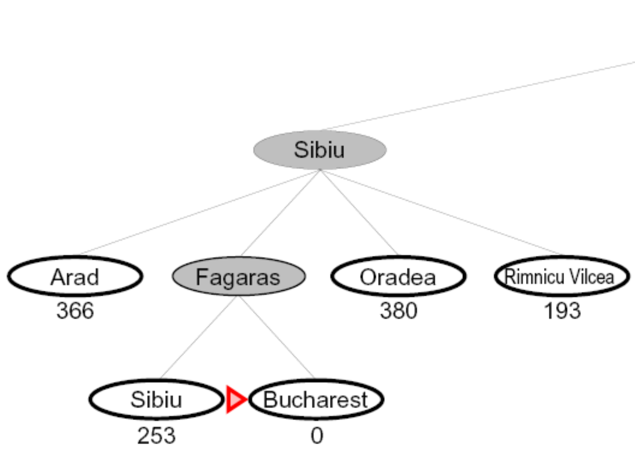
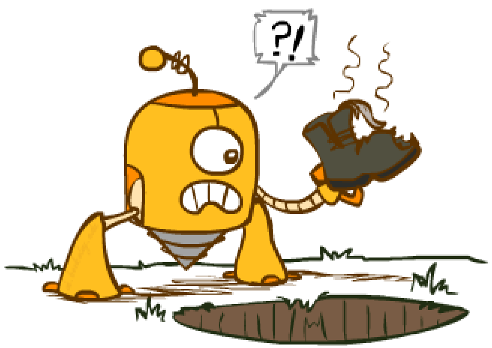
h(x)

Greedy Search



Expand the node that seems closest…

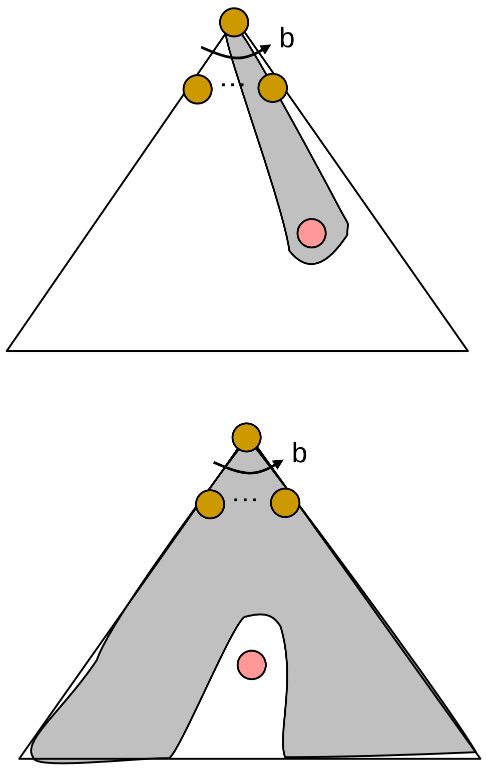




What can go wrong?

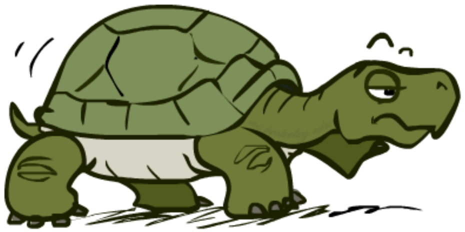
Greedy Search

- Strategy: expand a node that you think is closest to a goal state
  - Heuristic: estimate of distance to nearest goal for each state
- A common case:
  - Greedy's Best-first approach takes you straight to the (wrong) goal, but towards a goal nonetheless
- Worst-case: like a badly-guided DFS, explore everything except where you need to be

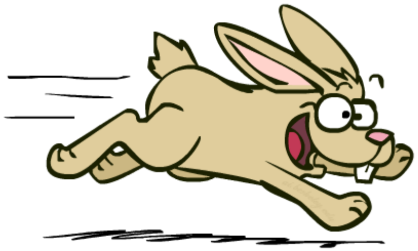


A* Search

A* Search



UCS

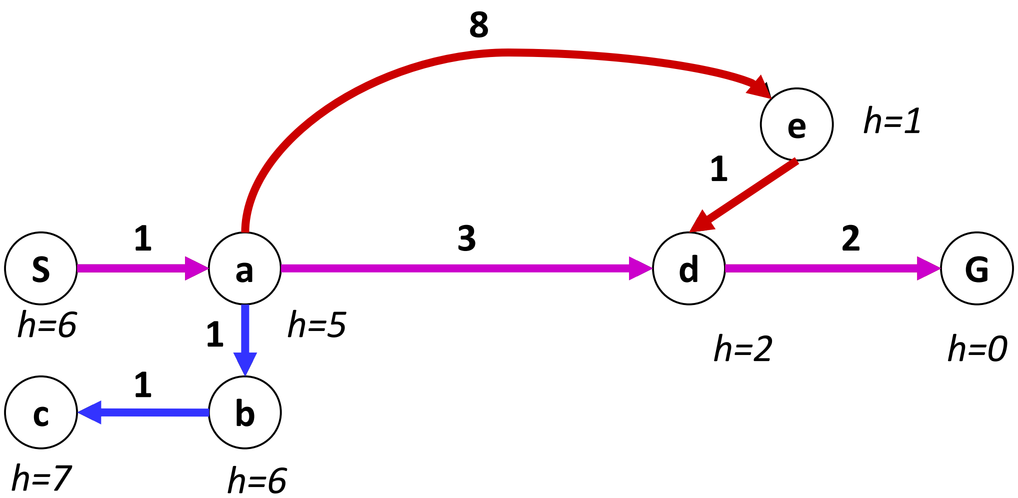


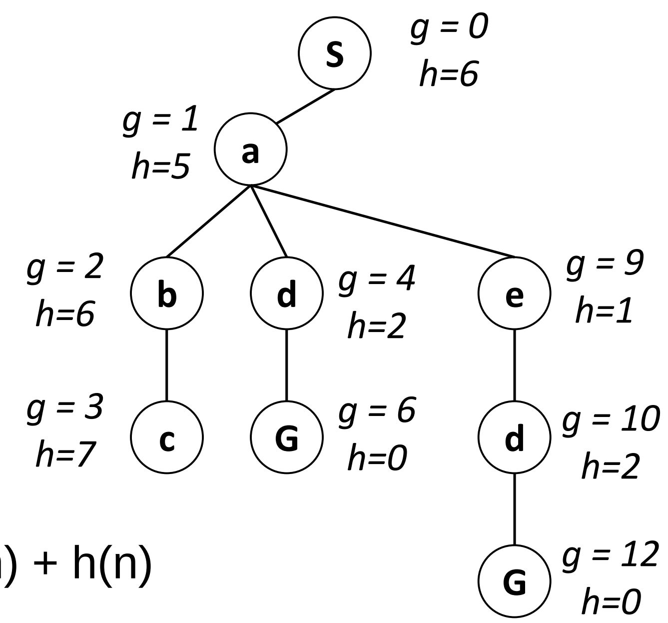
Greedy

A*

Combining UCS and Greedy

- Uniform-cost orders by path cost, or *backward cost* g(n)
- Greedy orders by goal proximity, or *forward cost* h(n)

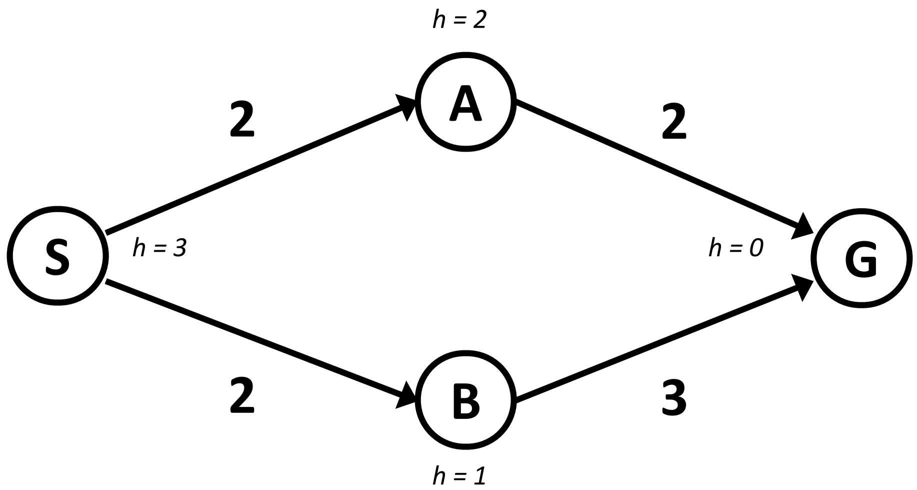




- A* Search orders by the sum: $f(n) = g(n) + h(n)$

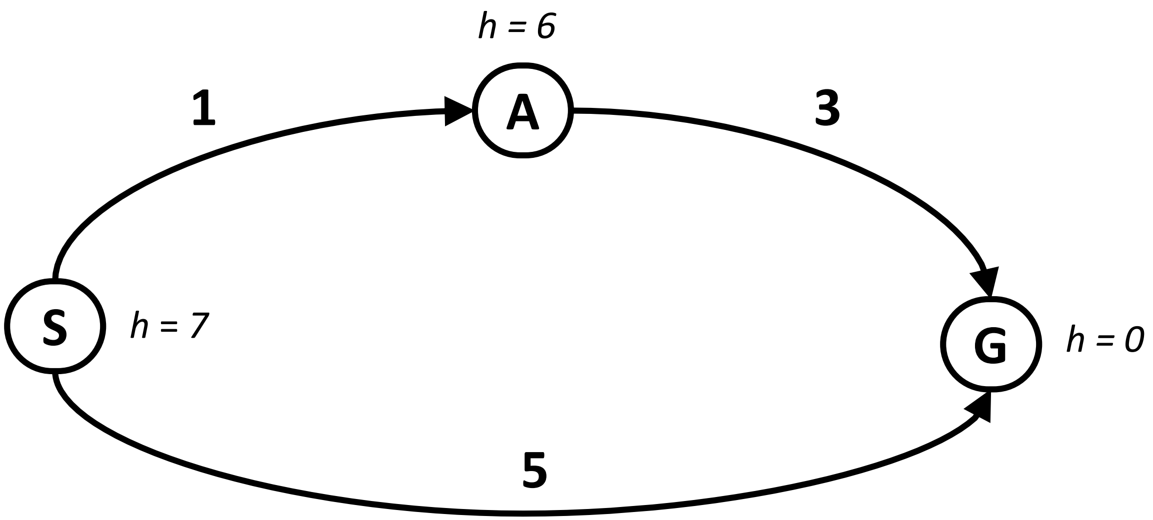
When should A* terminate?

- Should we stop when we enqueue a goal?



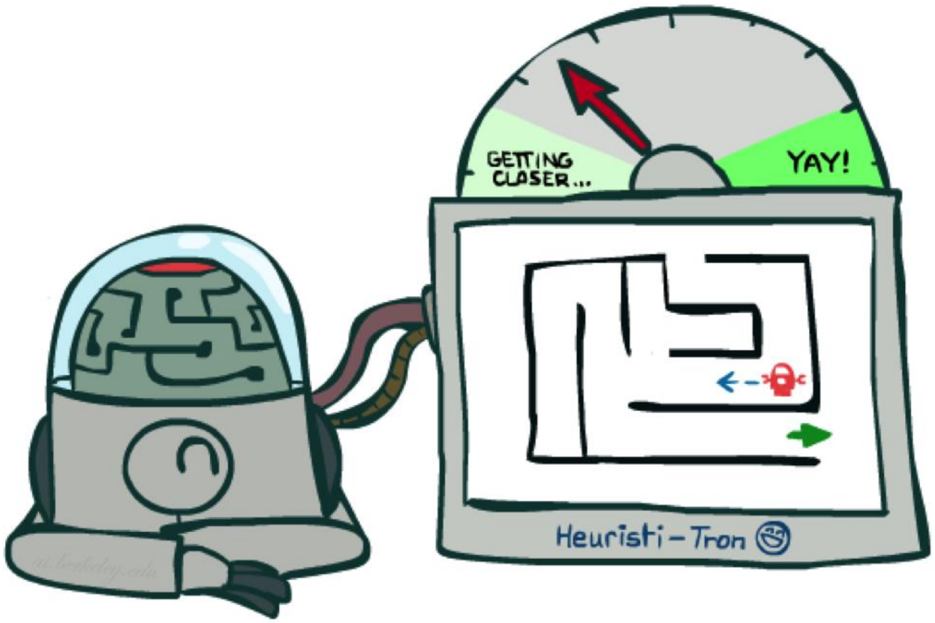
- No: only stop when we dequeue a goal

Is A* Optimal?

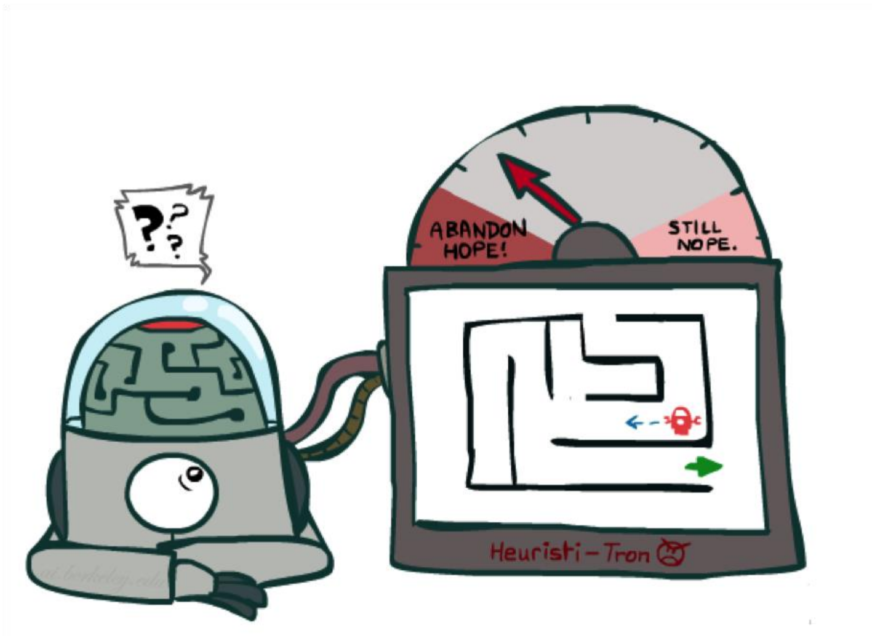


- What went wrong?
- Actual bad goal cost < estimated good goal cost
- We need estimates to be less than actual costs!

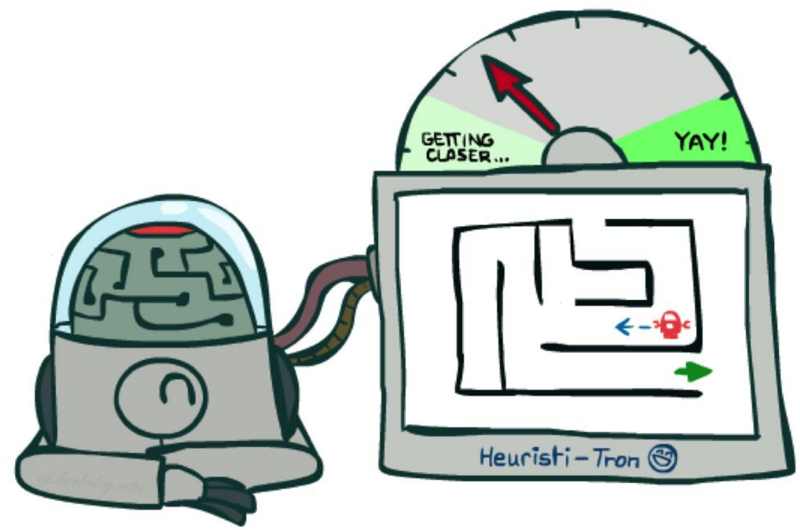
Admissible Heuristics



Idea: Admissibility



Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the fringe



Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs

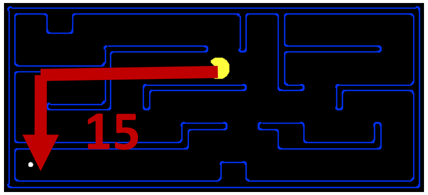
Admissible Heuristics

- A heuristic h is *admissible* (optimistic) if:

$$0 \leq h(n) \leq h^*(n)$$

where $h^*(n)$ is the true cost to a nearest goal

- Examples:



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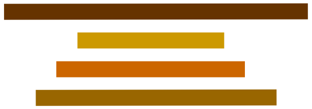
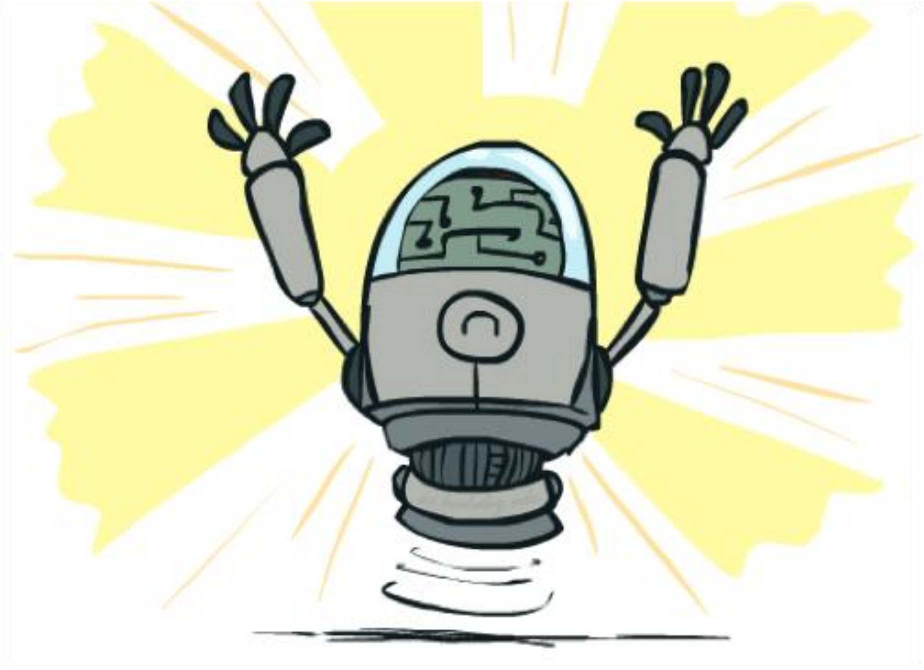


- Coming up with admissible heuristics is most of what's involved in using A* in practice.

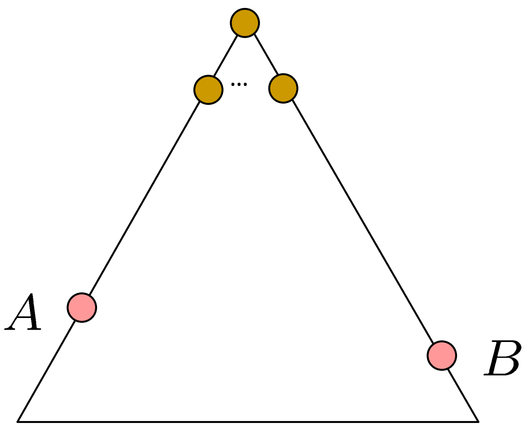
Optimality of A* Tree Search



Optimality of A* Tree Search

Assume:

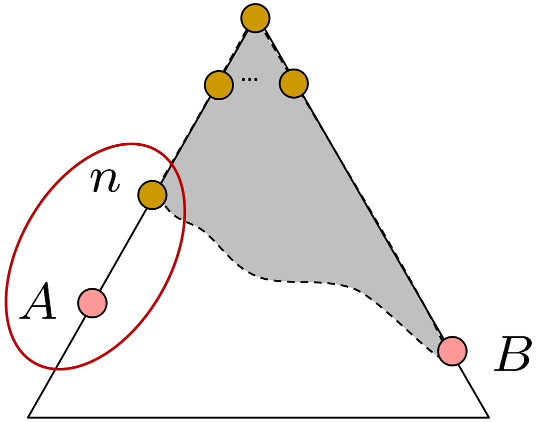
- A is an optimal goal node
- B is a suboptimal goal node
- h is admissible



Claim:

- A will exit the fringe before B

Optimality of A* Tree Search: Blocking



Proof:

- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)

$f(n) = g(n) + h(n)$	Definition of f-cost
$f(n) \leq g(A)$	Admissibility of h
$g(A) = f(A)$	h = 0 at a goal

Optimality of A* Tree Search: Blocking

1. f(n) is less than or equal to f(A)

- Definition of f-cost says:

  f(n) = g(n) + h(n) = (path cost to n) + (est. cost of n to A)

  f(A) = g(A) + h(A) = (path cost to A) + (est. cost of A to A)

- The admissible heuristic must underestimate the true cost

  h(A) = (est. cost of A to A)= 0

- So now, we have to compare:

  f(n) = g(n) + h(n) = (path cost to n) + (est. cost of n to A)

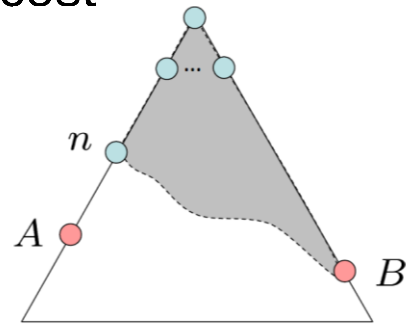
  f(A) = g(A) = (path cost to A)

- h(n) must be an underestimate of the true cost from n to A

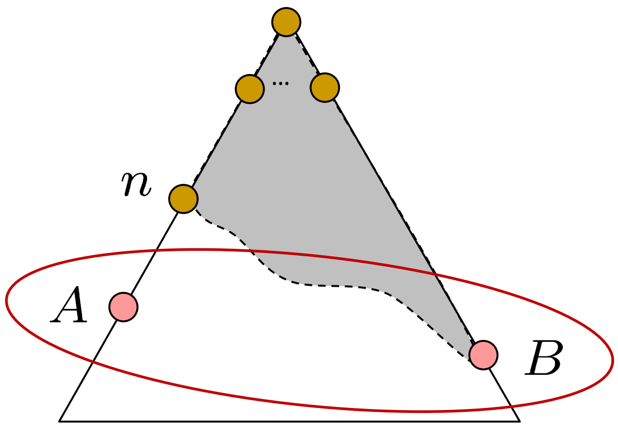
  (path cost to n) + (est. cost of n to A) ≤ (path cost to A)

  g(n) + h(n) ≤ g(A)

  f(n) ≤ f(A)



Optimality of A* Tree Search: Blocking



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- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)
  2. f(A) is less than f(B)

$g(A) < g(B)$ B is suboptimal

$f(A) < f(B)$ h = 0 at a goal

Optimality of A* Tree Search: Blocking

1. f(A) is less than f(B)

- Definition of f-cost says:

  f(A) = g(A) + h(A) = (path cost to A) + (est. cost of A to A)

  f(B) = g(B) + h(B) = (path cost to B) + (est. cost of B to B)
- The heuristic must underestimate the true cost

  h(A) = h(B) = 0
- So now, we have to compare:

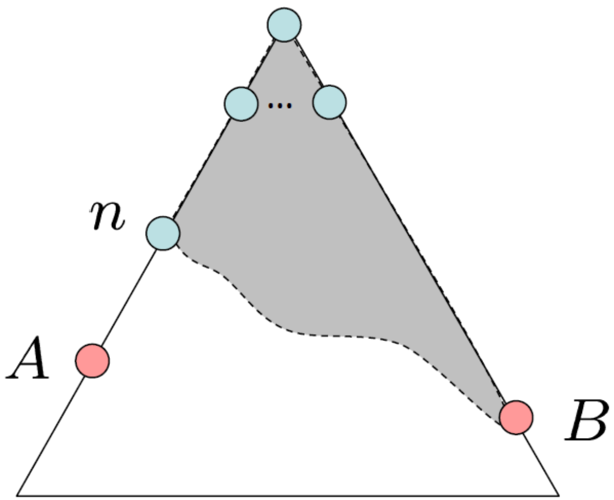
  f(A) = g(A) = (path cost to A)

  f(B) = g(B) = (path cost to B)
- We assumed that B is suboptimal! So

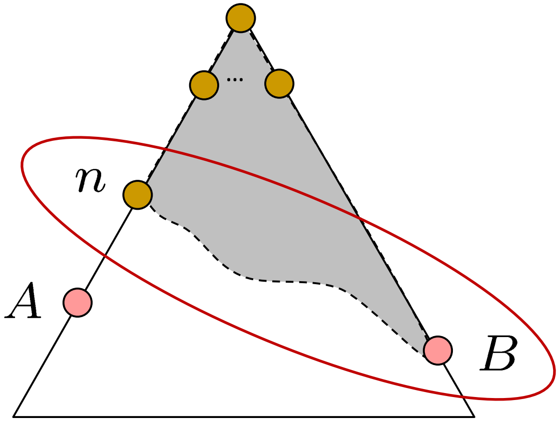
  (path cost to A) < (path cost to B)

  g(A) < g(B)

  f(A) < f(B)



Optimality of A* Tree Search: Blocking

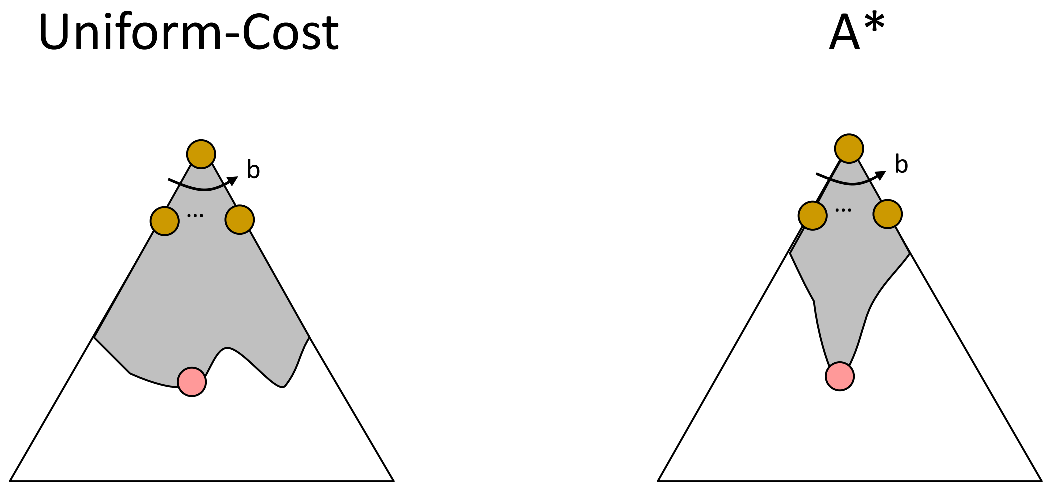


Proof:

- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)
  2. f(A) is less than f(B)
  3. *n* expands before B
- All ancestors of A expand before B
- A expands before B
- A* search is optimal

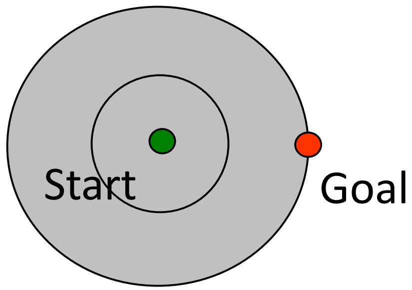
$$f(n) \leq f(A) < f(B)$$

Properties of A*

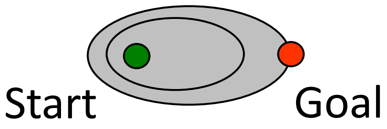


UCS vs A* Contours

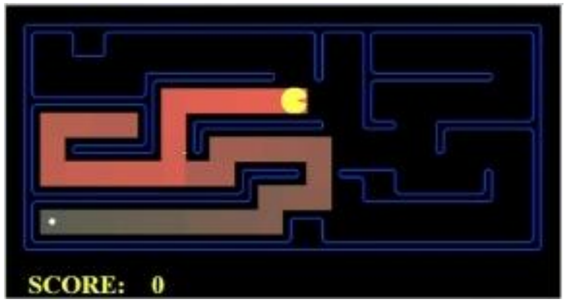
- Uniform-cost expands equally in all “directions”



- A* expands mainly toward the goal, but does hedge its bets to ensure optimality



Comparison



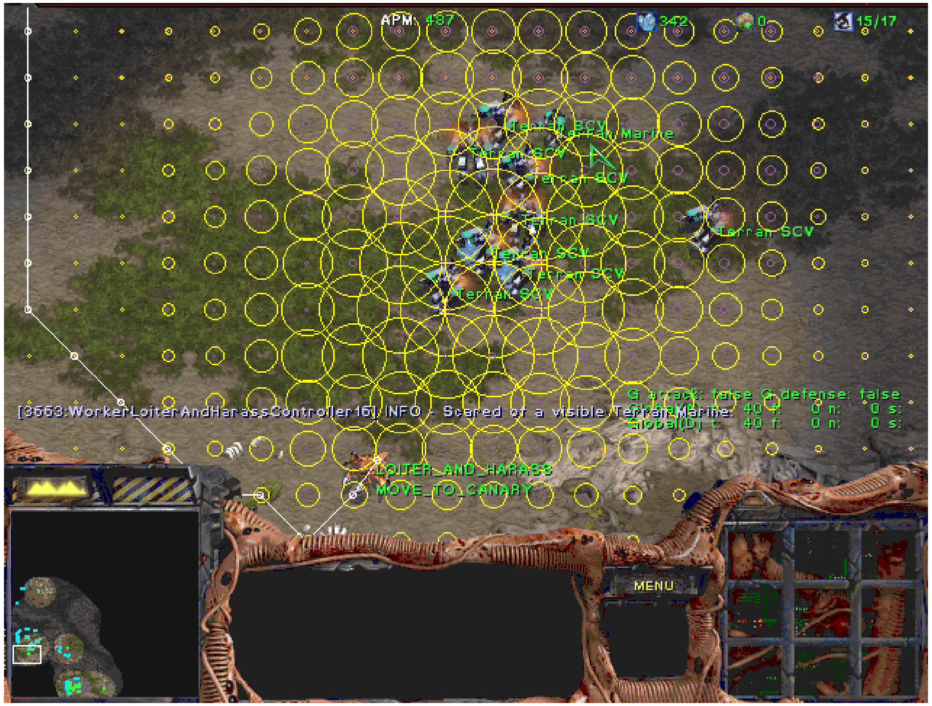
Greedy

Uniform Cost

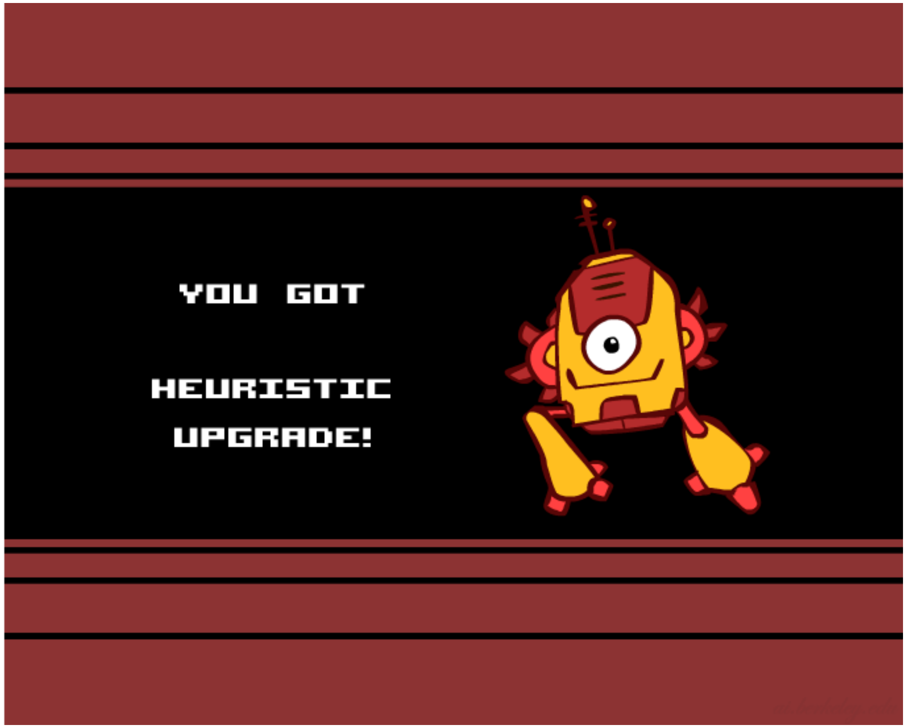
A*

A* Applications

- Video games
- Pathing / routing problems
- Resource planning problems
- Robot motion planning
- Language analysis
- Machine translation
- Speech recognition
- …



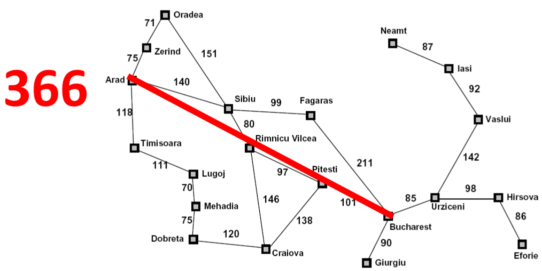
Creating Heuristics

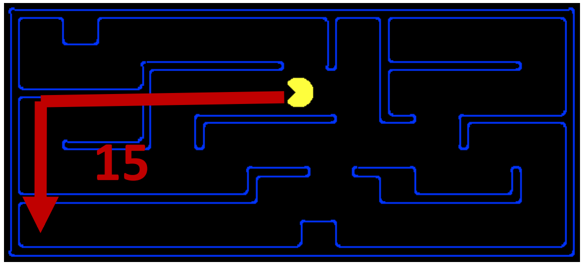


Creating Admissible Heuristics

- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics

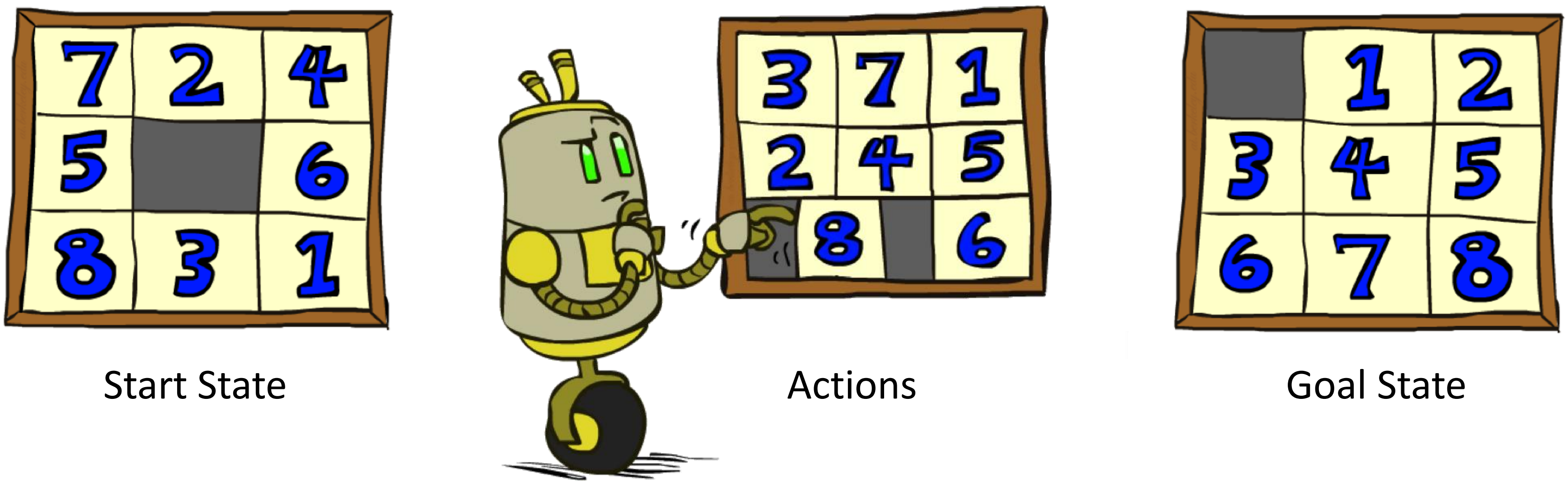
- Often, admissible heuristics are solutions to *relaxed problems,* where new actions are available





- Inadmissible heuristics are often useful too

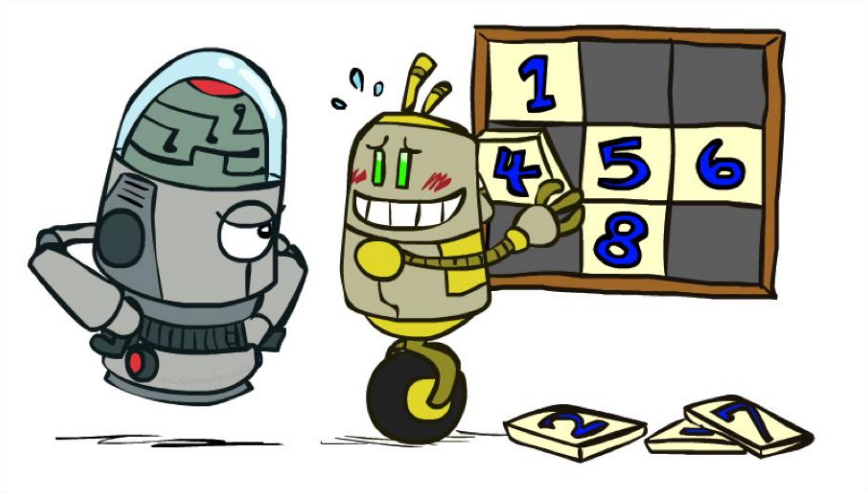
Example: 8 Puzzle

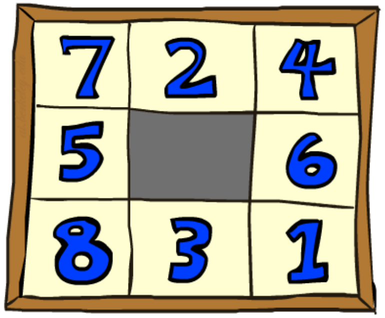


- What are the states?
- How many states?
- What are the actions?
- How many successors from the start state?
- What should the costs be?

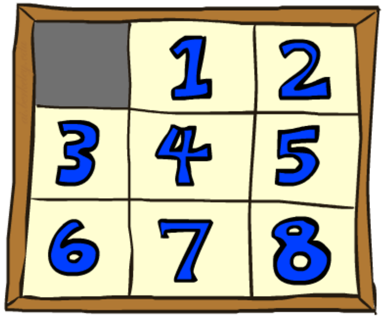
8 Puzzle I

- Heuristic: Number of tiles misplaced
- Why is it admissible?
- h(start) = 8
- This is a *relaxed-problem* heuristic





Start State



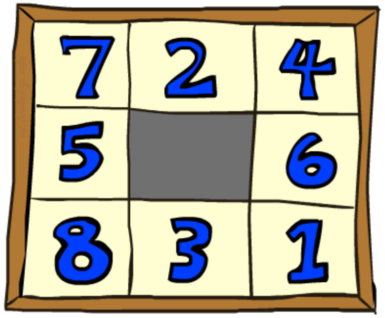
Goal State

	Average nodes expanded when the optimal path has…		
	…4 steps	…8 steps	…12 steps
UCS	112	6,300	3.6×10^6
TILES	13	39	227

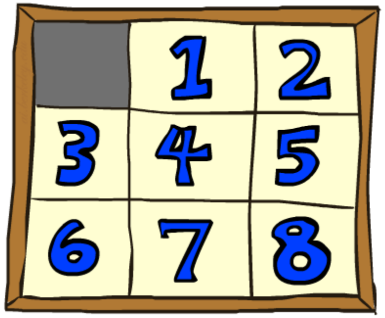
Statistics from Andrew Moore

8 Puzzle II

- What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?
- Total *Manhattan* distance
- Why is it admissible?
- h(start) = 3 + 1 + 2 + … = 18



Start State

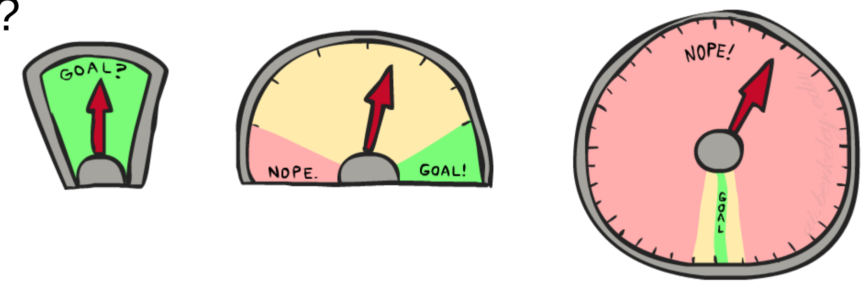


Goal State

	Average nodes expanded when the optimal path has…		
	…4 steps	…8 steps	…12 steps
TILES	13	39	227
MANHATTAN	12	25	73

8 Puzzle III

- How about using the *actual cost* as a heuristic?
  - Would it be admissible?
  - Would we save on nodes expanded?
  - What's wrong with it?



- With A*: a trade-off between quality of estimate and work per node
  - As heuristics get closer to the true cost, you will expand fewer nodes but usually do more work per node to compute the heuristic itself

Trivial Heuristics, Dominance

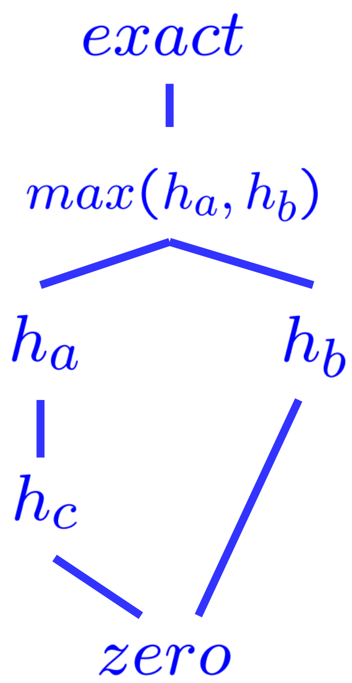
- Dominance: $h_a \geq h_c$ if

  $$\forall n : h_a(n) \geq h_c(n)$$

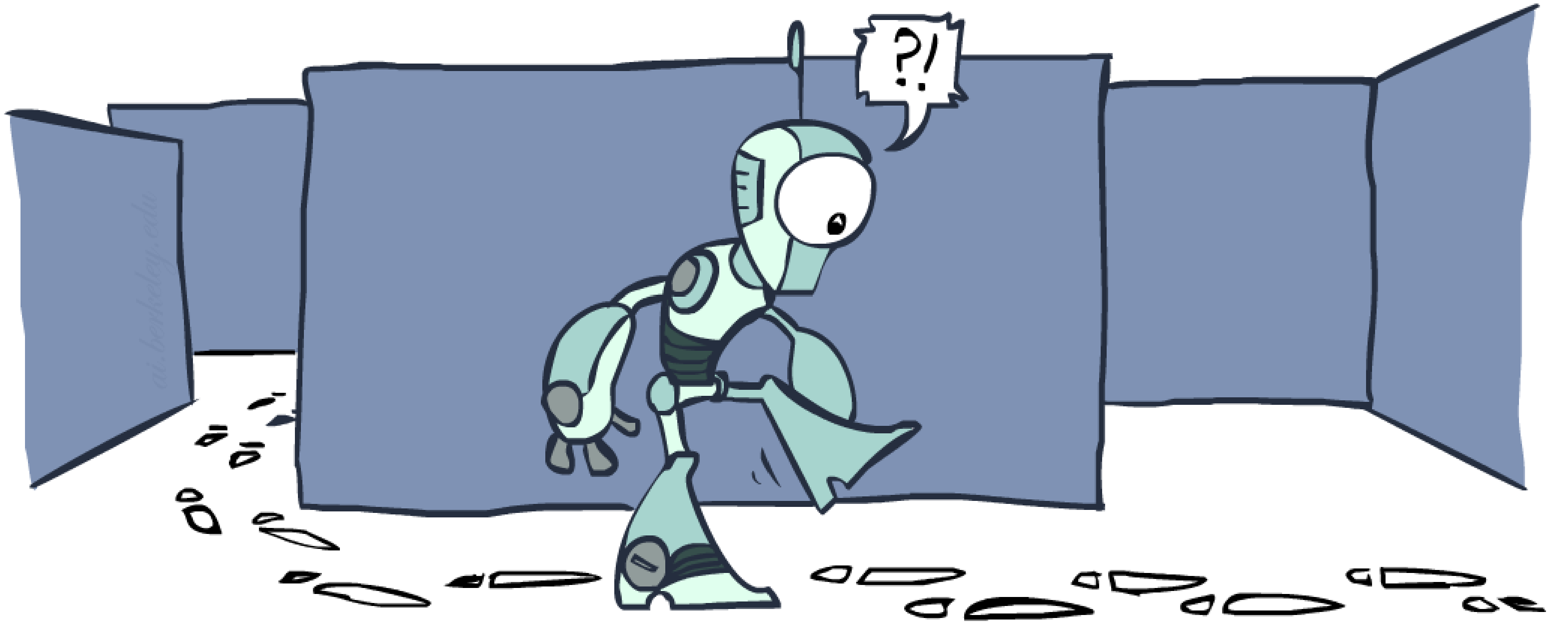
- Heuristics form a semi-lattice:
  - Max of admissible heuristics is admissible

  $$h(n) = max(h_a(n), h_b(n))$$

- Trivial heuristics
  - Bottom of lattice is the zero heuristic
  - Top of lattice is the exact heuristic

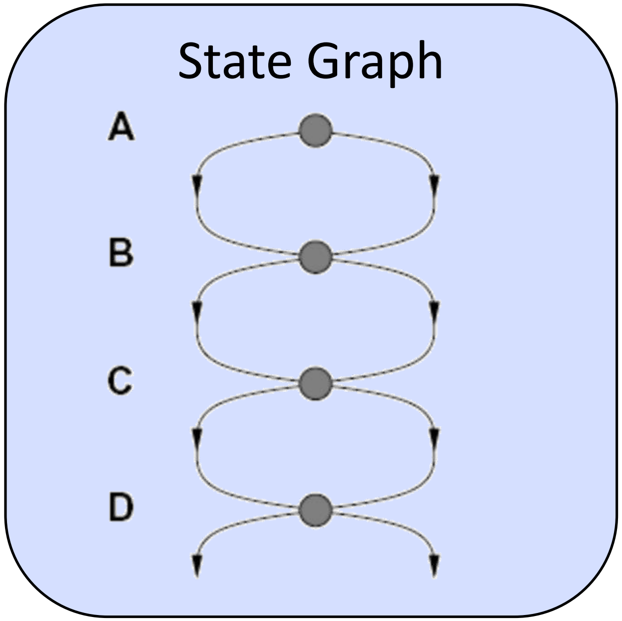


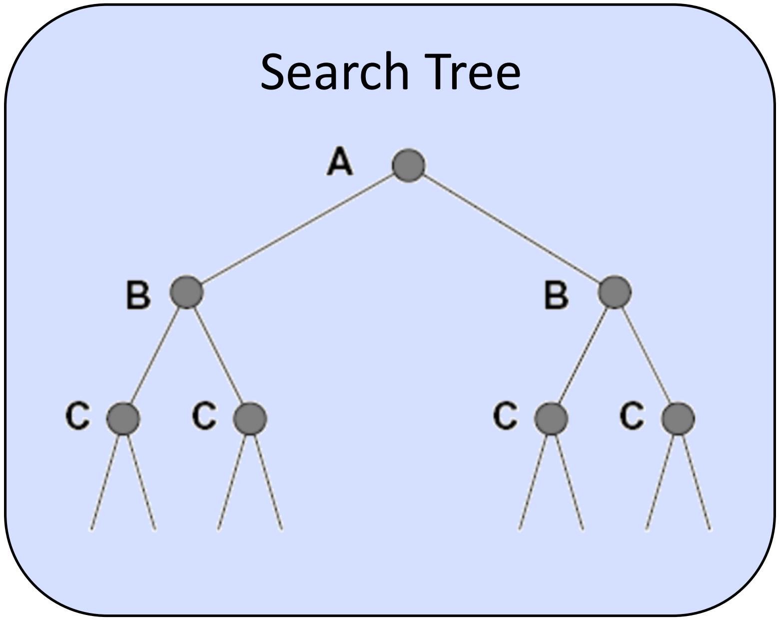
Graph Search



Tree Search: Extra Work!

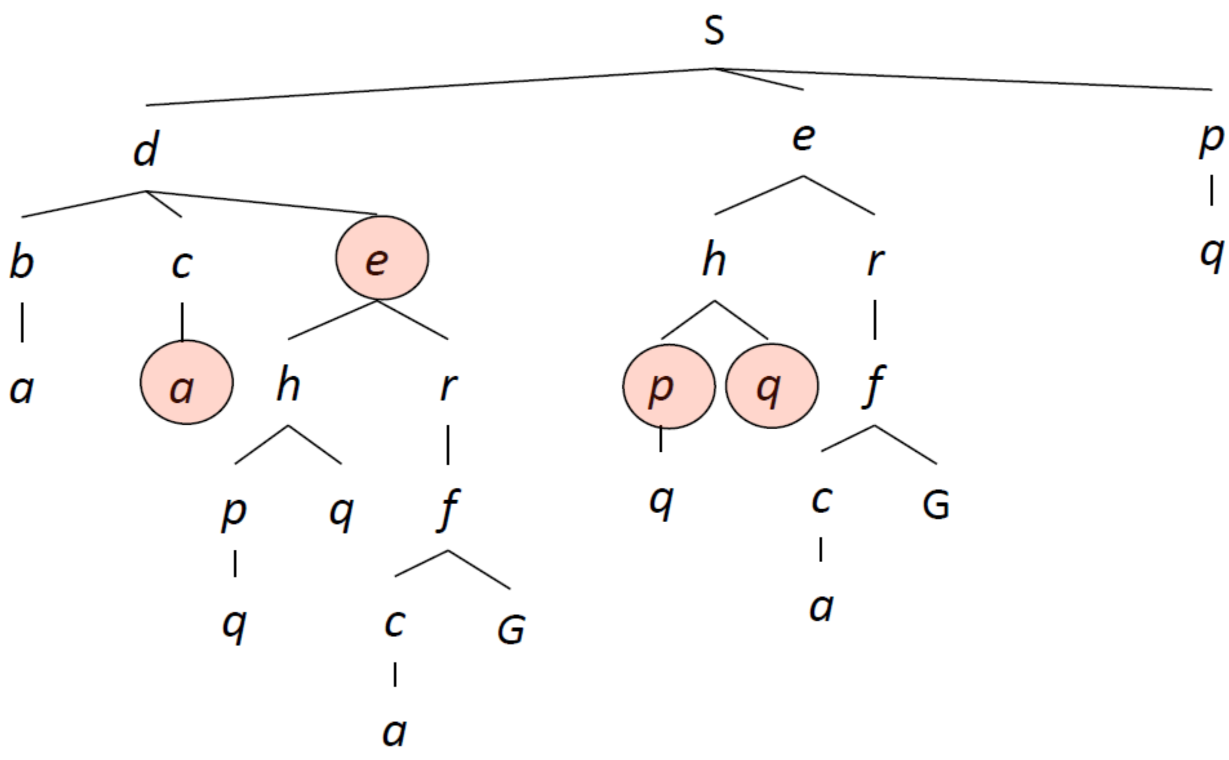
- Failure to detect repeated states can cause exponentially more work.





Graph Search

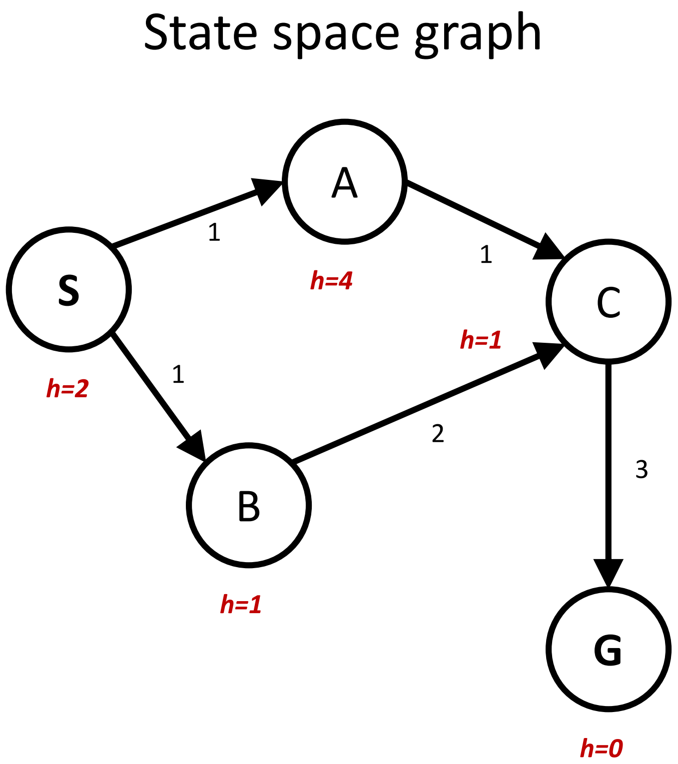
- In BFS, for example, we shouldn't bother expanding the circled nodes (why?)

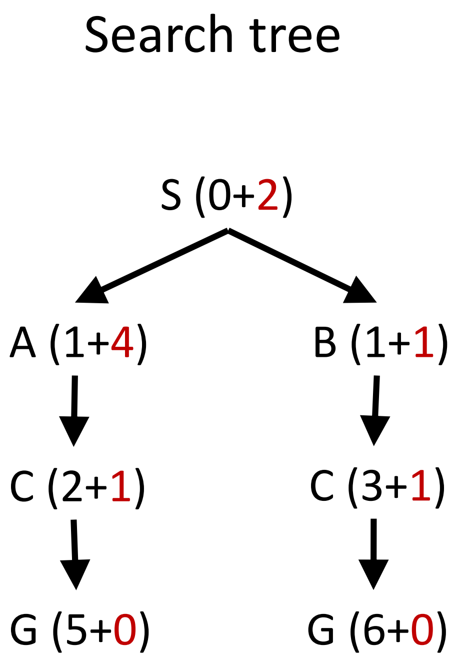


Graph Search

- Idea: never expand a state twice
- How to implement:
  - Tree search + set of expanded states (“closed set”)
  - Expand the search tree node-by-node, but…
  - Before expanding a node, check to make sure its state has never been expanded before
  - If not new, skip it, if new add to closed set
- Important: store the closed set as a set, not a list
- Can graph search wreck completeness? Why/why not?
- How about optimality?

A* Graph Search Gone Wrong?





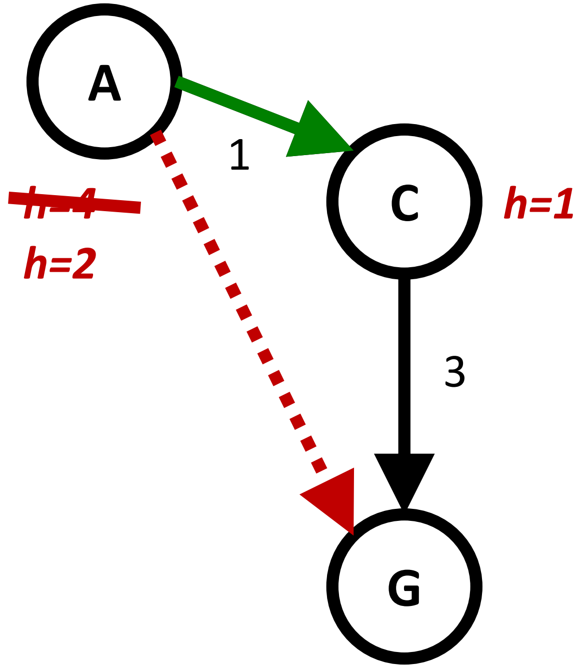
Consistency of Heuristics

- Main idea: estimated heuristic costs ≤ actual costs
  - Admissibility: heuristic cost ≤ actual cost to goal

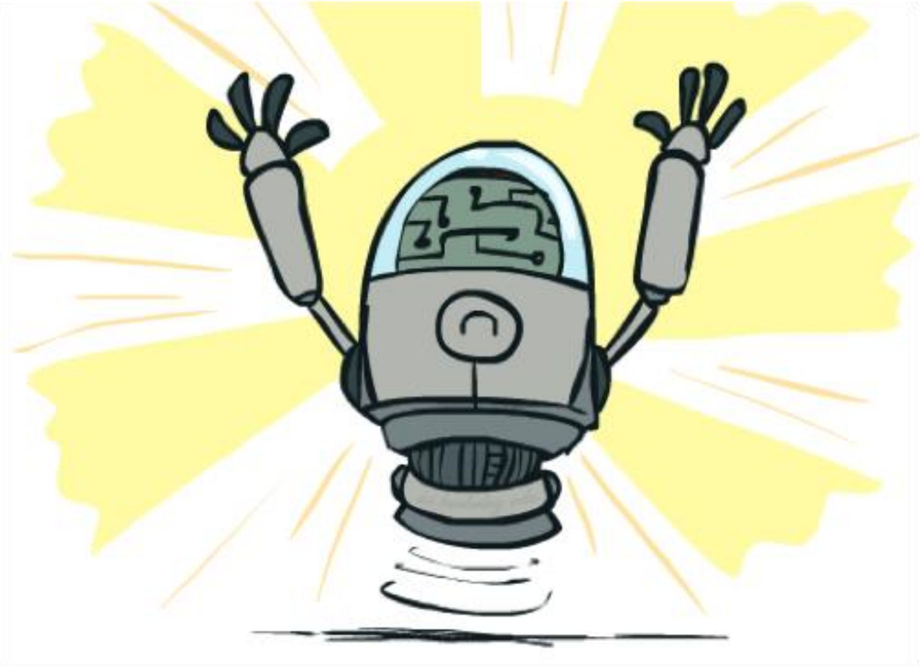
    h(A) ≤ actual cost from A to G
  - Consistency: heuristic "arc" cost ≤ actual cost for each arc

    h(A) – h(C) ≤ cost(A to C)
- Consequences of consistency:
  - The f value along a path never decreases

    h(A) ≤ cost(A to C) + h(C)
  - A* graph search is optimal

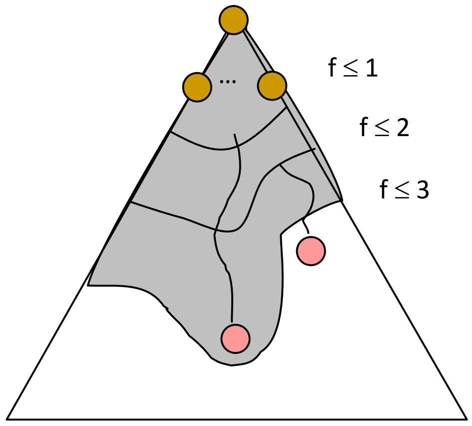


Optimality of A* Graph Search



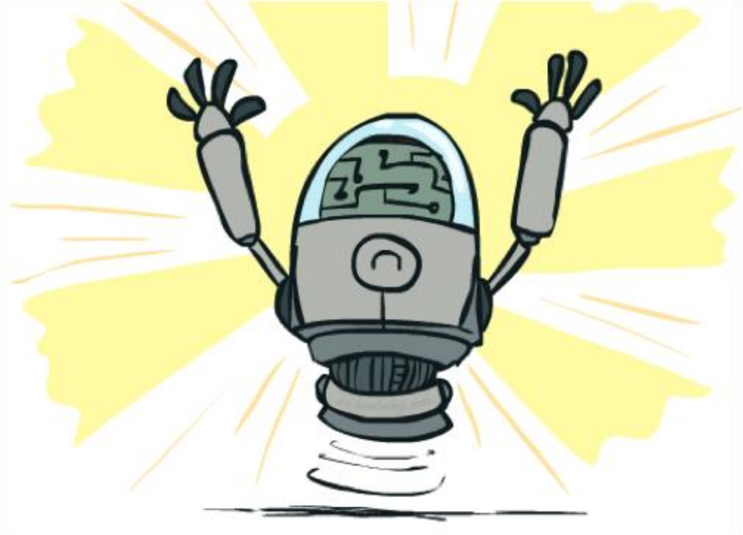
Optimality of A* Graph Search

- Sketch: consider what A* does with a consistent heuristic:
  - Fact 1: In tree search, A* expands nodes in increasing total f value (f-contours)
  - Fact 2: For every state s, nodes that reach s optimally are expanded before nodes that reach s suboptimally
  - Result: A* graph search is optimal



Optimality Properties

- Tree search:
  - A* is optimal if heuristic is *admissible*
  - UCS is a special case (h = 0)



- Graph search:
  - A* optimal if heuristic is *consistent*
  - UCS optimal (h = 0 is consistent)

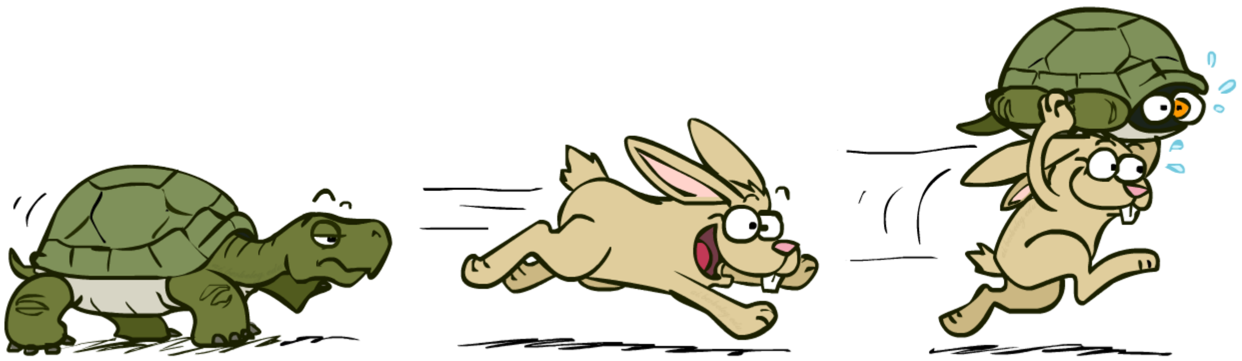
- Consistency implies admissibility

- In general, most natural admissible heuristics tend to be consistent, especially if from relaxed problems

A*: Summary

A*: Summary

- A* uses both backward costs and (estimates of) forward costs
- A* is optimal with admissible / consistent heuristics
- Heuristic design is key: often use relaxed problems



Tree Search Pseudo-Code

```
function TREE-SEARCH(problem, fringe) return a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        for child-node in EXPAND(STATE[node], problem) do
            fringe ← INSERT(child-node, fringe)
        end
    end
```

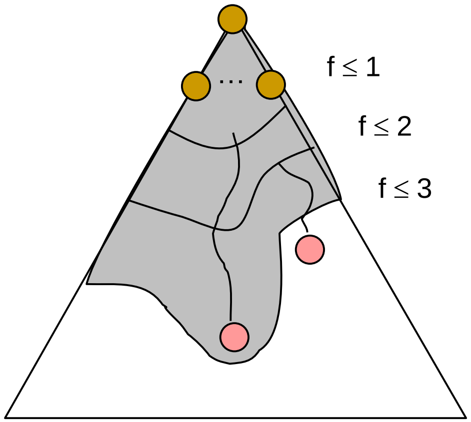
Graph Search Pseudo-Code

```
function GRAPH-SEARCH(problem, fringe) return a solution, or failure
    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        if STATE[node] is not in closed then
            add STATE[node] to closed
            for child-node in EXPAND(STATE[node], problem) do
                fringe ← INSERT(child-node, fringe)
            end
    end
```

Optimality of A* Graph Search

- Consider what A* does:
  - Expands nodes in increasing total f value (f-contours)
    Reminder: f(n) = g(n) + h(n) = cost to n + heuristic
  - Proof idea: the optimal goal(s) have the lowest f value, so it must get expanded first

There's a problem with this argument. What are we assuming is true?



Optimality of A* Graph Search

Proof:

- New possible problem: some *n* on path to G* isn't in queue when we need it, because some worse *n'* for the same state dequeued and expanded first (disaster!)
- Take the highest such *n* in tree
- Let *p* be the ancestor of *n* that was on the queue when *n*' was popped
- *f(p)* < *f(n)* because of consistency
- *f(n)* < *f(n')* because *n'* is suboptimal
- *p* would have been expanded before *n*'
- Contradiction!