POKHARA UNIVERSITY

Level: Bachelor Semester: Fall
Programme: BE
Course: Engineering Mathematics IV

Y : 2021
F : 100
F s: 45
T : 3hrs.

Candidates are required to give their answers in their *s as far as practicable.*

The figures in the margin indicate full marks.

Attempt all the questions.

. a) Define harmonic function. Is a function $v = 2xy - \dfrac{y}{x^2 + y^2}$ is harmonic? If yes, find a corresponding harmonic conjugate and the analytic function. 8

b) State Cauchy Integral formula for derivative. Evaluate $\oint_c \dfrac{z^6}{(2z-1)^6}$ dz, where c is the unit circle $|z| = 1$, counterclockwise 7

2. a) Integrate f(z) = $\dfrac{e^z + z}{z^3 - z} dz$ around a unit circle : $|z| = \dfrac{\pi}{2}$ using Cauchy's Residue theorem. 7

b) Define a bilinear transformation. Find the bilinear transformation which maps the points $z = 0, -1, i$ onto the points $w = i, 0, \infty$. Also find image of the unit circle $|z| = 1$. 8

3. a) Define Fourier integral. Choosing a suitable function, show that 7

$$\int_0^\infty \frac{\sin \pi\omega}{\omega} \sin wx \, dw = \begin{cases} \dfrac{\pi \sin \pi x}{2} & if \quad 0 \le x \ge \pi \\ 0 & if \; x > \pi \end{cases}.$$

b) Find the Fourier Transform of the function $f(x) = e^{\frac{-x^2}{2}}$ 8

4. a) State and prove first shifting theorem of Z transform. Using it evaluate the Z transform of $a^n \cos bt$ and $a^n \sin bt$. 7

b) Solve the difference equation by using Z-transform:

$$y_{n+2} - 3y_{n+1} + 2y_n = 4^n \text{ with } y_0 = y_1 = 1$$

5. a) Derive one dimensional wave equation with solution.

b) A tightly stretched string of length L, fixed at its ends, is initially in a position given by $u(x,0) = u_0 \sin^3\left(\frac{\pi x}{L}\right)$. If it is released from the rest from this position, find the displacement.

6. a) A homogeneous rod of conducting material of length 100cm has its ends kept at zero temperature and the temperature initially is

$$f(x) = \begin{cases} x, & 0 \le x \le 50 \\ 100 - x, & 50 \le x \le 100 \end{cases}$$

find the temperature distribution on the rod at any time.

b) Express the Laplacian $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ in polar co-ordinates. 8

7. Attempt all questions: 10

a) Find tangent vector on the curve $\vec{r} = \cos t\vec{i} + 2\sin t\vec{j}$, at $P(\frac{1}{2}, \sqrt{3}, 0)$

b) Verify that $u = x^2 + t^2$ is a solution of one dimensional wave equation.

c) Express f(z) = sinhz in terms of u+iv.

d) Solve $u_{xx} - u = 0$ by using separation of variables

Programme: BE
Course: Engineering Mathematics IV

Year : 2022
Full Marks: 100
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

a) Define Laplace equation and harmonic function. Determine a and b such that $u = ax^3 + by^3$ is harmonic and also find the harmonic conjugate. 7

b) State and prove Cauchy Integral Formula. Evaluate the integral $\oint_c \left(\frac{e^{5z}}{(z+i)^4} \right) dz$, where c: $|z| = 2$ 8

a) Show that the transformation $w = \frac{2z+3}{z-4}$ maps the circle $x^2 + y^2 - 4x = 0$ on to the straight line $4u + 3 = 0$ in w plane. 7

b) Find the series expansion of the function $f(z) = \frac{7z-2}{(z+1)z(z-2)}$ in the regions by using Laurentz series. 8

$i)\ 0 < |z+1| < 1 \qquad ii)\ 1 < |z+1| < 3.$

a) State and prove first shifting theorem of Z-transform. Using it find $Z[te^{bt}]$ 7

b) Solve the difference equation by using Z transform: 8

$$y_{n+2} - 4y_{n+1} + 4y_n = 2^n, where\ y_0 = 0, y_1 = 1.$$

a) Show that $\int_0^\infty \frac{w \sin xw}{a^2 + w^2} dw = \frac{\pi}{2} e^{-ax}$ *where* $x > 0, a > 0$ 7

b) Find Fourier cosine transform of $f(x) = e^{-mx}$ for m> 0. Then prove that $\int_0^\infty \frac{\cos kx}{1+x^2} dx = \frac{\pi}{2} e^{-k}$ 8

5. a) Find the solution of the differential equation $y^2u_x - x^2u_y = 0$ by using separating of variables. 7

b) Find the temperature in a laterally insulated bar of length L = 20cm whose ends are kept at a zero temperature, assuming that the initial temperature is $f(x) = \begin{cases} x & \text{if } 0<x<\frac{L}{2} \\ L-x & \text{if } \frac{L}{2}<x<L \end{cases}$ 8

6. a) Express the Laplacian $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ in polar co-ordinates. 7

b) Derive the solution of one dimensional wave equation for a vibrating sting.by using D Alembert's method. 8

7. Attempt all questions 2.5

a) Check analyticity of $f(z) = z^2$

b) Show that the Z transform is linear operator.

c) Solve the partial differential equation $u_{xx}+9u=0$.

d) Find the unit tangent vector to the curve

$$\vec{r}(t) = 2\cos t\vec{i} + \sin t\vec{j} \quad at \quad (\sqrt{2},\sqrt{2},0).$$

Level: Bachelor Semester:Fall
Programme:BE
Course:Engineering Mathematic IV

Year : 2020
Full Marks: 100
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

a) Check $u = \sin x \cosh y$ is harmonic or not? If yes, find corresponding harmonic conjugate v of u 8

b) Evaluate $\oint_C \frac{\cot z}{\left(z - \frac{\pi}{2}\right)^2} dz$, where C is the ellipse $4x^2 + 9y^2 = 36$. 7

a) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent series valid for 7

$(i)\ 1 < |z| < 3\ (ii)\ |z| > 3\ (iii)\ |z| < 1\ (iv)\ 0 < |z+1| < 2$

b) State and prove Cauchy residue theorem. Using it evaluate 8

$\int_C \left(\frac{z^2 \sin z}{4z^2 - 1}\right) dz$ where C is the circle $|z| = 2$.

a) Find the Z transform of (i) $r^n \cos n\theta$ (ii) $\frac{1}{n+2}$. 7

b) Solve the differential equation $y_{k+2} + 2y_{k+1} + y_k = k$ where $y_0 = 0$, $y_1 = 0$ using Z-transform. 8

a) Find the solution of the differential equation, $y^2 u_x - x^2 u_y = 0$, by using separating of variables. 7

b) Find the solution of one dimensional equation with boundary $u(0, t) = 0 = u(l, t)$ and initial condition $u(x, 0) = \left(\frac{100x}{l}\right)$. 8

5. a) Derive one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ with necessary assumptions. 7

b) What is Helmboltz's equation on F(x, y) and solve it subjected to F(0, y) = 0 = F(x, 0) = F(x, b). 8

"OR"

Find the deflection u(x, y, t) of the square membrane with a= b = 1 and c = 1, if the initial velocity is zero and the initial deflection is (0, 1). Sin3 π xsin4 π y.

6. a) Find the Fourier transform of $f(x) = \begin{cases} |x| & \text{if } -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$ 7

b) Define Fourier cosine integral. Hence, show that 8

$$\int_0^\infty \frac{\sin \omega \cos \omega x}{\omega} d\omega = \begin{cases} \pi/2 & \text{if } 0 \le x < 1 \\ \pi/4 & \text{if } x = 1 \\ 0 & \text{if } x > 1 \end{cases}$$

7. Write short notes on: 10

a) Write down Laplacian in cylindrical coordinate systems.

b) Prove $Z(a^n) = \frac{z}{z-a}$

c) Show that the transformation $w = e^z$ is conformal

d) Show that the fourier cosine transform satisfied linearity property.

Level: Bachelor
Programme: BE
Course: Engineering Mathematics IV

Semester: Spring

Year : 2019
Full Marks: 100
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) What do you mean by analyticity of function f(z). State Cauchy Remann equation and hence show that it is the necessary condition for the functions to be analytic. 8

   b) State Cauchy Integral formula. Evaluate $\int \frac{1}{z^2+4}\,dz$, where integration is along the ellipse $4x^2 + (y-2)^2 = 4$ 7

2. a) Find the image of infinite strip $1/4 < y < ½$ under the transformation $w = 1/z$. 8

   b) Define Singularities of a function f(z). Find the residues of 7
   $$f(z) = \frac{z+2}{(z+1)(z^2+1)^2}.$$

3. a) Find the inverse z- transform of $f(z) = \frac{2z}{(z-1)(z^2+1)}$ 7

   b) Using Z-transform solve the difference equation 8
   $y_{k+2} + 2y_{k+1} + y_k = k$, where $y_o = 0$, $y_1 = 0$

4. a) Derive Fourier integral of f(x) from Fourier series. Show that: 7
   $$\int_0^\infty \frac{\cos xw}{1+w^2}\,dw = \begin{cases} \frac{\pi}{2}\sin x & if \quad 0 \le x \le \pi \\ 0 & if \quad x > \pi \end{cases}$$

   b) Find Fourier cosine transform of $f(x) = e^{-mx}$ for m > 0. Then prove that $\int_0^\infty \frac{\cos kx}{1+x^2}\,dx = \frac{\pi}{2}e^{-k}$ 8

5. a) Using the method of separation of variable solve P.D.E.: $y^2u_x - x^2u_y = 0$ 7

$y_{n+2} - 3y_{n+1} + 2y_n = 4$, where $y_0 = 0$ and $y_1 = 1$.

4. a) Using Fourier cosine integral, show that

b) Find u (x, t) from one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, with boundary condition. $u(0,t) = 0 = u(L,t)$, initial deflection $f(x)$ and initial velocity $\left.\frac{\partial u}{\partial t}\right|_{t=0} = g(x)$.

6. a) A homogeneous rod of conducting material of length 100 cm has its ends kept at zero temperature and the temperature initially is 8

$$f(x)\begin{cases} x, & 0 \le x \le 50 \\ 100 - x, & 50 \le x \le 100 \end{cases}$$

Find the temperature u(x, t)

b) Find two-dimensional Laplace equation in polar co-ordinates. 7

7. Attempt all the questions. 4×2

a) Verify that: U= x^2+t^2 is the solution of one dimensional wave equation 5

b) Find the Z- tranform of na^{n-1}

c) Check analyticity of $f(z) = z^3$

d) Find the unit tangent vector to the curve $\vec{r}(t) = 2\cos t\vec{i} + \sin t\vec{j}$ at $(\sqrt{2}, \sqrt{2}, 0)$.

a) Solve $U_{xx}+U_{yy} = 0$ by the method of separation of variables 8

b) What is Helmholtz equation? Find its solution. 7

a) State and prove Initial and Final value theorems in Z-transform. Find the value of $Z(a^n \cos bt)$ and $Z(a^n \sin bt)$ 8

b) Solve $y_{n+2} - 3y_{n+1} + 2y_n = 0, \quad y_0 = 0, \; y_1 = 1$ 7

Answer all of the following questions. 4×2.5

a) Express the parametric equation of the hyperbola $x^2 - y^2 = 1, z = 0$.

b) Check the analyticity of the function $f(z) = \operatorname{Arg} z$

c) Find the z-transform of $f(n) = na^n$.

d) Find the residue of $f(z) = \dfrac{1}{z^2 - 1}$ at $z = 1$.

Level: Bachelor
Programme: BE
Course: Engineering Mathematics IV

Semester spring

Year : 2018
Full Marks: 100
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Define analytic function. Show that the function $u(x, y) = 3x^2y + x^2 - y^3 - y^2$ is a harmonic function. Find a function $v(x, y)$ such that $u + iv$ is an analytic function.

   b) Define Pole and Zeroes of a function. State Cauchy's residue theorem and evaluate $\oint_C \frac{e^z}{\cos z} dz$ where $C : |z| = 3$.

2. a) Find the expansion of $\frac{7z-2}{(z+1)z(z-2)}$ in the region given by

   i) $0 < |z+1| < 1$. ii) $1 < |z+1| < 3$.

   b) Given the bilinear transformation $w = \frac{3-z}{2z+1}$, find the mapping of the circle $|z|=1$ in the w- plane

3. a) Show that $\int_0^\infty \frac{\cos wx + w \sin wx}{1+w^2} dx = \begin{cases} 0 & if\ x < 0 \\ \frac{\pi}{2} & if\ x = 0 \\ \pi e^{-x} & if\ x > 0 \end{cases}$

   b) Find the Fourier sine and cosine transform of the function $f(x) = 2e^{-5x} + 5e^{-2x}$

4. a) Derive and find the solution of one dimensional wave equation.

   b) Find the temperature in a laterally insulated bar of length L whose ends are kept at a zero temperature, assuming that the initial temperature is $f(x) = \begin{cases} x & if\ 0 < x < \frac{L}{2} \\ L - x & if\ \frac{L}{2} < x < L \end{cases}$.

POKHARA UNIVERSITY

Level: Bachelor
Programme: BE
Semester: Spring
Course: Engineering Mathematics IV

Year : 2021
Full Marks: 100
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Define harmonic function. Prove that the function $v = \arg z$ is harmonic. Also, find its conjugate and the corresponding analytic function. 8

   b) State Cauchy's integral formula and using it integrate $\oint_c \frac{z^2}{(z^4-1)} dz$ where c is the circle $|z+i|=1$ in counter clockwise 7

2. a) Find the image of triangular region of the z-plane bounded by the lines x = 0, y = 0 and $\sqrt{3}\,x + y = 1$ under the transformation of $w = e^{i\pi/3}z$ and show the sketch in the diagram. 7

   b) Define Singularities of a function f(z). Find the residues of $f(z) = \frac{z+2}{(z+1)(z^2+1)^2}$. 8

OR

Find Laurent series of the function $f(z) = \frac{z^2-1}{z^2+5z+6}$ in the region i) when $|z| < 2$ ii) when $2 < |z| < 3$ and iii) when $|z| > 3$.

3. a) State and prove second shifting theorem of z-transform. Find z-transform of e^{-iat} and hence find Z(cos at) 7

   b) Use z-transform to solve the difference equation: $y_{n+2} - 3y_{n+1} + 2y_n = 4^n$, where $y_0 = 0$ and $y_1 = 1$. 8

4. a) Using Fourier cosine integral, show that 7

$$\int_0^\infty \frac{\sin\omega\cos\omega x\, d\omega}{\omega} = \begin{cases} \pi/2 & if\ 0 \le x < 1 \\ \pi/4 & if\ x = 1 \\ 0 & if\ x > 1 \end{cases}$$

b) Find Fourier cosine transform of $f(x) = e^{-mx}$ for m > 0. Then prove that $\int_0^\infty \frac{\cos kx}{1+x^2}\,dx = \frac{\pi}{2}e^{-k}$ 8

5. a) A tightly stretched string of length L, fixed at its ends, is initially in a position given by $u(x,0) = u_0 \sin^3\left(\frac{\pi x}{L}\right)$. If it is released from the rest from this position, find the displacement at any point x at time t. 8

b) Find the temperature in a laterally insulated bar of length L=10cm whose ends are kept at a zero temperature, assuming that the initial temperature is $f(x) = \begin{cases} x & if\ 0 < x < \frac{L}{2} \\ L - x & if\ \frac{L}{2} < x < L \end{cases}$ 7

6. a) Find the solution of differential equation $y^2u_x - x^2u_y = 0$ using separating of variables. 7

b) Find the solution of one-dimensional wave equation by D'Alembert's method. 8

OR

Express the Laplacian $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ into polar co-ordinates.

7. Attempt all questions:

a) Find a tangent vector and the corresponding unit tangent vector $\vec{u(t)}$ at a given point $r(t) = 2\cos t\,\vec{i} + \sin t\,\vec{j}$ at P $(\sqrt{2}, \sqrt{2}, 0)$ 2.5

b) Check analyticity of $f(z) = z^2$ 2.5

c) Find the poles of the function $f(z) = \frac{\sinh z}{(z - i\pi)}$. 2.5

d) Find z–transform of $Z(n^2)$ 2.5

POKHARA UNIVERSITY

Level: Bachelor
Semester: Fall
Year : 2017
Programme: BE
Full Marks: 100
Course: Engineering Mathematics IV
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Define harmonic function. If v = argz is harmonic? If yes, find a corresponding harmonic conjugate. 7

   b) State and prove Caucht's integral formula. Evaluate the integral 8

   $\oint_c \frac{\cos z}{(z-\pi i)^2} dz$ where c is unit circle emclosing the point πi.

   OR

   Find the fixed points and the normal form of the bilinear transformation $w = \frac{z-1}{z+1}$. Also determine the nature of this transformation.

2. a) Define singularity of a function. Evaluate the following integrals: 8

   i. $\int_c \frac{e^z}{\cos z} dz, \qquad C: |z| = 3$

   ii. $\int_c \frac{z+1}{z^4 - 2z^3} dz, \qquad C: |z| = \frac{1}{2}$

   b) State and prove first shifting theorem for z-transform using it to find $z(\cosh at \sin bt)$ 7

3. a) Find $Z^{-1}\left[\frac{2z^2+3z}{(z+2)(z-4)}\right]$ 7

   b) Solve the difference equation $y_{n+2} - 3y_{n+1} + 2y_n = 0$, where $y_0 = 0$ and $y_1 = 1$; by using z-transform. 8

4. a) Derive one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ with necessary 7

assumptions. 8

b) A homogeneous rod of conducting material of length 100 cm has its ends kept at zero temperature and the initial temperature be defined by $f(x) = \begin{cases} x & for\ 0 \le x \le 50 \\ 100 - x & for\ 50 < x \le 100 \end{cases}$. Find the temperature u(x,y) at any time t. 7

5. a) Starting from Fourier series, obtain the Fourier integral in complex form.

b) Show that $\int_0^\infty \frac{\cos wx + w \sin wx}{1+w^2} d\omega = \begin{cases} 0\ if\ x<0 \\ \frac{\pi}{2}\ if\ x=0 \\ \pi e^{-x}\ if\ x>0 \end{cases}$ 8

6. a) Solve $U_{xx}+U_{yy} = 0$ 7

b) Obtain the solution of one dimensional heat equation completely. 8

7. Attempt all 4×2.5

a) Find the parametric representation of the surface $x^2 + 4y^2 = 9$, $z = 3$

b) Find the tangent on the curve C with position vector $\vec{r} = cosht\vec{i} + sinht\vec{j}$, at $P\left(\frac{5}{3}, \frac{4}{3}, 0\right)$

c) Evaluate $\oint_c \frac{dz}{z}$ where c is the unit disk $|z| = 1$.

d) Find poles with their order of function $f(z) = \frac{1}{(z^2+a^2)^2}$.

POKHARA UNIVERSITY

Level: Bachelor | Semester: Spring | Year : 2017
Programme: BE | | Full Marks: 100
Course: Engineering Mathematics IV | | Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Define harmonic function. If u = sinhx siny, show that u is harmonic. Also, find its harmonic conjugate and the corresponding analytic function. 8

OR

Define an analytic function. Show that the Cauchy-Riemann equations are necessary for a function to be analytic.

b) State and prove Cauchy Integral Formula. Evaluate the integral $\oint_c \frac{z+1}{z^3-4z}dz$, where c is the unit circle $|z+2|=\frac{3}{2}$, counterclockwise. 7

2. a) Determine the region $w=e^{i\pi/4}$ in the w-plane corresponding to the triangular region bounded by the lines $x=0$, $y=0$ and $x+y=1$ in the z plane. 7

b) State Residue theorem. Integrate $\oint_c \frac{z-23}{z^2-4z-5}dz$ where $c:|z|=6$ using residue theorem. 8

3. a) State and prove second shifting theorem of Z- transform. Evaluate $Z(e^{-at}\sin wt)$ 7

OR

Find $Z^{-1}\left[\frac{z}{(z+1)^2(z-1)}\right]$

b) Solve the difference equation: $y_{n+2}-3y_{n+1}+2y_n=4^n$, where $y_0=0$ and $y_1=1$, by applying Z–transform. 8

4.

a) Show that $\int_0^\infty \left[\frac{\cos^{\pi}/_2\omega \cos x\omega}{1-\omega^2}\right] d\omega = \begin{cases} \pi/2 \cos x & if\ |x| < \pi/2 \\ 0 & if\ |x| > \pi/2 \end{cases}$ 7

b) Find Fourier sine transform of $f(x) = e^{-x}$, $x > 0$ and then show that $\int_0^\infty \frac{x \sin mx}{x^2+1} dx = \frac{\pi}{2} e^{-m}$ for $M > 0$. 8

5. a) Solve $xu_{xy} + 2yu = 0$ by using separating variables. 7

b) Find the solution of one Dimensional wave equation by D'Alemberts method. 8

6. a) Find the temperature u(x, t) in a laterally insulated bar of length L whose ends are kept at temperature 0, assuming that the initial temperature is $f(x) = \begin{cases} x & \text{if } 0 < x < L/2 \\ L - x & \text{if } L/2 < x < L \end{cases}$ 8

b) Derive Laplace equation in polar co-ordinate and also write the expression for cylindrical co-ordinates. 7

OR

Define potential function and then find the solution of potential function. by spherical membrane.

7. Attempt all questions 5×2.5

a) Evaluate $\oint_c \frac{dz}{z-3i}$, where c is the circle, |z-2i| = 2 counter clockwise direction

b) Find z–transform of Z (n^2)

c) Solve $u_{xx} - u = 0$

d) Write the equation of hyperbolid of two sheet and then sketch

POKHARA UNIVERSITY

Level: Bachelor
Programme: BE
Course: Engineering Mathematics IV

Semester: Fall

Year : 2018
Full Marks: 100
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Define harmonic function. Show that the function $u = 3x^2y + x^2 - y^3 - y^2$ is a harmonic function. Find the analytic function for which the given function is a real part. 8

   b) Evaluate $\oint_C \left(\frac{\cos(\pi z^2)}{z^2 - 3z + 2}\right) dz$ where C: $|z| = 3$. 7

2. a) Let the rectangular region R in the z- plane be bounded by lines x=0, y=0, x=2, y=3. Find the region R' of the w-plane into which R is mapped under the transformation $W = \sqrt{2}\ e^{\frac{i\pi}{4}} z$. 7

   b) Find the Taylor's and Laurent's series of the function $f(z) = \dfrac{z^2 - 1}{(z+2)(z+3)}$ 8

   OR

   State Cauchy Residue Theorem. By applying Cauchy Residue Theorem, evaluate $\oint_C \left(\frac{4 - 3z}{z(z-1)(z-2)}\right) dz$ where C: $|z| = \frac{3}{2}$.

3. a) State & prove first shifting theorem on Z-transform. Find the Z-transform of $e^{-at}t$. 8

   b) Solve the differential equation $y_{k+2} + 2y_{k+1} + y_k = k$ where $y_0 = 0$, $y_1 = 0$ using Z-transform. 7

4. a) Show that $\displaystyle\int_0^\infty \frac{\sin \pi w \sin xw}{1 - w^2} dw = \begin{cases} \frac{\pi}{2} \sin x & if\ \ 0 \le x \le \pi \\ 0 & if\ \ x > \pi \end{cases}$ 7

b) Find the Fourier cosine transform of $f(x) = e^{-x}$ $(x > 0)$ and hence by using Parseval's identity, show that that $\int_0^\infty \frac{dx}{(1+x^2)^2} = \frac{\pi}{4}$. 8

5. a) Define partial differential equation with suitable example. By separating the variables solve $u_{xx} + u_{yy} = 0$ 7

b) Find u(x, t) from one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, with boundary condition. $u(0,t) = 0 = u(L,t)$, initial deflection $f(x)$ and initial velocity $\left.\frac{\partial u}{\partial t}\right|_{t=0} = g(x)$. 8

6. a) Find the temperature in a laterally insulated bar of length L whose ends are kept at a zero temperature, assuming that the initial temperature is 7

$$f(x) = \begin{cases} x & \text{if } 0 < x < \frac{L}{2} \\ L - x & \text{if } \frac{L}{2} < x < L \end{cases}$$

b) Express the laplacian $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ in polar co-ordinates. 8

7. Attempt all questions. 4×2.5

a) If $u = y^3 - 3x^2 y$ show that u is harmonic.

b) Find z–transform of $z(a^n)$

c) If $\vec{r} = (3\cos t, 4\sin t, t)$ be the position vector of the curve. Find its curve.

d) Solve the partial differential equation $u_{yy} = u$.

POKHARA UNIVERSITY

Level: Bachelor
Programme: BE
Course: Engineering Mathematics IV

Semester: Fall

Year : 2019
Full Marks: 100
Pass Marks: 45
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Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) State and prove the necessary condition for analyticity. Test the analyticity of the function $f(z) = \log z$ 8

   b) State Cauchy Integral formula for derivative. Evaluate $\oint_c \frac{z^6}{(2z-1)^6}$ dz, where c is the unit circle $|z|=1$, counterclockwise 7

2. a) Integrate f(z) = $\frac{e^z + z}{z^3 - z} dz$ around a unit circle : $|z| = \frac{\pi}{2}$ using Cauchy's Residue theorem. 7

   b) Find the fixed points and the normal form of the bilinear transformation w = $\frac{z-1}{z+1}$. Also, determine the nature of this transformation. 8

3. a) Define Fourier integral. Choosing a suitable function, show that 8

$$\int_0^\infty \frac{\sin \pi\omega}{\omega} \sin wx\, dw = \begin{cases} \frac{\pi \sin \pi x}{2} & if \quad 0 \le x \ge \pi \\ 0 & if\, x > \pi \end{cases}.$$

   b) Find the Fourier Transform of the function $f(x) = e^{\frac{-x^2}{2}}$ 7

4. a) Define Z - transform. State and prove First shifting theorem of Z-transform. Evaluate $Z(t^2 e^{-bt})$ 8

OR

Find $Z^{-1}\left[\dfrac{z^3}{(z-1)^2(z+1)}\right]$

b) Solve the difference equation by using Z-transform: 7

$y_{n+2} - 3y_{n+1} + 2y_n = 4^n \ with \ y_0 = y_1 = 1$

5\. a) Derive one dimensional wave equation with solution. 8

b) Find the temperature u(x, t) which is distributed laterally in a insulated copper bar (c^2 = 1.158 cm^2/sec), 100 cm long and of constant cross section whose end points at x = 0 and x = 100 are kept at 0°C and its initial temperature is f(x) = $\sin^3(0.01)\,\pi x$ 7

6\.

a) Express the Laplacian $\nabla^2 u = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2}$ in polar co-ordinates. 8

b) Define partial differential equation with suitable example. By separating the variables solve $u_{xx} + u_{yy} = 0$ 7

7\. Attempt all questions: 10

a) Find the unit tangent vector to the curve

$\vec{r}(t) = 2\cos t\vec{i} + \sin t\vec{j} \quad at \quad (\sqrt{2}, \sqrt{2}, 0).$

b) Express f(z) = sinhz in terms of u+iv.

c) Solve $u_{xx} - u = 0$ by using separation of variables

d) Find z–transform of $n4^n$

Level: Bachelor
Programme: BE
Course: Engineering Mathematics IV

Semester: Fall

Year : 2014
Full Marks: 100
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1\. a) Define analyticity of a complex valued function f(z). Show that the function $u = \frac{x}{x^2+y^2}$ is harmonic. Find harmonic conjugate of u such that f(z) = u + iv is analytic. 8

b) State and Prove Cauchy Residue theorem. Evaluate $\oint_C \frac{dz}{z^8(z+4)}$, where C is $|z + 2| = 3$ in anticlockwise direction. 7

2\. a) Define conformal mapping. If u = $2x^2+y^2$ and v = $\frac{y^2}{x}$ show that the curves u = constant and v=constant cut orthogonally at all intersections but the transformation w = u+iv is not conformal. 7

OR

State and prove Cauchy–integral formula and hence evaluate

$$\oint_C \frac{2x^2+4z}{z-2}dz;\ c: |z| = 1$$

b) Define Z transform. State and Prove first shifting theorem on Z transform. Using it find Z transform of cosat and sinat. Also evaluate $Z(a^n cosbt)$ and $Z(a^n sinbt)$. 8

3\. a) Solve the difference equation: $y_{n+2} - 3y_{n+1} + 2y_n = 0$, where $y_0 = 0$ and $y_1 = 1$. 8

b) Find the temperature in a laterally insulated bar of length L whose ends are kept at a zero temperature, assuming that the initial temperature is $f(x) = \begin{cases} x & \text{if } 0 < x < \frac{L}{2} \\ L - x & \text{if } \frac{L}{2} < x < L \end{cases}$ 7

4\. a) Write one-dimensional wave equation and solve it. 8

b) Using the method of separation of variable solve the partial 7 differential equation $y^2u_x - x^2u_y = 0$.

5. a) Express Laplacian in polar co-ordinate system from Cartesian co-ordinate system. 8

OR

Find u (x, y, t) for the rectangular membrane with sides a and b with c =1, if the initial velocity is zero and initial deflection is $\sin\frac{2\pi x}{a}\sin\frac{3\pi y}{b}$.

b) Define Fourier sine and cosine integrals. Show that 7

$$\int_0^\infty \frac{\cos wx + w\sin wx}{1+w^2}dx = \begin{cases} 0 \text{ if } x<0 \\ \frac{\pi}{2} \text{ if } x=0 \\ \pi e^{-x} \text{ if } x>0 \end{cases}$$

OR

Show that: $\int_o^\infty \left(\frac{1-\cos\pi w}{w}\right)\sin wx\,dw = \begin{cases} \pi/2 & \text{if } 0<x<\pi \\ 0 & \text{if } x>\pi \end{cases}$.

6.

a) Find the Fourier transform of $f(x) = xe^{-x^2}$. 8

b) State and prove initial and final value theorem on Z transform. 7

7. Answer the followings: 4×2.5

a) Sketch the paraboloid $z = x^2 + y^2$

b) Find the parametric representation of the surface $y^2+(z-3)^2-9$, x=2

c) Find the unit tangent vector of $\vec{r}(t) = \cos t\,\vec{i} + 2\sin t\,\vec{j}$ at $\left(\frac{1}{2},\sqrt{3},0\right)$

d) Show that $\oint_C \frac{dz}{z} = 2\pi i$, where C is the circle $|z| = 1$ in anticlockwise direction.

Level: Bachelor
Programme: BE
Course: Engineering Mathematics IV

Semester: Spring

Year : 2014
Full Marks: 100
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Define analytic function f(z). State Cauchy Riemann equation and hence show that it is the necessary condition for the function to be analytics. 7

   b) State and prove Cauchy's integral formula. Hence using it integrate 8

   $\oint_c \frac{z^2}{(z^4-1)} dz$ where c is the circle $|z+i|=1$ in counter clockwise.

   OR

   Evaluate $\oint \frac{z^3 + Sinz}{c(z-i)^3} dz$, where 'c' is the boundary of the square with vertices ± 2, $\pm 2i$.

2. a) Expand the function $f(z) = \frac{z+3}{z(z^2-z-2)}$ in the region given by 8

   i. $/z/ < 1$,
   ii. $1 < /z/ < 2$,
   iii. $/z/ > 2$.

   b) Find the deflection u(x, t) of the vibrating string of length $L = \pi$, c2 = 1 and its initial velocity is zero and initial deflection is given by 7

$$f(x) = \begin{cases} 0.1x, & for \ 0 < x < \frac{\pi}{2} \\ 0.01(\pi - x), & for \ \frac{\pi}{2} < x < \pi \end{cases}$$

3. a) Find the solution of one dimensional wave equation by D' Alembert's 7

method.

b) Express the Laplacian $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ in polar co-ordinates. 8

4. a) State and prove first shifting theorem for Z-transform. Use it to find Z(coshat sinbt). 8

b) Find the inverse Z-transform of $\frac{2z}{(z-1)(z^2+1)}$ 7

5.

a) Show that . $Z(y_{n+k}) = z^k \left[\bar{y} - y_0 - \frac{y_1}{z} - \dots\dots\dots - \frac{y_{k-1}}{z^{k-1}} \right]$ where $\bar{y}$ = $Z(y_n)$. Using it solve $y_{k+1}+y_k=1$ if $y_0=0$. 7

b) Solve $u_{xx} + u_{yy} = 0$ by using separation method. 8

6. a) Define convolution of two functions. State and prove convolution theorem on Fourier transform. 7

OR

Define Fourier transform and evaluate Fourier transform of

$$f(x) = e^{\frac{-x^2}{2}}$$

b) Derive Fourier integral of f(x) from Fourier series. Show that. 8

$$\int_0^\infty \left[\frac{\cos xw + wsiinxw}{1 + w^2} \right] dw = \begin{cases} 0 & if \quad x < 0 \\ \pi / 2 \; if & x = 0 \\ \pi e^{-x} \quad if & x > 0 \end{cases}$$

7. Write short notes on: 4×2.5

a) If **r**(t) = (a + 2cos2t, b - 2sin2t, 0) be the position vector of any curve, find its equation in Cartesian form.

b) Verify that u = $x^2 + t^2$ is the solution of one dimensional wave equation.

c) Define the types of singularity of a complex function with examples.

d) Find Z(1) and $Z(-1)^n$

Level: Bachelor Semester: Fall Year : 2015
Programme: BE Full Marks: 100
Course: Engineering Mathematics IV Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) Define analytic function f(z). State Cauchy Riemann equation and hence show that it is the necessary condition for the function to be analytics. 7

   b) State and prove Cauchy's integral formula. Evaluate where c is the ellipse $4x^2+9y^2=36$. 8

   $$\oint_c \frac{\cot z}{\left(z-\frac{\pi}{2}\right)^2}dz$$

   OR

   Evaluate $\oint_c \frac{z^3+\sin z}{(z-i)^3}dz$, where 'c' is the boundary of the square with vertices ±2, ±2i.

2. a) State Laurent's theorem. Find the Laurent's series for 7

   $f(z) = \frac{1}{(z^2 - z^3)}$ in the region $0<|z|<1$.

   b) Define singularity, zeros and poles of a function. Evaluate 8

   $\oint_c f(z)dz$ where $f(z) = \frac{e^{2z}}{(z+1)^3}$ where c is the ellipse $4x^2+9y^2=16$.

3. a) State and prove convolution theorem on Z transform. 7

   b) Solve the difference equation: $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$, where $y_0 = 0$ and $y_1 = 0$. 8

4. a) Find the Fourier integral of the function; 7

   $$f(x) = \begin{cases} \pi/2 & if\ 0 \le x < 1 \\ \pi/4 & if\ x = 1 \\ 0 & if\ x > 1 \end{cases}$$

b) Find Fourier cosine transform of $f(x) = e^{-mx}$ for m>0,and then show that $\int_0^\infty \left(\frac{\cos kx}{1+x^2}\right) dx = \frac{\pi}{2} e^{-k}$ 8

5. a) Derive one dimensional wave equation of a string of length L which is fixed in two end points with required assumptions. 7

OR

Find the solution of one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, with initial temperature f(x) and boundary conditions is u(0,t)=0=u(L,t).

b) Derive two dimensional heat equations with necessary assumptions. 8

6. a) Find $Z^{-1}\left[\frac{2z^2+3z}{(z+2)(z-4)}\right]$ 7

b) A homogeneous rod of conducting material of length 100 cm has its end kept at zero temperature and temperature initially is

$$f(x) = \begin{cases} x; & 0 \le x \le 50 \\ 100 - x, & 50 \le x \le 100. \end{cases}$$ 8

7. Write short notes on: **(Any two)** 2.5×4=10

a) Find z–transform of na^{n-1}

b) Evaluate $\oint_c \frac{z^3}{2z-i} dz$ where /z/=1.

c) Solve the partial differential equation: $u_x + u_y = 0$, by separation of variables method.

d) Write equation of ellipsoid. Sketch it with center and axes of symmetry.

Level: Bachelor
Programme: BE
Course: Engineering Mathematics IV

Semester: Spring

Year : 2015
Full Marks: 100
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Show that the necessary condition for analyticity of f(z) = u+iv ,is $u_x = v_y$ and $u_y = -v_x$. 7

   b) Define Laplace equation. Test $u = \cos x \cosh y$ is harmonic or not. If yes, find the harmonic function and the corresponding analytic function f (z). 8

2. a) State and Prove Cauchy Residue theorem. Evaluate $\oint_C \frac{e^{5z}}{(z+i)^4} dz$, where C is a circle $|z| = 3$ along anticlockwise direction. 7

   b) Determine the region of $w = e^{\frac{i\pi}{4}}$ in the w-plane corresponding to the triangular region bounded by the lines x=0, y=0, and x+y=1 in the z-plane. 8

   Or

   Integrate: $\oint_C \frac{dz}{z^2+4}$; $c: 4x^2 + (y-2)^2 = 4$

3. a) Find the Z–transform of f(t) = a^n and hence find Z $\left\{Sin(\frac{n\pi}{2}\right\}$ and $Z\left\{Cos(\frac{n\pi}{2})\right\}$. 7

   b) Find the inverse of z-transform of $\frac{3z^3+2z}{(z+3)^2(z-2)}$ 8

4. a) Solve the difference equation: $y_{n+2} + 6y_{n+1} + 9y_n = 4^n$, where $y_0 = 0$ and $y_1 = 0$. 7

   b) Find Fourier sine and cosine integral representation of the function

$f(x) = e^{-x} + e^{-2x}$, for $x > 0$. 8

Or

Find Fourier transform of $f(x) = \begin{cases} 1 & for \ |x| < 1 \\ 0 & for \ |x| > 1 \end{cases}$

5. a) Define partial differential equation with suitable example. By separating the variables solve $u_{xx} + u_{yy} = 0$

b) Define partial differential equation, with suitable example. By separating the variables solve $u_{xx} + u_{yy} = 0$

Or

Find the solution of one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, with initial temperature f(x) and boundary conditions is u(0,t)=0=u(L,t).

6.

a) Express the Laplacian $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ in polar co-ordinates. 8

Or

Solve one dimerisional wave equation Completely.

b) Define Fourier integral. Choosing a suitable function, show that 7

$$\int_0^\infty \frac{\sin \pi\omega}{\omega} \sin wx \, dw = \begin{cases} \frac{\pi \sin \pi x}{2} & if \quad 0 \le x \le \pi \\ 0 & if \ x > \pi \end{cases}$$

Or

Find the Fourier cosine transform of e^{-x}.

7. Write short notes on: (**Any two**) 2×5

a) Solve the partial differential equation $u_x = 2xy$.

b) Write equation of an ellipsoid. Sketch it with centre and axis of symmetry.

c)

Verify that $u = x^2 + t^2$ is the solution of one dimensional wave equation

d) Derive Z inverse of $X(z) = \frac{z}{(z+1)(z-3)}$.

Level: Bachelor | Semester: Fall | Year : 2016

Programme: BE | Full Marks: 100

Course: Engineering Mathematics IV | Pass Marks: 45

Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Prove that if the function f(z) is analytic then show that Ux=Vy and Uy=-Vx 8

   b) Integrate the followings along the unit circle counterclockwise 7

      i. $\oint \frac{z^6}{(2Z-1)^6}$

      ii. $\oint \frac{z+1}{z^3-2z^2}$

2. a) Find the singular points and residues of the function 8

   $$f(z) = \frac{z+2}{(z-2)(z^2+1)^2}$$

   b) State Laurent's theorem. Find the Laurent's series for 7

   $$f(z) = \frac{1}{(z-z^3)}$$ in the region $01<|z+1|<2$.

3. a) Find the Z- transform of the function f(t) = e^{-iat} and hence deduce the value of Z(cosat) and Z(sinat). 7

   b) Using Z-transform solve the difference equation $y_{n+2}+6y_{n+1}+9y_n = 2^n \; when \; y_0 = y_1 = 0.$ 8

4. a) Define Z - transform. State and prove Second shifting theorem of Z-transform. Evaluate $Z\,(t^2e^{-bt})$ 7

   OR

   Find $Z^{-1}\,\frac{z^2+1}{z^2-2z+2}$.

   b) Choosing a suitable function show that $\int_0^\infty \left[\frac{\cos\omega x+\omega\sin\omega x}{1+\omega^2}\right] d\omega =$ 8

$$\begin{cases} 0 & \text{if } x < 0 \\ \pi/2 & \text{if } x = 0 \\ \pi e^{-x} & \text{if } x > 0 \end{cases}$$

5. a) Find the Fourier cosine and sine transform of $f(x) = e^{-ax}, a > 0$. 7

OR

Verify the convolution theorem for the functions $f(x) = e^{-x^2}$ and $g(x) = e^{-x^2}$.

b) Find u(x, t) of the string of length $l = \pi$ when $c^2 = 1$, the initial velocity is zero and the initial deflection is $0.1(\pi - x)$. 8

6. a) What is Helmholtz's equation on F(x, y) and solve it subject to F(0, y) = 0 = F(a, y) = F(x, 0) = F(x, b). 8

OR

Find the deflection u(x, y, t) of the square membrane with a = b= 1 and c = 1, if the initial velocity is zero and the initial deflection is (0.1) sin $3\pi x$ sin $4\pi y$.

b) Derive one dimensional heat equation with required assumptions. 7

7. Attempt all 4×2.5

a) Solve by using separation of variables $u_x - u_y = 0$

b) Examine whether $\bar{z}$ is analytic or not?

c) Find the unit tangent vector to the curve $\vec{r}(t) = 2\cos t\vec{i} + \sin t\vec{j}$ *at* $(\sqrt{2}, \sqrt{2}, 0)$.

d) Sketch the paraboloid $z = x^2 + y^2$.

POKHARA UNIVERSITY

Level: Bachelor | Semester: Spring | Year : 2016
Programme: BE | Full Marks: 100
Course: Engineering Mathematics IV | Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1.
a) Define analyticity of the complex valued function f(z). If f(z)= $z+\frac{1}{z}$, check analyticity of f(z) by using Cauchy Riemann equation. 8

b) State and prove Cauchy integral formula. Integrate $\oint_c \frac{1}{z^2+4}dz$, C: $4x^2+(y-2)^2=4$ counter clock wise. 7

2. a) Obtain the Taylor series and Laurent series of the function $f(z)=\frac{1}{(Z+2)(z^2+1)}$ when $1<|z|<2$. 7

OR

Define conformal mapping. Name the types of conformal mappings. Translate the rectangular region ABCD in Z plane bounded by x=1, x=3, y=0 and y=3 under the transformation w=z+(2+i). Illustrate with figure also.

b) State Cauchy Residue Theorem and hence evaluate $\oint \frac{z-23}{z^2-4z-5}dz$ where C: $|z-2|=4$. 8

3. a) Obtain the Fourier integral formula from the Fourier series assuming the required conditions. 7

OR

Show that: $\int_o^\infty \left(\frac{1-\cos \pi w}{w}\right)\sin wx dw = \begin{cases} \pi/2 & \text{if } 0<x<\pi \\ 0 & \text{if } x>\pi \end{cases}$

b) Find the Fourier transform of the function 8

$$f(x) = \begin{cases} 1 - x^2 & for\ |x| < 1 \\ 0 & for\ |x| > 1 \end{cases}$$

4. a) Find the solution of one dimensional wave equation by using D' Alembert's method. 8

b) Find the temperature distribution in a laterally insulated thin copper bar ($c^2 = 1.158 cm^2/sec$), 100cm long and of constant thickness whose end points at x = 0 and x = 100 are kept at 0^0c and initial temperature is $f(x) = \sin^3(0.01)\pi x$ 7

5. a) A string of length 20cm is fastened at both ends is displaced from its position of equilibrium by imparting to its points an initial velocity 7

$$g(x) = \begin{cases} x\ if\ 0 \le x \le 10 \\ 20 - x\ if\ 10 \le x \le 20 \end{cases}$$

Find the deflection U(x, t)

b) Derive two dimensional heat equation and solve completely. 8

6. a) State and prove first and second shifting theorems in Z-transform. Find the value of $Z(a^n \cos bt)$ and $Z(a^n \sin bt)$. 8

b) Using Z-transform, solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ when $y_0 = y_1 = 0$. 7

7. Attempt all: 2.5×4

a) If z = u + iv is an analytic function then prove that u and v both satisfy Laplace equation

b) Represent the curve $y^2 - (z-3)^2 = 9$, x = 0 parametrically

c) Evaluate $\oint \frac{z^3 \sin z}{3z-1}$ dz along a unit circle

d) State and prove the linear property on Z-transform