LUMBINI ENGINEERING COLLEGE

Internal examination

Year: 2024

Full Marks:100

Level: Bachelor

Pass Marks:45

Programme: BE (Computer)

Time :3hrs.

Course: Calculus II

Candidates are required to give their answers in their own words as for as practicable. The figures in the margin indicate full marks.

6x15=90

Attempt all the questions.

1a) Evaluate the given integral.

$\int_0^{a/\sqrt{2}} \int_y^{\sqrt{a^2-y^2}} \log(x^2+y^2)\, dxdy$ for a>0, (by changing into polar form) **OR**

$\iiint (x^2z^2+z^2y^2+y^2x^2)\, dxdydz$ throughout the volume of the sphere $x^2+y^2+z^2=b^2$.

b) Find the volume bounded by xy-plane ,the paraboleid $2z=x^2+y^2$ and the cylinder $x^2+y^2=4$.

2a) Solve by using power series method. $(1+x^2)y''+xy'-y=0$.

b) What is Legendre's equation? Find it's general solution.

3a) Define Laplace transforms. State first and second shifting theorem of Laplace transform. Find i) L(t coswt) ii) $L^{-1}\left(\frac{se^{-2s}}{s^2+1}\right)$

b) Solve the given initial value problem by using Laplace transform.

$y'' + 2y' - 3y = 6e^{-2t}, y(0) = 2, y'(0) = -14$

4a) Show that the value under the integral sign is exact in the plane and evaluate

$\int_{(0,1,1/2)}^{(\pi/2,3,2)} [y^2 cosx\, dx + (2ysinx + e^{2z})dy + 2ye^{2z}dz]$

b)Evaluate $\iint \vec{F}.\vec{n}dA$, Where $\vec{F}$= (xy, yz, xy) and S is the part of the surface $x^2+y^2+z^2=1$, which lies in the first octant.

5a) Verify stoke's theorem for the vector function $\vec{F}=(y^3,0,x^3)$ and C is the boundary of the triangle with vertices (1, 0, 0), (0, 1, 0) and (0, 0, 1).

b) Write the Fourier coefficient of a function f(x).Find the Fourier expansion of the function $f(x)=x+|x|$ $(-\pi < x < \pi)$.

6a) Find the Fourier sine as well as cosine series representation of the half range of the function $f(x) = x^2$ for $0<x<L$.

b) Find the breaking time for $u_t + uu_x = 0$, $u(x,0)=e^{-x^2}$.

4x2.5=10

7a) Find f(t) for $\frac{2s+6}{s^2+4}$

b) Find the unit tangent vector at any point on the curve x=3cost,y=3sint,z=4t.

c) Define even and odd function and find it for xcosnx .

d) Define linear first order partial differential equation.

POKHARA UNIVERSITY
FACULTY OF SCINCE AND TECHNOLOGY
SCHOOL OF ENGINEERING

Exam	Final Internal Examination 2080		
Level	B. E.	F M	100
Programme	Computer	PM	45
Year/Part	2nd Year (3rd Semester)	Time	3 hours

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.
Attempt all questions.

1. a) Evaluate the integral $\int_0^{4a}\int_y^{\frac{y^2}{4a}} \frac{x^2-y^2}{x^2+y^2}dx\,dy$ by changing into polar coordinates. 8
   b) Find the volume of the solid cut from the first octant by the surface $z = 4 - x^2 - y$. 7
2. a) Find the solution of the differential equation $y'' + 9y = 0$, by using power series method. 7
   b) i) Express $x^3 - 5x^2 + x + 2$. in term of Legendre polynomials. 4
   ii) Define Bessel's function of the first kind. Show that:
   $xJ'_v(x) = v\,J_v(x) - x\,J_{v+1}(x)$. 4
3. a) Define Laplace Transform of a function. Using Laplace transform prove the following: 8
   i. $\mathcal{L}\{t\,e^{-4t}\sin 3t\} = \frac{6(s+4)}{[(s+4)^2+9]^2}$.
   ii. $\mathcal{L}^{-1}\left\{\frac{1}{s^2(s^2+w^2)}\right\} = \frac{1}{w^2}\left(t - \frac{\sin wt}{w}\right)$

   OR

   State and prove second shifting theorem of Laplace transform. Use it to find the inverse of $\frac{e^{-2s}}{s^2}$.
   b) Solve by using the Laplace transform. 7
   $y'' + 4y' + 4y = e^{-t}$, where $y(0) = 0, y'(0) = 0$.

4. a) A particle moves on the curve $x = t^3 + 1, y = t^2, z = 2t + 5$ where t is the time. Find the component of its velocity and acceleration at t = 1 in the direction $\vec{i} + \vec{j} + 3\vec{k}$ 7
   b) Define Divergence and Curl of a vector. If $\phi = \log(x^2 + y^2 + z^2)$ find div (grad ϕ) and curl (grad ϕ). 8
5. a) Verify Green's theorem for $\vec{F} = (x - y)\vec{i} + (x + y)\vec{j}$ over the region R bounded by $y = x^2$ and $x = y^2$. 7
   b) State Gauss divergent theorem. Use divergent theorem to evaluate, $\iint_S \vec{F}.\vec{n}\,dA$, where (e^x, e^y, e^z), S is the surface of the cube $|x| \le 1$, $|y| \le 1$, and $|z| \le 1$. 8

   OR

   State Stoke's theorem. Find $\oint_C \vec{F}.d\vec{r}$ counter clockwise around the boundary C of the region R, where $\vec{F} = (\sin y, \cos x)$, R is the triangle with vertices $(0,0)$, $(\pi, 0)$ and $(\pi, 1)$
6. a) Obtain the Fourier series for the function $f(x) = x^2$ in the interval $0 < x < 2\pi$. 8
   b) Define non-linear advection equation. Find the breaking time for $u_t + 2u.u_x = 0, u(x, 0) = e^{-x^2}$. 7
7. Attempt all the questions: $[4 \times 2.5 = 10]$
   a) Define unit step function and find its Laplace transform.
   b) Evaluate the integral $\int_{(0,0)}^{(1,\frac{\pi}{2})}[e^x(\cos y\,dx - \sin y\,dy)]$.
   c) Define periodic function. Find period of the function f(x) = cos 5x.
   d) Find the Fourier sine series of function $f(x) = 1$ for $0 < x < L$.

*********The End*********

POKHARA UNIVERSITY FACULTY OF SCINCE AND TECHNOLOGY **SCHOOL OF ENGINEERING**	Exam	**Final Internal Examination 2080**		
	Level	B. E.(Civil)	F M	100
	Programme	Bachelor	PM	45
	Year/Part	2^{nd} year /3^{rd} semester	Time	3 Hrs

Subject: Calculus II Set B (Civil B)

Candidates are required to give answers in their own words as far as practicable.
The figure in the margin indicates full marks.
Attempt all the questions

1. a. **Evaluate the given integral:** $\int_0^2\int_{y^2}^4 y\cos(x^2)dxdy$ 7

b. **Find the volume in the first octant bounded by the co-ordinate planes, the cylinder $x^2 + y^2 = 4$ and the plane $z + y = 3$.** 8.

2. a. Solve by using power series: $y'' - 4xy' + (4x^2 - 2)y = 0$. 7.

b. (i) Define the Legendre's Equation of order n. Write the formula of $P_n(x)$ and sketch the graph of $p_2(x)$.

(ii) Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$. 4+4

3. a. State and prove the Second shifting theorem of Laplace Transform. Find the Laplace transform of $te^{2t}\sin t$. 4+3

b. Solve the given initial value problem using the Laplace transform

$y'' + 4y' + 3y = e^{-t}$. $y(0) = y'(0) = 1$ 8

4. a. If $\vec{V} = \frac{x\vec{i} + y\vec{j} + z\vec{k}}{\sqrt{x^2 + y^2 + z^2}}$ Show that: $\nabla.\vec{V} = \frac{2}{\sqrt{x^2 + y^2 + z^2}}$ and $\nabla \times \vec{V} = 0$. 8

b. Prove that: $\vec{F} = r^2\vec{r}$, Show that $\vec{F}$ is a conservative vector field and scalar potential is $\phi = \frac{r^4}{4}$ + Constant ..

OR

Evaluate: $\int_{\left(0,1,\frac{1}{2}\right)}^{\left(\frac{\pi}{2},3,2\right)} [y^2\cos x dx + (2y\sin x + e^{2z})dy + 2ye^{2z}dz]$ 8

5. a. Evaluate by using Green's Theorem of $\oint_C [(y - \sin x)dx + \cos x dy]$ where C is the triangle with vertices (0,0), $\left(\frac{\pi}{2},0\right)$ and $\left(\frac{\pi}{2},1\right)$. 7

b. Find $\iint_s (\vec{F}.\vec{n})ds$, for $\vec{F} = x^2\,\vec{i} + y^2\,\vec{j} + z^2\,\vec{k}$, $\vec{r} = (u\cos v,\ \ u\sin v,\ 3v); 0 \le u \le 1, 0 \le v \le 2\pi$.

OR

Evaluate $\oint_c \vec{F}.\vec{dr}$ by using Stoke's Theorem where $\vec{F} = y\,\vec{i} + xz^3\,\vec{j} - zy^3\,\vec{k}$, and

C: $x^2+y^2 = 4$, $z = -3$. 8

6. a. Find the Fourier cosine series of the function $\vec{F} = x^2$ for $0 < x < \pi$ in half range .Also

show that $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \ldots\ldots\ldots\ldots\ldots\ldots = \frac{\pi^2}{6}$. 8

b. Derive one dimensional traffic flow model using conservation law. 7

7. Attempt the following $4\times 2.5 = 10$

a. Find the inverse of Laplace transforms of $\frac{1}{(s^2 + w^2)s^2}$.

b. Using the divergence theorems to find $\iint_s (\vec{F}.\vec{n})ds$, where $\vec{F} = e^x\,\vec{i} + \vec{j} + e^z\,\vec{k}$ and

S: $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$

c. Find the Fourier series of the function $f(x) = x$ in the interval $-\pi < x < \pi$

d. Find the a_n of the Fourier Series of $f(x) = \frac{\pi - x}{2}$ for $0<x<2$.

The end

POKHARA UNIVERSITY
FACULTY OF SCIENCE AND TECHNOLOGY
SCHOOL OF ENGINEERING

Exam	Internal Examination 2080		
Level	B.E. (CIVIL and CE/BE)	F.M.	30
Programme	Bachelor	P.M.	15
Year/Part	2nd year 1st semester	Time	1 Hrs

Subject: Calculus II   Set C (Civil A)

Candidates are required to give answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a. Evaluate the given integral: $\int\int \frac{x\,dy\,dx}{\sqrt{x^2+y^2}}$ 7 7

b. Find the volume bounded by the xy- planes, the cylinder $x^2+y^2=1$ and the plane $x+y+z=3$. 8.

8

2. a. Solve the Bessel's equation: $x^2y'' + xy' + (x^2 - n^2)y = 0$. By using Frobenius method. 7

b. (i) Solve by power Series: $y' = 3y$. 4

(ii) If $P_n(x) = \frac{1}{2^n n!}\frac{d^n (x^2-1)}{dx^n}$ then show that: $P_n(1) = 1$ and $P_n(-1) = (-1)^n$. 4+4

3. a. Define Convolution of two functions. State and prove Convolution Theorem.

OR

(i) Define Unit step function. Find the Laplace transform of L {sin2t.u $(t-\pi)$ }.

(ii) Find the inverse Laplace transform of $\frac{s+1}{s^2(s^2+1)}$. 4 5 7.

b. Solve the given initial value problem using the Laplace transform

$y'' + y' - 2y = t$. $y(0) = 1$, $y'(0) = 0$. 8 8.

a. The necessary and sufficient condition for the vector value function $\vec{a}$ of the scalar variable t to have a constant direction is $\vec{a} \times \frac{d\vec{a}}{dt} = 0$. 7 7

b. If $\phi = \log(x^2 + y^2 + z^2)$ then find div(grad ϕ) and curl(grad ϕ). 8 8

a. Calculate $\oint_c \vec{F}.\vec{dr}$ where $\vec{F}$ = (cosh x, sinh y, e^z) and C be a path given by

$\vec{r}$ = (t, t^2, t^3) From (0,0,0) to (2,4,8).

OR Evaluate by using Green's Theorem of $\oint$ [illegible] where C is [illegible] with vertices (1,1), (2,2) and [illegible] 7

7

b. Evaluate $\oint \vec{F}\cdot d\vec{r}$ by using Stoke's Theorem where $\vec{F} = y^2\,\vec{i} + x^2\,\vec{k}$, and C is the boundary of the triangle with vertices (1,0,0), (0,1,0), (0,0,1). 8

8

6. a. Find the Fourier cosine series of the function $f = \begin{cases} x & \text{if } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \pi - x & \text{if } \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$ 8.

7

b. Define advection equation, non-linear advection equation and Breaking time. Find the breaking time for $u_t + u\,u_x = 0$, $u(x,0) = e^{-x^2}$ 7

7. Attempt the following $4 \times 2.5 = 10$

a. Find the inverse of Laplace transforms of $\dfrac{s^2}{(s^2 + a^2)(s^2 + b^2)}$.

8

b. Evaluate: $\iiint_v (x - y - z)\,dx\,dy\,dz$, where v: $1 \le x \le 2$, $2 \le y \le 3$, $1 \le z \le 3$.

8

c. Find the Fourier sine series of the function $f(x) = x$ in the interval $0 < x < \pi$

d. Find the a_n of the Fourier Series of $f(x) = \dfrac{\pi - x}{2}$ for $-\pi < x < \pi$.

The end

4. a) A particle moves on the curve $x = t^3 + 1, y = t^2, z = 2t + 5$ where t is the time. Find the component of its velocity and acceleration at t = 1 in the direction $\vec{i} + \vec{j} + 3\vec{k}$ 7

b) Define Divergence and Curl of a vector. If $\phi = \log(x^2 + y^2 + z^2)$ find div (grad ϕ) and curl (grad ϕ). 8

5. a) Verify Green's theorem for $\vec{F} = (x - y)\vec{i} + (x + y)\vec{j}$ over the region R bounded by $y = x^2$ and $x = y^2$. 7

b) State Gauss divergent theorem. Use divergent theorem to evaluate, $\iint_s \vec{F}.\vec{n}\, dA$, where (e^x, e^y, e^z), S is the surface of the cube $|x| \leq 1$, $|y| \leq 1$, and $|z| \leq 1$. 8

OR

State Stoke's theorem. Find $\oint_c \vec{F}.d\vec{r}$ counter clockwise around the boundary C of the region R, where $\vec{F} = (\sin y, \cos x)$, R is the triangle with vertices $(0,0)$, $(\pi, 0)$ and $(\pi, 1)$

6. a) Obtain the Fourier series for the function $f(x) = x^2$ in the interval $0 < x < 2\pi$. 8

b) Define non-linear advection equation. Find the breaking time for $u_t + 2u.u_x = 0, u(x, 0) = e^{-x^2}$. 7

7. Attempt all the questions: $[4 \times 2.5 = 10]$

a) Define unit step function and find its Laplace transform.

b) Evaluate the integral $\int_{(0,0)}^{(1,\frac{\pi}{2})} [e^x(\cos y\, dx - \sin y\, dy)]$.

c) Define periodic function. Find period of the function f(x) = cos 5x.

d) Find the Fourier sine series of function $f(x) = 1$ for $0 < x < L$.

*********The End*********

POKHARA UNIVERSITY
FACULTY OF SCINCE AND TECHNOLOGY
SCHOOL OF ENGINEERING

Exam	Final Internal Examination 2080		
Level	B. E.	F M	100
Programme	Computer	PM	45
Year/Part	2nd Year (3rd Semester)	Time	3 hours

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.
Attempt all questions.

1. a) Evaluate the integral $\int_0^{4a}\int_y^{\frac{y^2}{4a}} \frac{x^2-y^2}{x^2+y^2}dx\,dy$ by changing into polar coordinates. 8

b) Find the volume of the solid cut from the first octant by the surface $z = 4 - x^2 - y$. 7

2. a) Find the solution of the differential equation $y'' + 9y = 0$, by using power series method. 7

b) i) Express $x^3 - 5x^2 + x + 2$. in term of Legendre polynomials. 4

ii) Define Bessel's function of the first kind. Show that:

$x{J'}_v(x) = v\,J_v(x) - x\,J_{v+1}(x)$. 4

3. a) Define Laplace Transform of a function. Using Laplace transform prove the following: 8

i. $\mathcal{L}\{t\,e^{-4t}sin3t\} = \frac{6(s+4)}{[(s+4)^2+9]^2}$.

ii. $\mathcal{L}^{-1}\left\{\frac{1}{s^2(s^2+w^2)}\right\} = \frac{1}{w^2}\left(t - \frac{\sin wt}{w}\right)$ $\Rightarrow \frac{s^2+w^2-s^2}{s^2w^2(s^2+w^2)}$

OR

State and prove second shifting theorem of Laplace transform. Use it to find the inverse of $\frac{e^{-2s}}{s^2}$.

$\Rightarrow \frac{1}{s^2w^2} - \frac{1}{w^2(s^2+w^2)}$

b) Solve by using the Laplace transform. 7

$y'' + 4y' + 4y = e^{-t}$, where $y(0) = 0, y'(0) = 0$.

$\Rightarrow \frac{1}{w^2}t - \frac{1}{w^2}\sin wt$

$\Rightarrow \frac{1}{w^2}\left(t - \frac{\sin wt}{w}\right)$

UNITED TECHNICAL COLLEGE
Semester-Fall

Level	:	Bachelor	Year	:	2023
Programme	:	BEE/Comp-III	Full Marks	:	100
Course	:	Calculus-II	Pass Marks	:	45
			Time	:	1.5hrs

Candidates are required to give their answers in their own words as far as practicable.
Attempt the Questions.

1 a Show that the volume of solid enclosed between the cylinder $x^2 + y^2 = 2ax$ and $z^2 = 2ax$ is $\frac{64a^3}{15}$ Cu.unit.

b Evaluate: $\iiint x^2yzdxdydz$ over the volume of tetrahedron bounded by x = 0, y= 0, z = 0 and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

2 a Change the Cartesian Integrals $\int_{-a}^{a}\int_{0}^{\sqrt{a^2-x^2}} e^{-(x^2+y^2)}\, dydx$ to polar form and Integrate the polar Integrals.

b Applying Power Series Method. Solve the differential equation $y'' - y' = 0$.

3 a Define Legendre Equation and find its solution.

b Show that, $xxJ'_n(x) = -nJ_n(x) + xJ_{n-1}{}^{(x)}$

4 a Find the Laplace Transform of

i. $\frac{e^{-at}-e^{-bt}}{t}$

ii. $t^2 Coshwt$

b Solve the initial value problem by Laplace Transform.

$$y'' + 2y' + 5y = 0, y(0) = 2, y'(0) = -4.$$

5 a Define Convolution of two functions. Find $L^{-1}\left(\frac{1}{s^2(s-1)}\right)$ by using Convolution Theorem.

b Find the Fourier series of $f(x) = |x| for -\pi < x < \pi$ and show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} \dots\dots\dots. = \frac{\pi^2}{8}$.

6 a Find the Fourier sine and Cosine series of the function $f(x) = x^2$ for $0 < x < L$..

b Determine the Constants a and b such that the curl of $\underset{F}{\rightarrow} = (2xy + 3yz)\vec{\imath} + (x^2 + axz - 4z^2)\vec{\jmath}\,\vec{k} - (3xy + byz)\vec{k}$ is zero

7 Write Shorts Notes (Any One)

a Find $\oint_c \vec{f}.d\vec{r}$,where $\overrightarrow{F = \left(y, \frac{z}{2}, \frac{3y}{2}\right)}$ C is circle $x^2 + y^2 + z^2 = 6z$ and $z = x + 3$.

b Find $\iint_s \vec{f}.\vec{n}dA$, where $\vec{F} = (1, xy, yz)$ and S is surface $x^2 + y^2 \le z, y \ge 0, z \le 4$.

8 a Derive the Conservation Law. Solve $u_t + 2u.u_x = 0$ where $u(x, 0) = e^{-x}$.

b Derive minimum time $t_b = -\frac{1}{f'(\lambda)}$. Find the breaking time for $u_t + 2u.u_x = 0, u(x, 0) = e^{-x^2}$

NEPAL COLLEGE OF INFORMATION TECHNOLOGY
Assessment Fall 2023

Level: Bachelor | Year : 2024

Programme: BE-IT_M&D_III | Full Marks : 100

Course: Calculus-II | Time : 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1 (a) Evalute $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}}(x^2+y^2)dydx$ By changing into polar coordinates. 5

(b) Evaute $\int_0^{log2}\int_0^{x}\int_0^{x+logy} e^{x+y+z}dz\,dy\,dx$ 5

(c) State Dirichlet's integral ,use it to find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. 5

2 (a) Solve $the\ following\ differential\ equation\ y''+4y=0$ by power series method. 5

(b) If P_n (x) and P_m(x) are legendre polynomials,then prove that $\int_{-1}^{1}P_n(x)P_m(x)dx=0, for$ m not equal to n. 5

OR

Express $5\,x^2$ +5x-3 into legendre polynomials.

(c) Befine Bessels function of first kind of order n J_n(x) and J_{-n}(x).Show that 5

$[\ J_{1/2})]^2+[J_{-1/2}(x).]^2=\frac{2}{\Pi x}$.

OR

Show that x $J_n'(x)=nJ_n$(x) - xJ_{n+1}(x)

3 (a) (I)Show that $\int_0^{\infty}\frac{e^{-2t}sin2t}{t}dt=\frac{\Pi}{2}$ By using Laplace Transform. 4

(II)Define convolution of two function.Find the inverse Laplace transform of $\frac{1}{s(s^2+4)}$ 4

(b) Solve the following initial value problem using Laplace Transform: 7

$y'' - 2y' + y = \quad , y(0) = 2, y'(0) = -1$

4 (a) A moving particle along the curve $\vec{r} = (t^3 - 4t)\vec{i} + (t^2 + 4t)\vec{j} + (8t^2 - 3t^3)\vec{k}$, where t is time, find the magnitude of tangential component of its acceleration at t=2. 5

(b) Define the gradiant of a scalar function with geometrical interpretation.Find the directional derivative of $x^2y^2z^2$ *at the point* $(1,1,-1)$ *in the direction of the tangen* $e^t, y = \sin 2t + 1, z = 1 - \cos t$ *at* $t = 0$. 5

(c) Define line integral.Determine whether the line integral $\int (2xyz^2)dx + (x^2z^2 + z\cos yz)dy + (2x^2yz + y\cos yz)dz$ is independent of the path of integration? If so,then evalute it from $(1,0,1)$ to $(0, \frac{\pi}{2}, 1)$. 5

OR

Show that the value under the integral sign $\int_{(\pi, \frac{\pi}{2}, 2)}^{(0,\pi,1)} (-z\sin xz dx + \cos y dy - x\sin xz)$ *is exact and evalute the integral.*

5 (a) Green's theorem for plane. Use the Green's theorem in the plane for $\oint (3x^2\text{State} - 8y^2)dx + (4 - 6xy)dy$, *where C is the boundary of the region defined by* $y^2 = x^2$.

(b) Define Stoke's theorem.Evalute $\oint_c \overrightarrow{\vec{F}.\vec{dr}}$ by Stoke's theorem where $\vec{F} = (x^2 + y - 4)\vec{i} + 3xy\vec{j} + (2xz + z^2)\vec{k}$ over the surface of hemisphere $x^2 + y^2 + z^2 = 16$ *above the xy* − *plane.* 5

OR

Evalute $\iint \overrightarrow{F.\hat{n}}$ ds where $\vec{F} = (x+y^2)\vec{i} - 2x\vec{j} + 2yz\vec{k}$ and S is the surface of the plane 2x+y+2z=6 in the first octant.

c) Define Gauss Divergence theorem. Evalute $\iint \vec{F}.\hat{n}\, ds$ by using Gauss divergence theorem where $\vec{F}=(e^{x}, e^{y}, e^{z})$, $S: -1 \le x, y, z \le +1$. 5

6 (a) Find the Fourier cosine series of the function f(x)= L-x for 0<x<L 7

(b) Show that the fourier series A periodic function f(x) of period 2Π is defined for -Π<x<+Π as follows: 8

$f(x)=2\cos x, for\ |x| < \frac{\pi}{2}$

0, otherwise

of f(x) is given by

$f(x)=\frac{4}{\pi}[\frac{1}{2}+\frac{\pi}{4}\cos x+\frac{1}{1.3}\cos 2x-\frac{1}{3.5}\cos 4x+\frac{1}{5.7}\cos 6x-\cdots]$

7 (a) Solve: 5

$y^2 u_x - xy\, u_y = x(u - 2y)$,

where x and y are independent variable and u is dependent

(b) Define non-linear Advection equation.Find the breaking time for 5

$$u_t + 2u\, u_x = 0, u(x,0) = e^{-x^2}$$

National Academy of Science and Technology

(Affiliated to Pokhara University)

Dhangadhi, Kailali

<u>Pre-University Examination</u>

Level: Bachelor | Semester : **III_Fall** | Year: 2023

Program: B.E. (Computer/Civil) | F.M.: 100

Course: **Calculus II** | P. M.: 45

Time : 3 Hrs.

Candidates are required to give their answer in their own words as for as practicable. The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Evaluate $\int_0^{\pi}\int_x^{\pi} \frac{\sin y}{y}\,dy\,dx$. [7]

   b} State Dirichlet's theorem. Use it to evaluate the triple integral $\iiint_V x^2\,dx\,dy\,dz$. V: $x \geq 0, y \geq 0, z \geq 0$ & $\frac{x}{a}+\frac{y}{a}+\frac{z}{a}=1$ [8]

2. a) Using series solution method, solve: $(1-x^2)y''-2xy'+2y = 0$. [8]

   b) Express x^3+2x^2-x-3 in terms of Legendre polynomials. [7]

   OR

   Show that $\frac{d}{dx}[x^{-n}J_n(x)] = -x^{-n}J_{n+1}(x)$.

3. a) Evaluate: [8]

   i) $L\left\{\frac{e^{-t}\sin at}{t}\right\}$ ii) $L^{-1}\left\{\frac{1}{(s-1)(s^2+9)}\right\}$

   b) Using Laplace transform, solve the initial value problem: $y''-2y'+y=e^{t}$, $y(0)=2$, $y'(0)=-1$. [7]

4. a) Show that the necessary and sufficient condition for the vector valued function $\vec{a}$ of the scalar variable t to have a constant direction is $\vec{a} \times \frac{d\vec{a}}{dt} = \vec{0}$. [7]

   b) A particle moves along the curve $x=t^3-4t$, $y=t^2+4t$, $z=8t^2-3t^3$, where t is the time. Find the component of its velocity and acceleration at time t=1 in the direction of $\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$. [8]

5. a) State Green's theorem in the plane. Use it to evaluate the integral $\oint_C [(y - \sin x)dx + \cos x\,dy]$, C is the boundary of the Δ with vertices (0,0), (π,0), (π,1) [7]

b) Evaluate the surface integral $\iint_S (\overrightarrow{F}.\hat{n})ds$, where $\vec{F} = (x^2, -e^y, 1), S: x + y + z = 1; x \geq 0, y \geq 0, z \geq 0.$ [8]

OR

Using Gauss Divergence theorem, Evaluate the surface integral $\iint_S (\overrightarrow{F}.\hat{n})ds$, where $\vec{F} = (2xy + z, y^2, -x - 3), S: 2x + 2y + z = 6;$ $x = 0, y = 0, z = 0.$

6. a) Find the Fourier series of the function $f(x) = x^2, -\pi \leq x \leq \pi.$ [8]

   Hence show that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}.$

   b) Write the general form of one dimensional conservation law. Write any two applications of conservation law. Solve the p.d.e, $u_t + 2u.u_x=0$, $u(x,0)=e^{-x}$. [1+1+5=7]

7. Attempt **Any four** [2.5×4=10]

   i) Find the Laplace transform of $e^{-3t}u(t-2)$.

   ii) Find the unit normal vector to the surface $f=x^2y+2xz-4$ at (2,-2,3).

   iii) Show that the value under the integral sign of $\int_{(0,\pi)}^{(3,\frac{\pi}{2})} e^x(cosydx - sinydy)$ is exact. Hence evaluate it.

   iv) Check whether the function is odd or even. Find it's period.

$$f(x) = \begin{cases} \frac{1}{2} + x & for \quad -\frac{1}{2} < x < 0 \\ \frac{1}{2} - x & for \quad 0 < x < \frac{1}{2} \end{cases}$$

   v) Evaluate $\int_0^{\pi} \int_0^{sinx} ydydx.$

Pokhara University

Level: Bachelor | Semester: xxxx | Year: xxxx
Program: BE | | Full Marks: 100
Course: Calculus II | Time: 3 hrs. | Pass Marks: 45

MODEL QUESTION

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

Attempt all questions.

1. a) Evaluate the integral: $\int_0^2 \int_{y^2}^4 y\cos(x^2)dxdy$. **5**

   b) Evaluate the integral: $\int_0^1 \int_0^{(1-x)} \int_0^{(x+y)} e^z dzdydx$. **5**

   c) Find the volume in the first octant bounded by the co-ordinate planes, the cylinder $x^2 + y^2 = 1$ and the plane z+ y = 3. 5

2. a) Solve by using power series: $y'' - 4xy' + (4x^2 - 2)y = 0$. **7**

   b) (i) Express: $2x^2 - 4x + 2$ as Legendre polynomial. **4+4**

   (ii) Show that: $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

   OR

   Find the solution of Bessel's Equation.

   $$x^2 y'' + xy' + (x^2 - n^2)y = 0$$

3. a) (i) State first shifting theorem of Laplace Transform and find the Laplace transform of $t\ cosat$. 4+4

   (ii) Find the inverse Laplace transform of the function $\frac{s+1}{s^2(s+3)}$.

   b) Apply Laplace transform to solve the initial value problem 7

   $y'' + 4y' + 3y = e^{-t}, \quad y(0) = y'(0) = 1$

4. a) A particle moves along the curve $(t^3+1, t^2, 2t+5)$. Find the component of the velocity and acceleration at t=1 along $\vec{i}+\vec{j}+3\vec{k}$. **5**

b) If $\phi = \ln(x^2+y^2+z^2)$, then find $grad\phi$ and $div(grad\phi)$. **5**

c) Evaluate $\oint_C (x^2-3y)dx+(x+\sin y)dy$ where C is the boundary of the triangle with vertices $(0,0), (1,0)$ *and* $(0,2)$. **5**

5. a) Find $\iint_s (\vec{F}.\vec{n})ds$, for $\vec{F} = x^2\vec{i}+y^2\vec{j}+z^2\vec{k}$, **7**

$\vec{r} = (u\cos v,\quad u\sin v,\ 3v); 0 \le u \le 1, 0 \le v \le 2\pi.$

b) Evaluate $\oint_C (\overrightarrow{F}.\overrightarrow{dr})$ by using Stoke's theorem, where $\vec{F} = (y, xz^3, -zy^3)$ and $C: x^2+y^2=4, z=3$. **8**

OR

State Gauss divergence theorem and use it to evaluate the surface integral $\iint_S \vec{F}.\hat{n}\,ds$ for $\vec{F} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$ and S is a cube $0 \le x \le 1, 0 \le y \le 1$ *and* $0 \le z \le 1$.

6. a) Find the Fourier series of $f(x) = \frac{x^2}{2}$ for $-\pi \le x \le \pi$ and deduce that **7**

$$1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\ldots\ldots\ldots\ldots\ldots\ldots = \frac{\pi^2}{6}.$$

b) Find Fourier half range cosine and sine sires of $f(x) = e^x$ *in* $(0, L)$. **8**

7. Attempt any two questions: $[2 \times 5 = 10]$

a) Derive one dimensional traffic flow model using conservation law.

b) Find the breaking time for $u_t + u\,u_x = 0$, $u(x,0) = e^{-x^2}$.

c) Evaluate $\int_{(1,0,2)}^{(-2,1,3)} [(6xy^3+2z^2)dx + 9x^2y^2dy + (4xz+1)dz]$.

COSMOS COLLEGE
OF MANAGEMENT AND TECHNOLOGY

Term Test II

Date: 2080/10/14		Full Marks
Level	BE	Time
Programme	BEIT, BCE, BCV	1.5 hrs
Semester	III	

Subject: - Calculus II

- ✓ *Candidates are required to give their answers in their own words as far as practicable.*
- ✓ *Attempt All questions.*
- ✓ *The figures in the margin indicate* ***Full Marks****.*
- ✓ *Assume suitable data if necessary.*

1. State Stoke's Theorem. Using stokes theorem evaluate $\int_c \vec{F}.d\vec{r}$ where $\vec{F} = (y, \frac{z}{2}, \frac{3y}{2})$ and C is the ellipse $x^2 + y^2 + z^2 = 6z, z = x + 3$. [7]
2. State Gauss Divergence Theorem. Using the divergence theorem find $\iint_S \vec{F}.\hat{n}\, ds$ where $\vec{F} = y^2 e^z \vec{i} - xy\vec{j} + x \tan^{-1} y \vec{k}$ and S is the portion of the plane $2x + 3y + z = 6$ in the first octant. [8]
3. Evaluate the integral $\int_0^2 \int_{y^2}^4 y \cos x^2 dx\, dy$ by changing the equivalent integral obtained by reversing the order of integration. [7]
4. Find the volume bounded by the xy –plane, the paraboloid $2z = x^2 + y^2$ and the cylinder $x^2 + y^2 = 4$. [8]

a, e^x

OR

Find the Fourier series of $f(x) = x - x^2$ if $-\pi < x < \pi$ and show that

$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots = \frac{\pi^2}{12}$. [7]

5. *Solve the equation* $y' = y$ by power series method.
6. *State Green's Theorem. Using it evaluate* $\oint_c [(x^3 - 3y]dx + (x + \sin y)dy]$, C: the boundary *of the triangle with vertices* (0,0), (1,0), (0,2). [8]

y

7. *Attempt any two.* $[2 \times 2.5 = 5]$
   a) Define Periodic function with period. Find the period of $\tan\left(\frac{3x}{5}\right)$.
   b) Define Dirichlet integral and state Dirichlet Theorem.
   c) Find the area of the circle $x^2 + y^2 = a^2$ by changing to polar co-ordinates using double integral.

$(x_1, y_1) : (1, 0)$

$(x_2, y_2) : (0, 2)$

$y - y_1 = m(x - x_1)$

$y = -2(x - 1)$

$y = 2 - 2x$

Pokhara University

Everest Engineering College

Final Internal Assessment

Fall-2023

Level: Bachelor	**F.M:** 100
Program: BE CMP	**P.M:** 45
Faculty: Science and Technology	**Time:** 3 hrs.
Subject: Calculus-II (3^rd^ *Semester*)	

Attempt all the questions.

1. (a) Find the volume in the first octant bounded by co-ordinates planes, the cylinder $x^2+y^2=4$ and plane $z+y=3$. [7]

   (OR)

   Find by triple integration, volume of sphere $x^2+y^2+z^2=a^2$. [7]

   (b) Evaluate the integral $\int_0^{4a}\int_{y^2/4a}^{y} \frac{x^2-y^2}{x^2+y^2}\,dxdy$ by changing into polar co-ordinates . [8]

2. (a) Solve by power series method: $(1-x)y' = y$. [7]

   (b) Define Legendre's equation. Also derive the solution of Legendre's equation. [1+7]

3. (a) Evaluate the following:

   (i) $\mathcal{L}(t^2\cos\omega t)$ [4]

   (ii) $\mathcal{L}^{-1}(\frac{1}{s^2(s^2+\omega^2)})$ by using convolution. [4]

   (b) Solve the differential equation $y''-2y'+y=e^t \quad y(0)=2, y'(0)=-1$ by using Laplace transform. [7]

4. (a) Find the directional derivative of $f = xy^2+yz^3$ at (2,-1,1) along the direction of the normal to the surface $x\log z - y^2+4=0$ at (-1,2,1). [7]

   (b) If $\vec{v} = x^2yz\vec{i} + xy^2z\vec{j} + xyz^2\vec{k}$ then find [4+4]

   (i) $curl(curl\ \vec{v})$

   (ii) $div(curl\ \vec{v})$

5. (a) State Stoke's theorem. Evaluate $\oint_C \vec{F}.\vec{dr}$, where $\vec{F}=(z,x,y)$ and S is the hemisphere $z=(a^2-x^2-y^2)^{\frac{1}{2}}$. [1+7]

   (b) Evaluate $\iint_S \vec{F}.\vec{n}ds$ where. $\vec{F}=(x^2,-e^y,1)$ and S is the surface of plane $x+y+z=1$ in first octant. [7]

   (OR)

   Evaluate $\iint_S \vec{F}.\vec{n}ds$ where. $\vec{F}=e^x\vec{i}+e^y\vec{j}+e^z\vec{k}$ and $S: |x|\le 1, |y|\le 1, |z|\le 1$ by using Gauss divergence theorem. [7]

6. (a) Define periodic function with example. Find the fourier series representation of the function $f(x) = \frac{x^2}{2}$ for $-\pi < x < \pi$ and then show that: [1+6+1]

$$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots = \frac{\pi^2}{6}.$$

(b) Derive the partial differential equation governing the conversation law. [7]

7. Attempt all the questions: $[4 \times 2.5 = 10]$

(a) Find the Laplace transform of $t^2 u(t-2)$, where u is unit function.

(b) Show that the value under the integral sign $\int_{(0,1)}^{(2,3)} [(2x + y^3)dx + (3xy^2 + 4)dy]$ is exact and evaluate the integral.

(c) Obtain the half-range fourier sine series of the function $f(x) = \pi - x$ for $0 < x < \pi$.

(d) Find the general solution of $u_x + u_y = u$.

The End

4. a) A particle moves along the curve $(t^3+1, t^2, 2t+5)$. Find the component of the velocity and acceleration at t=1 along $\vec{i}+\vec{j}+3\vec{k}$. 5

b) If $\phi = \ln(x^2+y^2+z^2)$, then find $grad\phi$ and $div(grad\phi)$. 5

c) Evaluate $\oint_C (x^2-3y)dx+(x+siny)dy$ where C is the boundary of the triangle with vertices $(0,0), (1,0)$ and $(0,2)$. 5

5. a) Find $\iint_s (\vec{F}.\vec{n})ds$, for $\vec{F} = x^2\vec{i}+y^2\vec{j}+z^2\vec{k}$, 7

$\vec{r} = (u\cos v,\quad u\sin v,\ 3v); 0 \le u \le 1, 0 \le v \le 2\pi.$

b) Evaluate $\oint_C (\overrightarrow{F}.\overrightarrow{dr})$ by using Stoke's theorem, where $\vec{F} = (y, xz^3, -zy^3)$ and $C: x^2+y^2=4, z=3$. 8

OR

State Gauss divergence theorem and use it to evaluate the surface integral $\iint_S \vec{F}.\hat{n}\, ds$ for $\vec{F} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\,\hat{k}$ and S is a cube $0 \le x \le 1, 0 \le y \le 1$ and $0 \le z \le 1$.

6. a) Find the Fourier series of $f(x) = \frac{x^2}{2}$ for $-\pi \le x \le \pi$ and deduce that 7

$$1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\dots\dots\dots\dots = \frac{\pi^2}{6}.$$

b) Find Fourier half range cosine and sine sires of $f(x) = e^x$ in $(0, L)$. 8

7. Attempt any two questions: $[2 \times 5 = 10]$

a) Derive one dimensional traffic flow model using conservation law.

b) Find the breaking time for $u_t + u\, u_x = 0$, $u(x,0) = e^{-x^2}$.

c) Evaluate $\int_{(1,0,2)}^{(-2,1,3)} [(6xy^3+2z^2)dx+9x^2y^2dy+(4xz+1)dz]$.

Pokhara University
School of Engineering

Level : Bachelor
Time: 3 hrs
Program: BEE — Full Marks : 100
Course: Calculus II — Pass Marks : 45

Candidates are requested to answer the in their own words as far as practicable. The marks in the margin Indicate full marks.

Attempt all the questions.

1. a. Solve: $xy'' + 2y' + xy = 0$ by using Frobenius method. 8

   b. If $P_n(x)$ is the Legendre ploynomial, then show that

   $$\int_{-1}^{1} [P_n(x)]^2\,dx = \frac{2}{2n+1}.$$ 7

   OR

   If $P_n(x)$ is the Legendre polynomial of degree n, prove that $(n+1)P_{n+1}(x) = (2n+1)x\,P_n(x) - nP_{n-1}(x)$.

2. a. Find the Laplace transform of the folllowings 8

   (i) $\int_0^t te^t \sin t\,dt$

   (ii) $t^2 e^{-t} \sin t$

   b. Solve the following initial value problem by using Laplace transform: 7

   $x'' + 2x' + 5x = e^t \sin t,\ x(0) = 0,\ x'(0) = 1$

3. a. Evaluate $\iint_R \sqrt{a^2 - x^2 - y^2}\,dx\,dy$ over the semicircle $x^2 + y^2 = ax$ in first quadrant. 7

   b. Find the volume of the solid whose based is the region in xy-plane that is bounded the Parabola $y = 3 - x^2$, $y = 2x$ white the top is bounced by the plane $z = x + 1$. 7

   OR

   Evaluate $\iiint_V (x^2 + y^2 + z^2)\,dx\,dy\,dz$ taken over the volume of the sphere $x^2 + y^2 + z^2 = 1$.

4. a. Find the Fourier series of the function $f(x) = |x|$ in the interval $-\pi < x < \pi$ and hence show that

   $$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots$$ 8

   b. Find the Fourier sine and Fourier cosine series of the function $f(x) = e^x,\ 0 \le x \le L$. 7

5. a. A particle moves along the curve $x = 4\cos t$, $y = 4\sin t$, $z = 6t$. Find the velocity and acceleration at time $t = 0$ and $t = \frac{\pi}{2}$ sec. 7

   OR

   Show that the vector field $\vec{F} = yz\,\vec{i} + zx\,\vec{j} + xy\,\vec{k}$ is conservative. Also, find the scalar potential ϕ.

   b. If $\vec{F} = y^2\,\vec{i} - x^2\,\vec{j}$ and C is the boundary of the triangle with the vertices (1, 0), (0, 1), (−1, 0), find $\int_C \vec{F}.\,d\vec{r}$ 8

   OR

   Define gradient of a scalar function and curl of a vector function. Show that curl (gradϕ) = 0, for any scalar point function ϕ.

6. a. State stoke's theorem. Find the value of normal surface integral $\iint_S \vec{F}.\,\vec{n}\,ds$ for $\vec{F} = x\vec{i} - y\vec{j} + (z^2 - 1)\vec{k}$, where S is the surface bounded by the cylinder $x^2 + y^2 = 4$ between the planes $z = 0$ and $z = 1$.

   OR

   State Gauss divergence theorem. If $\vec{F} = 4x\vec{i} + x^2y\,\vec{j} - x^2z\,\vec{k}$, where S is the surface of the tetrahedron with vertices (0, 0, 0), (1, 0, 0), (0, 1, 0) and (0, 0, 1), find $\iint_S \vec{F}.\,\vec{n}\,ds$ by using Gauss divergence.

   b. Derive the one dimensional traffic flow model using conservative laws.

7. Attempt all questions: 10

   a. If $P_n(x)$ is the Legendre polynomial, prove that $P_n(1) = 1$.

   b. Show that the product of two odd functions is an even function.

   c. Find the general solution of $2u_x + 2u_y - u = 0$.

   d. Define unit step function and find its Laplace transform.

The End

Pokhara University
School of Engineering

Level : Bachelor
Time: 3 hrs
Program: BEE | Full Marks : 100
Course: Calculus II | Pass Marks : 45

Candidates are requested to answer the in their own words as far as practicable. The marks in the margin Indicate full marks.

Attempt all the questions.

1. a. Define Legendere's equation. Also derive the solution of Legendre's equation. 8

   b. If $P_n(x)$ is the Legendre ploynomial, then show that
   $nP_n(x) = xP'_n(x) - P'_{n-1}(x)$ 7

   OR

   If $J_v(x)$ is the Bessel function , then prove that
   $xJ'_v(x) = v\,J_v(x) - xJ_{v+1}(x)$.

2. a. Find the Laplace transform of the folllowings 8
   (i) $\frac{\sin^2 t}{t}$
   (ii) $\frac{e^{-at} - e^{-bt}}{t}$

   b. Solve the following initial value problem by using Laplace transform: 7
   $y'' - y' - 2y = 3e^{2t}$ given that $y(0) = 0$ and $y'(0) = -2$

3. a. Evaluate $\int_0^1 \int_x^1 x^2\, e^{y^4}\, dx\, dy$ by changing the order of integration 7

   OR

   Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by using double integration.

   b. Evaluate $\int\int\int_V x\, dx\, dy\, dz$ taken over the region in the first octant bounded by $x^{2/3} + y^{2/3} + z^{2/3} = 1$. 8

4. a. Find the Fourier series of the function
   $f(x) = \begin{cases} 0 & \text{if } -\pi < x < 0 \\ 1 & \text{if } 0 < x < \pi \end{cases}$ and hence show that
   $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots$ 8

   b. Find the Fourier sine and Fourier cosine series of the function $f(x) = \pi x - x^2,\ 0 < x < \pi$. 7

5. a. Prove that the necessary and sufficient condition for a vector function $\vec{r} = \vec{r}(t)$ of scalar variable to have a constant magnitude is $\vec{r} \cdot \frac{d\vec{r}}{dt} = 0$ 7

   OR

   Show that the curl of linear velocity of a rigid body equals twice the magnitude of the angular velocity.

   b. If $\vec{F} = x^2y^2\,\vec{i} + y^3\,\vec{j}$ and C is the curve $y^2 = 4x$ in the xy plane from (0, 0), (4, 4), find $\int_c \vec{F}.\vec{dr}$. 8

6. a. Evaluate $\int_c (x + y)dx + (2x - z)dy + (y + z)dz$, where C is the boundary of the triangle with the vertices (2, 0, 0), (0, 3, 0) and (0, 0, 6) using stokes theorem.

   OR

   State Gauss divergence theorem. If $\vec{F} = (2x + 3z)\vec{i} - (xz + y)\vec{j} + (y^2 + 2z)\vec{k}$, where S is the surface of the sphere having center (3, -1, 2) and radius 3, find $\int\int_S \vec{F}.\vec{n}\, ds$ by using Gauss divergence theorem.

   b. Derive Rankine- Hugonoit condition for shockwaves.
   Define breaking time and derive the minimum time for breaking time.

7. Attempt all questions.
   a. If $P_n(x)$ is the Legendre polynomial, prove that $P_n(-1) = (-1)^n$.
   b. Define odd function. Show that the $f(x) = \sin x$ is an odd function.
   c. Solve: $u_t + 2u.u_x = 0,\ u(x, 0) = e^{-x}$.
   d. State and prove linearity property of Laplace transform.

The End

Pokhara University

MADAN BHANDARI MEMORIAL ACADEMY NEPAL

Internal Examination 2080

Level : Undergraduate Semester: Third Year :2nd(2024)

Program: BE(Computer)

Course : Calculus II (MTH 210)

Full Marks=100

Pass Marks=45

Time: 3 hrs

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

1. a) Prove that $\int_0^{\sqrt{2}} \int_{-\sqrt{4-2y^2}}^{\sqrt{4-2y^2}} y\dot{a}ydx = \frac{8}{3}$ (5)

   b) Evaluate the integral $\int_0^{\log 2} \int_0^{x} \int_0^{x+y} e^{\,x+y+z}\, dzdydx$ (5)

   c) Find the volume of the hemisphere $x^2+y^2+z^2=a^2$ by using triple integral. (5)

2. a) Solve by using power series: $y'' - 4xy' + (4x^2 - 2)y = 0$ (7)

   b) (i) Express $f(x)=x^3-5x^2+x+2$ in terms of Legendre polynomial. (4)

   (ii) Show that $J'_0(x)=-J_1(x)$ (4)

   OR

   State Bessel's equation and find its solution.

3. a) Define Laplace transform and inverse Laplace transform. Evaluate

   a) $L\{\sinh at\}$ b) $L^{-1}\{\frac{1}{(s-1)(s^2+1)}\}$ [2+4+4]

   b) Apply Laplace transform to solve the initial value problem (7)

   $y''+4y'+3y=e^{-t}$, $y(0)=y'(0)=1$

4. a) A particle moves along the curve $x=t^3+1$, $y=t^2$, $z=2t+5$. Find the velocity and acceleration at $t=1$ (5)

   b) Show that $\vec{F}= yz\vec{i} + zx\,\vec{j} + xy\,\vec{k}$ is irrotational and find its scalar potential. (5)

c) Find the angle between the surface $x^2+y^2+z^2=9$ and $z= x^2+y^2-3$ at the point (2,1,2). (5)

5. a) Evaluate $\iint_S (\vec{F}.\hat{n})ds$ if $\vec{F}= yz\vec{i} + 2y^2\vec{j} + zx^2\vec{k}$ and S is the surface of the cylinder $x^2+y^2=9$ in the first octant between z=0 and z=2. (7)

b) State Stock's theorem. Evaluate $\oint_c \vec{F}.d\vec{r}$ by using Stocks theorem where $\vec{F} = (4z, -2x, 2x)$, c is the circle $x^2+y^2=1$, $z=y+1$ (8)

OR

State Green's theorem. Evaluate $\oint_c [(3x^2-8y^2)dx + (4y-6xy)dy]$ where c is the boundary of the region defined by $y^2=x$, $y=x^2$. (8)

6. a) Define Fourier series. Obtain the Fourier series for $f(x) = \frac{x^2}{2}$ in the interval $-\pi < x < \pi$ and hence show that

$$\sum\frac{1}{n^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots = \frac{\pi^2}{6} \quad [1+6]$$

b) Define half range Fourier series. Express f(x)=x as half range sine series and half range cosine series in $0< x < 2$. [2+4+4]

7. Attempt any two questions. [2×5=10]

a) Find the solution of $u_x+u_y=u$ given that $u(x,0)=2$.

b) Solve $u_t+2u.u_x=0$, $u(x,0)=e^{-x}$

c) Evaluate $\int_{(0,1)}^{(1,2)} [(x^2+y)dx + (y^2 + x)dy]$ along a straight line from (0,1) to (1,2)

1

Pokhara University

MADAN BHANDARI MEMORIAL ACADEMY NEPAL

Internal Examination 2080

Level : Undergraduate | Semester: Third | Year : 2^{nd}(2024)

Program: BE(Computer) | Set "B" | Full Marks=100

Course : Calculus II (MTH 210) | Pass Marks=45

Time: 3 hrs

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

1\. a) Claculate $\iiint_R (x^2 + y^2)\, dx\, dy$ over the region bounded by x=0, y=0 and x+y=1 (5)

b) Show that the area between the parabola y^2=4ax and x^2=4ay is $\frac{16}{3}a^2$ (5)

c) Evaluate $\int_0^y \int_0^{1-x} \int_0^{x+y} e^z\, dzdydx$. (5)

2\. a) Solve by using power series; $y'' + 9y = 0$. (7)

b) (i) Express f(x)=$2x^2$-4x+2 in terms of Legendre polynomial. (4)

(ii) Show that $J_{-n}(x)=(-1)^n J_n(x)$ (4)

OR

State Bessel's equation and find its solution.

3\. a) Define Laplace transform and inverse Laplace transform. Evaluate

a) L{ t cos 2t} b) $L^{-1}\{\frac{1}{((s^2+1)(s^2+1)}\}$ [2+4+4]

b) Apply Laplace transform to solve the initial value problem (7)

y'+3y=10 sint, y(0)=1

4\. a) A particle moves along the curve x=4cost, y= 4sint , z= 6. Find the velocity and acceleration at t=0 and $t = \frac{\pi}{2}$ (5)

b) Show that $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ is irrotational and find its scalar potential. (5)

c) The necessary and sufficient condition for a vector function $\vec{r} = \vec{r}(t)$ to have constant direction is $\vec{r} \times \frac{d\vec{r}}{dt} = 0$ (5)

5. a) Evaluate $\iint_S (\vec{F}.\hat{n})ds$ if $\vec{F} = yz\vec{i} + 2y^2\vec{j} + zx^2\vec{k}$ and S is the surface of the cylinder $x^2+y^2=4$ in the first octant between z=0 and z=2. (7)

b) State Stock's theorem. Evaluate $\oint_C \vec{F}.d\vec{r}$ by using Stocks theorem where $\vec{F} = (4z, -2x, 2x)$, c is the circle $x^2+y^2=1$, $z=y+1$ (8)

OR

State Green's theorem. Evaluate $\oint_C [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$ where c is the boundary of the region defined by $y^2=x$, $y=x^2$. (8)

6. a). Define Fourier series. Obtain the Fourier series for $f(x) = \frac{x^2}{2}$ in the interval $-\pi < x < \pi$ and hence show that

$1-\frac{1}{4}+\frac{1}{9}-\frac{1}{16}+\ldots=\frac{\pi^2}{12}$ [1+6]

b) Define half range Fourier series. Express f(x)=x as half range cosine series and half range sine series in $0< x < 2$. [2+4+4]

7. Attempt any two questions. [2×5=10]

a) Find the solution of $u_x+u_y-u=0$ given that $u(x,0)=4$.

b) Solve $u_t+2u.u_x=0$, $u(x,0)=e^{-x}$

c) Evaluate $\int_{(0,1)}^{(1,2)}[(x^2 + y)dx + (y^2 + x)dy]$ along a straight line from (0,1) to (1,2)

POKHARA ENGINEERING COLLEGE
Internal Assessment Examination

Level: Bachelor Semester – Fall Year :2024
Programme: BE (Computer) Full Marks: 100
Course: Calculus II Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) Find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, by double integration. -8

   b) Compute $\iiint_v x^2yz\,dx\,dy\,dz$ over the volume of tetrahedron bounded by $x=0, y=0, z=0$ and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ -7

   OR

   Using the double integrals find the area of the region $y^2=4ax$ and the parabola $x^2=4ay$

2. a) Solve $y''+9y=0$ by power series method. -8

   b) Prove that:

   i. $\frac{d}{dx}[x^nJ_n(x)]=x^nJ_{n-1}$ -(4+3)

   ii. Express $2x^2-4x+2$ in terms of Legendre's polynomials.

3. a) Calculate the convolution $1*1$ by integrating. -8

   b) Solve the following initial value problem using Laplace transform

   $y''-2y'+10y=0$, $y(0)=3$, $y'(0)=3$

   OR -7

   Solve $y''+4y'+3y=e^{-t}$ using Laplace transform.

4. a) *Find the work done by the force $\vec{F}=(2y+3)\vec{i}+xz\vec{j}+(yz-x)\vec{k}$ when it moves a particle from the point $(0,0,0)$ to the point $(2,1,1)$ along the curve $x=2t^2, y=t, z=t^3$..* -8

   b) Show that the value under integral sign is exact in plane and evaluate the integral: $\int_{(0,1)}^{(2,3)}[(2x+y^3)dx+(3xy^2+4)dy]$ -7

5. a) Evaluate: $\iint_s \vec{F}.\vec{n}\,dA$ where $\vec{F}=(x-z, y-x, z-y)$. $s: \vec{r}=(u\cos v, u\sin v, u), 0\le u\le 3, -1\le v\le 2\pi$ -8

   b) State Gauss Divergence theorem and evaluate $\iint_s \vec{F}.\vec{n}\,dA$ by using it where $\vec{F}=(x^2, 0, z^2)$, s is the box $|x|\le 1, |y|\le 3, |z|\le 2$.. -7

6. a) Find the Fourier Series of $f(x)=\begin{cases}1 \text{ if } & -\pi\le x\le 0\\ -1 \text{ if } & 0\le x\le \pi\end{cases}$ -8

   b) Expand $f(x)=x^2$ for $0\le x\le\pi$ in a Fourier cosine series and deduce -7

   i. $\sum_{n=1}^{\infty}\left(\frac{1}{n^2}\right)=\frac{\pi^2}{6}$

   ii. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}=\frac{\pi^2}{12}$

7. Attempt any twoquestions: 2×5=10

   a) Find the general solution of:

   $$2u_x+2u_y-u=0$$

   b) Find the breaking time for

   $u_t+2uu_x=0, u(x,0)=e^{-x^2}$

   c) Derive integral form of conservation law.

NEPAL ENGINEERING COLEGE

Level: Bachelor
Program: BE
Course: Calculus II

Assessment-2024

Full Marks: 100
Pass Marks: 45
Time : 3 hrs.

ındidates are required to give their answers in their own words as far as 'acticable.
he figures in the margin indicate full marks.
ttempt all the questions.

1. a) Evaluate the integral: $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{dx\,dy}{(1+x^2+y^2)^2}$ $[5 \times 3 = 15]$

   b) Evaluate the integral: $\int_{0}^{1}\int_{0}^{3-3x}\int_{0}^{3-3x-y} dzdydx$

   c) Find the volume bounded by the paraboloid $2z = x^2 + y^2$ and the cylinder $x^2 + y^2 = 4$ above xy-plane.

2. a) Solve by power series method: $y'' - 4xy' + (4x^2 - 2)y = 0$ [7]

   b) i) Use Rodrigues formula to express $x^2 - 2x + 3$ as a Legendre polynomial [4]

   ii) Show that $j_{3/2}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{1}{x}\sin x - \cos x\right]$ [4]

   OR

   Solve the Bessel's equation $x^2y'' + xy' + (x^2 - n^2)y = 0$ [8]

3. a) i) Define Laplace transform of a function f(t) for $t > 0$ and find the Laplace transform of $e^{-2t}\sin n\pi t$. [4]

   ii) Find the inverse Laplace transform of $\frac{1}{s(s^2+4)}$ [4]

   b) Using Laplace transform, solve the initial value problem:
   $y'' - 2y' + y = e^t, y(0) = 2, y'(0) = -1$

4. a) If $\vec{a} = 9\cos 3t\vec{i} - 4\sin 2t\vec{j} + 6t\vec{k}$ be the acceleration of a moving particle at any time t, find its velocity $\vec{v}$ and displacement $\vec{r}$ of the particle at any time t, if the particle start to move from rest. [5]

   b) Find the directional derivative of $\phi = x^2 - y^2 + 2z^2$ at the point (1, 2, 3) in the direction of $\vec{a} = \vec{i} + \vec{j} + \vec{k}$. [5]

c) Find the work done by the force $\vec{F} = y\vec{i} + z\vec{j} + x\vec{k}$ when a particle is moving from point (0, 0, 0) to (2, 2, 8) along the curve $x = t$, $y = t^2$, $z = t^3$. [5]

5. a) Find the flux of $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ on the first octant portion of the sphere $x^2 + y^2 + z^2 = 1$ [7]

5. b) State Stokes theorem and use it to fine the line ntegral $\oint_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = 4y\vec{i} + 2z\vec{j} + 6y\vec{k}$ and C is the circle $x^2 + y^2 + z^2 = 6z$, $z = x + 3$. [8]

OR

Find $\iint_S \vec{F} \cdot \vec{n}\, ds$, where $\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ and S is the closed cylinder $x^2 + y^2 = 4$, $z = 0$, $z = 3$.

6. a) Find the Fourier series of $f(x) = x - x^2$ for $-\pi < x < \pi$ and hence show that $\dfrac{\pi^2}{12} = \dfrac{1}{1^2} - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \dfrac{1}{4^2} + \dots\dots$ [7]

b) Find the Fourier cosine series as well as Fourier sine series of the function $f(x) = e^x$ for $0 < x < \ell$ [8]

7. Attempt any TWO: [2×5 = 10]

a) Solve: $2u_x + 2u_y - u = 0$, $u(x, 0) = 3$

b) Find the breaking time for $u_t + 2uu_x = 0$, $u(x,0) = e^{-x^2}$

c) Evaluate the path independent integral

$$\int_{(-1,2)}^{(3,1)} [(y^2 + 2xy)\,dx + (x + 2 + 2xy)\,dy]$$

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