Gandaki College of Engineering and Science
Lamachaur, Pokhara

Final Assessment 2080

Program: SE/CE
Subject: Algebra and Geometry
Semester: 2nd
Attempt all the questions

Full Marks: 100
Pass Marks: 45
Time: 3hrs.

1. a. Solve
$$x + y + z = 6$$
$$x - y + z = 2$$
$$2x - y + 3z = 9$$
by using Gauss elimination method. 7

OR Check the consistency and solve the equation
$$x + 2y + 3z = 1$$
$$2x + 3y + 2z = 2$$
$$2x + 3y + 4z = 1$$

b. State Cayley Hamilton theorem and use it to find inverse of matrix 8
$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$

2. a. Define linearly dependent and linearly independent. 7
Check whether the vectors (1,1,0), (1,0,1), (3,1,1) forms a basis of R^3 or not?

b. Maximize $z = 20x_1 + 20x_2$ subject to 8
$x_1 \geq 0, \quad x_2 \geq 0, \; -x_1 + x_2 \leq 1, \; x_1 + 3x_2 \leq 15, \; 3x_1 + x_2 \leq 21$

3. a. Making dual minimize $z = 2x_1 + 9x_2 + x_3$ subject to 8
$x_1 + 4x_2 + 2x_3 \geq 5, \qquad 3x_1 + x_2 + 2x_3 \geq 4,$
$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

b. Define absolute convergence, find center radius and interval of convergence of infinite series 7
$$\sum_{n=1}^{\infty} \frac{(x-5)^n}{n5^n}$$

a. Define scalar triple product. Derive the expression of scalar triple product of vectors. Find the volume of the tetrahedron whose vertices are P(3,4,5), A(2,1,1),B(2,1,5),C(1,4,2) . 8

b. Show that $[\vec{a} \times \vec{b} \quad \vec{b} \times \vec{c} \quad \vec{c} \times \vec{a}] = [\vec{a}\ \vec{b}\ \vec{c}]^2$ 7

a. Define eccentricity of a conic section. Find the condition that the line $y = mx + c$ may be tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 8

b. Derive the equation of the hyperbola in its standard form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. 7

a. Find the image of point p (1, 3, 4) in the plane $2x - y + z + 3 = 0$. 7

OR

Find the length and equation of shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$. and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$

b. Find the equation of the sphere which passes through the circle $x^2+y^2+z^2 - 6x - 2z+5=0, y=0$ and touch the plane $3y+4z+5=0$. 8

Short question: (2.5 x 4 = 10)

a. What will be the equation of the curve $x^2 - 3y^2 + 4x + 6y = 0$ if the origin is shifted to (-2, 1) ?

b. Test the convergence of $\frac{1}{\sqrt{2}+1} - \frac{1}{\sqrt{3}+1} + \frac{1}{\sqrt{4}+1} - \frac{1}{\sqrt{5}+1} + \cdots$

c. If $\vec{a} = 2\vec{i} + 3\vec{j} - 4\vec{k}$ and $\vec{b} = \vec{j} + \vec{k}$, find the vector projection of $\vec{b}$ on $\vec{a}$.

d. Show that the mapping $T: R \rightarrow R$ defined by $T(x) = 3x$ is linear.

Best of Luck

Pokhara University
Everest Engineering College
Final Internal Assessment
Spring

Level: Bachelor | **Year:** 2023
Program: BE IT (2^{nd} *Semester*) | **F.M:** 100
Faculty: Science and Technology | **P.M:** 45
Subject: Algebra & Geometry (MTH 150) | **Time:** 3 hrs.

Attempt all the questions.

1. (a) Define the consistency of a system of linear equations. Check the consistency of the equations $2x - y + 3z = 9$, $x + y + z = 6$, $x - y + z = 2$. If it is consistence, find the solution. [1+3+3]

   (b) Construct the duality of the following linear programming problem and solve by using simplex method. [2+6]

$$\begin{gathered} minimize \quad z = 4x_1 + 3x_2 \\ \text{Subject to} \\ 2x_1 + 3x_2 \geq 1 \\ 3x_1 + x_2 \geq 4 \\ x_1, x_2 \geq 0 \end{gathered}$$

2. (a) Define eigen-value and eigen vector of the square matrix A. Find the eigen values and corresponding eigen vectors of the square matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$. [2+6]

   (b) Verify Caley-Hamilton theorem of the given matrix and find inverse of $A = \begin{pmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ -2 & -3 & 0 \end{pmatrix}$. [4+3]

3. (a) Define scalar triple product. Give its geometrical interpretation. If $\vec{a} = \vec{i} - 2\vec{j} - 3\vec{k}$, $\vec{b} = 2\vec{i} + \vec{j} - \vec{k}$ and $\vec{c} = \vec{i} + 3\vec{j} - 2\vec{k}$. verify that $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$. [1+1+6]

   (b) Define reciprocal system of vectors $\vec{a}, \vec{b}$ and $\vec{c}$. Show that: $\vec{a}' \times \vec{b}' + \vec{b}' \times \vec{c}' + \vec{c}' \times \vec{a}' = \dfrac{\vec{a} + \vec{b} + \vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$ where $[\vec{a}\ \vec{b}\ \vec{c}] \neq 0$. [1+6]

4. (a) State p-series test. Test the convergence and divergence of the infinite series: $\sum[\sqrt{n^3+1} - \sqrt{n^3-1}]$. [1+7]

   (b) Find the center, radius of convergence and interval of convergence of the power series $\sum_{n=0}^{\infty} \dfrac{(x-5)^n}{n5^n}$. [7]

5. (a) Define conic section. Derive standard equation of ellipse. [1+7]

OR

Find the center, vertices, eccentricity and foci of the ellipse:

$9x^2 + 6y^2 + 18x - 96y + 9 = 0.$ [8]

(b) Find the condition that the line $lx + my + n = 0$ may be tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$ [7]

6. (a) Find the shortest distance between the lines $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{1-y}{5} = \frac{z+2}{2}$. Also find the equation of the line of shortest distance. [4+4]

OR

Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. Also, obtain the equation of the plane containing them. [4+4]

(b) Find the equation of the sphere having the circle $x^2 + y^2 + z^2 = 9$, $x - 2y + 2z = 5$ as a great circle. Also determine its center and radius. [5+2]

7. Attempt all questions: [4 ×2.5=10]

(a) What does the equation $2x^2 + y^2 - 4x + 4y + 3 = 0$ becomes when it is transformed to parallel axis through the point $(1, -2)$.

(b) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (x, x + y)$. Check T is linear or not.

(c) Find the equation of cone whose vertex is (α, β, γ) and guiding curve is $y^2 = 4ax$, $z = 0$.

(d) Find the equation of right circular cylinder whose radius is 4 and axis is the line $x = 2y = -z$.

The End

UNITED TECHNICAL COLLEGE

QT Exam

Semester: Spring

Year : 2023

Full Marks: 50

Pass Marks: 23

Time : 1.5hrs.

Level: Bachelor

Programme: B.E(Computer)

Course: Algebra and geometry

Attempt only 50 marks questions.

1 a. Find S.D.and equation of plane between the lines $\frac{x}{4}=\frac{y+1}{3}=\frac{z-2}{2}$, $5x-2y-3z+6=0=x-3y+2z-3$. 8

b. Find the equation of sphere which passes through the points (0 ,-2 ,-4) (2,-1 ,-1)and whose centre lies on the lines $5y+2z=0=2x-3y$. 7

2 a. Plane through the OX and OY include an angle α .Show that their lines of intersection lie on the cone is $z^2(x^2+y^2+z^2)=x^2y^2\tan^2\alpha$. 8

b. Determine the nature of the conic section represented by $5x^2+6xy+5y^2+18x-2y-3=0$.Also find the Centre length and equation of axes, e, foci. 7

3 a. Obtain the vertices, Centre, coordinates of foci, eccentricity of the following ellipse: $9x^2+4y^2+36x-8y+4=0$. 8

b. Show that the line $lx+my+n=0$ touches the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ if $a^2l^2-b^2m^2=n^2$. 7

4 a. Solve the linear programming problem by simplex method.[constructing duality] 8
Minimize $Z=2x_1+9x_2+x_3$ subject to $x_1+4x_2+2x_3\geq 5$, $3x_1+x_2+2x_3\geq 4$, $x_1\geq 0, x_2\geq 0, x_3\geq 0$.

b. Solve (Big M- method) Maximize $z=-3x_1+7x_2$ subject to $2x_1+3x_2\leq 5$, $5x_1+2x_2\geq 3$, $x_2\leq 1$ 7

5 a. Find Eigen value and corresponding Eigen vector of the square matrix. 8

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

b. Define consistence and inconsistence of a system of linear equations. . Check consistence of given system of linear equation. 7

$$5x+5y-10z=0$$
$$2w-3x-3y+6z=2$$
$$4w+x+y-2z=4$$

6 a. Find the radius of convergence and interval of convergence of the series $\sum_{n=1}^{\infty}(-1)^n \frac{(x-5)^n}{n5^n}$ 8

b. b) Define scalar triple product and reciprocal vectors. 7
Find the reciprocal vector of

$$\overrightarrow{2i}+\overrightarrow{3j}-\vec{k},\ \vec{i}-\vec{j}-\overrightarrow{2k},\ \overrightarrow{-i}+\overrightarrow{2j}+\overrightarrow{2k}$$

7 a. Prove that of $r=\frac{de}{1+e\cos\theta}$. Sketch and describe the conic section of $r=\frac{12}{2-3\sin\theta}$ 8

b. State and prove the D'Alembert ratio test. Test the series is absolutely convergent or not, 7

$$1-\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}-\frac{1}{4\sqrt{4}}+\cdots$$

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LUMBINI ENGINEERING COLLEGE

Second Internal Exam (2023)

Level: Bachelor | Semester: Spring | Full Marks : 100

Programme : B.E. | Pass Marks :45

Course: Algebra and Geometry | Time :3hrs.

Attempt all the questions.

1. a) Check the consistency of the given system of equations and solve it: (7)

$5x + 3y + 7z = 4; \quad 3x + 26y + 2z = 9; \quad 7x + 2y + 10z = 5$

b) Define basis of a vector space over the field. Show that the vectors $(1,2,1)$, $(2,1,0)$, $(1,-1,2)$ form a basis of R^3. (8)

2. a) Find the eigen value and eigen vector of the matrix: $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. (8)

b) Use simplex method to find the minimize value of $Z = -x_1 + x_2 - 3x_3$ subject to the constraints:

$x_1 + x_2 + x_3 \leq 10, \quad -2x_1 + x_3 \geq -2, \quad 2x_1 - 2x_2 + 3x_3 \leq 0, \quad x_1, x_2, x_3 \geq 0.$ (7)

3. a) Show that the four points with position vectors $(4,5,1), (0,-1,-1), (3,9,4)$ and $(-4,4,4)$ are coplanar. (7)

b) If $\vec{a'}, \vec{b'}, \vec{c'}$ is a reciprocal system to $\vec{a}, \vec{b}, \vec{c}$ then show that $\vec{a}, \vec{b}, \vec{c}$ is also reciprocal system to $\vec{a'}, \vec{b'}, \vec{c'}$ (8)

OR

If $\vec{a}, \vec{b}, \vec{c}$ are coplanar and $\vec{a}$ is not parallel to $\vec{b}$, prove that

$$\begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} \end{vmatrix} \vec{c} = \begin{vmatrix} \vec{c}.\vec{a} & \vec{a}.\vec{b} \\ \vec{c}.\vec{b} & \vec{b}.\vec{b} \end{vmatrix} \vec{a} + \begin{vmatrix} \vec{a}.\vec{a} & \vec{c}.\vec{a} \\ \vec{a}.\vec{b} & \vec{c}.\vec{b} \end{vmatrix} \vec{b}.$$

4. a) Find the condition that the line $lx + my + n = 0$ may be a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Also find the point of contact. (8)

b) Find the equation of the tangents to the hyperbola $3x^2 - 4y^2 = 12$ which are perpendicular to the line $y = x + 2$. Also find the point of contact. (7)

5. a) Find the magnitude and equation of the shortest distance between the lines $\frac{x-5}{3} = \frac{7-y}{16} = \frac{z-3}{7}$ and $\frac{x-9}{3} = \frac{y-13}{8} = \frac{15-z}{5}$. (8)

b) Find the equation of the tangent planes to the sphere $x^2 + y^2 + z^2 + 6x - 2z + 1 = 0$ which passes through $\frac{16-x}{1} = z = \frac{2y+30}{3}$. (7)

OR

Show that the equation to a right circular cone whose vertex is O, axis OX and semi - vertical angle α is $y^2 + z^2 = x^2 tan^2\alpha$

6. a) Test the convergence or divergence of series : $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots\ldots$ (7)

b) Prove that for infinite series $\sum a_n$ to be convergent it is necessary that $\lim_{n\to\infty} (a_n) = 0$. By taking suitable example show that the converse may not be true. (8)

7. Attempt all the questions. (4 × 2.5 = 1(

a) Find the rank of matrix $A = \begin{bmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ -2 & 4 & -4 \end{bmatrix}$.

b) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$.

c) Show that the mapping $T: R^3 \to R^2$ defined by $T(x,y,z) = (x + y - z, 0)$ is linear.

d) Find the equation of hyperbola with vertex at $(\pm 2, 0)$ and foci at $(\pm 5, 0)$.

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National Academy of Science and Technology

(Affiliated to Pokhara University)

Dhangadhi, Kailali

Pre-University Examination

Level : Bachelor	Semester: **II_Spring**	Year : 2022
Programme: B.E.(Computer)		F.M. : 100
Course: **Algebra and Geometry**		P.M. : 45
		Time : 3hrs

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks

Attempt all the questions.

1.a) Define consistency of the system of the linear equations. Check consistency of: $-8x+3y+4z=20$, $3x+4y+5z=26$, $3x+5y+6z=31$. If it is consistence, find it solution by Gauss Elimination method. [8]

b) Find Eigen values and corresponding Eigen vectors of a matrix. [7]

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

2.a) Using simplex method maximize $Z = 15x_1+10x_2$ subject to $2x_1+2x_2 \leq 10$, $2x_1+3x_2 \leq 10$ *and* x, $x_1 \geq 0$, $x_2 \geq 0$ [7]

b) Find the dual of given Lpp and solve by using simple method

Minimize $z=4x_1+3x_2$ subject to $2x_1+3x_2 \geq 1$, $3x_1+x_2 \geq 4$, $x_1 \geq 0$, $x_2 \geq 0$ [8]

3.a) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) + (\vec{a} \times \vec{c}) \times (\vec{d} \times \vec{b}) + (\vec{a} \times \vec{d}) \times (\vec{b} \times \vec{c})$ is parallel to the vector $\vec{a}$. [5]

b) Find the volume of parallelepiped hose concurrent edges are $\vec{i} + \vec{j} + 4\vec{k}$, $2\vec{i} + 2\vec{j} - \vec{k}$ and $\vec{i} - \vec{j} + 2\vec{k}$ [5]

c) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}}$ [5]

4.a) Find the interval, centre and radius of convergence of thee series

$\sum_{n=1}^{\infty} \frac{(x-5)^n}{n10^n}$ [7]

b) Define eccentricity of a conic section, and derive the equation of a ellipse, in its standard form. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ [8]

5.a) Find the condition that the line $lx+my+n=0$ touches the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ Find the point of contact. |7|

b) Show that the lines $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$ and $\frac{x-8}{7}=y-4=\frac{z-5}{3}$ are coplanar. Find their common point and the equation of the plan which they lie |8|

6.a) Find the equation of sphere which passess through the circle $x^2+y^2+z^2-6x-2z+5=0$, $y=0$ and touch the plane $3y+4z+5=0$. |7|

b) Find the equation of the cone with vertex (α,β,γ) and base $y^2=4ax$, $z=0$ |8|

7. Attempt all questions. 4x2.5 = 10

i) Find the value of k so that the vectors $\vec{a}=2\vec{i}-\vec{j}+\vec{k}$, $\vec{b}=\vec{i}+2\vec{j}+3\vec{k}$ and $\vec{c}=3\vec{i}+k\vec{j}+5\vec{k}$ are coplanar

ii) Determine whether the sequence converges or diverges $u_n=\sin\left(\frac{2n\pi}{1+8n}\right)$

iii) Find centre and radius of the sphere: $x^2+y^2+z^2+2x+3y-4z+10=0$

iv) Identify the conic $r=\frac{6}{3+2\sin\theta}$ with focus at origin, the line of directrix and eccentricity

4. a) State the p-test for the convergence of an infinite series. Also test the convergence of the series $\sum \frac{n}{1+n\sqrt{n}}$ 7

b) Find the interval centre and radius of convergence of the series $\sum \frac{(x-5)^n}{n5^n}$ 8

5. a) Find the centre, vertices, foci, eccentricity and length of latus rectum of the ellipse $9x^2 - 16y^2 - 72x + 96y - 144 = 0$. 7

b) Find the condition of a line $x + ny + n = 0$ to be a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Also find the point of contact. 8

6. a) Prove that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{2} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect at a point. Also find the point of intersection and the plane containing them. 8

OR

Find the length and the equation of the line of shortest distance between the lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

b) Find the centre and the radius of the circle of intersection of the plane $x - 2y + 3z = 3$ and the sphere $x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0$. 7

OR

Prove that the second degree equation

$$2x^2 - 4yz - 8xy - 4x - 2y + 6z + 35 = 0$$

Represents a cone. Also find its vertex.

7. **Attempt ALL questions (2.5 × 4)** 10

a) Is the set {(1, 1,0), (1, 0, 1), (3, 1, 2)} forms a basis of R^3 ?

b) Find the equation of a line through the point (3, 2, – 6) and parallel to the line $\frac{x-2}{2} = \frac{y-4}{3} = \frac{z-6}{5}$.

c) Find the centre and the radius of the sphere $x^2 + y^2 + z^2 - 6x + 4y - 2z + 5 = 0$.

d) Find the radius of the sphere having (1, – 2, 4) and (3, 2. 2) as two ends of a diameters.

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NEPAL ENGINEERING COLLEGE

[Set 2]

Level: Bachelor — Assessment — Year : 2023
Programme: BE (Computer/CRE) — Full Marks: 100
Course: **Algebra and Geometry** — Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt any ALL questions.

1. a) Define consistency of the system of linear equations. Classify the system of linear equations according to the nature of its solution. Test the consistency of the system of equations. If consistent, solve by Gauss elimination method 1+2+5
   $x - 5y + 2z = -1,\ 2x + y + z = 1,\ x + 2y - z = 2.$

   b) i. Define linearly dependent and independent set of vectors. Are the vectors (1, 1, 1), (1, 2, 3), (– 1, 1, 3) linearly dependent? 4+4
   ii. Define linear transformation. Is the transformation T: $\mathbb{R}^2 \to \mathbb{R}$ defined by $T(x, y) = |x + y|$ a linear transformation?

a) State Cayley-Hamilton theorem. Verify the theorem for the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ and also find the inverse of the matrix. 1+3+3

OR

Find the eigen values and eigenvectors of the matrix $\begin{bmatrix} 3 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ 3+4

b) Constructing duality, solve the LPP: Minimize $z = 8x_1 + 9x_2$, subject to $x_1 + 3x_2 \geq 4,\ 2x_1 + x_2 \geq 5,\ x_1 \geq 0,\ x_2 \geq 0$ 8

3. a) Find the reciprocal system of the set of vectors: 7
   $$2\vec{i} + 3\vec{j} - \vec{k},\ \vec{i} - \vec{j} - 2\vec{k},\ -\vec{i} + 2\vec{j} + 2\vec{k}$$

   b) i) If $\vec{a}, \vec{b}, \vec{c}$ are three coplanar vectors, prove that $\vec{b}+\vec{c},\ \vec{c}+\vec{a}$ *and* $\vec{a}+\vec{b}$ are also coplanar 4+4
   ii) If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors, prove that $\vec{b}\times\vec{c},\ \vec{c}\times\vec{a}$ *and* $\vec{a}\times\vec{b}$ are also non- coplanar.

NEPAL COLLEGE OF INFORMATION AND TECHNOLOGY

1. Test convergence and divergence of following series

$\sum\left[\sqrt{n^2+1}-n\right]$. [8]

2. Find the radius of convergence and interval of convergence of the series

$\sum_{n=0}^{\infty}\frac{(x-2)^n}{10^n}$ 7

3. Find the equation to the line through (-1, 3, 3) and perpendicular to the plane $x + 2y + 2z = 3$. Also find the length of perpendicular and coordinates of its foot. 7

4. Find the shortest distance between the lines

$$\frac{x-3}{2}=\frac{y-4}{3}=\frac{z-5}{4} \text{ and } \frac{x-4}{3}=\frac{y-5}{4}=\frac{z-7}{5}$$

Also, find the equation of the shortest distance. 10

5. Find the equation of the sphere through the circle

$x^2+y^2+z^2=1, 2x+4y+5z=6$ and touching the plane $z=0$. 8

Find the equation to the line through (-1, 3, 3) and perpendicular to the plane $x + 2y + 2z = 3$. Also find the length of perpendicular and coordinates of its foot. 7

Find the shortest distance between the lines

$$\frac{x-3}{2}=\frac{y-4}{3}=\frac{z-5}{4} \text{ and } \frac{x-4}{3}=\frac{y-5}{4}=\frac{z-7}{5}$$

Also, find the equation of the shortest distance. 8

b) Find the equation of the sphere through the circle

$x^2+y^2+z^2=1, 2x+4y+5z=6$ and touching the plane $z=0$. 7

17. Test the convergence or divergence of the infinite series $\sum\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{\frac{3}{2}}}$ and $\sum\left(\frac{\arctan n}{1+n^2}\right)$.

18. Find the volume of a parallelepiped whose concurrent edges are represented by $\vec{i}+\vec{j}+\vec{k}$, $2\vec{i}+\vec{j}-2\vec{k}$ and $3\vec{i}+2\vec{j}-\vec{k}$.

19. Define vector triple product between three vectors. If $\vec{a}=\vec{i}-2\vec{j}-3\vec{k}, \vec{b}=2\vec{i}+\vec{j}-\vec{k}, \vec{c}=\vec{i}+3\vec{j}-2\vec{k}$. Find $\vec{a}\times(\vec{b}\times\vec{c})$. Also verify that $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}$

20. Prove that $2\vec{a}=\vec{i}\times(\vec{a}\times\vec{i})+\vec{j}\times(\vec{a}\times\vec{j})+\vec{k}\times(\vec{a}\times\vec{k})$

21. Show that the vectors $\vec{a}\times(\vec{b}\times\vec{c})$, $\vec{b}\times(\vec{c}\times\vec{a})$, $\vec{c}\times(\vec{a}\times\vec{b})$ are coplanar.

22. Define reciprocal system to the vectors $\vec{a},\vec{b},\vec{c}$. If $\vec{a'},\vec{b'},\vec{c'}$ are the reciprocal system to the vectors $\vec{a},\vec{b},\vec{c}$, prove that the vectors $\vec{a},\vec{b},\vec{c}$ also form reciprocal system to the vectors $\vec{a'},\vec{b'},\vec{c'}$.

23. Define slack and surplus variable. Using simplex method, Maximize $z=5x_1+3x_2$ subject to $x_1+x_2\le 2$, $5x_1+2x_2\le 10$, $3x_1+8x_2\le 12$, $x_1\ge 0$, $x_2\ge 0$.

24. Solve the linear programing problem, Minimize $z=8x_1+9x_2$ subject to $x_1+x_2\ge 5$, $3x_1+x_2\ge 21$, $x_1\ge 0$, $x_2\ge 0$. Using simplex method, ~~by constructing the duality.~~

25. Maximize $Z = 3x+2y$ Subject to $2x+y\le 9, x+2y\ge 9, x,y\ge 0$. Using Big-M method.

26. Transform the equation $2x^2+4xy+5y^2-4x-22y+7=0$ parallel axes through $(-2,3)$.

27. By what angle must the axes be rotated so that the term containing xy in the equation $ax^2+2hxy+by^2=0$ may be removed.

28. Find the centre, vertices, eccentricity and foci of the ellipse. $9x^2+16y^2+18x-96y+9=0$.

29. Find the condition that the line $lx+my+n=0$ to be tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

30. Define conic section. Obtain the equation of hyperbola in the standard form.

31. The foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$. Find the equation of hyperbola having eccentricity 2.

32. Find the equation of ellipse referred to its axes as the axes of coordinate and foci along x- axis with latus rectum of length 4 and distance between foci is $4\sqrt{2}$.

33. Identify the give conic with focus at origin, and find the line of directrix, and the eccentricity of $r=\frac{4\sec\theta}{\sec\theta+3}$.

34. Find the polar equation of conic with focus a pole ~~and~~ having eccentricity (e) $=\frac{1}{2}$ and directrix $y = 4$

ALGEBRA AND GEOMETRY

TUTERIAL-1

1. Prove that $\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$.

2. Using gauss elimination method solve the system;

$3w + 17x - y - 2z = -2,\ 10x + 4y - 2z = -4,\ -w + x + y = 6,\ 8w - 34x + 16y - 10z = 4.$

3. Check the following system of linear equation is consistent or not if consistent solve it.

$$2x + 5y + 6z = 13;\ 3x + y - 4z = 0;\ x - 3y - 8z = -10.$$

4. Check the transformation is linear or not?

$T: R^2 \to R^3$ be defined by $T(x, y) = (x, y, xy)$.

5. Let $V = R^3$ be a vector space. Show that $W = \{(x, y, z): x + 2y + z = 0\}$ is the vector subspace of V.

6. Define eigen value and eigen vector. Find the Eigen value(s) and Eigen vector(s) of the square matrix $A = \begin{bmatrix} 7 & -1 & 3 \\ 6 & 1 & 4 \\ 2 & 4 & 8 \end{bmatrix}$

7. Obtain the modal matrix of the matrix

$A = \begin{bmatrix} -1 & 1 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & -3 \end{bmatrix}$. Also find the A^{-1} using Cayley Hamiltan Theorem.

8. Are the vectors $(1,2,1), (2,1,0), (1,-1,2)$ forms the basis of R^3.

9. Find the interval, radius and centre of convergence of the infinite series

$$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$$

10. Test the convergence and find the interval of convergence of the series $\sum_{(n=1)}^{\infty} \frac{(x-5)^n}{n5^n}$.

11. Find the radius of the convergence and interval of convergence of the series

$$x + \frac{3}{5}x^2 + \frac{8}{5}x^3 + \frac{15}{17}x^4 + \cdots + \frac{n^2-1}{n^2+1}x^n + \cdots$$

12. Find the interval and radius of the convergence of the series

$$\frac{x}{1.3} + \frac{x^2}{2.5} + \frac{x^3}{3.7} + \cdots + \frac{x^n}{n(2n+1)} + \cdots$$

13. Check the convergence or divergence of the infinite series $\sum[(n^3+1)^{\frac{1}{3}} - n]$ And $\sum\left(\frac{(-100)^n}{n!}\right)$

14. Check the convergence or divergence of the series $\frac{2}{1} + \frac{2.5.8}{1.5.9} + \frac{2.5.8.11}{1.5.9.13} + \cdots$

15. State the Leibnitz's test for alternating series. Show that the infinite series $\sum \frac{(-1)^n}{n+3}$ is conditional convergent.

16. Test the convergence or divergence of infinite series $\frac{1}{e^2} + \frac{1}{e^4} + \frac{1}{e^6} + \ldots$ to ∞.

COSMOS COLLEGE
OF MANAGEMENT AND TECHNOLOGY

Term Test I

Date	2080-03-03		
Level	BE	Full Marks	50
Programme	BE IT, BCE, BCV	Time	
Semester	II	1.5 hrs	

Subject: - Algebra & Geometry

- Candidates are required to give their answers in their own words as far as practicable
- Attempt All questions
- The figures in the margin indicate **Full Marks**
- Assume suitable data if necessary

1. Define an ellipse. Derive the standard equation of the ellipse. [7]
2. Find the condition of tangency that the line $lx + my + n = 0$ may touch the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Also, find the point of contact. [8]
3. Find the equation of tangent to the parabola $y^2 = 8x$ which makes an angle $45°$ with the straight line $y = 3x + 5$. Also find the point of contact. [8]
4. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $4x - 3y + 1 = 0 = 5x - 3z + 2$ are coplanar. Also find the point of contact. [7]
5. Find the image of the point P (1,3,4) in the plane $2x - y + z + 3 = 0$. [7]
6. Find the equation of the straight line lying in the plane $x - 2y + 4z - 51 = 0$ and intersecting the straight line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-6}{7}$ at right angles. [8]
7. Short Questions [2*2.5=5]

   a) Find the equation of the plane through (1,1,1) and parallel to the plane $3x - 4y + 5z = 0$.

   b) Find the foci and eccentricity of the ellipse $x^2 + 9y^2 - 4x + 18y + 4 = 0$.

COSMOS COLLEGE
OF MANAGEMENT AND TECHNOLOGY

Term Test II

Date: 2080/05/03			
Level	BE	Full Marks	50
Programme	All	Time	
Semester	II	1.5 hrs	

Subject: - Algebra and Geometry

- ✓ *Candidates are required to give their answers in their own words as far as practicable.*
- ✓ *Attempt All question.*
- ✓ *The figures in the margin indicate Full Marks.*
- ✓ *Assume suitable data if necessary.*

1. a) Define eigen value and eigen vectors of the square matrix. Find eigen value and vectors of the square matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{pmatrix}$. [8]

   b) Find the center and radius of convergence and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n(2x-1)^n}{n6^n}$. [7]

2. a) Show that the plane $2x - y + 2z = 14$ touches the sphere $x^2 + y^2 + z^2 - 4x + 2y - 4 = 0$. Find their point of contact. [8]

   b) Find the length and equation of the S.D between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $2x - 3y + 27 = 0 = 2y - z + 20$. [7]

   OR

   Find the equation of right circular cylinder whose guiding curve is $x^2 + y^2 + z^2 = 9, x - y + z = 3$.

3. a) Construct the dual problem and solve by using simplex method of the LPP minimize $Z = 20x_1 + 30x_2$ subject to $x_1 + 4x_2 \geq 8, x_1 + x_2 \geq 5, 2x_1 + x_2 \geq 7, x_1 \geq 0, x_2 \geq 0$. [8]

   b) Define scalar triple product. Show that $(\vec{a} \times \vec{b}) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$ if and only if the vectors $\vec{a}$ and $\vec{c}$ are collinear. [7]

   OR

   Check consistency and solve $x + y + z = 1, x + 2y + 2z = 4, x + 3y + 7z = 13$.

Short Questions

Attempt any two [$2 \times 2.5 = 5$]

4. a) Test the convergence or divergence of the series $\sum \frac{n+1}{2n+3}$.

   b) Define Linear transformation. Check the following transformation is linear or not? $T: R^2 \to R$ defined by $T(x,y) = |x + y|$.

   c) Find the rank of the matrix $A = \begin{pmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{pmatrix}$.

NEPAL ENGINEERING COLLEGE
[Set 2]

Level: Bachelor Assessment Year : 2023
Programme: BE (Computer/CRE) Full Marks: 100
Course: **Algebra and Geometry** Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt any ALL questions.

1. a) Define consistency of the system of linear equations. Classify the system of linear equations according to the nature of its solution. Test the consistency of the system of equations. If consistent, solve by Gauss elimination method 1+2+5
$x - 5y + 2z = -1,\ 2x + y + z = 1,\ x + 2y - z = 2.$

b) i. Define linearly dependent and independent set of vectors. Are the vectors $(1, 1, 1), (1, 2, 3), (-1, 1, 3)$ linearly dependent? 4+4
ii. Define linear transformation. Is the transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}$ defined by $T(x, y) = |x + y|$ a linear transformation?

2. a) State Cayley-Hamilton theorem. Verify the theorem for the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ and also find the inverse of the matrix. 1+3+3

OR

Find the eigen values and eigenvectors of the matrix $\begin{bmatrix} 3 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ 3+4

b) Constructing duality, solve the LPP: Minimize $z = 8x_1 + 9x_2$, subject to $x_1 + 3x_2 \geq 4,\ 2x_1 + x_2 \geq 5,\ x_1 \geq 0,\ x_2 \geq 0$ 8

3. a) Find the reciprocal system of the set of vectors: 7
$2\vec{i} + 3\vec{j} - \vec{k},\ \vec{i} - \vec{j} - 2\vec{k},\ -\vec{i} + 2\vec{j} + 2\vec{k}$

b) i) If $\vec{a}, \vec{b}, \vec{c}$ are three coplanar vectors, prove that 4+4
$\vec{b} + \vec{c},\ \vec{c} + \vec{a}$ *and* $\vec{a} + \vec{b}$ are also coplanar

ii) If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors, prove that $\vec{b} \times \vec{c}, \vec{c} \times \vec{a}$ *and* $\vec{a} \times \vec{b}$ are also non- coplanar.

Marks: 100
Pass Marks
Time

4. a) State the p-test for the convergence of an infinite series. Also test the convergence of the series $\sum \frac{n}{1+n\sqrt{n}}$

b) Find the interval centre and radius of convergence of the series $\sum \frac{(x-5)^n}{n5^n}$ 8

5. a) Find the centre, vertices, foci, eccentricity and length of latus rectum of the ellipse $9x^2 - 16y^2 - 72x + 96y - 144 = 0$. 7

b) Find the condition of a line $x + ny + n = 0$ to be a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$. Also find the point of contact. 8

6. a) Prove that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{2}=\frac{y-4}{3}=\frac{z-6}{5}$ intersect at a point. Also find the point of intersection and the plane containing them. 8

OR

Find the length and the equation of the line of shortest distance between the lines $\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$

b) Find the centre and the radius of the circle of intersection of the plane $x - 2y + 3z = 3$ and the sphere $x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0$. 7

OR

Prove that the second degree equation

$$2x^2 - 4yz - 8xy - 4x - 2y + 6z + 35 = 0$$

Represents a cone. Also find its vertex.

7. **Attempt ALL questions (2.5 × 4)** 10

a) Is the set {(1, 1,0), (1, 0, 1), (3, 1, 2)} forms a basis of $\mathbb{R}^3$?

b) Find the equation of a line through the point (3, 2, – 6) and parallel to the line $\frac{x-2}{2}=\frac{y-4}{3}=\frac{z-6}{5}$.

c) Find the centre and the radius of the sphere $x^2 + y^2 + z^2 - 6x + 4y - 2z + 5 = 0$.

d) Find the radius of the sphere having (1, – 2, 4) and (3, 2. 2) as two ends of a diameters.

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POKHARA UNIVERSITY
FACULTY OF SCINCE AND TECHNOLOGY
SCHOOL OF ENGINEERING

Exam	**Final Internal Examination 2080**		
Level	B.E.	F.M	100
Programme	[illegible]	P.M	45
Year/Part	1^{st} year/2^{nd} semester	Time	3 Hrs

Subject: Algebra and Geometry

Candidates are required to give answers in their own words as far as practicable. The figure in the margin indicates full marks.

Attempt all the questions

1. a. Define the consistency of a system of equations. Check the consistency of the equations $x+y+z=-2$, $2x+8y+5z=5$, $x+2y-z=2$. If it is consistence find its solution. 7

   b. Verify Cayley-Hamilton theorem of given matrix and find inverse of A 8

$$A=\begin{bmatrix}1 & 2 & 1\\ 2 & 1 & 3\\ 0 & 1 & 3\end{bmatrix}$$

OR

Diagonalizable the matrix $A=\begin{bmatrix}2 & 1\\ 4 & -1\end{bmatrix}$

2. a. Minimize the given by dual problem in Simplex method. 7

   $Minimize Z = 3x_1 + 2x_2$ subject to $7x_1 + 2x_2 \ge 30, 5x_1 + 4x_2 \ge 20$, $2x_1 + 8x_2 \ge 16$ and $x_1 \ge 0, x_2 \ge 0$.

   b. Maximize the given LPP by Big M -method 8

   $Max Z = 3x_1 - x_2$ subject to $2x_1 + x_2 \ge 2$; $x_1 + 3x_2 \le 2$, $x_2 \le 4$ and $x_1 \ge 0, x_2 \ge 0$

3. a. If $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n \to \infty} a_n = 0$; converse may not be true. 7

   b. Find the radius, centre, interval of the infinite power series $\sum_{n=1}^{\infty} \frac{(3)^{2n}(x-2)^n}{n+1}$. 8

4. a. Define Hyperbola. Derive the standard equation of hyperbola. 7

   OR

   Find the condition that the line $lx+my+n=0$ may touch the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Find the point of contact.

b. Show that the equations $14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0$ represent a ellipse. Determine the centre, major axis and eccentricity of ellipse. 8

5. a. (i) Show that: $\left[\vec{a}\times\vec{b} \quad \vec{b}\times\vec{c} \quad \vec{c}\times\vec{a}\right] = \left[\vec{a} \quad \vec{b} \quad \vec{c}\right]^2$

(ii) Verify that. $\vec{a}\times\left(\vec{b}\times\vec{c}\right) = \vec{b}\left(\vec{a}.\vec{c}\right) - \vec{c}\left(\vec{a}.\vec{b}\right)$ if $\vec{a} = -\vec{i} + 2\vec{j} + 2\vec{k}$

$\vec{b} = 2\vec{i} + 3\vec{j} - \vec{k}$ $\vec{c} = -\vec{i} - \vec{j} - 2\vec{k}$. 4+4.

b. Find the two points on the line $\frac{x-2}{1} = \frac{y+3}{2} = \frac{z+5}{2}$ on either side of (2, -3, -5) and a distance 3 from it.

6. a. Show that the lines are $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. Also find the equation containing them.

OR

Find the shortest distance between lines $\frac{x+3}{-4} = \frac{y-6}{3} = \frac{z}{2}$ and $\frac{x+2}{-4} = \frac{y}{1} = \frac{z-7}{1}$ Also find the equations of the shortest distance. 8

b. Find the equation of tangent planes to the sphere $x^2 + y^2 + z^2 + 6x - 2z + 1 = 0$ which passes through $\frac{16-x}{1} = \frac{z}{1} = \frac{2y+30}{3}$.

$4\times 2.5 = 10$

7. Answer the following

a. Let $T: R^3 \to R^2$ be defined by $T(x, y, z) = (x + y - z, 0)$ check T is linear or not.

b. Show that: $\left(\vec{a}\times\vec{b}\right)\left(\vec{a}\times\vec{c}\right) + \left(\vec{a}.\vec{b}\right)\left(\vec{a}.\vec{c}\right) = \left(\vec{a}.\vec{a}\right)\left(\vec{b}.\vec{c}\right)$

c. Find the reciprocal of $\vec{a}$ of the system of vectors: $\vec{a} = -\vec{i} + 2\vec{j} + 2\vec{k}$, $\vec{b} = 2\vec{i} + 3\vec{j} - \vec{k}$

$\vec{c} = -\vec{i} - \vec{j} - 2\vec{k}$.

d. Find the equation of latus rectum of parabola $16x^2 - 24xy + 9y^2 - 104x - 172y + 44 = 0$.

The End

NEPAL ENGINEERING COLLEGE

Changunarayan, Bhaktapur

(Assessment Spring Semester 2023)

Level: Bachelor | Full Marks: 100
Programme: BE | Pass Marks: 45
Course: .Engineering Mathematics III | Time: 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) What do you mean by consistency of system of linear equations? When system of linear equations has infinitely many solutions? Is a system of linear equations $x+3y+6z=2$, $3x-y+4z=9$, $x-4y+2z=7$ consistent? If it is consistent system, find its solution. 2+1+4

   b) Using Cayley Hamilton theorem find the inverse of $\begin{bmatrix} 7 & -1 & 3 \\ 6 & 1 & 4 \\ 2 & 4 & 8 \end{bmatrix}$. 8

2. a) Prove that the infinite series $\sum_{n=1}^{\infty}(-1)^{n+1}u_n = u_1 - u_2 + u_3 - u_4 + \cdots$ are alternatively positive and negative and each term is numerically less than preceding term and $\lim_{n\to\infty} u_n = 0$, then the series $\sum_{n=1}^{\infty}(-1)^{n+1}u_n$ is convergent. Also check the convergence and divergence of the infinite series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}n}{5^n}$. 5+3

   b) Find the interval, centre and radius of convergence of the power series $\sum_{n=1}^{\infty}\frac{(x-5)^n}{n\,5^n}$.

   OR

   When Taylor series expansion reduce to Maclaurin series expansion? Find the Maclaurin series expansion of $\tan^{-1}x$ *for* $|x| \leq 1$. 7

3. a) Find the Fourier series representation of the periodic function $f(x) = |x|\sin x$, *for* $-\pi \leq x \leq \pi$. 7

b) Find the Fourier cosine series as well as Fourier sine series of the function $f(x) = e^x$ if $0 < x < L$. 4+4

4. a) Prove that the necessary and sufficient condition for the vector function $\vec{a}$ of a scalar variable t to have a constant direction is $\vec{a} \times \frac{d\vec{a}}{dt} = \vec{0}$. 7

b) Define Divergence and Curl of a vector. If $\phi = \log(x^2 + y^2 + z^2)$ find div (grad ϕ) and curl (gradϕ). 8

5. a) Define surface integral of $\vec{F}$ on the surface S. Evaluate $\iint_S \vec{F}.\hat{n}ds$ where $\vec{F} = (x^2, e^x, 1)$, where S is the surface $x + y + z = 1$, $x \geq 0, y \geq 0, z \geq 0$. 8

b) State Gauss divergence theorem. Using divergence theorem, evaluate the surface integral $\iint_S \vec{F}.\hat{n}ds$ where $\vec{F} = (x^3, y^3, z^3)$ and S is the sphere $x^2 + y^2 + z^2 = 9$. 7

6. a) Using simplex method, Maximize $z = 5x_1 + 3x_2$ subject to $x_1 + x_2 \leq 2$, $5x_1 + 2x_2 \leq 10$, $3x_1 + 8x_2 \leq 12$, $x_1 \geq 0$, $x_2 \geq 0$. 7

b) Solve the linear programing problem, Minimize $z = 8x_1 + 9x_2$ subject to $x_1 + x_2 \geq 5$, $3x_1 + x_2 \geq 21$, $x_1 \geq 0$: $x_2 \geq 0$, using simplex method, by constructing the duality. 8

7. Short Questions $(2.5 \times 4 = 10)$

a) Find the Eigen value of the matrix $\begin{bmatrix} 4 & 0 \\ 0 & -6 \end{bmatrix}$.

b) State limit comparison test. How can you choose $\sum v_n$? Give a suitable example.

c) Find the period of $\tan \frac{x}{3}$.

d) Evaluate the direction derivative of $f = x - y$ at (4,5) in the direction of $\vec{a} = 2\vec{i} + \vec{j}$.

THE END

NEPAL COLLEGE OF INFORMATION TECHNOLOGY
ASSESSMENT SPRING-2023

Level: Bachelor
Programme: BE_CE 2^{nd} sem

Course: **Algebra and Geometry**

Year : 2022
Full Marks : 100
Pass Mark : 45
Time : 3 hrs

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) Define Eigen value and Eigen vectors of a square matrix A. Find Eigen value and their corresponding Eigen vector of $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (8)

   b) By using Gauss elimination method examine whether the following system of linear equations has solution or not and solve if exist. (7)

   $5x + 3y + 7z = 4$
   $3x + 26y + 2z = 9$
   $7x + 2y + 10z = 5$

   OR

   Show that {(1,0,2), (2,-1,1), (0,1,1)} forms a basis for R^3 space.

2. a) Test the convergence of the given series: $\sum[\sqrt{n^2+1} - \sqrt{n^2-1}]$ (7)

   b) Find the radius of convergence and interval of convergence of the series

   $\sum(-1)^n \frac{(2x-1)^n}{n6^n}$ (8)

3. a) Define conic section. Derive the tangent equation of a ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ *at point* (x_1, y_1). Also find the condition for a line $y = mx + c$ to be a tangent of above ellipse.

b) Find the center, eccentricity, foci and equation of directrices of hyperbola $9x^2 - 16y^2 + 18x + 32y - 151 = 0$. (7)

4. a) Find equation of the cone with vertex at (0,0,3) and guiding curve is $x^2 + y^2 = 4, z = 0$. (7)

OR

Find the equation of right circular cylinder whose guiding curve is a circle $x^2 + y^2 + z^2 = 9, x - y + z = 3$.

b) Define skew lines and shortest distance between them. Find magnitude and equation of shortest distance between the lines $\frac{x-5}{3} = \frac{7-y}{16} = \frac{z-3}{7}$ and $\frac{x-9}{3} = \frac{y-13}{8} = \frac{15-z}{5}$. (8)

5. a) Find the dual of the primal problem and solve using simplex method:
minimize $z = 4x_1 + 3x_2$ subject to $2x_1 + 3x_2 \geq 1$, $3x_1 + x_2 \geq 4$, $x_1, x_2 \geq 0$. (8)

b) Apply simplex method to solve, maximize $z = 15x_1 + 10x_2$ *subject to*,
$2x_1 + 2x_2 \leq 10$, $2x_1 + 3x_2 \leq 10$. $x_1, x_2 \geq 0$ (7)

6. a) Show that i) $(b \times c) \times (c \times a) = [a, b, c]c$ (8)
ii) $[a + b \quad b + c \quad c + a] = 2[a \ b \ c]$

b) Find the center and radius of the circle in which the sphere $x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0$ *is cut by plane* $x - 2y + 2z = 3$. (7)

7. Answer all : (4*2.5=10)

a) Show that the transformation $T: R^2 \to R^2$ defined by $T(x,y) = (x+y, x-y)$ is linear.

b) Find the direction cosine of normal of a plane $x + 4y - 2z = 5$.

c) Find the rank of the given matrix $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \\ 0 & 2 & 5 \end{bmatrix}$

d) Define scalar triple product of three vectors and write its geometrical interpretation.

Try again

OXFORD
COLLEGE OF ENGINEERING & MANAGEMENT

First Terminal Exam, Spring 2022

Level: Bachelor   Semester: Spring   Year : 2022
Programme: BE   Full Marks: 60
Course: Algebra and Geometry   Time : 2hrs.

SET A

Candidates are required to give their answers in their own words as practicable. The figures in the margin indicate full marks.

Attempt all the questions.

1. a. Check whether the system of linear equations is consistent or not, if consistent solve it by using Gauss elimination method. 8

$$3x + y + z = 8$$
$$x + y + z = 6$$
$$x - y + z = 5$$

b. State Cayley -Hamilton Theorem and verify for the matrix 7

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{pmatrix}$$

2 a. Apply simplex method to solve, Maximize: $Z = 15x_1 + 10x_2$ 5

Subjected to : $2x_1 + 2x_2 \leq 10; 2x_1 + 3x_2 \leq 10;\ x_1, x_2 \geq 0$

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b. Apply simplex method for dual to solve, Minimize: 5

$Z = 21x_1 + 50x_2$ Subjected to :
$2x_1 + 5x_2 \geq 12; 3x_1 + 7x_2 \geq 17; x_1, x_2 \geq 0$

3. a. Define vector triple product. If $a = i - 2j - 3k$, $b = 2i + j - k$ & $c = i + 3j - 2k$ find $(a \times b) \times c$. Also verify that $a \times (b \times c) = (a \cdot c)b - (a \cdot b)c$. 7

b. What does scalar triple product give? Explain geometrically and prove that
$(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a}) = |\vec{a} \ \vec{b} \ \vec{c}|\vec{c}$. 8

4. a. State and prove D'Alemberts' Ratio Test for an infinite series of positive terms. 8

b. Find the centre, radius of convergence and interval convergence of power series $\sum_{n=0}^{\infty} \frac{(x-5)^n}{n5^n}$ 7

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NEPAL ENGINEERING COLLEGE

Changunarayan, Bhaktapur

UNIT TEST - 2023

Level: Bachelor | *Year: 2023*
Programme : Electrical and Electronics | *Full Marks: 40*
Course : Algebra and Geometry | *Pass Marks: 16*
Semester: Spring | *Time: 1.4 Hours*

Set-A

Candidates are required to give their answers in their own words as far as practicable.

Attempt ANY FIVE questions.
Note: Odd CRN odd number & even CRN even number questions

1. Find the equation of the plane containing the points $(1,-1,2)$ and is perpendicular to the planes $2x+3y-2z=5$ and $x+2y-3z=8$.
2. Find the equation of the plane, when the plane $x+3y+5z=7$ is rotated through a right angle about its intersection with the plane $x-2y-6z=8$.
3. Find the image of the point $P(1,3,4)$ in the plane(mirror) $2x-y+z+3=0$
4. Prove that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ & $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$
5. Find the magnitude and equation of the line of the shortest distance between the lines $\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7}$, $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$ are coplanar. Also, obtain the equation of plane containing them .
6. The necessary condition for the infinite series $\sum u_n$ to be convergent is $\lim_{n\to\infty} u_n = 0$ but the converse of this theorem may not be true.
7. State D'Alembert's Ratio test. Test the convergent and divergent of Test the convergence and divergence of the series

   (a) $\frac{2\cdot 5}{1\cdot 5}+\frac{2\cdot 5\cdot 8}{1\cdot 5\cdot 9}+\frac{2\cdot 5\cdot 8\cdot 11}{1\cdot 5\cdot 9\cdot 13}+\ldots$

   (b) $\sum_{n=1}^{\infty}\frac{1}{(2n+1)!}$

8. Define p-test. Test the convergence and divergence of

   (a) $\frac{1}{1\cdot 2\cdot 3}+\frac{3}{2\cdot 3\cdot 4}+\frac{5}{3\cdot 4\cdot 5}+\ldots$

   (b) $\sum_{n=1}^{\infty}\frac{n!}{n^2}$.

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9. Define p-test. Test the convergence and divergence of

(a) $\sum_{n=0}^{\infty}\left[(n^3+1)^{1/3}-n\right]$, (b) $\sum_{n=0}^{\infty}\left(\frac{n}{n+1}\right)^{n^2}$

10. Define ellipse. Derive the standard equation of ellipse.

11. Find the condition that the line $lx+my+n=0$ may touch the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. Also find the point of contact.

12. Find the condition that the line $y=mx+c$ may touch the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. Also find the point of contact.

13. Find the centre, vertices, eccentricity, foci and length of latus rectum of the ellipse $9x^2+4y^2+72x-8y-176=0$.

14. Find the centre, vertices, eccentricity, foci and length of latus rectum of the hyperbola $9x^2-16y^2-90x+64y+17=0$.

The End

$lx+my+n=0$

$y=\frac{-lx-n}{m}$

$\sqrt{(x-c)^2+y^2}$

$(x-c)^2+y^2$

National Academy of Science and Technology

(Affiliated to Pokhara University)

Accredited by University Grants Commission(UGC)

Dhangadhi, Kailali

First- Terminal Examination

Level : Bachelor | Semester: II_Spring
Programme: B.E. Computer | Year : 2022
Course: Algebra and Geometry | Full Marks:100
Pass Marks: 45
Time : 3hrs

Attempt all the questions.

1.a) Obtain the equation of plane, which passes through the point (-1,3,2) and is perpendicular to each of the plane x+2y+3z=5 and 3x+3y+2z=8. [7]

b) Prove that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ are coplanar. Also obtain the equation of plane containing them. [8]

2.a)Find the equation of the spheres which pass through the circle $x^2+y^2+z^2=5$, $x+2y+3z=3$ and touch the plane $4x+3y=5$. [7]

b) Define shortest distance between two skew lines is space. Find the length and equation of shortest distance between the lines.

$$\frac{x}{2}=\frac{y}{-3}=\frac{z}{1} \text{ and } \frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$$

[8]

3.a) A sphere has ends points (0,0,0) and (4,5,6) at opposite ends of a diameter. Find the equation of the sphere having the intersection of the circle with the plane x+y+z=3 as a great circle. [7]

b) Find the equation of sphere which passess through the circle $x^2+y^2+z^2-6x-2z+5=0$, y=0 and touch the plane 3y+4z+5=0. [8]

4.a) Define eccentricity of a conic section, and derive the equation of a ellipse in its standard form. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ [7]

b) Find the condition that the line lx+my+n=0 touches the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ Find the point of contact [8]

5.a) Find the center, vertices and foci of the ellipse :

$$x^2 + 8x + 16y^2 = 0$$

b) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) + (\vec{a} \times \vec{c}) \times (\vec{d} \times \vec{b}) + (\vec{a} \times \vec{d}) \times (\vec{b} \times \vec{c})$ is parallel to the vector $\vec{a}$. [7]

6.a) Find the volume of parallelepiped hose concurrent edges are $3\vec{i} - 4\vec{j} + 4\vec{k}$, $2\vec{i} + 2\vec{j} - \vec{k}$ and $\vec{i} - \vec{j} + 2\vec{k}$. [8]

b) Define vector triple product. If $\vec{a} = 2i - 3j - 3k$, $\vec{b} = 2i + 3j - k$ & $\vec{c} = i + 3j - 2k$ find $(a \times b) \times c$. Also verify that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} . \vec{c})\vec{b} - (\vec{a} . \vec{b})\vec{c}$. [7]

[8]

7. Attempt all questions [4x2.5=10]

i) Find the value of k so that the vectors

$\vec{a} = 2\vec{i} - \vec{j} + \vec{k}$, $\vec{b} = \vec{i} + 2\vec{j} + 3\vec{k}$ and $\vec{c} = 3\vec{i} + k\vec{j} + 5\vec{k}$ are coplanar

ii) Find the angle between following pair of planes:

$2x + 3y + 5z = 0$ and $x - 2y + z = 20$

iii) Find centre and radius of the sphere:

$$x^2 + y^2 + z^2 + 2x + 3y - 4z + 10 = 0$$

iv) Find the coordinates of vertices, eccentricity and foci of

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

LUMBINI ENGINEERING COLLEGE

First Internal Exam (2023)

Level : Bachelor

Semester: Spring

Full Marks : 50

Programme : B.E.

Pass Marks:22.5

Course : Algebra and Geometry

Time :1.5hrs.

Attempt all the questions.

1. a) Find the condition that the line $lx + my + n = 0$ may touch the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Also find the point of contact (7)

   b) Find the equation of tangents to the ellipse $x^2 + 3y^2 = 3$, which are parallel to the line $4x - y + 8 = 0$. Also find the point of contact. (8)

2. a) Find the magnitude and equation of the line of shortest distance between the lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-3}$ (7)

   b) Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 1$, $2x + 4y + 5z = 6$ and touching the plane $z = 0$ (8)

3. a) Find the centre and radius of the circle in which the sphere $x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0$ is cut by the plane $x - 2y + 2z = 3$. (8)

   b) Find the equation of the cone with vertex (α, β, γ) and base (guiding curve) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $z = 0$. (7)

4. Translate the axes so as to change the equation $3x^2 - 2xy + 4y^2 + 8x - 10y + 8 = 0$ into an equation with linear terms missing. (5)

OR

Find the equation of a cylinder whose generating lines have direction cosines (l, m, n) and which passes through the circle $x^2 + z^2 = a^2$, $y = 0$.

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Marks : 10
Mark : 45
: 3 hrs

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NEPAL COLLEGE OF INFORMATION TECHNOLOGY
ASSESSMENT SPRING-2023

Level: Bachelor
Programme: BE_CE 2nd sem

Course: **Algebra and Geometry**

Year : 2022
Full Marks : 100
Pass Mark : 45
Time : 3 hrs

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) Define Eigen value and Eigen vectors of a square matrix A. Find Eigen value and their corresponding Eigen vector of $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (8)

   b) By using Gauss elimination method examine whether the following system of linear equations has solution or not and solve if exist. (7)

   $5x + 3y + 7z = 4$
   $3x + 26y + 2z = 9$
   $7x + 2y + 10z = 5$

   OR

   Show that {(1,0,2), (2,-1,1), (0,1,1)} forms a basis for R^3 space.

2. a) Test the convergence of the given series: $\sum[\sqrt{n^2+1} - \sqrt{n^2-1}\,]$ (7)

   b) Find the radius of convergence and interval of convergence of the series

   $\sum(-1)^n \frac{(2x-1)^n}{n6^n}$ (8)

3. a) Define conic section. Derive the tangent equation of a ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $at\ point\ (x_1, y_1)$. Also find the condition for a line $y = mx + c$ to be a tangent of above ellipse.

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b) Find the center, eccentricity, foci and equation of directrices of hyperbola $9x^2 - 16y^2 + 18x + 32y - 151 = 0$. (7)

4. a) Find equation of the cone with vertex at (0,0,3) and guiding curve is $x^2 + y^2 = 4, z = 0$. (7)

OR

Find the equation of right circular cylinder whose guiding curve is a circle $x^2 + y^2 + z^2 = 9, x - y + z = 3$.

b) Define skew lines and shortest distance between them. Find magnitude and equation of shortest distance between the lines $\frac{x-5}{3} = \frac{7-y}{16} = \frac{z-3}{7}$ and $\frac{x-9}{3} = \frac{y-13}{8} = \frac{15-z}{5}$. (8)

5. a) Find the dual of the primal problem and solve using simplex method:

minimize $z = 4x_1 + 3x_2$ subject to $2x_1 + 3x_2 \geq 1$, $3x_1 + x_2 \geq 4$, $x_1, x_2 \geq 0$. (8)

b) Apply simplex method to solve, maximize $z = 15x_1 + 10x_2$ *subject to*,

$2x_1 + 2x_2 \leq 10, 2x_1 + 3x_2 \leq 10$. $x_1, x_2 \geq 0$ (7)

6. a) Show that i) $(b \times c) \times (c \times a) = [a, b, c]c$ (8)

ii) $[a + b \quad b + c \quad c + a] = 2[a \ b \ c]$

b) Find the center and radius of the circle in which the sphere $x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0$ *is cut by plane* $x - 2y + 2z = 3$. (7)

7. Answer all : (4*2.5=10)

a) Show that the transformation $T:R^2 \rightarrow R^2$ defined by $T(x,y) = (x+y, x-y)$ is linear.

b) Find the direction cosine of normal of a plane $x + 4y - 2z = 5$.

c) Find the rank of the given matrix $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \\ 0 & 2 & 5 \end{bmatrix}$

d) Define scalar triple product of three vectors and write its geometrical interpretation.