NATIONAL ACADEMY OF SCIENCE AND TECHNOLOGY

(Affiliated to Pokhara University)

Dhangadhi, Kailali

Pre-University Examination

Level: Bachelor | Semester: IV_Spring | Year : 2024

Programme: B.E. Computer | Full Marks : 100

Course: **Applied Mathematics IV** | Pass Marks : 45

Time : 3hrs.

Candidates are required to give their answer in their own words as for as practicable. The figures in the margin indicate full marks.

Attempt all the questions.

1.a) Prove that u=$y^3 - 3x^2y$ is a harmonic function. determine its harmonic conjugate and find the corresponding analytic function f(z) =u+iv **[8]**

b) Evaluate: f(z)=$\frac{z^4}{(z+1)(z-i)^2}$. where c is $9x^2 + 4y^2 = 36$ **[7]**

2.a) Find the Taylor's and Laurent's series which represents the function $f(z)=\frac{z^2-1}{(z+2)(z+3)}$ in the regions:

i) $|z|<2$ *ii)* $2<|z|<3$ *iii)* $|z|>3$. **[7]**

b) State Cauchy's residue theorem. Using residue theorem, evaluate the integrals **[8]**

$i)\oint_c \frac{1}{z^2+4}dz, where \quad C:|z-i|=2$ and

$ii)\oint_c \frac{4-3z}{z^2-z}dz, where \quad C:|z|=2.$

3.a) Define Z-transform of a function f(n). Find the Z-transform of $e^{in\pi/2}$ and hence find $Z[\cos(\frac{n\pi}{2})]$ *and* $Z[\sin(\frac{n\pi}{2})]$ **[8]**

b) Using Z-transform, solve the difference equation **[7]**

$y_{n+2} + 3y_{n+1} + 2y_n = 0, y_0 = 0, y_1 = 1.$

4.a) Show that: $\int_0^\infty \frac{\cos xw + w \sin xw}{1+w^2} dw = \begin{cases} 0 \; if \; x < 0 \\ \frac{\pi}{2} \; if \; x = 0 \\ \pi e^{-x} \; if \; x > 0 \end{cases}$ [7]

b) Find the Fourier sine transform of the function

$f(x)=\begin{cases} x^2 & for \; 0 < x < 1 \\ 0 & for \; x > 1 \end{cases}$ [8]

OR

Find the Fourier cosine transform of $f(x)=e^{-mx}$ for m>0 and then

Show that $\int_0^\infty \frac{\cos kx}{1+x^2} dx = \frac{\pi}{2} e^{-k}$.

5.a) A tightly stretched string of length 20 cm fastened at both ends is displaced its position of equilibrium , by imparting to each of its points at initial velocity $g(x) = x$, for $0 \le x \le 10$

$= 20-x$, for $10 \le x \le 20$, where x being distance from one end .Find the displacement at any time t. [7]

b) Derive one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ [8]

6.a) Express the Laplacian $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ in polar co-ordinates. [8]

b) Find the solution of one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ with the appropriate initial and boundary conditions. [7]

OR

Solve $u_{xx}+u_{yy}=0$ using separation of variables method.

7. Solve: [4X2.5 = 10]

i) Express $f(z) = \cos z$ in the form $f(z)=u+iv$.

ii) Show that the function $u = \sin x \cosh y$ is a solution of two dimensional Laplace equation.

iii) Show that Fourier cosine transforms satisfy linearity property.

iv) Find $Z(e^n)$

Final Internal Examination 2081

Faculty of Science and Technology

School of Engineering, Pokhara University

Full Marks: 100

Pass Marks: 45

Program: BE (Computer)

Course: Applied Mathematics Time: 3 hrs.

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

Attempt all questions.

Section A

1. a) Show that the function $u = \cos x \cosh y$ is harmonic. 2.5
   b) State the Cauchy Integral formula. 2.5
   c) Derive the formula of $Z[a^n]$. 2.5
   d) Write the Fourier cosine and sine integral formula for the function $f(x)$. 2.5

Section B

2. a) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent series for the region $0 < |z+1| < 2$. 5
   b) Find the poles and residues of $f(z) = \frac{z^2}{(z+2)(z-1)^2}$. 5
   c) Find the fixed point and the normal form of the bilinear transformation $w = \frac{z-1}{z+1}$. 5
3. a) Obtain the inverse Z-transform of $\frac{z}{z^2+9z+20}$. 5
   b) Solve the partial differential $u_{xy} - u = 0$ by separating the variable. 5
   c) Find Fourier cosine transform of $f(x) = e^{-mx}$, where $m > 0$. 5

Section C

4. a) Evaluate the integral $\oint_C \frac{4-3z}{z(z-1)(z-2)} dz$ where C is the circle $|z| = \frac{3}{2}$. 7
   b) What do you mean by analyticity of function $f(z)$. State Cauchy Riemann equation and show that it is the necessary condition for the function to be analytic. 2+4

50

5\. a) State and prove first shifting theorem for Z-transform using it to find the value of $Z(\cos hat \sin bt)$. 7

b) Solve the differential equation by using Z-transform. $y_{n+2} - y_n = 2^n$ with $y_0 = 0, y_1 = 1$. [2+3+2]

6\. a) Examine the suitable function show that: 7

$$\int_0^\infty \left[\frac{\cos \omega x + \omega \sin \omega x}{1+\omega^2}\right] d\omega = \begin{cases} 0 & if\ x < 0 \\ \frac{\pi}{2} & if\ x = 0 \\ \pi e^{-x} & if\ x > 0 \end{cases}$$

OR

Find the Fourier sine transform of $e^{-x}, x \geq 0$ and hence by Parseval's identity, show that $\int_0^\infty \frac{x^2}{(1+x^2)^2} dx = \frac{\pi}{4}$.

b) Find the temperature in a laterally insulated bar of length L whose ends are kept at temperature 0, assuming that the initial temperature is

$$u(x,t) = \begin{cases} x; & 0 \leq x \leq \frac{L}{2} \\ L - x; & \frac{L}{2} \leq x \leq L \end{cases}$$ 8

7\. a) Derive one dimensional wave equation of a string of length L which is fixed in two end points with required assumptions. 7

OR

Find the solution of one-dimensional heat equation, $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ with initial temperature f(x) and boundary conditions $u(x,0) = 0 = u(l,t)$.

b) Express Laplacian $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ in polar co-ordinate system. 8

THE END

CamScanner

NEPAL ENGINEERING COLLEGE

Changunarayan, Bhaktapur

(Assessment Spring Semester 2024)

Level: Bachelor | Full Marks: 100
Programme: BE | Pass Marks: 45
Course: .Applied Mathematics | Time: 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Show that $e^x(x\cos y - y\sin y)$ is a harmonic function. Find the analytic function for which $e^x(x\cos y - y\sin y)$ is imaginary part. 8

   b) State Cauchy integral formula. Evaluate $\oint_C \frac{z+1}{z^3-2z^2}\,dz$, where C is the $|z| = 1$,counterclockwise. 7

2. a) Find the bilinear transformation which maps the points $z = 0, -1, i$ onto $w = i, 0, \infty$. Also find the image of the unit circle $|z|=1$. 8

   OR

   Find the Laurent series for $f(z) = \frac{7z-2}{z^3-z^2-2z}$ in the region given by (i) $0 < |z+1| < 1$ $(ii)\ 1 < |z+1| < 3$

   b) State Cauchy Residue Theorem. By applying Cauchy Residue Theorem, evaluate $\oint_C \frac{4-3z}{z(z-1)(z-2)}\,dz$ where $C: |z| = \frac{3}{2}$. 7

3. a) State and prove first shifting theorem of Z transform. Obtain $Z^{-1}\left(\frac{3z^2-18z+26}{(z-2)(z-3)(z-4)}\right)$. 8

   b) Using Z transform and inverse Z transform solve the equation $y_{n+2} - 2y_{n+1} + y_n = 2^n; y_0 = 2, y_1 = 1$ 7

4.

a) Show that $\int_0^\infty \frac{\sin \pi w \sin xw}{1-w^2}\,dw = \begin{cases} \frac{\pi}{2}\sin x & if\ 0 \le x \le \pi \\ 0 & if\ x > \pi \end{cases}$ 7

b) Find the Fourier transform of $e^{-|x|}$. 8

Hence evaluate $\int_0^\infty \frac{x \sin mx}{1+x^2}\,dx$

5\. a) Derive the one dimensional heat equation with necessary assumptions. 8

OR

Using the method of separation of variables, solve

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u \text{ where } u(x,0) = 6e^{-3x}$$

b) A tightly stretched string with fixed end points $x = 0, and\ x = l$ is initially at rest in its equilibrium position. If it is set vibrating by giving to each of its points a velocity $kx(l-x)$, find the displacement of the string at any distance x from one end at any time t. 7

6\. a) Solve; $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ which satisfies the conditions 7

$u(0,y) = u(l,y) = u(x,0) = 0$ and $u(x,a) = \sin\frac{n\pi x}{l}$

b) Solve; $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0.$ 8

7\. Short Notes (2.5×4= 10)

a) Check analyticity of $f(z) = z^4$.

b) Find the fixed point of $f(z) = \frac{2-2z}{z+1}$.

c) Find the Z transform of (na^n).

d) Solve the partial differential $u_{yx} = u_y$.

THE END

NEPAL COLLEGE OF INFORMATION TECHNOLOGY [NCIT]
ASSESSMENT-SPRING

Level: Bachelor
Programme: BE IT/CE

Year : 2024
Full Marks : 100
Pass Mark : 45
Time : 3 hrs

Course: Applied Mathematics

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) Define harmonic function. Show that u = $\frac{x}{x^2+y^2}$ is harmonic function. If your answer is yes, find corresponding analytic function f(z) = u+iv. (8)

   b) State Cauchy residue theorem. Evaluate the integral
   $\oint_c \left(\frac{z^2 \sin z}{4z^2-1}\right) dz$, where , c: $|z| = 2$ (7)

2. a) Find Z-transform of $e^{\frac{in\pi}{2}}$ and then find $Z(\cos\frac{n\pi}{2})$ and $Z(\sin\frac{n\pi}{2})$

   b) Solve the difference equation by using Z transform:
   $y_{n+2} - 4y_{n+1} + 4y_n = 2^n$, where $y_0 = 0, y_1 = 1$. (8)

3. a) State Laurent's Theorem. Find the Laurent's Series for f(z) = $\frac{z^2-1}{z^2+5z+6}$ in the region i) $2<|z|<3$. ii) $|z|>3$.

   OR

   Find the bilinear transformation of w = $\frac{5-4z}{4z-2}$ mapping the circle $|z| = 1$ (4+4)

   b) State Cauchy integral formula and by using it evaluate
   $\oint_c \left(\frac{z^2}{z^4-1}\right) dz$, where c: $|z+i| = 1$. (7)

4. a. Show that

$$\int_0^\infty \frac{\cos xw + w\sin xw}{1+w^2}\,dw = \begin{cases} 0 \text{ if } x < 0 \\ \frac{\pi}{2} \text{ if } x = 0 \\ \pi e^{-x} \text{ if } x > 0 \end{cases}$$ (7)

b. Find the Fourier cosine transform of $f(x) = e^{-mx}$ for $m > 0$ and then show that (8)

$$\int_0^\infty \frac{\cos kx}{1+x^2}\,dx = \frac{\pi}{2}e^{-k}$$

5. a. Solve $u_{xx} + u_{yy} = 0$ by using separating of variables. (7)

b. Find the derivation of one dimensional wave equation.

6. a. Reduce Lapalacian operator in polar and cylindrical form (8)

b. Find the temperature in a literally insulated bar of length L whose ends are kept at temperature 0, assuming that the initial temperature is (8)

$$f(x) \begin{cases} x \text{ if } 0 < x < \frac{L}{2} \\ L-x \text{ if } \frac{L}{2} < x < L \end{cases}$$

7. Attempt all questions (4×2.5 = 10)

a. Show that the Fourier sine transform is linear operator.

b. Express the following function f(z) = sinz in the form u+iv.

c. Find $Z(a^n)$.

d. Show that u = arg z is harmonic function.

$\int u.v\,dx = u\int v\,dx - $ $uv_1 - u'v_2$

$u\int v\,dx - \int \frac{du}{dx}\int$

Pokhara University

Everest Engineering College

Pre-Board Exam

Semester - Spring

Level: Bachelor

Program: BE CMP/IT, 4th Semester

Faculty: Science and Technology

Subject: Applied Mathematics

Year: 2024

F.M: 50

P.M: 23

Time: 1.5 hrs.

*Attempt **all** the questions.*

1. (a) Define analyticity of the complex valued function $f(z)$. Does differentiability always imply analyticity? Justify with suitable example. State and prove the necessary condition for a complex valued function $f(z) = u + iv$ to be analytic in a domain D. [8]

   (b) Define harmonic function. Is $u = Sin\, x\ cosh\, y$ harmonic? If yes, find a corresponding harmonic conjugate. Also, find corresponding analytic function $f(z)$. [7]

2. (a) State and prove Cauchy Integral Theorem. Is it possible to apply Cauchy Integral Theorem to evaluate the integral $\oint\limits_c \frac{1}{z^2+4}\, dz$, where c is the ellipse $x^2+4y^2 = 4$. counterclockwise? Justify. [8]

   (b) Expand $f(z) = \frac{7z-2}{z(z+1)(z-2)}$ in Laurent's series valid for (i) $0 < |z+1| < 1$, (ii) $1 < |z+1| < 3$, (iii) $|z+1| > 3$ [7]

3. (a) State Cauchy's Residue Theorem. Use it to evaluate the following integral $\oint_C \frac{e^z+z}{z^3-z}\, dz$, where $c : |z| = \pi/2$, counter-clockwise. [8]

(b) Define Z-transform. State and Prove First Shifting Theorem of Z-transform. Also, prove that

$$Z(t^k) = -Tz\,\frac{d}{dz}[Z(t^{k-1})]$$

Then use it find $Z(t^2 e^{-ibt})$. [7]

(OR)

Define convolution of two discrete time function $f(t)$ and $g(t)$. State and prove Convolution Theorem of Z-transform.

4. (a) Solve the difference equation $y_{n+2} - 4y_{n+1} + 4y_n = 2^n$, $y_0 = 0$, $y_1 = 1$ by using Z-transform. [7]

(b) Derive one dimensional Wave equation with necessary assumptions. [8]

(OR)

Find the deflection $u(x,t)$ of the vibrating string of length L with $c^2 = 1$ and its initial deflection is zero and initial velocity is

$$\begin{cases} x & \text{if } 0 < x < \frac{L}{2} \\ (L-x) & \text{if } \frac{L}{2} < x < L \end{cases}$$

5. (a) Find the temperature function $u(x,t)$ in a laterally insulated thin copper bar of length L with constant cross section whose endpoints at $x = 0$ and $x = L$ are kept at $0°C$ and whose initial temperature is $f(x) = sin^3 \frac{\pi x}{L}$. Use $(c^2 = 0.175\ cm^2/sec)$ [7]

(b) Express the Laplacian $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ in polar coordinates. [8]

6. (a) Find the Fourier Integral of the function

$$f(x) = \begin{cases} 0 & \text{if } 0 < x \\ \pi/4 & \text{if } x = 0 \\ \pi e^{-2x} & \text{if } x > 0 \end{cases}.$$

[7]

(b) Find the Fourier sine transform of e^{-x} and by using Parseval's Identity, show that $\int_0^\infty \frac{x^2}{(1+x^2)^2}\, dx = \frac{\pi}{4}$. [8]

7. Answer the following questions: [4×2.5]

(a) Find the bilinear transformation $w = f(z)$ which maps $z_1 = \infty,\ z_2 = i,\ z_3 = 0$ onto the points $w_1 = 0,\ w_2 = i,\ w_3 = \infty$. Also, find its fixed points.

(b) Find $Z(3n^2 - 2n + 1)$

(c) State and prove Initial Value Theorem of Z-transform.

(d) Let $f(x)$ be continuous on $\mathbb{R}$ and $f(x) \to 0$ as $x \to \infty$ and $f'(x)$ is absolutely integrable on $\mathbb{R}$, then show that $\mathcal{F}c\{f'(x)\} = w\,\mathcal{F}s\{f(x)\} - \sqrt{\frac{2}{\pi}}\, f(0)$.

105

UNITED TECHNICAL COLLEGE

Semester-Fall

1. a show that the necessary condition for analytic of f(z) = u+iv is $u_x = v_y$ and $u_y = -v_x$.

b Define Laplace equation . If $u = 3x^2y+x^2-y^3-y^2$ show that there exist a function v(x, y) such that f(z) = u+iv is analytic .

2. a State and prove Cauchy's Integral formula. Integrate $\oint_C \frac{d_z}{z^2+4}$ where C: $4x^2+ (y-2)^2 = 4$

OR State and prove Cauchy's Residue theorem .Evaluate $\oint_C \frac{\sinh z d_z}{2z-1}$ where c: $|z-i| = 1$.

b Define conformal mapping .Determine the region of Transformation w =3z in the w-plane where the region in z-plane enclosed by the lines y= 1 , y = 2 , x = 1 , x = 2 .

3. a Solve: $Y_{n+2}+ 6Y_{n+1}+9Y_n=2^n$. When $Y_0 = Y_1 = 0$. b) Find $Z^{-1}\left(\frac{z^2+1}{z^2-2z+2}\right)$. $(1+i)(1-i)$

4. a if f(n)= 0 for n<0 such that Z(f(t)) =F(z) then Z(f(t- kT)) = z^{-k} F(z) for n>0 ,k>0 .

b Derive two dimensional wave equation with necessary assumptions .

OR A uniform rod of length L has its ends maintained at a temp. 0° and initial temp. Of rod is $u(x, 0) = 3\sin\frac{\pi x}{L}$ for $0< x < L$.

5. a find the deflection u(x ,y ,t) of the square membrane with a = b = c =1. If initial velocity is zero and initial deflection is k $\sin\pi x \sin\pi y$.

OR Find the solution of one –dimensional wave equation under the boundary and initial condition.

b Solve : $U_{xx}-U_{yy} = 0$ separating variables.

6. a State and Prove Convolution Theorem for Fourier transform .

b. find the Fourier integral of $f(x) = e^{-x} + e^{-2x}$ $x > 0$.

7. Short questions.

a. Express the function f(z) = coshz in term of u+iv .

b. Find the Z- transform of na^n

c. show that $F_c(af(t) + bg(t)) = a F_c(f) + bF_c(g)$.

d. Solve the PDE $u_{xy} = u_x$

Universal Engineering & Science College

Affiliated to Pokhara University

Chakupat, Lalitpur

Level: Bachelor	Semester: IV	Year: 2024
Programme : BE Computer	Time: 3 hours	Full Marks: 100
Course:Applied Mathematics		Pass Marks: 45

Pre-Board Examination - 2081 (Spring 2023)

Candidates are requested to give their answers in their own words as far as practicable. Figure in the margin indicates full marks.

Attempt **All** the questions:

1. a. Define the Laplace equation and harmonic function. Is $v = (x^2 - y^2)^2$ harmonic? If yes find its harmonic conjugate. 8

   b. State Cauchy Integral formula for derivative. Evaluate $\oint_c \frac{z^6}{(2z-1)^6} dz$, where c is the unit circle $|z| = 1$, counter clockwise 7

2. a. State Laurent's theorem. Find Laurent's series for 7

   $$f(z) = \frac{1}{(z - z^3)} \text{ in the region } 01 < |z+1| < 2.$$

   b. Define singularity, zeros, and poles of a function. Evaluate $\oint_c f(z)dz$ where $f(z) = \frac{e^{2z}}{(z+1)^3}$ where c is the ellipse $4x^2+9y^2=16$. 8

3. a. Define Z - transform. State and prove the Second shifting theorem of Z- transform. Evaluate $Z(t^2 e^{-bt})$ 7

   OR

   Find $Z^{-1} \frac{z^2+1}{z^2-2z+2}$.

   b. Solve $U_{n+2} - 2\cos\alpha\, U_{n+1} + U_n = 0$ where, by using z-transform 8

4. a. Find the Fourier integral of the function 7

$$f(x) = \begin{cases} \pi/2 & \text{if } 0 \le x < 1 \\ \pi/4 & \text{if } x = 1 \\ 0 & \text{if } x > 1 \end{cases}$$

b. Define the convolution of the two functions. State and prove the convolution theorem on Fourier transform. 8

5. a. Solve one-dimensional wave equation with initial deflection is $0.01\sin 3x$ and initial velocity is zero and $L = \pi$, $c^2 = 1$ 7

b. Find the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, having zero temperature in endpoints and initial temperature 8

6. a. Derive two-dimensional heat equation with required assumption. 7

b. Find the deflection u(x,y,t) of a square membrane with a = b=1 with c=1if the initial velocity is zero and the initial deflection is $0.1 \sin 3\pi x \ \sin 4\pi y$ 8

7. Attempt all.

a. Express f(z) = sinz in the form u + iv 2

b. Find z–transform of na^n 2

c. solve $u_{xx} - u_{yy} = 0$ 2

d. Find the unit tangent vector to the curve $\vec{r}(t) = 2\cos t\vec{i} + \sin t\vec{j}$ at $(\sqrt{2}, \sqrt{2}, 0)$. 2

e. Sketch the paraboloid $z = x^2 + y^2$. 2

Pokhara Engineering College

Level : Bachelor **Semester : 2024 Spring**

Programme : BE Computer **Full Mark : 100**

Course : Applied Maths **Pass Mark :40**

Time : 3 hrs.

Candidates are required to give their answers in their own words as far as practicable. Figures in margin indicate full marks.

<u>Attempt all the questions</u>

1) a) Define harmonic function. Verify that $u = \cos x \cosh y$ is harmonic and its harmonic conjugate. (8)

b) Integrate the function $f(z) = \frac{1}{z^2+4}$ over the given contour c counterclockwise where c is the ellipse $4x^2 + (y-2)^2 = 4$. (7)

2) a) State Cauchy Residue theorem. Evaluate $\int_c \frac{z+1}{z^4-2z^3} dz$ where $c: |z| = \frac{1}{2}$ counterclockwise. (8)

b) Find the Taylor and Laurent's series of $f(z) = \frac{2z-3i}{z^2-3iz-2}$ in the region

(i) $0 < |z| < 1$ (ii) $|z| > 2$ (7)

3) a) Find the Z-transform of $e^{\frac{in\pi}{2}}$ and then deduce the value of $Z\left(\cos\frac{n\pi}{2}\right)$ and $Z\left(\sin\frac{n\pi}{2}\right)$. (8)

b) State and prove first shifting theorem of Z-transform. Use it to evaluate $Z(na^n)$ and $Z(e^{-at})$. (7)

20

4) a) Find the inverse z-transform of $F(z) = \frac{3z^2+2z+1}{z^2+3z+2}$ (8)

OR

Solve using Z-transform

$y_{n+2} - 3y_{n+1} + 2y_n = 4^n, y_0 = 0, y_1 = 1$

b) Derive one dimensional wave equation of a string of length L which is fixed in two end points with necessary assumptions. (7)

OR

Find the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ with initial temperature $f(x)$ and boundary conditions $u(0,t) = 0$ and $u(L,t) = 0$.

5) a) Solve by using separation of variables

(i) $u_x + u_y = 0$ (ii) $u_{xy} - u = 0$. (8)

b) Express the Laplacian in polar coordinatesystem from cartesian co-ordinate system. (7)

OR

Find the temperature distribution in a laterally insulated thin copper bar ($c^2 = 1.158\ cm^2/sec$). 100cm long and of constant cross section whose end points at $x = 0$ and $x = 100$ are kept at $0°C$ and whose initial temperature is

(i) $f(x) = \sin(0.01)\,\pi x$ (ii) $f(x) = \sin^3(0.01)\,\pi x$

6) a) Show that:

$$\int_0^\infty \left[\frac{\cos x\omega + \omega \sin x\omega}{1+\omega^2}\right] d\omega = \begin{cases} 0 \ if \ x < 0 \\ \frac{\pi}{2} \ if \ x = 0 \\ \pi e^{-x} \ if \ x > 0 \end{cases}$$ (8)

b) Find the Fourier cosine transform of $f(x) = e^{-mx}; m > 0$ and then show that $\int_0^\infty \frac{\cos kx}{1+x^2} dx = \frac{\pi}{2} e^{-k}$ (7)

7) Attempt anytwo (2×5=10)

a) Show that $z\bar{z}$ is not an analytic function.

b) Find the location & order of zeros of $(z^2+1)(e^z - 1)$.

c) Verify $u = e^{-t} \sin x$ to satisfy one dimensional heat equation.

MadanBhandari College of Engineering

Urlabari-3, Morang

Final Internal Examination

Level: Bachelor
Programme: BE Computer
Year/Part: II/II

Full Marks: 100
Pass Marks: 45
Time: 3 hrs

Subject: - Applied Mathematics

Attempt all the questions:

1.	a) Define harmonic function. Is a function $v = x^3 - 3xy^2$ harmonic? If yes, find a corresponding harmonic conjugate and the analytic function	8
	b) State Cauchy integral formula for derivative. Evaluate $\oint_c \frac{z^6}{(2z-1)^6}\, dz$, where c is the unit circle $\|z\|=1$ counter clockwise.	7
2.	a) Find the Laurent series for $f(z) = \frac{z+3}{z(z^2-z-2)}$ in the region i) $0<\|z\|<1$ (ii) $1<\|z\|<2$ (iii) $\|z\|>2$	8
	b) Define Singularity, zero and poles of function. Evaluate $\oint_c f(z)\ dz$, where $f(z) = \frac{e^{2z}}{(z+1)^3}$ where c is the ellipse $4x^2 + 9y^2 = 16$	7
3.	a) Define Fourier integral. By choosing a suitable function, show that $\int_0^\infty \frac{\cos xw + w\sin xw}{1+w^2}\, dw = \begin{cases} 0 & if\ x<0 \\ \frac{\pi}{2} & if\ x=0 \\ \pi e^{-x} & if\ x>0 \end{cases}$	8
	b) Find the Fourier transform of the function $f(x) = e^{-x^2/2}$	7
4.	a) State and prove first shifting theorem of z transform. Evaluate the Z transform of $a^n\ cosbt$ and $a^n sinbt$	8
	b) Solve the difference equation by using Z-transform $y_{n+2} - 4y_{n+1} + 4y_n = 2^n$ with $y_0 = 0, y_1 = 1$	7
5.	a) What do you mean by analyticity of function f(z). State Cauchy Remann equation and hence show that it is the necessary condition for the function to be analytic.	8
	b) State Cauchy Residue theorem. By applying Cauchy Residue theorem, Evaluate	

	$\oint_C \frac{4-3z}{z(z-1)(z-2)} dz$ where C: $\lvert z \rvert = \frac{3}{2}$	7
6.	a) Find Fourier sine transform of $f(x) = e^{-x}$ for x>0. Then prove that $\int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m}$ for m>0	8
	b) State and prove initial theorem and find the inverse of z-transform of $F(z) = \frac{z^2-3z}{(z-5)(z+2)}$	7
7.	Attempt all the questions: a) Prove $Z(a^n) = \frac{z}{z-a}$ b) Check analyticity of $f(z) = z^3$ c) Find Z-transform of Sin($\frac{n\pi}{2}$) and Cos ($\frac{n\pi}{2}$) d) Show that $z\,\bar{z}$ is not an analytic function	10

****Best of luck*****

Lumbini Engineering, Management & Science College

Final Internal Assessment Exam

Level: Bachelor	**Year: 2024**
Program: BE	**Full Marks: 100**
Course: Engineering Mathematics IV	**Time: 3 hrs.**

Attempt all the questions:

1.a) Define analytic function. State and prove the necessary condition for f(z) to be analytic. Also check whether sinz is analytic or not? (8)

OR

Define Harmonic function. If a function f(z) is analytic then show that $U_x = V_y$ and $U_y = -V_x$.

b) Write Define conformal mapping. Name the type of conformal mapping. Translate the rectangular region ABCD in Z Plane bounded by x=1, x=3, y=0, and y=3 under the transformation w= z+ (2+i). Show with figure. .... (7)

2.a) State Cauchy Integral formula for derivative. Evaluate $\oint_c \frac{e^z}{(z+1)^3(z-2)}dz$, where c:|z-1|=3 (8)

b) Find the Laurent Series expansion of $\frac{z^2-1}{z^2+5z+6}$ in the region i) |z|<2 ii) 2<|z|<3 (7)

3.a) State and prove the second shifting theorem of Z-transform. Obtain z-transform of $(1-e^{-at})$, a>0 (7)

b) Solve the difference equation using z-transform.
$y_{n+2}-3y_{n+1}+2y_n=4^n$, $y_0=0$, $y_1=1$ (8)

4.a) Find the Fourier cosine transform of $f(x)=e^{-x}$, x>0 and hence show that $\int_0^\infty \frac{1}{(1+x^2)^2}dx = \frac{\pi}{4}$ (7)

b) What are the Fourier sine and cosine integrals of a function f(x)?
Show that $\int_0^\infty \frac{\cos\frac{\pi}{2}w \cos xw}{1-w^2}dw = \begin{cases} \frac{\pi}{2}\cos x\,; & |x| < \frac{\pi}{2} \\ 0 & |x| > \frac{\pi}{2} \end{cases}$

5.a) Solve the wave equation $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$ with given boundary conditions u(0,t)=0, u(L,t)=0 and initial deflection u(x,0)=f(x), initial velocity $\frac{\partial u}{\partial t}$=g(x). (8)

b) A homogeneous rod of conducting material of length 100 cm has its ends kept at zero temperature and temperature initially is
$f(x) = \begin{cases} x\,; & 0 \le x \le 50 \\ 100-x\,; & 50 \le x \le 100 \end{cases}$
Find the temperature U(x,t) at any time (7)

6.a) Solve the partial differential equation $y^2u_x - x^2u_y = 0$ by separating the variables. (8)

OR

Derive the solution of two-dimensional wave equation under the condition when a circular membrane of radius R is vibrating.

b) Show that U= e^{2x}(x cosy − y siny) is a harmonic function. Find an analytic function for U. If U is the real part. (7)

7. **Solve the following:** **(5x2=10)**

i) Find u and v if f(z)=tanz

ii) Prove $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$ if f(x) is even

iii) Solve the PDE u_{yy}=u

iv) Find the Z-transform of discrete unit time impulse δ(n).

v) Obtain the image of the strip 0<y< ½ under the transformation $w=\frac{1}{z}$

The End

Gandaki College of Engineering and Science

Pokhara - 16, Lamachour

Level: Bachelor | **Year 2024**

Programme : BE Computer | **Full Marks: 100**

Course: Applied Mathematics | **Pass Marks: 45**

Semester: IV (Spring) | **Time: 3 hrs**

Candidates are required to give their answer in their own words as far as practicable.

Attempt all the questions.

Q. 1 a) Define harmonic function. check

$U(x, y) = 3x^2y + x^2 - y^3 - y^2$ is harmonic or not?. If yes, find a function $V(x, y)$ such that $u + iv$ is an analytic function. (7)

b) Define bilinear transformation. find a bilinear transformation which maps the points $i, -i, 1$ of the z - plane into $0, 1, \infty$ of the w - plane respectively. (8)

Q. 2 a) State Cauchy integral formula for derivative. Evaluate $\oint_C \frac{\cot z}{(z-\frac{\pi}{2})^2} dz$

Where C is the ellipse $4x^2 + 9y^2 = 36$ (7)

or

Find the Laurent's or Taylor's series which represents the function. $f(z) = \frac{z^2-1}{(z+2)(z+3)}$

i) When $|Z| < 2$ ii) When $2 < |Z| < 3$ iii) When $|Z| > 3$

Q. 2 b) State Cauchy residue theorem.; Integrate $\oint_C \left(\frac{Z-23}{Z^2-4Z-5}\right) dz$; where: $|Z-2| = 4$ (8)

3. a) State and prove first shifting theorem of Z - transform. Using it evaluate the Z transform of $a^n \cos bt$ and $a^n \sin bt$ (7)

b) Solve the difference equation by using Z- transform.

$U_{x+2} - 2\cos\alpha\, U_{x+1} + U_x = 0$; Where $U_0 = 0, U_1 = 1$ (8)

4. Show that : a. $\int_0^{\infty} \left[\frac{\cos xw + w \sin xw}{1+w^2}\right] dw = \begin{cases} 0 & if\ x < 0 \\ \frac{\pi}{2} & if\ x = 0 \\ \pi e^{-x} & if\ x > 0 \end{cases}$ (7)

b) Find the Fourier transform of the function $f(x) = \begin{cases} e^{x} & \text{if } x < 0 \\ e^{-x} & \text{if } x < 0 \end{cases}$ (8)

5 a) Derive one dimensional wave equation; $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ with necessary assumption. (7)

b) Define partial differential equation find the solution of one dimensional heat equation under certain initial and boundary condition. (8)

6 a) Find the temperature in a laterally insulted bar of length L whose ends are kept at temperature O^0 C, assuming that the initial temperature is

$$f(x) = \begin{cases} x & \text{if} \quad 0 < x < \frac{L}{2} \\ L - x & \text{if} \quad \frac{L}{2} < x < L \end{cases}$$ (7)

b) Express the Laplacian; $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ in polar co-ordinates. (8)

7. Attempt all the questions: (4×2.5=10)

a) Find Z - transform of n.

b) Show that $f(z) = z^2$ is an analytic function.

c) Solve the partial differential equation $u_{xy} = u_x$

d) Expand $f(z) = \frac{2-z}{(1-z)^2}$ by using Maclaurin's series expansion.

-Best of Luck-

COSMOS COLLEGE
OF MANAGEMENT AND TECHNOLOGY

Term Test II

Date: 2081/03/06			
Level	BE	Full Marks	50
Programme	BEIT, BCE	Time	
Semester	IV		1.5 hrs

Subject: - *Applied Mathematics*

- ✓ *Candidates are required to give their answers in their own words as far as practicable.*
- ✓ *Attempt* ***All*** *questions.*
- ✓ *The figures in the margin indicate* ***Full Marks.***
- ✓ *Assume suitable data if necessary.*

1 a) State Cauchy Residue's theorem. Use it to evaluate $\oint Tan\, z\, dz$, where c: $|z| = 2$ [7]

b) Define harmonic function. Prove that $u = y^3 - 3x^2y$ is a harmonic function. Determine its harmonic conjugate v of function f(z). [8]

2 a) Evaluate $\oint_c \frac{z^2}{(z^4-1)} dz$ where c is the circle $|z+1|=1$ in counter clockwise. [8]

b) State Taylor series. Expand the function $f(z) = \frac{1}{z-z^3}$ in the region $1< /z+1/ <2$. [7]

3 a) Find Fourier cosine transform of $f(x) = e^{-mx}$ for m > 0. [8]
Then prove that $\int_0^\infty \frac{\cos kx}{1+x^2} dx = \frac{\pi}{2} e^{-k}$

OR

Define convolution of the two functions. If f(x) and g(x) are piecewise continuous, bounded absolutely integrable on the x- axis. Prove that $F(f*g) = \sqrt{2\pi}\ F(f).F(g)$ [7]

b) Show that $\int_0^\infty \frac{\cos wx + w \sin wx}{1+w^2} dw = \begin{cases} 0 \ if \ x < 0 \\ \frac{\pi}{2} \ if \ x = 0 \\ \pi e^{-x} \ if \ x > 0 \end{cases}$ [7]

4 a) Find Maclaurin expansion of the function $f(z) = \frac{z+2}{1-z^2}$. [2.5]

b) Show that $\oint_C \frac{dz}{z} = 2\pi i$, where C is the unit circle, counter-clockwise. [2.5]

College of Engineering and Management

Nepalgunj – 10, Banke

Level : Bachelor
Programmer : BE [redacted] IT
Course : Engineering Mathematics IV

Semester :- Spring (4)

Year : 2022
Full Marks : 100
Pass Marks : 45
Time : 3 hrs.

Candidate are required to give their answers in their own words as far as practicable.
The figure in the box indicates marks.
Attempt all questions

1. a. Define harmonic function. Show that the function $u = 3x^2y+x^2-y^3-y^2$ is a harmonic function, Find the analytic function for which the given function is a real part. 8

   b. Evaluate $\oint \left(\frac{\cos(\pi z^2)}{z^2-3z+2}\right) dz$ where $C:|z| = 3$ 7

2. a. Let the rectangular region R in the z- plane be bounded by lines x=0, y=0, x=2, y=3. Find the region R' of the w-plane into which the given R is mapped under transformation $W = \sqrt{2}\, e^{\frac{i\pi}{4}} z$. 7

   b. Find the Taylor's and Laurent's series of the function $f(x) = \frac{z^2-1}{(z+2)(z+3)}$ 8

   OR

   State Cauchy Residue Theorem. By applying Cauchy Residue Theorem, evaluate $\oint \left(\frac{4-3z}{z(z-1)(z-2)}\right) dz$ where $C : |z| = \frac{3}{2}$.

3. a. State & prove first shifting theorem on Z-transform. Find the Z-transform of $e^{-at}\, t$. 8

   b. Solve the differential equation $y_{k+2} + 2y_{k+1}+y_k = k$ where $y_0=0$, $y_1 = 0$ using Z-transform. 7

4. a. Show that $\int_0^\infty \frac{\sin \pi w \sin xw}{1-w^2} dw = \begin{cases} \frac{\pi}{2}\cos x & if\ 0 \le x \le \pi \\ 0 & if x \ge \pi \end{cases}$ 7

   b. Find Fourier sine transform of $f(x) = e^{-x}$, $x > 0$ and then show that $\int_0^\infty \frac{x \sin nx}{x^2+1} dx = \frac{\pi}{2} e^{-m} for\ M > 0$ 8

5. a. Solve $xu_{xy}+2yu = 0$ by using separating variables. 7

   b. Find the solution of one Dimensional wave equation by D' Alembert's method. 7

6. a. Find the temperature u(x,t) in a laterally insulated bar of length L, whose ends are kept at temperature 0, assuming that the initial temperature is $f(x) = \begin{cases} x & if\ 0 < x < L/2 \\ L - x & if \frac{L}{2} < x < L \end{cases}$ 8

   b. Derive Laplace equation in polar co-ordinate and also write the expression for cylindrical co-ordinates. 7

   OR

   Define potential function and then find the solution of potential function by spherical membrane.

7. Attempt all questions. 2.5×4

   a. Evaluate $\oint_c \frac{dz}{z-3i}$, where c is the circle, $|z - 2i|=2$ counter clockwise direction

   b. Find z-transform of $Z(n^2)$

   c. Solve ~~uxx – u = 0~~ $u_{xx} - u = 0$

   d. Write the equation of hyperbolid of two sheet and then sketch.