# POKHARA UNIVERSITY

Level: Bachelor | Semester : Spring | Year : 2023
Programme: BE | | Full Marks: 100
Course: Algebra and Geometry | | Pass Marks: 45
| | Time : 3hrs.

*Candidates are required to give their answers in their own words as far as practicable.*

*The figures in the margin indicate full marks.*

***Attempt all the questions.***

1. a) Check consistency and solve by Gauss elimination method $x + y + z = 6, x - y + z = 2, 2x + y - z = 1.$ 7

   b) Define eigen value and vector. Find the eigen value, eigenvector and diagonalize of the matrix $A = \begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix}$ 8

2. a) Solve the linear programming problem by simplex method.[constructing duality] 7

   Minimize $Z = 2x_1 + 9x_2 + x_3$ subject to $x_1 + 4x_2 + 2x_3 \geq 5$, $3x_1 + x_2 + 2x_3 \geq 4$, $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$.

   **OR**

   Using simplex method maximize $Z = 150x_1 + 300x_2$ subject to $2x_1 + x_2 \leq 16$, $x_1 + x_2 \leq 8$ *and* $x_2 \leq 3.5$, $x_1 \geq 0$, $x_2 \geq 0$.

   b) Solve (Big M- method) Maximize $z = -3x_1 + 7x_2$ subject to $2x_1 + 3x_2 \leq 5$, $5x_1 + 2x_2 \geq 3$, $x_2 \leq 1$ 8

3. a) State D' Alembert's Ratio test show that 7

   i. $\sum \frac{1}{n}$ is divergent.

   ii. $\sum \frac{1}{n^2}$ is convergent.

   iii. $\sum \frac{(-1)^n}{n}$ is conditional convergent.

   b) Find the center, radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x)^{2n+1}}{(-4)^n}$. 8



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4. a) Define eccentricity of a conic section, and derive the equation of the ellipse in its standard form, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ 7

6

OR

Show that the line $lx + my + n = 0$ touches the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ if $a^2l^2 - b^2m^2 = n^2$. Also find the point of contact.

b) Show that the equations: $16x^2 - 24xy + 9y^2 - 104x - 172y + 44 = 0$ represents a parabola. Also find the equation of axis and vertex. 8

5. a) Define scalar triple product and reciprocal vectors. Find the reciprocal vector of 7

5

$\overrightarrow{2i}+\overrightarrow{3j}-\vec{k},\ \vec{i}-\vec{j}-\overrightarrow{2k},\ -\vec{i}+\overrightarrow{2j}+\overrightarrow{2k}$.

b) Find the distance of the point (3,-4,5) from the plane 2x+5y+6z=16 measured along a line with direction cosines proportional to 2,1,-2 8

6. a) Find the shortest distance between the lines $\frac{x+3}{-4}=\frac{y-6}{3}=\frac{z}{2}$ and $\frac{x+2}{-4}=\frac{y}{1}=\frac{z-7}{1}$ Also Find the equation of shortest distance 7

6

b) Find the equation of the sphere for which the circle $x^2+y^2+z^2+7y-2z+2=0, 2x+3y+4z=8$ is a great circle. 8

5

7. Attempt all the questions 7 2.5×4

a) Show that $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]=[\vec{a}\ \vec{b}\ \vec{c}]^2$

b) Find the centre the hyperbola:

$5x^2-4y^2+20x+8y=4$.

c) Show that the mapping $T: R^2 \rightarrow R^2$ define by

$T(x, y) = (x, x+y)$ is linear.

d) Plane through the OX and OY include an angle $\alpha$. Show that their lines of intersection lie on the cone is $(z^2+x^2)(y^2+z^2)\cos^2\alpha=x^2y^2$.



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