POKHARA UNIVERSITY

Level: Bachelor
Programme: BE
Course: Engineering Mathematics I

Semester: Fall

Year : 2014
Full Marks: 100
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

a) Define continuity and differentiability of a function. Show that differentiability of a function f(x)=a, implies continuity but converse may not be always true. 7

OR

If $\log y = tan^{-1}x$, show that

i. $(1 + x^2)y_2 + (2x - 1)y_1 = 0$

ii. $(1 + x^2)y_{n+2} + (2nx + 2x - 1)y_{n+1} + n(n + 1)y_n = 0$

b) State and prove Rolle's theorem. Is Rolle's theorem applicable to the function $f(x) = tanx$ in the interval? 8

a) Define indeterminate forms. State L Hopital rule and using it, show that $\lim_{x \to 0} \left(\frac{sinx}{x}\right)^{1/x} = 1$. 8

b) Find the altitude of the right circular cylinder of maximum volume that can be inscribed in a given right circular cone of height h. 7

OR

Define the asymptotes of a curve and classify them. Find the asympotes of the curve :

$x^4 - y^4 + 3x^2y + 3xy^2 + xy = 0$

Integrate Any Three 3×5

a) $\int \frac{dx}{4-5\,Sin^2x}$

b) $\int_a^b x^m\,dx$ (by summation method)

c) $\int \frac{e^x dx}{e^x - 3e^{-x} + 2}$

d) $\int_0^{\frac{\pi}{4}} \log(1+\tan x)dx$

(.a) Find the area bounded by $x^2 = 4y$ and $y = |x|$.

OR

Find the volume of the solid in the region in first quadrant bounded by the parabola $y = x^2$, the y – axis and the line y=1 revolving about the line x = 3/2. 7

b) Use Trapezoidal and Simpson's rule with n = 6 to approximate the area between the curve $y = (2x+1)^2$ ordinates x = 1, x = 4 and x axis. Compare the result with exact value. 8

a) Define vector triple product. If $\vec{a} = \vec{i} - 2\vec{j} - 3\vec{k}$, $\vec{b} = 2\vec{i} + \vec{j} - \vec{k}$ and $\vec{c} = \vec{i} + 3\vec{j} - 2\vec{k}$ find $(\vec{a} \times \vec{b}) \times \vec{c}$. Also verify that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$. 8

b) Find the equation of the plane through the point (2,4,5) and perpendicular to the line x=5+t, y=1+3t, z=4t. 7

a) Define eccentricity of a conic section, and derive the equation of a hyperbola in its standard form. 8

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

b) Find the condition that the line lx+my n=0 touches the parabola y^2=4ax. Find the point of contact. 7

7. Answer the followings: 4×2.5

a) Find the radius of curvature of the curve $y^2 = 4ax$ at (x,y).

b) Integrate $\int x \sin^2 x \, dx$

c) Evaluate improper integral $\int_0^{\infty} \frac{1}{x^2+9}dx$

d) If $\vec{a} = \vec{i} + 2\vec{j} + \vec{k}$, $\vec{b} = \vec{i} + \vec{j} + \vec{k}$, find unit vector along $\vec{a} \times \vec{b}$.

POKHARA UNIVERSITY

Level: Bachelor
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Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Define continuity and differentiability of a function. Show that the function: 8

$$f(x) = x^2+2 \quad \text{for} \quad x \le 1$$
$$= 3x \quad \text{for} \quad x > 1$$

is continuous at x=1 but not differentiable at x=1

OR

If $y = (x^2-1)^n$ show that;

i. $(x^2-1)y_2+2(1-n)xy_1-2ny=0$

ii. $(x^2-1)y_{n+1}+2xy_{n+1}-n(n+1)y_n=0$

b) State and prove that Lagrange's Mean value Theorem with its geometrical interpretation. 7

2. a) State the L 'Hospital Rules and evaluate the limit: 7

$$\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{sin^2x}\right)$$

b) Define the asymptotes of a curve and classify them. Find the asymptotes of the curve 8

$$x^2(x-y)^2 - a^2(x^2+y^2) = 0$$

OR

Find the altitude of the right circuler cone of maximum value that can be inscribed in a sphere of radius a.

3. Integrate (Any three) 3×5

a) $\int \frac{dx}{2-3\sin 2x}$

b) $\int_a^b e^{mx}dx$ (by summation method)

c) $\int \frac{x^3 dx}{(x-2)(x-3)}$

d) $\int_0^1 cot^{-1}(1-x-x^2)dx$

4. a) Find the area inside the circle $x^2 + y^2 = 1$ and outside the parabola $y^2 = 1 - x$. Also sketch the region. 7

OR

Find the volume of the solid in the region in the first quadrant bounded above by the curve $y = x^2$, below by the x – axis and on the right by the line $x = 1$, about the line $x = -1$.

b) Find approximate value of $\int_1^2 \frac{1}{x} dx$ using Trapezoidal and simpson's rule with $n = 10$ and then compare the results with the exact value of the integral. 8

5. a) Define eccentricity of a conic section and derive the equation of a ellipse in its standard form. 7

b) Find the equation of the tangents to the parabola $y^2 = 7x$ which is perpendicular to the line $4x + y = 0$. Also, find the point of contact. 8

6. a) Define scalar and vector triple product vectors. Show that $(\vec{a} \times \vec{b}) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$ if the vectors $\vec{a}$ and $\vec{c}$ are collinear. 7

b) Find the equation of the plane through (3,2,1) and (1,2,3) which is perpendicular to the plane 4x-y+2z=7. 8

7. Write short notes on: 4×2.5

a) Find the arc length of the curve $y = x^{3/2} from\ x = 0\ to\ x = 2$

b) Evaluate the improper integral $\int_0^\infty \frac{dx}{1+x^2}$.

c) Find the radius of curvature at $(s, \Psi) for\ the\ curve\ s = 8a\ sin^2 \frac{\Psi}{6}$

d) If $\vec{a} = 2\vec{i} + \vec{j} - \vec{k}$ and $\vec{b} = \vec{i} + 2\vec{j} + \vec{k}$, find a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$.

POKHARA UNIVERSITY

Level: Bachelor　　　Semester: Fall　　　Year : 2015
Programme: BE　　　Full Marks: 100
Course: Engineering Mathematics I　　　Pass Marks: 45
　　　Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Examine the continuity and differentiability at x =2 of the function f(x) defined as follows　$f(x) = 2-x$　for $0 < x < 2$　8

$= -2+3x-x^2$　for $2 \leq x < 4$

OR

If $y = \sin^{-1} x$ show that

i. $(1-x^2)y_2 - xy_1 = 0$

ii. $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$

b) State Lagrange's Mean Value theorem. Is Lagrange's mean value theorem applicable to the function $f(x) = |x|$ in the interval $[-1,1]$? Give reasons.　7

2. a) Find the asymptotes of the curve　8

$$x^3 - 2y^3 + 2x^2y - xy^2 + xy - y^2 + 1 = 0$$

OR

A cylindrical tin can closed at both ends with given capacity has to be constructed. Show that the amount of tin required will be minimum when the height is equal to the diameter.　7

b) Evaluate : $\lim_{x \to 0} \left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$

3. Integrate (**Any three**)　3×5

a) $\int \frac{dx}{5 + 4\cos x}$

b) $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\, dx$

c) $\int \frac{x^3}{(x-2)(x-3)}$

d) $\int_0^{\infty} e^{-x^2}\,dx$

a) Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line x=3 about the line x=3. 7

b) Using Trapezoidal and Simpson's rule, estimate the integral 8

$\int_0^4 \frac{1}{x^2+4}\,dx$ with n=4 subintervals.

. a) Find the volume of a tetrahedron whose one vertex is at the origin and the other three vertices are (3,2,1), (2,3,-1) and (-1,2,3). 7

b) Find the equation of plane passing through (2, 4, 5), (1, 5, 7) and (-1, 6, 8). 8

6. a) Find the condition that the line lx + my + n =0, may touch to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ 7

b) Define conic section and derive the standard equation of Ellipse. 8

7. Do the followings: 4×2.5

a) Evaluate $\int_1^e x \log x\,dx$

b) Find the radius of curvature of $y = x^2+4$ at (0, 4).

c) Evaluate $\int \frac{x}{(x-3)(x+1)}\,dx$

d) Find the scalar projection of $\bar{a} = i - 2\bar{j} + \bar{k}$ on $\bar{b} = \bar{i} + 2\bar{j} - \bar{k}$.

POKHARA UNIVERSITY

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Course: Engineering Mathematics I

Semester: Spring

Year : 2015
Full Marks: 100
Pass Marks: 45
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Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) If a function f(x) is defined by 7

$$f(x) = \begin{cases} x-2 & \text{for } \geq 2 \\ 4-x^2 & \text{for } x<2 \end{cases}$$

Show that it is continuous at x=2 but not differentiate at x=2.

OR

If, $y=\sin^{-1}x$, show that

i. $(1-x^2)y_2-xy_1=0$

ii. $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$

b) State L 'Hopital Rule for indetermianteform. 8

Evaluate $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x}$.

2. a) An oil tank is to be made in the form of a right circular cylinder to contain one quart of oil. What dimension of the can will require the least amount of materials. 7

OR

Find the asymptotes of the curve:

$x^2(x-y)^2-a^2(x^2+y^2)=0$

b) State and prove that Lagrange's Mean value theorem. What is its geometrical meaning? 8

3. Integrate any **THREE** of the following: 3×5

a) $\int \frac{x^3}{(x-2)(x-3)}dx$

b) $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^2 x\, dx$

c) $\int_{0} x^{3}\, dx$ (by summation method)

d) $\int \frac{dx}{4+5\sin x}$

a) Find the reduction formula for $\int \cos^{n}x\, dx$ and then evaluate $\int \cos^{7}x\, dx$

OR

Approximate the integral $\int_1^4 \frac{1}{1+x}\, dx$ with n = 4, using Trapezoidal and Simpson's rule.

b) Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x- axis and the line $y = x - 2$.

5. a) Find by vector method the equation of the plane through the points (2,4,5), (1,5,7) and (-1,6,8).

b) Define vector triple product. If $a = i - 2j - 3k$, $b = 2i + j - k$ & $c = i + 3j - 2k$ find $(a \times b) \times c$.

Also verify that $a \times (b \times c) = (a \cdot c)\, b - (a \cdot b)\, c$.

6. a) Define eccentricity of a conic section, and derive the equation of a hyperbola in its standard form. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

b) Find the condition for the line $y = mx + c$ to be tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

7. Write short notes on:

a). Find the radius of curvature of curve $y^2 = 4x$ at (0, 0)..

b) Find center and vertices of theconicsection $x^2 - y^2 - 2x + 4y = 4$.

c) Evaluate: $\int \frac{dx}{x + \sqrt{x}}$

d) Let two functions $f: R \to R$ and $g: R \to R$ defined as $f(x) = x + 2$, $g(x) = 3x^2$, $x \in R$. Find fog(x) and gof(x).

POKHARA UNIVERSITY

Level: Bachelor
Programme: BE
Course: Engineering Mathematics I

Semester: Fall

Year : 2016
Full Marks: 100
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) Prove that the continuity of a function at a point is the necessary but not the sufficient condition for the existence of the derivative of the function at that point. 8

OR

State Leibnitz theorem. If $y = e^{x^2}$, show that $y_{n+1} - 2xy_n - 2ny_{n-1} = 0$

b) State and prove Cauchy's mean value theorem. 7

2. a) Evaluate $\lim_{x \to 0}\left(\frac{Sinx}{x}\right)^{1/x^2}$ 7

b) An open tank of a given volume with a square base and vertical sides has to be constructed. Show that the amount of tin required will be minimum when the height of the tank is half the side of the square base. 8

OR

Find the asymptotes of the curve $x^2(x-y)^2 - a^2(x^2+y^2) = 0$.

3. Integrate **any three** of the following 3×5

a) $\int \frac{e^x(1+\sin x)}{1+\cos x}dx$

b) $\int \frac{1}{5-13\sin x}dx$

c) $\int_0^{\pi/4} \log(1+\tan\theta)d\theta$

d) $\int_0^{\infty} \frac{xdx}{(1+x^2)^2} dx$

4. a) Find the area of the region of the circle $x^2 + y^2 = 4$ cut off by the line $x - 2y = -2$ in the first two quadrants. 7

OR

Find the volume of the solid in the region bounded by the parabola $y = x^2$ and the line $y = 2x$ in the first quadrant about y-axis.

b) Find the approximate area using Simpson's and Trapezoidal rules for the area bounded by curve $y = x^2+4$, the x-axis and the lines $x=1$ and $x=4$ (using $n=6$) and compare these results with exact values. 8

5. a) Define eccentricity of a conic section, and derive the equation of a hyperbola in its standard form. 8

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

b) Find the centre, vertices, foci and equation of directrix of the ellipse $25x^2 + 9y^2 - 100x + 54y - 44 = 0$. 7

6. a) Find by vector method the equation of plane perpendicular to $4x-y+z=0$ and passing through $(3,2,1)$and$(1,2,3)$. 7

b) Define scalar triple product. Give its geometrical interpretation. If the vectors $2\vec{i} - \vec{j} + 2\vec{k}$, $5\vec{i} + \lambda\vec{j} + 2\vec{k}$ and $\vec{i} + 6\vec{k}$ are coplanar, Find the value of λ. 8

7. Attempt all 4×2.5

a) $\int \tan^{-1} x dx$

b) Find all horizontal and vertical asymptotes of $y = \frac{x^2 - 4}{x^2 - 1}$

c) Find the eccentricity foci and vertices of the hyperbola $9(x-2)^2 - 4(y+3)^2 = 36$

d) Evaluate $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^4 x \, dx$

POKHARA UNIVERSITY

Level: Bachelor | Semester: Spring | Year : 2016
Programme: BE | Full Marks: 100
Course: Engineering Mathematics I | Pass Marks: 45
Time : 3hrs.

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The figures in the margin indicate full marks.
Attempt all the questions.

1. a) Define continuity and differentiability of a function f(x) at x = a. Find 8
$f'(0)$ if it exists, where $f(x) = \begin{cases} x^2 \cos\frac{1}{x} & for\, x \neq 0 \\ 0 & for\, x = 0 \end{cases}$

OR

If $y = tan^{-1}x$, then show that
$(1 + x^2)y_{n+1} + (2nx)y_n + n(n-1)y_{n-1} = 0.$

b) State L 'Hopital Rule for indetermiante forms. 7
Evaluate $\lim_{x \to 0} \left(\frac{tanx}{x}\right)^{1/x^2}$.

2. a) State and prove Cauchy's Mean value Theorem. 8
b) A cylindrical tin can closed at both ends with given capacity has to be constructed. Show that the amount of tin required will be minimum when the height is equal to the diameter.

OR

Define asymptotes. Find the asymptotes of the curve
$(x^2 - y^2) - 2(x^2 + y^2) + x - 1 = 0$

3. Integrate (**Any three**) 3×5

a) $\int \frac{1}{5 - 13\sin x} dx$

b) $\int \frac{xe^x}{(x+1)^2} dx$

c) $\int_0^{\frac{\pi}{4}} \log(1 + \tan\theta)\, d\theta$

d) $\int_0^1 \sqrt{x}\, dx$, by using limit as a summation methods

4. a) Find the area bounded by $x^2 = 4y$ and $y = |x|$. 7

b) Use Trapezoidal and Simpson's rule, estimate the Integral 8

$\int_0^4 \frac{1}{x^2+4}\, dx$ with n = 6.

5. a) Derive the standard equation of ellipse with centre (0, 0). 8

b) Show that the line $lx + my + n = 0$ touches the hyperbola 7

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if $a^2l^2 - b^2m^2 = n^2$.

6. a) Find the equation of plane passing through (1, 2, 3), (3, 2, 1) which is 7

perpendicular to the plane 4x-y+2z=7. (Use vector method.)

b) Define vector triple product of three vectors. Derive the expression of 8

vector triple product of vectors.

7. Attempt all: 4×2.5

a) Find the radius of curvature of curve $y^2 = 4x$ at (0, 0)

b) Integrate $\int x\sin^2 x\, dx$

c) If $\vec{a} = i - 2\vec{j} + \vec{k}$ and $\vec{b} = \vec{i} - 2\vec{j} - \vec{k}$ find unit vector of $\vec{a} \times \vec{b}$

d) Find the arc length of the curve $y = x^2 + 1$ from x= 1 and x = 2.

POKHARA UNIVERSITY

Level: Bachelor | Semester: Fall | Year : 2017
Programme: BE | Full Marks: 100
Course: Engineering Mathematics I | Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1\. a) Examine the continuity and derivability at $x = 0$ *and* $x = \frac{\pi}{2}$ of the function $f(x) = \begin{cases} 1 & when \quad (-\infty, 0) \\ 1 + \sin x & when \quad x \in [0, \pi/2) \\ 2 + (x - \pi/2)^2 & when \quad x \in [\pi/2, \infty) \end{cases}$ 8

OR

State Leibnitz theorem for successive derivative of the product of two functions. If $y = \sin^{-1} x$ then show that

i. $(1 - x^2)\, y_2 - xy_1 = 0$

ii. $(1 - x^2)\, y_{n+2} - (2n + 1)xy_{n+1} - n^2 y_n = 0.$

b) State and prove Rolle's Theorem. Verify the theorem for the function $f(x) = \log\left(\frac{x^2 + ab}{(a+b)x}\right)$ in $[a, b]$ where $0 < a < b$. 7

2\. a) A cone is inscribed in a sphere of radius r, prove that it's volume as well as its curved surface is greatest when the altitude is $\frac{4r}{3}$. 8

OR

Find the asymptote to the curve $y^2x^2 - 3yx^2 - 5xy^2 + 2x^2 + 6y^2 - x - 3y + 2 = 0$.

b) Show that $\lim_{x \to \infty}\left(x - x^2 ln\left(1 + \frac{1}{x}\right)\right) = \frac{1}{2}$. 7

3. Evaluate *Any Three*

a) $\int \frac{\cos x\, dx}{\sqrt{2\sin^2 x + 3\sin x + 4}}$

b) $\int \frac{2\sin x + 3\cos x}{3\sin x + 4c\cos x} dx$

c) $\int_0^1 \frac{\log(1+x)\, dx}{1+x^2}$

d) Evaluate $\int_a^b e^{-x} dx$ by summation method.

4. a) Find the volume of the solid generated by revolving the region in the first quadrant bounded by the parabola $y=x^2$, below by the x-axis and on the right by the line $x=2$ about y-axis.

b) Find approximate values of $\int_2^5 (x^2+1)dx$ using Simpson's and Trapezoidal rules with $n = 6$. Also compare the results with exact value.

5. a) Find the plane through $A(1,1,1)$ and perpendicular to the line of intersection of the planes $2x+y+3z = 5$ and $3x+2y+z=7$.

b) Prove that the four points having position vectors $-\mathrm{i} + 2\mathrm{j} - 4\mathrm{k}, 2\mathrm{i} - \mathrm{j} + 3\mathrm{k}, 6\mathrm{i} + 2\mathrm{j} - \mathrm{k}$ and $-12\mathrm{i} - \mathrm{j} - 3\mathrm{k}$ are coplanar.

6. a) Define conic section by their eccentricity and classify them. Derive standard equation of parabola $y^2 = 4ax$.

b) Find the condition that the line $y = mx + c$ may be tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

7. Attempt all 4×

a) Find the vertical asymptotes to the curve $x^2 + xy + 4y + 3 = 0$

b) Find the radius of curvature at the origin of the curve $x^3+y^3=3axy$

c) Evaluate $\int x^5 e^x dx$

d) Transform to parallel axis through the point $(3, -4)$ the equation $x^2 - y^2 + 2x - 3y = 0$.

POKHARA UNIVERSITY

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Course: Engineering Mathematics I

Semester: Spring

Year : 2017
Full Marks: 100
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Examine continuity and differentiability at $x = 2$ of the function 8

$$f(x) = -2 + 3x - x^2 \quad \text{when } 0 \le x \le 2$$
$$= 2 - x \quad \text{when } 2 < x < 4$$

OR

State Leibniz theorem. If $y = a\cos(\log x) + b\sin(\log x)$, show that $x^2 y_{n+2} + (2n+1)\,x\,y_{n+1} + (n^2+1)\,y_n = 0$.

b) State and prove Lagrange's mean value theorem. Verify it for $f(x)=x^2+3x+2$ at $[0,2]$ 7

2. a) State L'Hospital theorem and evaluate the limit: 7

$$\lim_{x \to 0} (\cos x)^{\cot^2 x}$$

b) Find the asymptotes of the curve, $x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$. 8

3. Integrate *any three* 3×5

a) $\int \frac{dx}{13+3\cos x+4\sin x}$

b) $\int_0^a \frac{dx}{x+\sqrt{a^2-x^2}}$

c) $\int_0^{\pi/2} \log(\cos\theta).dx$

d) $\int_0^2 x^2 dx$. (by summation method)

4. a) Find the area bounded between the curve $y = x^2+1$ and the line $x - y + 3 = 0$ 7

OR

Find the volume of paraboloid formed by revolving the parabola $y^2=4x$ and the line $x=1$ about x-axis.

b) Evaluate; $\int_0^{\pi} \sin x \, dx$ by using trapezoid rule, simpson's rule and compare the result with the exact value taking $n = 6$.

5. a) Find the centre, vertices, eccentricity and foci of the ellipse $9x^2 + 6y^2 + 18x - 96y + 9 = 0$.

b) Find the equation of tangents to the hyperbola $3x^2-4y^2=12$, which are perpendicular to the line $y=x+2$.

6. a) Find by vector method the equation of the plane through A (2,1,-1) and perpendicular to the line of intersection the planes $2x + y - z = 3$, $x + 2y + z = 2$

b) Define Scalar and Vector Triple Product. If $[\vec{a}, \vec{b}, \vec{c}] = 0$, show that $[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = 0$.

7. Attempt all questions

a) Find the radius of curvature of $x = r\cos\theta$, $y = r\sin\theta$

b) Find the arc length of the curve $y = x^2$ from $x = -1$ to $x = 2$

c) Find the center, vertex of the hyperbola $9(x-2)^2 - 4(y+3)^2 = 36$

d) Determine the value of λ, so that $\vec{a} = 2\vec{i} + \lambda\vec{j} + \vec{k}$ and $b = 4\vec{i} - 2\vec{j} - 2\vec{k}$ are perpendicular.

POKHARA UNIVERSITY

Level: Bachelor
Programme: BE
Course: Engineering Mathematics I

Semester: Fall

Year : 2018
Full Marks: 100
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) Prove that the differentiability of a function at a point implies the continuity of the function at that point. Give an example to show that the converse may not be true. 8

OR

If $y = a\cos(\log x) + b\sin(\log x)$ show that

i. $x^2y_2 + xy_1 + y = 0$ and

ii. $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$.

b) State and prove Lagrange's Mean value theorem. 7

Show that $\frac{b-a}{b} < \log(\frac{b}{a}) < \frac{b-a}{a}$ by using Lagrange's mean value theorem.

2. a) Evaluate $\lim_{x\to0}(\cos x)^{\cot^2x}$ 7

b) Find the asymptotes of the curve $x^2(x-y)^2 - a^2(x^2+y^2) = 0$. 8

OR

A square piece of tin of side 18 cm is to be made into a box without lid, cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the value of box is maximum possible?

3. Evaluate the following integrals (Any three) 5×3

a) $\int \frac{x^3}{(x-2)(x-3)}dx$

b) $\int \frac{1}{2+\cos x+\sin x}dx$

c) $\int_a^b e^{-x}\,dx$ by summation method

d) $\int_0^{\frac{\pi}{2}} \frac{x\,dx}{\sin x+\cos x} = \frac{\pi}{2\sqrt{2}} \log(\sqrt{2}+1)$.

4. a) Find the volume of the solid in the region in the first quadrants bounded by the parabola $x = \sqrt{y}$ and the line y=x is revolved about y-axis. 8

b) Find approximate value of $\int_0^3 (x^2+1)\,dx$ by Simpson's and Trapezoidal Rule with n = 6. Compare the result with exact value 7

5. a) Find the condition that the line $lx + my + n = 0$ may be a tangent to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. 8

b) Define conic section and derive the standard equation of Ellipse. 7

6. a) Find the equation of the plane through the points (2,4,5) and perpendicular to the line $\frac{x-5}{1} = \frac{y-1}{3} = \frac{z}{4}$ by vector method. 8

b) Define vector triple product. If

$\vec{a} = \vec{i} - 2\vec{j} - 3\vec{k}$, $\vec{b} = 2\vec{i} + \vec{j} - \vec{k}$ and $\vec{c} = \vec{i} + 3\vec{j} - 2\vec{k}$

Also verify that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}\,.\,\vec{c})\,\vec{b} - (\vec{a}\,.\,\vec{b})\,\vec{c}$. 2.5+4

7. Attempt all questions.

a) Find the radius of curvature at any point (r, θ) for the curve $r = ae^{\theta\cot\alpha}$.

b) Find the center, vertices and foci of the ellipse

$$x^2 + 10x + 25y^2 = 0$$

c) Evaluate $\int \frac{x}{(x-3)(x+1)}\,dx$

d) Find the value of p so that the vectors

$\vec{a} = 2\vec{i} - \vec{j} + \vec{k}$, $\vec{b} = \vec{i} + 2\vec{j} + 3\vec{k}$ and $\vec{c} = 3\vec{i} + p\vec{j} + 5\vec{k}$ are coplanar.

POKHARA UNIVERSITY

Level: Bachelor
Programme: BE
Course: Engineering Mathematics I

Semester: Spring

Year : 2018
Full Marks: 100
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) Show that the function $f(x)$ defined by 8

$$f(x) = \begin{cases} -x \text{ when } x \le 0 \\ x \text{ when } 0 < x < 1 \\ 2 - x \text{ when } x \ge 1 \end{cases}$$

is continuous at $x = 0$ and $x = 1$, but is not differentiable at $x = 1$.

OR

If $y=\sqrt{\frac{1+x}{1-x}}$ prove that

(i) $(1-x)y^2 = 1+x$

(ii) $(1-x^2)y_n - \{2(n-1)x+1\}y_{n-1} - (n-1)y_{n-2} = 0$

b) State and prove that Cauchy's Mean Value theorem. Is the theorem applicable to the functions $f(x)= x$ and $g(x)= x^2-2x$ in the interval [0,2]? Why? 7

2.
a) Evaluate: $\lim_{x \to 0} \left(\frac{1}{x^2}\right)^{\tan x}$ 7

b) A cone is inscribed in a sphere of radius r, prove that it's volume as well as its curved surface is greatest when the altitude is $\frac{4r}{3}$ 8

OR

Find the asymptote to the curve $x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$

3. Integrate (Any Three): 3×5

a. $\int \frac{(x+2)}{\sqrt{4x-x^2}}dx$

b. $\int \frac{1}{1-\cos x+\sin x}dx$

c. Prove: $\int_0^1 \cot^{-1}(1-x-x^2)dx = \frac{\pi}{2} - \log 2$

d. $\int_0^1 \sqrt{x}dx$ by summation method.

4. a) Find the volume of the solid generated by revolving the region bounded by y = $\sqrt{x}$ and the lines y=1,x=4 about the line y=1. 7

b) Approximate the area by using Trapezoidal and Simpson's rule to the integral $\int_1^4 \frac{1}{1+x}dx, n = 6$. Also compare with exact. 8

5. a) Define hyperbola. Derive the standard equation of hyperbola. 7

b) Find the condition that the line $y = mx + c$ may touch the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Also find the point of contact. 8

6. a) Define scalar and vector product of three vectors. Prove that the scalar triple product of three vectors represent the volume of parallelepiped. What conclusion can be drawn if $\vec{a}.(\vec{b} \times \vec{c}) = 0$? 7

b) Show that the vectors $\vec{a} \times (\vec{b} \times \vec{c}), \vec{b} \times (\vec{c} \times \vec{a})$ and $\vec{c} \times (\vec{a} \times \vec{b})$ are coplanar. 8

7. Attempt all the questions: 2.5×4

a) Find the radius of curvature at (s,ψ) $for\ the\ curve\ s = 8a\sin^2\frac{\psi}{6}$

b) Find centre, vertices and foci of the ellipse: $x^2+10x+25y^2=0$

c) Find the volume of a parallelepiped whose concurrent edges are represented by $\vec{i}+\vec{j}+\vec{k},\ 2\vec{i}+\vec{j}-2\vec{k}$ and $3\vec{i}+2\vec{j}-\vec{k}$.

d) $\int x^3 \log x\, dx$.

POKHARA UNIVERSITY

Level: Bachelor
Semester: Fall
Programme: BE
Course: Engineering Mathematics I

Year : 2019
Full Marks: 100
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) State Leibnitz's theorem for successive derivative of product of two functions y = u.v If $y=\left(x+\sqrt{1+x^2}\right)^m$, show that $\left(1+x^2\right)y_{n+2}+\left(2n+1\right)xy_{n+1}+\left(n^2-m^2\right)y_n=0$ 8

OR

Show that the function $f(x)=\begin{cases} x & for \quad x<1 \\ 2-x & for \quad 1\le x\le 2. \\ x-\dfrac{x^2}{2} & for \quad x>2 \end{cases}$

is continuous at x = 1 & x=2. Does $f^1(x)$ exists at these points.

b) State Rolle's theorem with its geometrical meaning. Also verify the theorem for the function $f(x)=(x-a)^m(x-b)^n$ 7

2. a) State L'Hospital rule. Prove that: $\lim_{x\to 0}(cotx)^{sin2x}=1$. 7

b) Find the asymptotes of given curve $x^2(x-y)^2-a^2(x^2+y^2)=0$ 8

OR

Find the total surface area of the right circular cylinder of greatest surface that can be inscribed in a given sphere of radius r.

3. Integrate (Any Three) 15

a) $\int\dfrac{dx}{4+5\sin x}$

b) $\int_0^a\dfrac{dx}{x+\sqrt{a^2-x^2}}$

c.) $\int_0^1 \frac{dx}{(1-x^6)^{\frac{1}{6}}}$

d) $\int_0^2 x^2 dx$. (by summation)

4. a) Find the area between the curves $x = y^2$ and $2y^2 = -x+3$

OR

Show that the volume of sphere of radius r is $\frac{4}{3}\pi r^3$

b) Find the approximate area using Simpson's and Trapezoidal rule for the area bounded by the curve $y = \sin x$, the x- axis and the lines $x = \pi/2$ and $x = 2\pi$ (using n = 6) and compare these results with exact value.

OR

Obtain the reduction formula for $\int_{\pi/4}^{\pi/2} \cot^n x dx$ and evaluate $\int_{\pi/4}^{\pi/2} \cot^7 x dx$

5. a) Define eccentricity of a conic section, and derive the equation of a hyperbola in its standard form. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ 7

b) Find the equation of tangent to the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$, which is parallel to the line x=y+5. 8

6. a) Define scalar and vector product of three vectors. Prove that the scalar triple product of three vectors represent the volume of parallelepiped. What conclusion can you draw about these vectors if $\vec{a}.(\vec{b}\times\vec{c}) = 0$. 7

b) Find the equation of the plane through (1, 0, -1) and (-1, 2, 1) and parallel to the line of intersection of the planes $3x + y - 2z = 0$ and $4x - y + 3z = 0$ 10

7. Attempt All questions

a) Find the domain and range for $y = \sqrt{4 - x^2}$

b) Find the equation of parabola having focus (-3,0) and directrix x+5=0

c) If $\vec{a} = i - 2\vec{j} + \vec{k}$ and $\vec{b} = \vec{i} + 2\vec{j} - \vec{k}$ find the projection of $\vec{a}$ on $\vec{b}$

d) Evaluate $\int \tan^{-1} x$

POKHARA UNIVERSITY

Level: Bachelor
Programme: BE

Semester – Spring

Year : 2019
Full Marks: 100
Pass Marks : 45
Time : 3hrs.

Course: Engineering Mathematics I

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) Test the continuity and differentiability of the function 8

$$f(x) = \begin{cases} x & for \quad 0 < x < 1 \\ 2 - x & for \quad 1 \le x \le 2 \\ x - \frac{x^2}{2} & for \quad x > 2 \end{cases}$$

at x = 1 and x = 2.

OR

If $y = (\sin^{-1} x)^2$, show that

i. $(1 - x^2)\, y_2 - xy_1 - 2 = 0$

ii. $(1 - x^2)\, y_{n+2} - (2n+1)\, xy_{n+1} - n^2 y_n = 0$

b) State and prove Lagrange's Mean value theorem. Write its geometrical meaning. Also state how does it differ from Roll's theorem 7

2. a) A conical tank of the given capacity has to be constructed. Find the ratio of the height to the radius of the base for the minimum amount of the canvas required for the tent. 8

OR

Define asymptotes of the curves with different types. Find asymptotes of

$x^3+3x^2y-xy^2-3y^3+x^2-2xy+3y^2+4x+5$

b) Evaluate $\lim_{x \to e} (\log x)^{\frac{1}{1-\log x}}$. 7

3. Integrate (any three) 5

a) $\int \frac{dx}{2 + \cos x + \sin x}$

b) $\int_1^2 \frac{dx}{(x+1)\sqrt{x^2-1}}$ 5

c) $\int_0^1 \frac{\log(1+x)dx}{1+x^2}$ 5

d) $\int_a^b sinxdx$ by using summation method

4. a) Find the area of the region in the first quadrant bounded by the line y=x, x=2 and the curve $y=1/x^2$ and the x- axis. 7

OR

Find the volume of the solid revolution of the triangular region bounded by 2x+3y=6, y=x and x=0 about the x- axis.

b) Find approximate area bounded by given curves $y=\sqrt{x}+3$ from x = 1 to x = 4,by using Simpson's and Trapazoidal rule with n = 4. Compare these values with exact value. 8

5. a) Find by vector method the equation of the plane which passes through the points (1, 1, -1), (2, 0, 2) and (0, -2, 1). 3

b) What does scalar triple product give ? Also prove $\vec{a}$ x($\vec{b}x\vec{c}$) = $(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}$ 7

6. a) Define eccentricity of a conic section and classify conic sections. Find the condition that the line lx + my + n = 0 may be a tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. 8

b) Derive the equation of the ellipse in its standard form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. 7

7. **Attempt all :** 2.5 x4

a) Find domain and range of the function $y=\sqrt{9-x}$.

b) Find radius of curvature of the curve x= a(θ+ sinθ), y= a(1-cosθ) at θ = 0.

c) What will be the equation of the curve $x^2+y^2-10x-12y+36=0$ if the origin is shifted to (5, 6) ?

d) If $\vec{a}=\vec{c}+2\vec{j}+\vec{k}$, $\vec{b}=\vec{i}+\vec{j}+\vec{k}$, find projection of $\vec{a}$ on $\vec{b}$.