POKHARA UNIVERSITY

Level: Bachelor
Programme: BE
Course: Calculus I

Semester: Fall

Year : 2022
Full Marks: 100
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Define continuity and differentiability of a function. Show that Differentiability of a function f(x) at x=a implies continuity but Converse may not be always true. 7

OR

If $\log y = \tan^{-1}x$, show that

i) $(1+x^2)y_2+(2x-1)y_1=0$

ii) $(1+x^2)y_{n+2}+(2nx+2x-1)y_{n+1}n(n+1)y_n=0$

b) State and prove Lagrange's Mean value theorem. Interpret it geometrically. 8

2. a) Find the asymptotes of the curve: 8

$x^3+3x^2y-xy^2-3y^3+x^2-2xy+3y^2+4x+5=0$

b) Find the perimeter of the asteroid: $x^{2/3}+y^{2/3}=a^{2/3}$ 7

3. Integrate (Any **Three**) of the following: 3×5

a) $\int \frac{x^3}{(x-2)(x-3)}dx$

b) $\int \frac{1}{4-5\sin x}dx$

c) $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}$

d) $\int_0^{\frac{\pi}{2}} \sin^3 x\cos^4 x\,dx$

4. a) Find the volume of the solid in region bounded by the curve $y=x^2+1$ and the line $y=-x+3$ revolved bout the x-axis. 8

b) State and prove Euler's theorem on homogeneous function of two independent variables of degree n. 7

If $u=\cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+\frac{1}{2}\cot u=0.$

5. a) Find the extreme values of the function $f(x,y,z)=x^2+y^2+z^2$ subject to the constraints $x+y+z=3a$. 7

b) Show that the substitution $y=y_1+u$ where y_1 is a solution of Riccati's equation, reduces the Riccati's equation to a Bernoulli's equation. 8

6. a) Find the general solution of the differential equation 7

$y''-y'-2y=3e^{2x}$, $y(0)=0$, $y'(0)=-2$

OR

Solve Second order differential equation of the series RLC circuit

$L\frac{d^2Vc}{dt^2}+R\frac{dVc}{dt}+\frac{1}{C}Vc=\frac{Vin}{C}$, where

$R=10\Omega$, $L=1$ H, $C=16\times10^{-4}F$ $V_{in}=0$, $V_C(0)=6V$, $V_c'(0)=6A$

b) Find the general solution of the differential equation by using method of variation of Parameters: $y''+9y=\text{cosec}3x$. 8

7. Attempt all the questions: 4×2.5

a) Find y_n if $y=x^n$, where n is positive integer

b) Find the radius of curvature of the curve $y^2=4x$ at (0,0).

c) Show that the function $f(x,y)=x^3+y^3-3xy$ has a saddle point at (0,0).

d) Solve: $\frac{dy}{dx}+\frac{1-\cos 2y}{1-\cos 2x}=0$