Vector Notes

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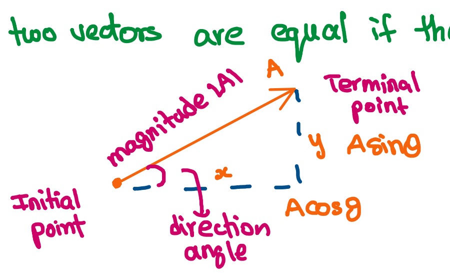
PHYSICAL QUANTITIES

SCALARS	VECTORS
(has magnitude only)	(has magnitude and direction)
E.g:- speed, pressure, temperature, energy, etc	E.g:- velocity, acceleration, force, displacement, etc.

- two vectors are equal if they have the same direction and magnitude.



- if initial point = (a, b) and terminal point = (c, d), then the resultant vector will be $< c-a, d-b >$
- a vector can be resolved into horizontal and vertical components.
- 1D vectors cannot be resolved into components
- representation of a vector quantity → $\vec{A} = A_x\hat{i} + A_y\hat{j}$, where $A_x = A\cos\theta$, and $A_y = A\sin\theta$
- $|A| = \sqrt{A_x^2 + A_y^2}$, and $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$ to find θ

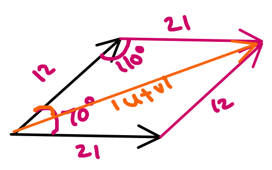
NEGATIVE OF A VECTOR :- a negative vector is a vector going in the opposite direction. $(v + (-v) = 0)$

ADDITION OF VECTORS :- • head to tail rule
• parallelogram law

PROPERTIES OF VECTOR ADDITION :- • commutative law $(\vec{a} + \vec{b} = \vec{b} + \vec{a})$
• associative law $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$

- to find the vector sum of two vectors using only their magnitudes and the angle between them, you use cosine rule.

Q- $|\vec{u}| = 12$
$|\vec{v}| = 21$
angle = 70°



$a^2 = b^2 + c^2 - 2bc\cos\theta$

$|\vec{u} + \vec{v}| = \sqrt{12^2 + 21^2 - 2(12)(21)\cos 110°}$

$|\vec{u} + \vec{v}| = 27.5$

- to find angle you use sin rule.

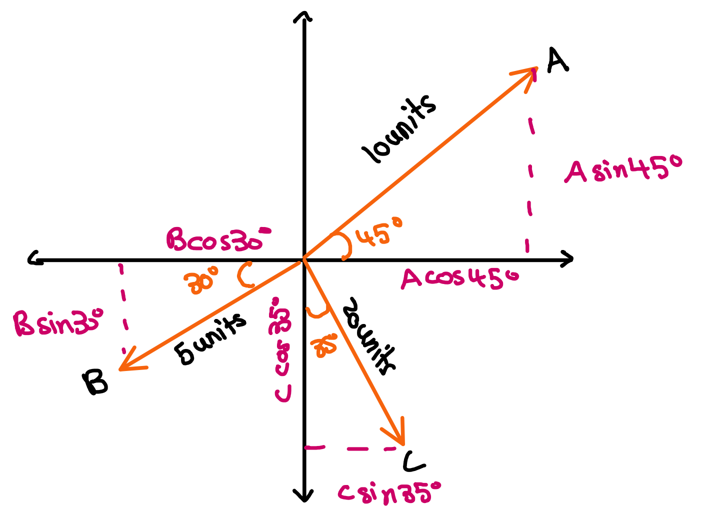
$\frac{x}{\sin x} = \frac{y}{\sin y} \longrightarrow \frac{27.5}{\sin 110} = \frac{21}{\sin y}$

$y = 46°$, 46° to $\vec{u}$

SUBTRACTION OF VECTORS :- $\vec{d} = \vec{a} + (-\vec{b})$

RESULTANT VECTOR BY COMPONENTS

Q-



$$\boxed{\vec{R} = R_x\hat{i} + R_y\hat{j}}$$

$R_x = 10\cos 45° - 5\cos 30° + 20\sin 35°$

$R_y = 10\sin 45° - 5\sin 30° - 20\cos 35°$

$\vec{R} = 14.2i - 11.8j$

$|R| = 18.46$

UNIT VECTOR :- • has magnitude of exactly 1, and points in a particular direction.
- unit vector notation → $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$
- $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$
- unit vector → $\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{|\vec{r}|}$

MULTIPLICATION OF VECTORS

SCALAR MULTIPLICATION :- • multiplying a vector with a real number c
- means multiplying its magnitude by c
- a neg real number reverses the direction
- means to take a vector and add together that many times.
- COMMUTATIVE LAW $(m\vec{A} = \vec{A}m)$
- ASSOCIATIVE LAW $(m(n\vec{A}) = \vec{A}(mn))$
- DISTRIBUTIVE LAW $((m+n)\vec{A} = m\vec{A} + n\vec{A})$
  $(m(\vec{A} + \vec{B}) = m\vec{A} + m\vec{B})$

DOT PRODUCT :- • $A \cdot B = AB\cos\theta$
- two vectors with an angle between them, and we wish to find the projection of one onto the direction of the other
- if the two vectors are perpendicular, then the dot product = 0
- $A \cdot B = A_xB_x + A_yB_y + A_zB_z$
- dot product maximum when $\theta = 0°$
- can use θ or $360 - \theta$ as cosine of both angles is same
- is commutative $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$
- dot product can be regarded as product of magnitude of one vector and scalar component of second vector along the direction of first vector

CROSS PRODUCT :- • $A \times B = AB\sin\theta\,\hat{n}$
- $\hat{n}$ is a unit vector perpendicular to both $\vec{A}$ and $\vec{B}$, defined by the right hand rule
- θ is the smallest of the two angles between the two vectors as sin of θ and $360 - \theta$ are different
- cross product of $\vec{A}$ and $\vec{B}$ produces a third vector $\vec{C}$ with the magnitude $AB\sin\theta\,\hat{n}$
- the direction of $\vec{C}$ is perpendicular to both A and B, determined using right hand rule
- if A and B are parallel, then cross product = 0
- if A and B are perpendicular, then cross product is maximum
- not commutative → $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$
- distributive → $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$
- $\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{i} - (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$

ADDITIONAL NOTES :- • area of parallelogram = $|A \times B|$, where A and B are any 2 sides
- area of triangle = $\frac{1}{2} \times |A \times B|$, where A and B are any 2 sides

Q- Two sides of a Δ expressed as $\vec{A} = 5\hat{i} - 4\hat{j} + 3\hat{k}$ and $\vec{B} = 3\hat{i} - 2\hat{j} - \hat{k}$. Calculate area

area = $\frac{1}{2}|\vec{A} \times \vec{B}|$

$$\vec{A} \times \vec{B} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -4 & 3 \\ 3 & -2 & -1 \end{bmatrix}$$

$= [(-4 \times -1) - (-2 \times 3)]\hat{i} - [(5 \times -1) - (3 \times 3)]\hat{j} + [(5 \times -2) - (3 \times -4)]\hat{i}$

$= 10\hat{i} + 14\hat{j} + 2\hat{k}$

$|\vec{A} \times \vec{B}| = \sqrt{10^2 + 14^2 + 2^2} = 10\sqrt{3}\ m^2$

area $= \frac{1}{2} \times 10\sqrt{3} = 5\sqrt{3}\ m^2$