Oscillation Notes

24/10/2023 8:47 PM

OSCILLATORY MOTION:- motion that is periodic in time, for example, power lines oscillations due to wind, earthquake oscillations, etc.
(repeats itself in a regular pattern over time)

SIMPLE HARMONIC MOTION:-

- acceleration $\propto$ displacement from equilibrium position (particle oscillates back and forth about a stable equilibrium position under restoring force)
- acceleration and displacement are in opposite directions
- e.g $\longrightarrow$ pendulum, bob attached to spring, plucked guitar string, etc
- $F_s = -kx$

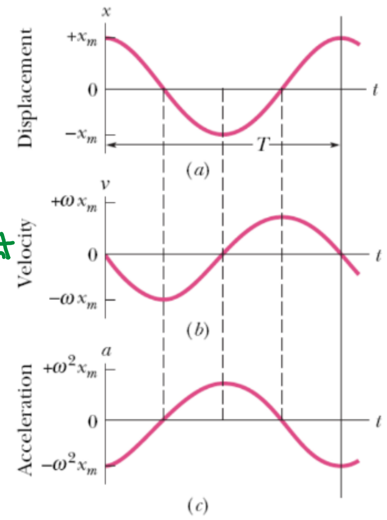
  $-kx = ma$

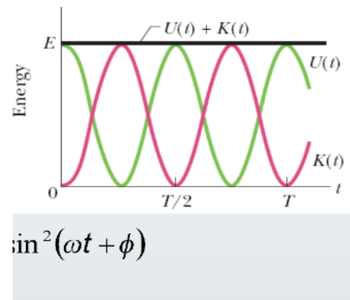
  $a = -\frac{k}{m}x$

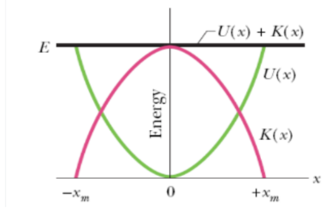
  $a = -\omega^2 x$
- frequency $\longrightarrow$ num of oscillations per second
- $T = \frac{1}{f}$
- $x(t) = x_m \cos(\omega t + \phi)$
- $v(t) = -x_m \omega \sin(\omega t + \phi)$
- $a(t) = -x_m \omega^2 \cos(\omega t + \phi)$
- $\omega = \frac{2\pi}{T} = 2\pi f$
- $\omega = \sqrt{\frac{k}{m}}$

speed at certain displacement

- $v = \omega\sqrt{A^2 - x^2}$
- v_{max} at equilibrium
  0 at extreme



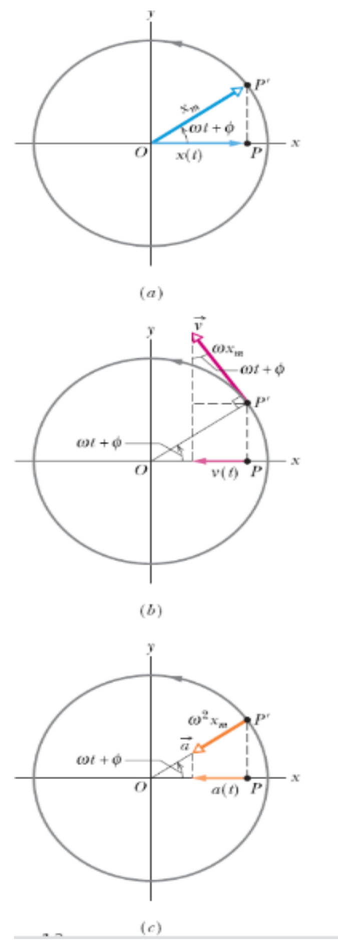




- $U(t) = \frac{1}{2}kx^2 = \frac{1}{2}kx_m^2 = \frac{1}{2}kx_m^2\cos^2(\omega t + \phi)$
- $K(t) = \frac{1}{2}mv^2 = \frac{1}{2}m x_m^2 \omega^2 \sin^2(\omega t + \phi) = \frac{1}{2}k x_m^2 \sin^2(\omega t + \phi)$
- $E = U + K = \frac{1}{2}k x_m^2$

PROPERTIES OF SHM:-

- acc $\propto$ - displacement
- displacement from equilibrium position, velocity, & acceleration all vary sinusoidally with time but are not in phase
- f and T of motion are independent of amplitude



SHM & UNIFORM CIRCULAR MOTION:-

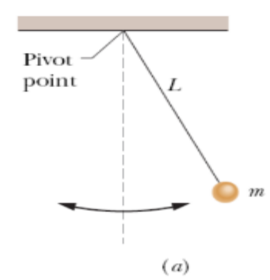
- SHM is projection of uniform circular motion on diameter of circle in which cm occurs
- UCM can be considered as combination of 2 SHMs, one along x-axis and one along y-axis, with phase difference of 90°

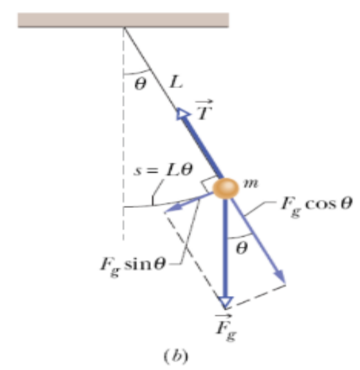
PENDULUMS:-

- $\tau = -L(F_g \sin\theta) = I\alpha$
- $\alpha = \frac{-mgL}{I}\theta$
- $T = 2\pi\sqrt{\frac{L}{g}}$

where I = inertia, L = length, α = angular acceleration, θ = angular displacement

true for small θ





DAMPED OSCILLATIONS:-

- motion of the oscillator is reduced by an external force
- $x(t) = x_0 e^{-\frac{bt}{2m}}\cos(\omega' t + \phi)$
- $\omega' = \sqrt{\omega_0^2 - \frac{b^2}{4m^2}} = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$
- $b > 2m\omega_0 \longrightarrow$ overdamped
- $b = 2m\omega_0 \longrightarrow$ critical damping
- $b < 2m\omega_0 \longrightarrow$ underdamped (should always be $<$ to find T)
- R (retarding force) $= -bv$
- $\Sigma F_x = -kx - bv = ma_x$
- $x = Ae^{-\frac{bt}{2m}}\cos(\omega' t + \phi)$

RESONANCE:-

- when oscillator subjected to external, periodic force $\longrightarrow$ forced oscillation
- when a system is disturbed by a periodic driving force with freq = natural freq of system, the system will oscillate with large amp, resonance occurs.
- $x(t) = A\cos(\omega_e t + \delta)$
- $A_0 = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega_e^2)^2 + \left(\frac{b}{m}\omega_e\right)^2}}$

  where ω_e = angular freq, ω_0 = natural freq, δ = phase angle
- $\tan\delta = \frac{b}{m}\frac{\omega_e}{\omega_0^2 - \omega_e^2}$, $\omega_0 = \sqrt{\frac{k}{m}}$