Assignment#02

24/10/2023 9:37 AM

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QUESTION#01

$x = 4\cos(3\pi t + \pi)$

a) $2\pi f = 3\pi$ $\quad T = \frac{2\pi}{\omega} = \frac{2\pi}{3\pi} = 0.667s$

$f = 1.5Hz$

b) amp = 4m

c) phase constant = π rad

d) $x = 4\cos(3\pi(0.28) + \pi)$

$x = 2\sqrt{2}$

$x = 2.83m$

QUESTION#02

a) $\frac{12}{5} = 2.4s$

b) $f = \frac{1}{2.4} = 0.417\ Hz$

c) $\omega = \frac{2\pi}{T} = \frac{5\pi}{6} = 2.62\ rad/s$

QUESTION#03

a) $\omega = \sqrt{\frac{k}{m}}$

$\omega^2 m = k$

$\left(\frac{2\pi}{0.25}\right)^2 (0.2) = k$

$k = 126.3\ N/m$

b) $E = \frac{1}{2}kx_m^2$

$2 = \frac{1}{2}(126.3)x_m^2$

$x_m = 0.178m$

QUESTION#04

mass = 2kg

force for rest = 20N

displacement from equilibrium = 0.2m

initial position = 0.2m

$g = 9.81 m/s^2$

a) $f = -kx$

$k = \frac{f}{x} = \frac{20}{0.2} = 100\ N/m$

b) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100}{2}} = \sqrt{50} = 7.07\ rad/s$

$\omega = 2\pi f$

$f = \frac{\sqrt{50}}{2\pi} = 1.125\ Hz$

c) $v_{max} = x_m\omega$

$V_{max} = 0.2 \times \sqrt{50} = \sqrt{2} = 1.414\ m/s$ (v_{max} occurs at equilibrium position)

d) $a_{max} = x_m\omega^2$

$a_{max} = 0.2(\sqrt{50})^2 = 10\ m/s^2$ (a_{max} occurs at extreme positions)

e) $E = \frac{1}{2}kx_m^2$

$E = \frac{1}{2}(100)(0.2)^2 = 2J$

f) $v = \omega\sqrt{A^2 - x^2}$

$v = \sqrt{50}\sqrt{0.7^2 - \left(\frac{0.3}{2}\right)^2}$

$v = 1.33\ m/s$

g) $a = x_m\omega^2$

$a = (0.0667)(\sqrt{50})^2$

$a = 3.335\ m/s^2$

QUESTION#05

mass = 2kg

$F = 3\sin(2\pi t)$

$k = 20\ N/m$

a) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{20}{2}} = \sqrt{10} = 3.16\ rad/s$

$\omega = \frac{2\pi}{T}$

$T = \frac{2\pi}{\sqrt{10}} = 1.987\ s$

b) $A = \frac{F/m}{\sqrt{(\omega_0^2 - \omega^2)^2 + \left(\frac{b}{m}\omega\right)^2}}$

$A = \frac{3/2}{\sqrt{(10)^2 - (3.16)^2)^2}}$

$A = 0.0509\ m$

QUESTION#06

a) $\omega' = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$

$2\pi f = \sqrt{\frac{2.05\times10^4}{10.6} - \left(\frac{3}{2(10.6)}\right)^2}$

$f = 7.04Hz$

b) $e^{-\frac{bT}{2m}} = e^{-\frac{3(0.14332)}{2\times10.6}}$

$\left(1 - e^{-\frac{3(0.14332)}{2(10.6)}}\right) \times 100 = 2.01\ \%$

c) $E = \frac{1}{2}k\left(x_m e^{-\frac{bt}{2m}}\right)^2$

$0.05\left(\frac{1}{2}kx_m^2\right) = \frac{1}{2}kx_m^2 e^{-\frac{bt}{m}}$

$0.05 = e^{\frac{-3(t)}{10.6}}$

$\ln(0.05) = \frac{-3t}{10.6}$

$\frac{-\ln(0.05)\times(10.6)}{3} = t$

$t = 10.6s$

QUESTION#07

$v = -x_m\omega\sin(\omega t + \phi)$

$4 = -5\sin(\omega(0) + \phi)$

$\sin^{-1}\left(\frac{-4}{5}\right) = \phi$

$\phi = -0.927\ rad$

QUESTION#08

m = 2kg

U = 2J at x = 20cm

U = 0.5J at x = 10cm

$U(x) = bx^2$

a) $2 = b(0.2)^2$

$b = 50\ J/m^2$

$0.5 = b(0.1)^2$

$b = 50\ J/m^2$

$M.E = \frac{1}{2}mv^2$ initial M.E at x = 0 cm

$M.E = \frac{1}{2}(2)(0.85)^2$

$M.E = 0.7225J$

$P.E = bx^2$ P.E at x = 15 cm

$P.E = (50)(0.15)^2$

$P.E = 1.125J$

$E = K.E + P.E$ initial E at x = 0

$E = 0.7225 + 0 = 0.7225J$

since total energy at equilibrium < potential energy at x = 15 cm, the particle will turn back before it reaches x = 15cm

b) $P.E_{max} = bx^2$

$0.7225 = 50x^2$

$x = 0.12m \rightarrow 12cm$