IEOR HW#3

① a) $Cimp(T_{old}) - Cimp(T_{new})$

$$= \sum_{n=1}^{m} N Q_n(T_{old}) - \sum_{m+1}^{m+1} \hat{N}_m \hat{Q}_m(T_n) = \Delta$$

$$= \sum_{1}^{m} N_m \left(\frac{1}{N_m}\right) \sum (y - \hat{y}_m)^2 - \sum_{m+1}^{m+1} (y_i - \hat{y}_m)^2$$

$$= \sum_{i=R} (y_i - \hat{y}_m)^2 - \sum_{n=m}^{m+1} (y_i - y_m)^2$$

$$\Delta = \sum (y_i - \hat{y}_m)^2 - \sum (y_i - \hat{y}_m)^2 - \sum (y_i - \hat{y}_m)^2$$

$$\boxed{\Delta = \sum_{i=x \in M_{old}} \left(y_i - \frac{1}{N_m} \sum y_i\right)^2 - \sum \left(y_i - \frac{1}{N_m} \sum y_i\right)^2 - \sum \left(y_i - \frac{1}{N_{m+1}} \sum y_i\right)^2}$$

b) $\Delta = \left[\sum (y_i - \hat{y}_m)^2\right] - \left[\sum (y_i - \hat{y}_m)^2\right] - \left[\sum (y_i - \hat{y}_{m-1})^2\right]$

$$= \left[\sum_{i=m_i \in m} (y_i - \hat{y}_m)^2 + \sum (y_i - \hat{y}_m)^2 - \left[\sum (y_i - \bar{y}_m)^2\right]\right.$$

$$- \left[\sum (y_i - y_{m+1})^2\right]$$

$$\underbrace{\sum (y_i - \hat{y}_m)^2 - \left[\sum (y_i - \bar{y}_m)^2\right]}_{\min \hat{R}_m = \hat{y}_m \geq 0} + \underbrace{\sum (y_i - \hat{y}_m)^2 - \left[\sum (y_i - \bar{y}_{m+1})^2\right]}_{\min \hat{R}_{m+1} = \hat{y}_{m+1} \geq 0}$$

(c) $C_\alpha(T_{new}) = C_{imp}(T_{new}) + \alpha SST \cdot |T_{new}|$

$C\alpha(T_{old}) = C_{imp}(T_{old}) + \alpha SST \cdot |T_{old}|$

$= C_{imp}(T_{old}) + \alpha \sum_{i=1}^{n} (y_i - \hat{y})^2 \cdot |T_{old}|$

$= C_{imp}(T_{old}) + \alpha \sum (y_i - \hat{y})^2 - M$

$C\alpha(T_{old})\ C\alpha(T_{new}) = C_{imp}(T_{old}) - C_{imp}(T_{new}) - \alpha(\sum(y_i - \hat{y})^2)$

$= \Delta - \alpha(\sum(y_i - \hat{y})^2)$

$C_\alpha(T_{old}) - C_\alpha(T_{new}) = \sum(y_i - \hat{y}_{m_{old}})^2 - \sum(y_i - \hat{y}_{m_{new}})^2$

$- \sum(y_i - \hat{y}_{m_{new}})^2 - \alpha \sum(y_i - \bar{y})^2$

$\alpha \leq R^2_{new} - R^2_{old}$

then

$\sum(y_i - \hat{y}_{m_{old}})^2 - [\sum(y_i - \hat{y}_{m_{new}})^2 + \sum(y_i - \hat{y}_{m_{new}})^2$

$$\boxed{C_\alpha(T_{new}) \leq C_\alpha(T_{old})}$$