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IEOR 142 HW #2

① a) It makes more sense to apply a log regression model to a seperable dataset because if we use seperable data we risk a logistical regression model that doesn't converge, which would prevent us from being able to improve our model.

b) $f(t) = \log(1+e^{-t})$

$g(t) = \log(1+e^{t}) - t$

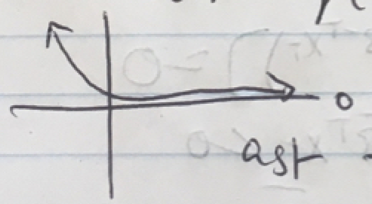
$t = \log(e^{t})$

$$\log\left(1+\frac{1}{e^{t}}\right) = \log(1+e^{t}) - \log(e^{t})$$

$$= \log\left(\frac{e^{t}}{e^{t}} + \frac{1}{e^{t}}\right) = \log\left(\frac{1+e^{t}}{e^{t}}\right)$$

$$= \log\left(\frac{1+e^{t}}{e^{t}}\right) = \log\left(\frac{1+e^{t}}{e^{t}}\right)$$

c) $f(t) = \log(1+e^{-t})$



as $t \to \infty$ $f(t) = 0$

d) $\min_{\beta} \left\{ \sum^{n} \left[\log\left(1+e^{(\beta^T x_i)}\right) - y_i \beta^T x_i \right] \right\}$

$\rightarrow \lim_{t \to \infty} \log(\text{loss}(t\bar{\beta})) = 0$

i) $y_i = 1$

$\lim_{t \to \infty} \sum \left[\log\left(1+e^{\infty \bar{\beta}^T x_i}\right) - \beta^T x_i \right] = 0$

we know

$\log(1+e^{-t}) = \log(1+e^{t}) - t \Rightarrow$

$\log(1+e^{-(t\bar{\beta} x_i)}) = \log(1+e^{t\bar{\beta} x_i}) - t\bar{\beta}^T x_i$

Above is true when $\beta^T x_i > 0$

$a_1 x_{i1} \ldots x_n x_{in} - \infty > 0$ for $y_i = 1$

so $\bar{\beta}$ exists such that $\bar{\beta}^T x_i > 0$

ii) $y_i = 0$

$\lim_{t \to \infty} \sum \left[\log\left(1+e^{(t\bar{\beta} x_i)}\right) \right]$

$\lim_{t \to \infty} \sum \left[\log\left(1+e^{\infty \beta^{-T} x_i}\right) \right] = 0$

Above is true if $\bar{\beta}^T x_i < 0$

so there exists a $\bar{\beta}$ such that

$\bar{\beta}_1 x_{i1} + \ldots \bar{\beta}_n x_{in} < 0$

1e) we saw from the work that we did that β can always increase which means that our loss function can approach 0. This means that our error can always decrease, which means that the accuracy and quality of our model can increase. Since there are infinite values of β that can make our model better our $f(x)$ will never converge, therefore it makes more sense to work w/ a non seperable dataset bc seperable datasets converge. Ultimately the bad behavior of the optimization problem does align with this intuition.

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2)

```
library(readr)
library(caTools)
df <- read_csv("C:/Users/Murtz.Kizilbash/Desktop/ieor142/hw2/framingham.csv")
```

```
## Parsed with column specification:
## cols(
##   male = col_double(),
##   age = col_double(),
##   education = col_character(),
##   currentSmoker = col_double(),
##   cigsPerDay = col_double(),
##   BPMeds = col_double(),
##   prevalentStroke = col_double(),
##   prevalentHyp = col_double(),
##   diabetes = col_double(),
##   totChol = col_double(),
##   sysBP = col_double(),
##   diaBP = col_double(),
##   BMI = col_double(),
##   heartRate = col_double(),
##   glucose = col_double(),
##   TenYearCHD = col_double()
## )
```

```
set.seed(123)



sample = sample.split(df$male, SplitRatio = .7)
train = subset(df, sample == TRUE)
test = subset(df, sample == FALSE)
```

i)

```
model <- glm(TenYearCHD~ glucose + heartRate + BMI + diaBP +sysBP + totChol + diabetes + prevalentHyp +

summary(model)
```

```
##
## Call:
## glm(formula = TenYearCHD ~ glucose + heartRate + BMI + diaBP +
##     sysBP + totChol + diabetes + prevalentHyp + prevalentStroke +
##     BPMeds + cigsPerDay + currentSmoker + age + education + male,
##     data = train)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
```

```
## -0.70155  -0.18441  -0.10427  -0.01103   1.10208
##
## Coefficients:
##                                          Estimate Std. Error t value
## (Intercept)                            -6.399e-01  9.348e-02  -6.846
## glucose                                 9.817e-04  3.838e-04   2.558
## heartRate                              -3.124e-04  5.818e-04  -0.537
## BMI                                     1.911e-03  1.832e-03   1.043
## diaBP                                  -2.158e-03  9.806e-04  -2.201
## sysBP                                   3.462e-03  5.901e-04   5.867
## totChol                                 7.501e-05  1.604e-04   0.468
## diabetes                                2.686e-02  5.204e-02   0.516
## prevalentHyp                            6.274e-03  2.077e-02   0.302
## prevalentStroke                         7.060e-02  9.130e-02   0.773
## BPMeds                                  1.889e-02  4.241e-02   0.445
## cigsPerDay                              2.126e-03  9.208e-04   2.309
## currentSmoker                           2.200e-02  2.164e-02   1.017
## age                                     6.581e-03  9.233e-04   7.127
## educationHigh school/GED               -9.063e-03  2.374e-02  -0.382
## educationSome college/vocational school -4.464e-03  2.635e-02  -0.169
## educationSome high school               1.376e-02  2.311e-02   0.595
## male                                    5.491e-02  1.485e-02   3.697
##                                        Pr(>|t|)
## (Intercept)                            9.49e-12 ***
## glucose                                0.010591 *
## heartRate                              0.591336
## BMI                                    0.297195
## diaBP                                  0.027821 *
## sysBP                                  5.02e-09 ***
## totChol                                0.640008
## diabetes                               0.605791
## prevalentHyp                           0.762611
## prevalentStroke                        0.439435
## BPMeds                                 0.656060
## cigsPerDay                             0.021030 *
## currentSmoker                          0.309456
## age                                    1.33e-12 ***
## educationHigh school/GED               0.702712
## educationSome college/vocational school 0.865498
## educationSome high school              0.551607
## male                                   0.000223 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 0.1146586)
##
##     Null deviance: 325.00  on 2559  degrees of freedom
## Residual deviance: 291.46  on 2542  degrees of freedom
## AIC: 1740.5
##
## Number of Fisher Scoring iterations: 2
```

$$tenyearCHD = -.06 - .0003(heartRate) - .002(diaBP) + .00008(totChol) + .0063(prevalentHyp) + .01(BPmeds) + .022(curren$$

ii) According to the summary of the significance of the features, the most important risk factors in predecting whether or not someone will have CHD in 10 years is their age. When it comes to age, every increase in age increases the log odds of 10yearCHD by .007

iii)

$$560000(p/4) + 60000(1 - p/4) = 500000(p)$$

$p = .16$

iv)

```
test$prediction = predict(model, newdata = test, type = 'response')

high_risk <- subset(test, prediction >= .16)
low_risk <- subset(test, prediction < .16)

tp = nrow(subset(test, prediction >= .16 & TenYearCHD == 1))
fn = nrow(subset(test, prediction < .16 & TenYearCHD == 1))
fp = nrow(subset(test, prediction >= .16 & TenYearCHD == 0))
tn = nrow(subset(test, prediction < .16 & TenYearCHD == 0))

tpr = tp/(tp + fn)
fpr = fp/(fp+tn)
accuracy = (tp+tn)/(tp+fp+fn+tn)

tpr
```

```
## [1] 0.68
```

```
fpr
```

```
## [1] 0.3770314
```

```
accuracy
```

```
## [1] 0.6320583
```

the true positive rate is .68 the false positive rate is .377 the accuracy is .63

The tpr tells us the number of people who contracted CHD in 10 years that were correctly identified.

The fpr tells us the proportion of negative cases incorrectly identified by the model

the accuracy tells us the proportion of the data that was correctly identified.

v)

if chd is not affected by treatment:

$$EXPECTEDCOST = \frac{36(500000) + 131(560000) + 423(60000)}{1507 + 423 + 131 + 136}$$

Which equals 106417.5 dollars.

This assumption does not make much sense because if taking medecine does not have an affect on the development of the condition, then the premise of this study is invalid.

if taking preventative medicines does reduce the outcome of CHD:

$$EC = \frac{35(500000) + 13(.08)(560000) + 23(1.2)(60000)}{1097}$$

so the expected cost in this case is 97670

vi)

```
predTest = predict(model, test, type = 'response')
table(test$TenYearCHD, predTest > .999)
```

```
##
##     FALSE
##   0   923
##   1   175
```

vii)

```
new <- data.frame(male=0, age=51, education = 'College', currentSmoker=1, cigsPerDay = 20, BPMeds = 0, 

predict(model, newdata = new, type = 'response')
```

```
##         1
## 0.1692132
```

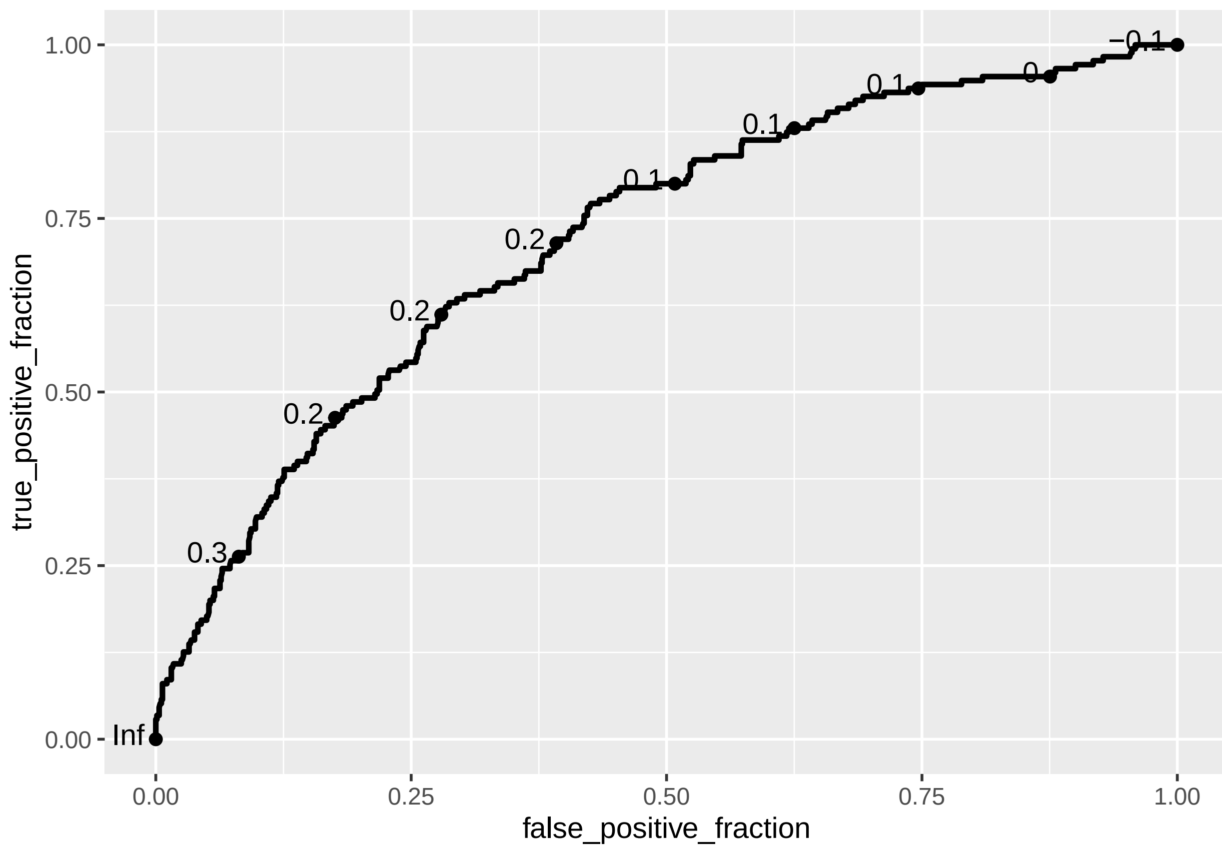
the predicted probability that this patient will contract CHD in the next 10 years is .17, we should prescribe the medecine bc the patent probabiity exceeds the threshold.

b)

```
library(plotROC)
```

```
## Loading required package: ggplot2
```

```
ggplot(test, aes(d = test$TenYearCHD, m = test$prediction)) + geom_roc()
```



the ROC curve looks at the tradeoff between fpr and tpr. We want a high tpr and low fpr, if we decrease the threshold to allow for more positive predictions, the classifier will have a higher fpr as we classify more predictions as yes. The ROC curve looks almost like a step function that increases and plateaus at intervals.

AUC = .739

c)

To break even, the formula is

$$(300000 + c)(.0294) + .09706c = .1176(300000)$$

c = 26460

d) There are many issues with this study and this analysis, prescribing a medication when we do not even know if it will have a preventitive affect is both alarming and could introduce some horrible side effects. One way to fix this is to run a small case study or do some analysis that would allow us to holistically look at the sorts of things going on