Structural Limits of Threshold-Driven Logic Grids

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Abstract. This late-breaking abstract probes the structural limits of a minimal binary cellular automaton inspired by Spencer-Brown's idea of re-entry. We simulate three local update regimes—Classic (direct copy), Self (probabilistic inertia), and Threshold CA (formerly "Re-Entry")—on 100×100 grids driven by constant, impulse, random and sinusoidal inputs. All variants converge to homogeneous states within fewer than 50 steps, except the threshold model, which forms short-lived clusters under sinusoidal drive. The negative result quantifies a lower bound: binary self-reference and static neighborhoods alone cannot sustain complex dynamics. We outline imminent extensions—memory-bearing self-reference, multivalued states and adaptive connectivity—to overcome these limits and move toward richer unconventional computation.

Keywords: Unconventional Computation · Cellular Automata · Laws of Form · Self-reference

1 Introduction

Unconventional computing seeks substrates where *structure*, rather than external control, performs the computation. Spencer-Brown's *Laws of Form* ties such power to two primitives: drawing a distinction and letting that distinction *re-enter* itself, i.e. act on its own result [1, 3]. Re-entry thus epitomises self-reference and has been linked to memory, recursion, and autonomy in living and technical systems [2]. Reviewers of our draft noted that our update rule lacks explicit self-reference and is better described as a *threshold cellular automaton* (T-CA). We adopt this term. This late-breaking abstract asks: *How much dynamical structure can a minimal T-CA sustain on a binary* 100×100 *grid under simple drives?* We show that all variants collapse to homogeneity within <50 steps—establishing a quantitative lower bound—and sketch three extensions (memory-bearing self-reference, multivalued states, adaptive connectivity) that will be explored next.

2 Model

We study a binary grid $X(t) \in \{0, 1\}^{100 \times 100}$ with synchronous updates. At each time step t, all cells receive the same external drive $s(t)$ and update in parallel based on one of three local rules:

$$x_i(t+1) = \begin{cases} \textbf{Classic:} & s(t) \\ \textbf{Self:} & \begin{cases} s(t) & \text{with probability } p \\ x_i(t) & \text{with probability } 1-p \end{cases} \\ \textbf{Threshold:} & \begin{cases} s(t) & \text{if } \#\text{neigh}_i\big(s(t)\big) \geq k \\ x_i(t) & \text{otherwise} \end{cases} \end{cases}$$

Unless noted otherwise, we fix $p = 0.4$, $k = 2$, and use a Moore-8 neighbourhood. We probe four global driving signals: constant $[s(t) \equiv 1]$, impulse $[s(t) = \delta_{t,10}]$, random $[s(t) \sim \text{Bernoulli}(0.5)]$, and sinusoidal $[s(t) = \text{sgn}(\sin(2\pi t/20))]$. We track two global metrics:

- The number of cell updates per step: $\Delta n(t) = \sum_{i,j}[X_{i,j}(t) \neq X_{i,j}(t-1)]$.
- The Shannon entropy: $H(t) = -\sum_{b\in\{0,1\}} P_b(t) \log_2 P_b(t)$, where $P_b(t)$ is the fraction of cells in state b.

3 Results

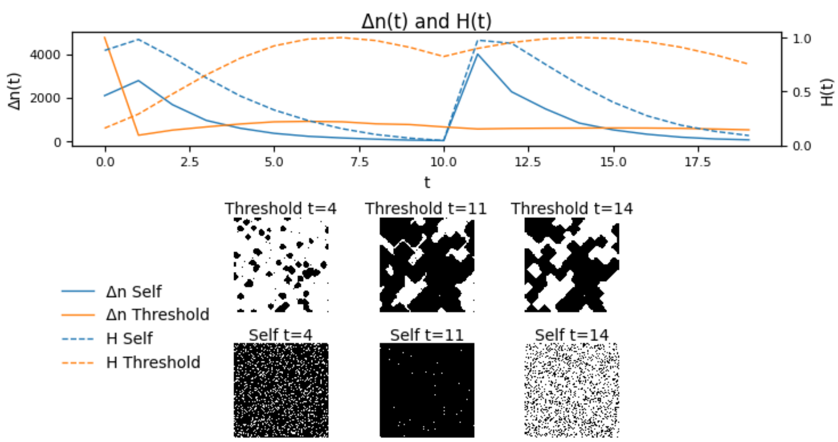

Fig. 1. Dynamics of binary logic grids under sinusoidal input. Top: Global metrics over time for the self (blue) and threshold (orange,k=2, 8-neighbourhood) models: number of state changes per step n(t) (solid) and Shannon entropy H(t) (dashed). Middle: Snapshots of the threshold model at selected time steps (t=4,11,14), illustrating transient cluster formation and coarsening. Bottom: Corresponding snapshots for the self model at the same time points. All simulations use a 100×100 grid, p=0.4, sinusoidal drive, and random initial state.

All tested configurations—Classic, Self, and Threshold (re-entry-inspired) models—collapsed into static or homogeneous states within fewer than 50 steps, even under sinusoidal input and permissive update thresholds. This rapid convergence was most pronounced in the Classic and Self models, where state-change (Δn) and entropy (H) both fell sharply to zero. The Threshold model (Moore-8, $k = 2$) briefly produced small clusters or bands under oscillating input, but these patterns disappeared within 50 cycles. No propagating, oscillating, or self-organizing behaviour was observed. Figure 1 summarizes the rapid loss of complexity and the absence of persistent structure in all regimes. Importantly, this is a negative result: our implementation, based on minimal distinction and a local threshold proxy for re-entry, is insufficient for sustaining complex or emergent dynamics. Full self-reference or recursive distinction—as in Spencer-Brown's original conception—was not realized in these models.

4 Lessons Learned and Outlook

Our experiments yield two core insights. First, minimal threshold-driven binary automata fail to sustain persistent diversity or pattern formation: all models collapse to homogeneity within a few dozen steps. Second, the initial configuration decisively shapes transient dynamics, aligning with classic findings on initial condition sensitivity in spatial and collective systems [5]. A promising next step is the introduction of explicit self-reference and memory, where a cell's past directly influences its update. For example:

> If $s(t)$ matches $x_i(t)$, the cell remains unchanged unless at least 6 of 8 neighbors have the opposite state—then it flips.
> If $s(t)$ and $x_i(t)$ differ, the cell stays unchanged unless at least 5 neighbors support the signal—then it flips.

While this rule adds persistence and collective switching, it only realizes basic memory. True form-based re-entry, as envisioned in the *Laws of Form*, would involve recursive self-invocation:

$$x_i(t{+}1) = f\big(x_i(t), s(t), \{x_j(t)\}_{j \in N(i)}, x_i(t{-}\tau), \ldots, f(\cdots)\big)$$

Such nested update logic could allow emergent dynamics beyond simple thresholds. Exploring these ideas—conceptually and computationally—remains a key challenge for future research.

References

1. Spencer-Brown, G.: *Laws of Form*. Allen & Unwin (1969)
2. Kauffman, L.H.: Imaginary Values in Mathematical Logic. Proc. MVL (1987)
3. Varela, F.J.: A Calculus for Self-Reference. IJGS **2**(1) (1975)
4. Xu, Y. et al.: Emergent Dynamics in Neural Cellular Automata. arXiv:2404.06406 (2024)
5. Nowak, M.A., May, R.M.: Evolutionary games and spatial chaos. *Nature* **359**, 826–829 (1992)