# advanceddynamics Advanced Dynamics Notation<sup>1</sup> IAT<sub>E</sub>X Package

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 $<sup>^1</sup>$ Based on the notation from Dr. Mazzoleni's MAE 511/789 courses at North Carolina State University.

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# 1 Introduction

This document serves as the documentation for the advanced dynamics notation IATEX package advanceddynamics which provides a set of macros and commands to easily create text in the notation of the NCSU graduate courses MAE~511 - Advanced~Dynamics~II and MAE~789 - Advanced~Dynamics~II taught by Dr. Mazzoleni.

This document will outline all of the available commands and provide some examples of the typesetting.

# 1.1 Required Packages

There are 6 required packages to use advanceddynamics. They are:

- 1. accents for custom bar [1]
- 2. amsmath for math notation [4]
- 3. amssymb for math symbols [6]
- 4. graphicx for scaling subscripts and superscripts [2]
- 5. mathtools for additional math functionality [3]
- 6. tensor for prescripts [5]

These packages will be automatically imported when using advanceddynamics. However, importing these packages first is a good way to ensure that they load correctly, especially since this is my first LATEX package. Note that mathtools already includes the amsmath package, so you can omit its import if desired.

This can be done with the following lines before your document's \begin{document}:

```
\usepackage{accents}
\usepackage{amsmath}
\usepackage{amsym}
\usepackage{graphicx}
\usepackage{mathtools}
\usepackage{tensor}
```

# 1.2 Obtaining the Package

The advanceddynamics package and documentation are available for download at:

```
https://github.com/nacanega/advanceddynamics
```

# 1.3 Using the Package

You can use advanceddynamics by including the following line before your document's \begin{document}:

```
\usepackage{advanceddynamics}
```

Note that you will need to ensure that the file advanceddynamics.sty is either located in the same location as your ".tex" files or placed with your other LATEX distribution's local packages. More detailed information can be found in Section 10.1.

# 2 Frames of Reference

The commands in this section help the user to define reference frames and their corresponding sets of orthonormal unit vectors.<sup>2</sup> Note that all commands in this section should be used inside of a math environment.

### Normal Reference Frames

### $2.1 \ \text{fr}$

### $\mathbf{fr}\{\langle point \rangle\}$

Frame. This command is used to add a bar above a character to allow it to be read as a frame attached to a specified point  $\langle point \rangle$ . Case-insensitive.

 $\langle point \rangle = Point; letter (a-z, A-Z)$ 

### Example

Say we want to define an inertial reference frame about the point O.

This is accomplished with the LATEX below:

 $fr{0}$ 

Display/Inline Mode Output

 $\bar{O}$ 

### 2.2 \frsc

### $\frac{\langle frame \rangle}$

Small-caps frame. This command is used to define frame  $\langle frame \rangle$  in a subscript or superscript. Case-insensitive.

 $\langle frame \rangle = Frame; letter (a-z, A-Z)$ 

### Example

Say we want to define x-component of the vector  $\vec{r}$  expressed in the  $\vec{O}$  frame.

This is accomplished with the LATEX below:

 $\r_{x\frsc{0}}$ 

Display/Inline Mode Output

 $r_{x\bar{o}}$ 

<sup>&</sup>lt;sup>2</sup>Note that my notation differs slightly from the original notation since unit vectors are always marked with a "hat" rather than a plain arrow.

# 2.3 \ihat

### 

I-hat. This command is used to display the  $\hat{\imath}$  unit vector for a specified frame  $\langle frame \rangle$ . The bar is automatically added. Case-insensitive.

 $\langle frame \rangle = Frame; letter (a-z, A-Z)$ 

### Example

Say we want to express  $\hat{i}$  for the  $\bar{A}$  frame.

This is accomplished with the LaTeX below:

\ihat{a}

Display/Inline Mode Output

 $\hat{\imath}_{\!\scriptscriptstyle \bar{A}}$ 

# 2.4 \jhat

# 

J-hat. This command is used to display the  $\hat{j}$  unit vector for a specified frame  $\langle frame \rangle$ . The bar is automatically added. Case-insensitive.

 $\langle point \rangle = Point; letter (a-z, A-Z)$ 

### Example

Say we want to express  $\hat{j}$  for the  $\bar{B}$  frame.

This is accomplished with the LATEX below:

\jhat{b}

Display/Inline Mode Output

 $\hat{\jmath}_{\scriptscriptstyle ar{B}}$ 

### 2.5 \khat

# 

K-hat. This command is used to display the  $\hat{k}$  unit vector for a specified frame  $\langle frame \rangle$ . The bar is automatically added. Case-insensitive.

 $\langle point \rangle = Point; letter (a-z, A-Z)$ 

### Example

Say we want to express  $\hat{k}$  for the  $\bar{C}$  frame.

This is accomplished with the LATEX below:

\khat{c}

Display/Inline Mode Output

 $\hat{k}_{\scriptscriptstyle ar{C}}$ 

### $2.6 \ \text{frDef}$

### $\mathbf{frDef}\{\langle point \rangle\}$

Define frame. This command is used define a frame as a point  $\langle point \rangle$  and three orthonormal unit vectors. Case insensitive.

 $\langle point \rangle = Point; letter (a-z, A-Z)$ 

### Example

Say we want to define the frame about the point P.

This is accomplished with the LATEX below:

\frDef{P}

Display/Inline Mode Output

$$\bar{P} = \{P, \hat{\imath}_{\scriptscriptstyle \bar{P}}, \hat{\jmath}_{\scriptscriptstyle \bar{P}}, \hat{k}_{\scriptscriptstyle \bar{P}}\}$$

# 2.7 \frExp

# $\frac{free}{\langle frame \rangle} {\langle value \rangle}$

Expressed in frame. This command is used to show that a matrix or vector quantity  $\langle value \rangle$  is expressed in a given frame  $\langle frame \rangle$ .

```
\langle frame \rangle = Frame; letter (a-z, A-Z)
\langle value \rangle = Vector or matrix quantity
```

### Example

Say we want to show that  $\vec{v}$  is expressed in the  $\bar{B}$  frame.

This is accomplished with the LATEX below:

\frExp{B}{\vec{v}}

Display/Inline Mode Output

 $[\vec{v}]_{\!\scriptscriptstyle ar{B}}$ 

### 2.8 \frVec

# $\frue{frue}{\langle frame \rangle}{\langle icomp \rangle}{\langle jcomp \rangle}{\langle kcomp \rangle}$

Vector frame components. This command is used to express a vector in terms of its components in each of the frame's unit vectors.

```
\langle frame \rangle = Frame; letter (a-z, A-Z)
\langle icomp \rangle = i-hat-component expression
\langle jcomp \rangle = j-hat-component expression
\langle kcomp \rangle = k-hat-component expression
```

### Example

Say we want to express  $[\vec{r}]_{\bar{o}} = \langle x, y, z \rangle$  in its  $\bar{O}$  frame components.

This is accomplished with the  $\LaTeX$  below:

\frVec{0}{x}{y}{z}

Display/Inline Mode Output

$$(x)\hat{\imath}_{\scriptscriptstyle \bar{O}} + (y)\hat{\jmath}_{\scriptscriptstyle \bar{O}} + (z)\hat{k}_{\scriptscriptstyle \bar{O}}$$

# 2.9 \frTen

# $\mathbf{fren}(\langle frame \rangle) \{\langle ii \rangle\} \{\langle ij \rangle\} \{\langle ik \rangle\} \{\langle ji \rangle\} \{\langle jj \rangle\} \{\langle jk \rangle\} \{\langle ki \rangle\} \{\langle ki \rangle\} \{\langle kk \rangle\} \{\langle ii \rangle\}$

Tensor frame components. This command is used to express a tensor in terms of its components in each of the frame's unit vector combinations. Must be inside a align or align\* environment.

 $\langle frame \rangle = Frame; letter (a-z, A-Z)$ 

 $\langle ii \rangle = \text{i-hat-i-hat-component expression}$ 

 $\langle ij \rangle = \text{i-hat-j-hat-component expression}$ 

 $\langle ik \rangle = \text{i-hat-k-hat-component expression}$ 

 $\langle ji \rangle$  = j-hat-i-hat-component expression

 $\langle jj \rangle = \text{j-hat-j-hat-component expression}$ 

 $\langle jk \rangle = \text{j-hat-k-hat-component expression}$ 

 $\langle ki \rangle = \text{k-hat-i-hat-component expression}$ 

 $\langle kj \rangle = \text{k-hat-j-hat-component expression}$ 

 $\langle kk \rangle = \text{k-hat-k-hat-component expression}$ 

### Example

Say we want to express  $[\tilde{I}]_{\bar{o}}$  in its  $\bar{O}$  frame components (say a-i).

This is accomplished with the LATEX below:

 $frTen{0}{a}{b}{c}{d}{e}{f}{g}{h}{i}$ 

### Display Mode Output

$$(a)\hat{i}_{\bar{o}}\hat{i}_{\bar{o}} + (b)\hat{i}_{\bar{o}}\hat{j}_{\bar{o}} + (c)\hat{i}_{\bar{o}}\hat{k}_{\bar{o}} \cdots + (d)\hat{j}_{\bar{o}}\hat{i}_{\bar{o}} + (e)\hat{j}_{\bar{o}}\hat{j}_{\bar{o}} + (f)\hat{j}_{\bar{o}}\hat{k}_{\bar{o}} \cdots + (g)\hat{k}_{\bar{o}}\hat{i}_{\bar{o}} + (h)\hat{k}_{\bar{o}}\hat{j}_{\bar{o}} + (i)\hat{k}_{\bar{o}}\hat{k}_{\bar{o}}$$

### $2.10 \frSub$

```
\frac{frSub}{\langle frame \rangle} {\langle value \rangle} {\langle subsc \rangle}
```

Subscript and frame. This command is used to add a pre-superscript frame  $\langle frame \rangle$  and specified subscript  $\langle subsc \rangle$  to a value  $\langle value \rangle$ .

```
\langle frame \rangle = Frame; letter (a-z, A-Z)
```

 $\langle value \rangle$  = Value to apply frame and subscript to

 $\langle subsc \rangle = \text{Subscript value}$ 

### Example

Say we want to define the velocity  $\vec{v}$  of a satellite expressed in the  $\bar{O}$  inertial reference frame.

This is accomplished with the LATEX below:

\frSub{0}{\vec{v}}{satellite}

Display/Inline Mode Output

 ${}^{\scriptscriptstyle{\bar{O}}} \vec{v}_{satellite}$ 

### 2.11 \frx

# $\frac{frame}{fx{\langle frame \rangle}}{\langle value \rangle}$

Frame x-component. This command defines the x-component of value  $\langle value \rangle$  in a specified frame  $\langle frame \rangle$ .

```
\langle frame \rangle = Frame; letter (a-z, A-Z)
```

 $\langle value \rangle$  = The value that we want the x-component of

### Example

Say we want to define the x-component of the vector  $\vec{a}$  expressed in the  $\vec{D}$  frame.

This is accomplished with the LATEX below:

 $\frx{D}{\vec{a}}$ 

Display/Inline Mode Output

 $\vec{a}_{x\bar{\scriptscriptstyle D}}$ 

# 2.12 \fry

# $\frac{frame}{fry}{\langle frame}{}\}{\langle value}{}$

Frame y-component. This command defines the y-component of value  $\langle value \rangle$  in a specified frame  $\langle frame \rangle$ .

```
\langle frame \rangle = Frame; letter (a-z, A-Z)
\langle value \rangle = The value that we want the y-component of
```

### Example

Say we want to define the y-component of the vector  $\vec{a}$  expressed in the  $\vec{D}$  frame.

This is accomplished with the LATEX below:

 $fry{D}{\vec{a}}$ 

Display/Inline Mode Output

 $\vec{a}_{y\bar{\scriptscriptstyle D}}$ 

### 2.13 \frz

# $\frac{frame}{fvalue}$

Frame z-component. This command defines the z-component of value  $\langle value \rangle$  in a specified frame  $\langle frame \rangle$ .

```
\langle frame \rangle = Frame; letter (a-z, A-Z)
\langle value \rangle = The value that we want the z-component of
```

# Example

Say we want to define the z-component of the vector  $\vec{a}$  expressed in the  $\vec{D}$  frame.

This is accomplished with the LATEX below:

 $\frz{D}{\vec{a}}$ 

Display/Inline Mode Output

 $\vec{a}_{z\bar{D}}$ 

# Numbered Normal Reference Frames

This type of frame is similar to a normal frame, except the unit vectors and point also have an associated number.

### 2.14 \frn

### $\mathbf{frn}(\langle point \rangle) \{\langle num \rangle\}$

Numbered frame. This command is used to add a bar above a character and a numeric subscript  $\langle num \rangle$  to allow it to be read as a frame attached to a specified point  $\langle point \rangle$ . Case-insensitive.

```
\langle point \rangle = Point; letter (a-z, A-Z)
\langle num \rangle = Number of point
```

### Example

Say we want to define an inertial reference frame about the point  $A_1$ .

This is accomplished with the LATEX below:

 $fr{A}{1}$ 

Display/Inline Mode Output

 $\bar{A}_1$ 

### 2.15 \frnsc

# $\fract{\langle point \rangle} {\langle num \rangle}$

Small-caps numbered frame. This command is used to define a numbered frame in a subscript or superscript. Case-insensitive.

```
\langle point \rangle = Point; letter (a-z, A-Z) numNumber of point
```

### Example

Say we want to define x-component of the vector  $\vec{r}$  expressed in the  $\bar{A}_1$  frame.

This is accomplished with the LATEX below:

 $r_{x\frnsc{A}{1}}$ 

Display/Inline Mode Output

 $r_{xar{A}_{\scriptscriptstyle 1}}$ 

### 2.16 \ihatn

# $\displaystyle \begin{array}{l} {\bf \hat{\langle}} & {\bf$

I-hatn. This command is used to display the  $\hat{\imath}$  unit vector for a specified frame  $\langle frame \rangle$  with number  $\langle num \rangle$ . The bar is automatically added. Case-insensitive.

```
\langle frame \rangle = Frame; letter (a-z, A-Z)
\langle num \rangle = Number of point
```

# Example

Say we want to express  $\hat{i}$  for the  $\bar{A}_1$  frame.

This is accomplished with the LATEX below:

 $\displaystyle \frac{a}{1}$ 

Display/Inline Mode Output

 $\hat{\imath}_{ar{A}_1}$ 

# 2.17 \jhatn

# 

J-hatn. This command is used to display the  $\hat{j}$  unit vector for a specified frame  $\langle frame \rangle$  with number  $\langle num \rangle$ . The bar is automatically added. Case-insensitive.

```
\langle point \rangle = Point; letter (a-z, A-Z)
\langle num \rangle = Number of point
```

### Example

Say we want to express  $\hat{j}$  for the  $\bar{B}_2$  frame.

This is accomplished with the LATEX below:

Display/Inline Mode Output

 $\hat{\jmath}_{\scriptscriptstyle{ar{B}_{\!2}}}$ 

### 2.18 \khatn

# 

K-hatn. This command is used to display the  $\hat{k}$  unit vector for a specified frame  $\langle frame \rangle$  with number  $\langle num \rangle$ . The bar is automatically added. Case-insensitive.

```
\langle point \rangle = Point; letter (a-z, A-Z)
\langle num \rangle = Number of point
```

### Example

Say we want to express  $\hat{k}$  for the  $\bar{C}_3$  frame.

This is accomplished with the LATEX below:

Display/Inline Mode Output

 $\hat{k}_{ar{\scriptscriptstyle C}_{\scriptscriptstyle 3}}$ 

### $2.19 \ \text{frnDef}$

# $\mathbf{frnDef}(\langle point \rangle) \{\langle num \rangle\}$

Define numbered frame. This command is used define a frame as a numbered point and three orthonormal unit vectors. Case insensitive.

```
\langle point \rangle = Point; letter (a-z, A-Z)
\langle num \rangle = Number of point
```

### Example

Say we want to define the frame about the point  $P_6$ .

This is accomplished with the LATEX below:

\frnDef{P}{6}

Display/Inline Mode Output

$$\bar{P}_6 = \{P_6, \hat{\imath}_{\bar{P}_6}, \hat{\jmath}_{\bar{P}_6}, \hat{k}_{\bar{P}_6}\}$$

# $2.20 \ \text{frnExp}$

# $\frame \frame \frame$

Expressed in numbered frame. This command is used to show that a matrix or vector quantity is expressed in a given numbered frame  $\langle frame \rangle$ .

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z)  $\langle num \rangle$  = Number of point

 $\langle value \rangle = \text{Vector or matrix quantity}$ 

### Example

Say we want to show that  $\vec{v}$  is expressed in the  $\bar{B}_2$  frame.

This is accomplished with the LATEX below:

 $frnExp{B}{2}{vec{v}}$ 

Display/Inline Mode Output

 $[ec{v}]_{ar{\mathit{B}}_{\!\scriptscriptstyle 2}}$ 

### $2.21 \ \text{frnVec}$

# 

Vector numbered frame components. This command is used to express a vector in terms of its components in each of the numbered frame's unit vectors.

 $\langle frame \rangle = Frame; letter (a-z, A-Z)$ 

 $\langle num \rangle = \text{Number of point}$ 

 $\langle icomp \rangle = i$ -hat-component expression

 $\langle jcomp \rangle = \text{j-hat-component expression}$ 

 $\langle kcomp \rangle = \text{k-hat-component expression}$ 

### Example

Say we want to express  $[\vec{r}]_{\bar{A}_1} = \langle x, y, z \rangle$  in its  $\bar{A}1$  frame components.

This is accomplished with the  $\LaTeX$  below:

 $\frnVec{A}{1}{x}{y}{z}$ 

Display/Inline Mode Output

$$(x)\hat{\imath}_{\bar{A}_{1}} + (y)\hat{\jmath}_{\bar{A}_{1}} + (z)\hat{k}_{\bar{A}_{1}}$$

### 2.22 \frnTen

# 

Tensor numbered frame components. This command is used to express a tensor in terms of its components in each of the numbered frame's unit vector combinations. Must be inside a align or align\* environment.

 $\langle frame \rangle = Frame; letter (a-z, A-Z)$ 

 $\langle num \rangle = \text{Number of point}$ 

 $\langle ii \rangle = \text{i-hat-i-hat-component expression}$ 

 $\langle ij \rangle = \text{i-hat-j-hat-component expression}$ 

 $\langle ik \rangle$  = i-hat-k-hat-component expression

 $\langle ji \rangle = \text{j-hat-i-hat-component expression}$ 

 $\langle ij \rangle = \text{j-hat-j-hat-component expression}$ 

 $\langle jk \rangle$  = j-hat-k-hat-component expression

 $\langle ki \rangle = \text{k-hat-i-hat-component expression}$ 

 $\langle kj \rangle = \text{k-hat-j-hat-component expression}$ 

 $\langle kk \rangle = \text{k-hat-k-hat-component expression}$ 

### Example

Say we want to express  $[\tilde{I}]_{\bar{A}_1}$  in its  $\bar{A}_1$  frame components (say a-i).

This is accomplished with the LATEX below:

 $frnTen{A}{1}{a}{b}{c}{d}{e}{f}{g}{h}{i}$ 

### Display Mode Output

$$(a)\hat{\imath}_{\bar{A}_{1}}\hat{\imath}_{\bar{A}_{1}}+(b)\hat{\imath}_{\bar{A}_{1}}\hat{\jmath}_{\bar{A}_{1}}+(c)\hat{\imath}_{\bar{A}_{1}}\hat{k}_{\bar{A}_{1}}\cdots\\+(d)\hat{\jmath}_{\bar{A}_{1}}\hat{\imath}_{\bar{A}_{1}}+(e)\hat{\jmath}_{\bar{A}_{1}}\hat{\jmath}_{\bar{A}_{1}}+(f)\hat{\jmath}_{\bar{A}_{1}}\hat{k}_{\bar{A}_{1}}\cdots\\+(g)\hat{k}_{\bar{A}_{1}}\hat{\imath}_{\bar{A}_{1}}+(h)\hat{k}_{\bar{A}_{1}}\hat{\jmath}_{\bar{A}_{1}}+(i)\hat{k}_{\bar{A}_{1}}\hat{k}_{\bar{A}_{1}}$$

### $2.23 \ \text{frnSub}$

```
\frac{frnSub}{\langle frame \rangle} {\langle num \rangle} {\langle value \rangle} {\langle subsc \rangle}
```

Subscript and numbered frame. This command is used to add a pre-superscript frame and specified subscript to a value.

```
\langle frame \rangle = Frame; letter (a-z, A-Z)
\langle num \rangle = Number of point
\langle value \rangle = Value to apply frame and subscript to
\langle subsc \rangle = Subscript value
```

# Example

Say we want to define the velocity of a satellite expressed in the  $\bar{A}_1$  inertial reference frame.

This is accomplished with the LaTeX below:

\frnSub{A}{1}{\vec{v}}{satellite}

Display/Inline Mode Output

 ${}^{ar{A}_{\!\scriptscriptstyle 1}}ec{v}_{\!\scriptscriptstyle satellite}$ 

### $2.24 \ \text{frnx}$

```
\frac{\langle frame \rangle}{\langle num \rangle} {\langle value \rangle}
```

Numbered frame x-component. This command defines the x-component of value  $\langle value \rangle$  in a specified frame  $\langle frame \rangle$ .

```
\langle frame \rangle = Frame; letter (a-z, A-Z)
\langle num \rangle = Number of point
\langle value \rangle = The value that we want the x-component of
```

### Example

Say we want to define the x-component of the vector  $\vec{a}$  expressed in the  $\bar{D}_4$  frame.

This is accomplished with the LATEX below:

 $\frac{D}{4}{\vec{a}}$ 

Display/Inline Mode Output

 $\vec{a}_{xar{\scriptscriptstyle D}_{\scriptscriptstyle A}}$ 

# 2.25 \frny

# $\frac{frame}{frny}{\langle frame}{} {\langle num \rangle} {\langle value \rangle}$

Numbered frame y-component. This command defines the y-component of value  $\langle value \rangle$  in a specified frame  $\langle frame \rangle$ .

```
\langle frame \rangle = Frame; letter (a-z, A-Z)
```

 $\langle num \rangle = \text{Number of point}$ 

 $\langle value \rangle$  = The value that we want the y-component of

### Example

Say we want to define the y-component of the vector  $\vec{a}$  expressed in the  $\bar{D}_4$  frame.

This is accomplished with the LATEX below:

 $\frac{1}{4}{\vec{a}}$ 

Display/Inline Mode Output

 $ec{a}_{y_{ar{D}_4}}$ 

### $2.26 \ \text{frnz}$

# $\frac{frame}{fnz}{\langle fnum\rangle}{\langle value\rangle}$

Numbered frame z-component. This command defines the z-component of value  $\langle value \rangle$  in a specified frame  $\langle frame \rangle$ .

```
\langle frame \rangle = Frame; letter (a-z, A-Z)
```

 $\langle num \rangle = \text{Number of point}$ 

 $\langle value \rangle$  = The value that we want the z-component of

### Example

Say we want to define the z-component of the vector  $\vec{a}$  expressed in the  $\bar{D}_4$  frame.

This is accomplished with the LATEX below:

 $\frac{D}{4}{\vec{a}}$ 

Display/Inline Mode Output

 $ec{a}_{zar{\scriptscriptstyle D}_{\scriptscriptstyle A}}$ 

# **Special Reference Frames**

This type of frame is similar to a normal frame, except the unit vectors are numbered lowercase versions of the specified point.

### 2.27 \uveca

### $\uveca{\langle frame \rangle}$

Unit vector 1. This command is used to display the first unit vector for a specified frame  $\langle frame \rangle$ . The bar is automatically added. Case-insensitive.

 $\langle frame \rangle = Frame; letter (a-z, A-Z)$ 

### Example

Say we want to express the first unit vector for the  $\bar{A}$  frame.

This is accomplished with the LATEX below:

\uveca{a}

Display/Inline Mode Output

 $\hat{a}_{\scriptscriptstyle 1}$ 

### 2.28 \uvecb

### $\uvecb{\langle frame \rangle}$

Unit vector 2. This command is used to display the second unit vector for a specified frame  $\langle frame \rangle$ . The bar is automatically added. Case-insensitive.

 $\langle point \rangle = Point; letter (a-z, A-Z)$ 

### Example

Say we want to express the second unit vector for the  $\bar{B}$  frame.

This is accomplished with the LATEX below:

\uvecb{b}

Display/Inline Mode Output

 $\hat{b}_{2}$ 

### 2.29 \uvecc

### $\uvecc{\langle frame \rangle}$

Unit vector 3. This command is used to display the third unit vector for a specified frame  $\langle frame \rangle$ . The bar is automatically added. Case-insensitive.

$$\langle point \rangle = Point; letter (a-z, A-Z)$$

### Example

Say we want to express the third unit vector for the  $\bar{C}$  frame.

This is accomplished with the LATEX below:

\uvecc{c}

Display/Inline Mode Output

 $\hat{c}_{\!\scriptscriptstyle 3}$ 

### $2.30 \ \text{fruDef}$

### $\frac{fruDef{\langle point \rangle}}$

Define special frame. This command is used define a special frame as a point and three orthonormal unit vectors. Case insensitive.

$$\langle point \rangle = Point; letter (a-z, A-Z)$$

### Example

Say we want to define the special frame about the point P.

This is accomplished with the LATEX below:

\fruDef{P}

Display/Inline Mode Output

$$\bar{P} = \{P, \hat{p}_1, \hat{p}_2, \hat{p}_3\}$$

### 2.31 \fruVec

# 

Vector special frame components. This command is used to express a vector in terms of its components in each of the special frame's unit vectors.

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z)  $\langle comp1 \rangle$  = 1-component expression  $\langle comp2 \rangle$  = 2-component expression  $\langle comp3 \rangle$  = 3-component expression

### Example

Say we want to express  $[\vec{r}]_{\bar{F}} = \langle x, y, z \rangle$  in its  $\bar{F}$  frame components.

This is accomplished with the LATEX below:

 $fruVec{F}{x}{y}{z}$ 

Display/Inline Mode Output

$$(x)\hat{f_1} + (y)\hat{f_2} + (z)\hat{f_3}$$

### 2.32 \fruTen

 $\label{lem:lem:condition} $$ \operatorname{C11}} {\langle c11 \rangle} {\langle c12 \rangle} {\langle c12 \rangle} {\langle c21 \rangle} {\langle c22 \rangle} {\langle c23 \rangle} {\langle c31 \rangle} {\langle c32 \rangle} {\langle c32 \rangle} {\langle c33 \rangle} = 0$ 

Tensor special frame components. This command is used to express a tensor in terms of its components in each of the special frame's unit vector combinations. Must be inside a align or align\* environment.

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z)  $\langle c11 \rangle$  = 1-1-component expression  $\langle c12 \rangle$  = 1-2-component expression  $\langle c13 \rangle$  = 1-3-component expression  $\langle c21 \rangle$  = 2-1-component expression  $\langle c22 \rangle$  = 2-2-component expression  $\langle c23 \rangle$  = 2-3-component expression  $\langle c31 \rangle$  = 3-1-component expression  $\langle c32 \rangle$  = 3-2-component expression  $\langle c33 \rangle$  = 3-3-component expression

### Example

Say we want to express  $[\tilde{I}]_{\bar{F}}$  in its  $\bar{F}$  frame components (say a-i).

This is accomplished with the LaTeX below:

 $fruTen{F}{a}{b}{c}{d}{e}{f}{g}{h}{i}$ 

### Display Mode Output

$$(a)\hat{f}_{1}\hat{f}_{1} + (b)\hat{f}_{1}\hat{f}_{2} + (c)\hat{f}_{1}\hat{f}_{3} \cdots + (d)\hat{f}_{2}\hat{f}_{1} + (e)\hat{f}_{2}\hat{f}_{2} + (f)\hat{f}_{2}\hat{f}_{3} \cdots + (g)\hat{f}_{3}\hat{f}_{1} + (h)\hat{f}_{3}\hat{f}_{2} + (i)\hat{f}_{3}\hat{f}_{3}$$

# 3 General Terms

This section includes the commands needed to typeset most of the terms from *MAE 511*. Note that all commands in this section should be used inside of a math environment. Note that when inputting strings, you will have to enclose in \mathit if longer than one character and in \smca when dealing with capital letters.

### Translational Terms

### $3.1 \ \text{traV}$

```
\trav{\langle frame \rangle} {\langle point \rangle}
```

Translational velocity. This command is used to express the translational velocity of a point  $\langle point \rangle$  in a specified frame  $\langle frame \rangle$ .

```
\langle frame \rangle = \text{Letter (a-z, A-Z)}
\langle point \rangle = \text{Name of point; expression}
```

### Example

Say we want to define the velocity of particle B in its body frame  $\bar{B}$ .

This is accomplished with the L<sup>A</sup>T<sub>E</sub>X below:

\traV{b}{\smca{b}}

Display/Inline Mode Output

 $^{ar{B}}ec{v}_{\!\scriptscriptstyle B}$ 

### $3.2 \ \text{traA}$

# $\track {\langle frame \rangle} {\langle point \rangle}$

Translational acceleration. This command is used to express the translational acceleration of a point  $\langle point \rangle$  in a specified frame  $\langle frame \rangle$ .

```
\langle frame \rangle = Letter (a-z, A-Z)
\langle point \rangle = Name of point; expression
```

### Example

Say we want to define the acceleration of particle B in an inertial reference frame  $\bar{O}$ .

This is accomplished with the LATEX below:

\traA{o}{\smca{b}}

Display/Inline Mode Output

 $^{\scriptscriptstyle ar{\scriptscriptstyle O}}ec{a}_{\scriptscriptstyle B}$ 

### 3.3 \relR

# $\mathbf{relR}(\langle point \rangle) \{\langle refpt \rangle\}$

Relative displacement. This command is used to express the position of  $\langle point \rangle$  relative to  $\langle refpt \rangle$ .

```
\langle point \rangle = Name of point; expression \langle refpt \rangle = Name of reference point; expression
```

### Example

Say we want to define the displacement of a particle A relative to the point O.

This is accomplished with the LATEX below:

 $\left\{ \left( A\right) \right\}$ 

Display/Inline Mode Output

 $ec{r}_{\scriptscriptstyle A/O}$ 

### $3.4 \ \text{relV}$

# 

Relative velocity. This command is used to express the velocity of  $\langle point \rangle$  relative to  $\langle refpt \rangle$  in a specified frame  $\langle frame \rangle$ .

 $\langle frame \rangle$  = Frame that velocity is expressed in

 $\langle point \rangle = \text{Name of point; expression}$ 

 $\langle refpt \rangle$  = Name of reference point; expression

### Example

Say we want to define the velocity of a particle B relative to the particle A in the  $\bar{O}$  frame.

This is accomplished with the LATEX below:

 $\relV{o}{\smca{B}}{\smca{A}}$ 

Display/Inline Mode Output

 $^{\scriptscriptstyle ar{O}}ec{v}_{\scriptscriptstyle B/\!A}$ 

### $3.5 \ \text{relA}$

# $\mathbf{relA}(\langle frame \rangle) \{\langle point \rangle\} \{\langle refpt \rangle\}$

Relative acceleration. This command is used to express the acceleration of  $\langle point \rangle$  relative to  $\langle refpt \rangle$  in a specified frame  $\langle frame \rangle$ .

 $\langle frame \rangle$  = Frame that velocity is expressed in

 $\langle point \rangle = \text{Name of point; expression}$ 

 $\langle refpt \rangle$  = Name of reference point; expression

### Example

Say we want to define the acceleration of a particle B relative to the particle A in the  $\bar{O}$  frame.

This is accomplished with the LATEX below:

 $\relA{o}{\smca{B}}{\smca{A}}$ 

Display/Inline Mode Output

 $^{\scriptscriptstyle \bar{O}}\vec{a}_{\scriptscriptstyle B/\!A}$ 

# 3.6 \Forc

### 

Force. This command is used to express a force with a specified subscript  $\langle sub \rangle$ .

 $\langle sub \rangle = \text{Subscript}; \text{ expression}$ 

### Example

Say we want to define a gravitational force with g-shorthand and a frictional force with friction written out.

This is accomplished with the LATEX below:

\Forc{g} \text{ and } \Forc{\mathit{friction}}

Display/Inline Mode Output

 $\vec{F}_g$  and  $\vec{F}_{friction}$ 

# **Rotational Terms**

### $3.7 \setminus angV$

### $\aggV{\langle frame1 \rangle} {\langle frame2 \rangle}$

Angular velocity. This command is used to express the angular velocity of  $\langle frame2 \rangle$  relative to  $\langle frame1 \rangle$ .

 $\langle frame1 \rangle$  = First frame; letter (a-z, A-Z)  $\langle frame2 \rangle$  = Second frame; letter (a-z, A-Z)

### Example

Say we want to define the angular velocity of the  $\bar{A}$  frame relative to the  $\bar{O}$  frame.

This is accomplished with the LATEX below:

 $\angV{0}{A}$ 

Display/Inline Mode Output

 $\bar{O}_{\vec{\omega}}^{\bar{A}}$ 

# $3.8 \setminus angA$

```
\aggA{\langle frame1\rangle}{\langle frame2\rangle}
```

Angular acceleration. This command is used to express the angular acceleration of  $\langle frame2 \rangle$  relative to  $\langle frame1 \rangle$ .

```
\langle frame1 \rangle = First frame; letter (a-z, A-Z) \langle frame2 \rangle = Second frame; letter (a-z, A-Z)
```

### Example

Say we want to define the angular acceleration of the  $\bar{A}$  frame relative to the  $\bar{O}$  frame.

This is accomplished with the LATEX below:

 $\angA{0}{A}$ 

Display/Inline Mode Output

 $\bar{\alpha}^{\bar{A}}$ 

# $3.9 \setminus angMp$

```
\label{eq:lambda} $$ \agMp{\langle frame \rangle} {\langle point1 \rangle} {\langle point2 \rangle} {\langle mass \rangle} $$
```

Particle angular momentum. This command is used to express the angular momentum of a particle with mass  $\langle mass \rangle$  with respect to  $\langle point2 \rangle$  with respect to  $\langle point1 \rangle$ , relative to the frame  $\langle frame \rangle$ .

```
\langle frame \rangle = \text{Letter (a-z, A-Z)}
\langle point1 \rangle = \text{Name of point1; expression}
\langle point2 \rangle = \text{Name of point2; expression}
\langle mass \rangle = \text{Name of mass; expression}
```

### Example

Say we want to define the angular momentum of a particle of mass m with respect to B with respect to A expressed in the  $\bar{O}$  frame.

This is accomplished with the LATEX below:

 ${\bf Display/Inline\ Mode\ Output}$ 

 ${}^{\scriptscriptstyle ar{\scriptscriptstyle O}}_{\scriptscriptstyle A}\!\vec{h}_{m/\!\scriptscriptstyle B}$ 

# 3.10 \angMs

# $\aggms{\langle frame \rangle} {\langle point1 \rangle} {\langle point2 \rangle}$

System angular momentum. This command is used to express the angular momentum of a system with respect to  $\langle point2 \rangle$  with respect to  $\langle point1 \rangle$ , relative to the frame  $\langle frame \rangle$ .

```
\langle frame \rangle = Letter (a-z, A-Z)
\langle point1 \rangle = Name of point1; expression
\langle point2 \rangle = Name of point2; expression
```

### Example

Say we want to define the angular momentum of a system with respect to P with respect to Q expressed in the  $\bar{F}$  frame.

This is accomplished with the LATEX below:

 $\angMs{f}{\smca{Q}}{\smca{P}}$ 

Display/Inline Mode Output

 ${}^{ar{F}}_{Q} \vec{h}_{P,\mathrm{sys}}$ 

# $3.11 \setminus angM$

# $\aggM{\langle frame \rangle}{\langle point1 \rangle}{\langle spec \rangle}$

General angular momentum. This command is used to express the angular momentum of a specified particle, body, or system with respect to a specified point with respect to  $\langle point1 \rangle$ , relative to the frame  $\langle frame \rangle$ .

```
\langle frame \rangle = \text{Letter (a-z, A-Z)}
\langle point1 \rangle = \text{Name of point1; expression}
\langle spec \rangle = \text{Specified particle, body, or system and specified point; expression}
```

### Example

Say we want to define the angular momentum of a rigid body with respect to its center of mass with respect to Q expressed in the  $\bar{F}$  frame.

This is accomplished with the LATEX below:

 $\angM{f}{\smca{Q}}{\CM}$ 

Display/Inline Mode Output

 ${}^{\bar{F}}_{\it Q} \vec{h}_{\scriptscriptstyle 
m CM}$ 

# 3.12 \torq

# $\operatorname{torq}\{\langle sub \rangle\}$

Torque. This command is used to express a torque with a specified subscript  $\langle sub \rangle$ . Note that \mathit may be needed if the subscript is a word.

 $\langle sub \rangle = \text{Subscript}; \text{ expression}$ 

### Example

Say we want to define a torque due to gravity with the g-shorthand and a torque due to a *motor*.

This is accomplished with the L<sup>A</sup>T<sub>E</sub>X below:

\torq{g} \text{ and } \torq{\mathit{motor}}

Display/Inline Mode Output

 $\vec{\tau_g}$  and  $\vec{\tau}_{motor}$ 

### **Matrix Terms**

### $3.13 \setminus rotM$

# 

Rotation matrix. This command is used to express the rotation matrix to convert from  $\langle frame2 \rangle$  to  $\langle frame1 \rangle$ .

```
\langle frame1 \rangle = First frame; letter (a-z, A-Z) \langle frame2 \rangle = Second frame; letter (a-z, A-Z)
```

### Example

Say we want to define the rotation matrix to convert from the  $\bar{O}$  frame to the  $\bar{A}$  frame.

This is accomplished with the LATEX below:

\rotM{A}{0}

Display/Inline Mode Output

 $^{\bar{\mathbf{A}}}\![C]^{\bar{\mathbf{O}}}$ 

### $3.14 \ \text{rotMd}$

# $\t \operatorname{rotMd} \{\langle frame1 \rangle\} \{\langle frame2 \rangle\}$

Rotation matrix derivative. This command is used to express the derivative of the rotation matrix that converts from  $\langle frame2 \rangle$  to  $\langle frame1 \rangle$ .

```
\langle frame1 \rangle = First frame; letter (a-z, A-Z) \langle frame2 \rangle = Second frame; letter (a-z, A-Z)
```

### Example

Say we want to define the derivative of the rotation matrix that converts from the  $\bar{O}$  frame to the  $\bar{A}$  frame.

This is accomplished with the LATEX below:

 $\t \Lambda(A){0}$ 

Display/Inline Mode Output

 ${}^{\bar{{\scriptscriptstyle A}}}\![\dot{C}]^{\!\bar{{\scriptscriptstyle O}}}$ 

### 3.15 \Rx

### $\Rx{\langle angle \rangle}$

X rotation matrix. This command is used to express the rotation matrix about the x-axis (positive CCW).

```
\langle angle \rangle = Angle name; expression
```

### Example

Say we want to define the x rotation matrix using the angle  $\phi$ .

This is accomplished with the LATEX below:

 $\Rx{\phi}$ 

Display/Inline Mode Output

 $[R_x(\phi)]$ 

# 3.16 \Ry

## $\Ry\{\langle angle \rangle\}$

Y rotation matrix. This command is used to express the rotation matrix about the y-axis (positive CCW).

 $\langle angle \rangle$  = Angle name; expression

#### Example

Say we want to define the y rotation matrix using the angle  $\theta$ .

This is accomplished with the LATEX below:

 $\Ry{\theta}$ 

Display/Inline Mode Output

 $\left[\mathbf{R}_{\mathbf{y}}\!(\boldsymbol{\theta})\right]$ 

#### 3.17 \Rz

## $\Rz\{\langle angle \rangle\}$

Z rotation matrix. This command is used to express the rotation matrix about the z-axis (positive CCW).

 $\langle angle \rangle$  = Angle name; expression

## Example

Say we want to define the z rotation matrix using the angle  $\psi$ .

This is accomplished with the LATEX below:

 $\Rz{\psi}$ 

Display/Inline Mode Output

 $[R_z(\psi)]$ 

#### $3.18 \setminus \text{omcrMat}$

# $\colonerright \colonerright \colonerright$

Omega cross matrix. This command is used to express the omega cross matrix, a matrix which when multiplied by a column vector is equivalent to taking the cross product of the angular velocity of  $\langle frame2 \rangle$  with respect to  $\langle frame1 \rangle$  and that vector.

```
\langle frame1 \rangle = First frame; letter (a-z, A-Z) \langle frame2 \rangle = Second frame; letter (a-z, A-Z)
```

## Example

Say we want to define the omega cross matrix based on the relative angular velocity of the  $\bar{B}$  frame with respect to the  $\bar{A}$  frame.

This is accomplished with the LaTeX below:

\omcrMat{A}{B}

Display/Inline Mode Output

$$[\bar{\alpha}\vec{\omega}^{\bar{B}}\times]_{\bar{B}}$$

#### $3.19 \setminus \text{omcrx}$

# $\operatorname{\mathtt{\comcrx}} \langle frame1 \rangle \} \{ \langle frame2 \rangle \}$

Omega cross x-term. This command is the x-term of the omega cross matrix, a matrix which when multiplied by a column vector is equivalent to taking the cross product of the angular velocity of  $\langle frame2 \rangle$  with respect to  $\langle frame1 \rangle$  and that vector.

```
\langle frame1 \rangle = First frame; letter (a-z, A-Z) \langle frame2 \rangle = Second frame; letter (a-z, A-Z)
```

## Example

Say we want to define the x-term of the omega cross matrix based on the relative angular velocity of the  $\bar{B}$  frame with respect to the  $\bar{A}$  frame.

This is accomplished with the LATEX below:

 $\operatorname{\operatorname{Nomcrx}}\{A\}\{B\}$ 

$${}^{ar{A}}\omega_{xar{B}}^{ar{B}}$$

# 3.20 \omegamery

```
\colone{line} \colone{line}
```

Omega cross y-term. This command is the y-term of the omega cross matrix, a matrix which when multiplied by a column vector is equivalent to taking the cross product of the angular velocity of  $\langle frame2 \rangle$  with respect to  $\langle frame1 \rangle$  and that vector.

```
\langle frame1 \rangle = First frame; letter (a-z, A-Z) \langle frame2 \rangle = Second frame; letter (a-z, A-Z)
```

#### Example

Say we want to define the y-term of the omega cross matrix based on the relative angular velocity of the  $\bar{B}$  frame with respect to the  $\bar{A}$  frame.

This is accomplished with the LaTeX below:

\omcry{A}{B}

Display/Inline Mode Output

 ${}^{ar{A}}\omega^{ar{B}}_{yar{B}}$ 

#### 3.21 \omegamerz

```
\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\colone{1}\col
```

Omega cross z-term. This command is the z-term of the omega cross matrix, a matrix which when multiplied by a column vector is equivalent to taking the cross product of the angular velocity of  $\langle frame2 \rangle$  with respect to  $\langle frame1 \rangle$  and that vector.

```
\langle frame1 \rangle = First frame; letter (a-z, A-Z) \langle frame2 \rangle = Second frame; letter (a-z, A-Z)
```

#### Example

Say we want to define the z-term of the omega cross matrix based on the relative angular velocity of the  $\bar{B}$  frame with respect to the  $\bar{A}$  frame.

This is accomplished with the LATEX below:

\omcrz{A}{B}

Display/Inline Mode Output

 ${}^{ar{A}}\omega_{zar{B}}^{ar{B}}$ 

## Inertia Terms

## 3.22 \inerTen

# $\iner{Ten}{\langle sub \rangle}$

Inertia tensor. This command is used to express a generic inertia tensor with a specified subscript  $\langle sub \rangle$ .

 $\langle sub \rangle = \text{Subscript}; \text{ expression}$ 

#### Example

Say we want to define the inertia tensor about the center of mass.

This is accomplished with the LATEX below:

\inerTen{\CM}

Display/Inline Mode Output

 $\widetilde{I}_{\scriptscriptstyle ext{CM}}$ 

#### 3.23 \inerMat

## $\inerMat{\langle frame \rangle}{\langle sub \rangle}$

Inertia tensor expressed as a matrix. This command is used to express a generic inertia tensor with a specified subscript  $\langle sub \rangle$  as a matrix in a specified frame  $\langle frame \rangle$ .

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z) subSubscript; expression

#### Example

Say we want to define the inertia tensor about the center of mass and express it as a matrix in the  $\bar{A}$  frame.

This is accomplished with the LATEX below:

\inerMat{a}{\CM}

Display/Inline Mode Output

 $[\tilde{I}_{\scriptscriptstyle ext{CM}}]_{\!ar{A}}$ 

## $3.24 \setminus Ixx$

# $\label{line} \label{line} $$ \xi {\langle frame \rangle} {\langle sub \rangle} $$

Inertia tensor xx-component. This command is used to express the xx-component of the inertia tensor about a specified point in a specified frame  $\langle frame \rangle$ .

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z) subSubscript; expression

#### Example

Say we want to define the inertia tensor's xx-component about the center of mass in the  $\bar{A}$  frame.

This is accomplished with the LATEX below:

 $Ixx{a}{CM}$ 

Display/Inline Mode Output

 ${}^{\bar{A}}I_{xx/\!{\rm CM}}$ 

## 3.25 \Ixy

# $\t xy{\langle frame \rangle} {\langle sub \rangle}$

Inertia tensor xy-component. This command is used to express the xy-component of the inertia tensor about a specified point in a specified frame  $\langle frame \rangle$ .

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z) subSubscript; expression

## Example

Say we want to define the inertia tensor's xy-component about the center of mass in the  $\bar{A}$  frame.

This is accomplished with the LATEX below:

 $Ixy{a}{CM}$ 

Display/Inline Mode Output

 ${}^{\bar{A}}I_{xy/\!{\rm CM}}$ 

## $3.26 \setminus Ixz$

# 

Inertia tensor xz-component. This command is used to express the xz-component of the inertia tensor about a specified point in a specified frame  $\langle frame \rangle$ .

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z) subSubscript; expression

#### Example

Say we want to define the inertia tensor's xz-component about the center of mass in the  $\bar{A}$  frame.

This is accomplished with the LATEX below:

 $Ixz{a}{CM}$ 

Display/Inline Mode Output

 ${}^{\bar{A}}I_{xz/\!\mathrm{CM}}$ 

# 3.27 \Iyx

# $\t x{\langle frame \rangle} {\langle sub \rangle}$

Inertia tensor yx-component. This command is used to express the yx-component of the inertia tensor about a specified point in a specified frame  $\langle frame \rangle$ .

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z) subSubscript; expression

## Example

Say we want to define the inertia tensor's yx-component about the center of mass in the  $\bar{A}$  frame.

This is accomplished with the LATEX below:

 $Iyx{a}{CM}$ 

Display/Inline Mode Output

 ${}^{\bar{A}}I_{yx/\!\mathrm{CM}}$ 

# 3.28 \Iyy

# $\langle Iyy \{ \langle frame \rangle \} \{ \langle sub \rangle \}$

Inertia tensor yy-component. This command is used to express the yy-component of the inertia tensor about a specified point in a specified frame  $\langle frame \rangle$ .

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z) subSubscript; expression

#### Example

Say we want to define the inertia tensor's yy-component about the center of mass in the  $\bar{A}$  frame.

This is accomplished with the LATEX below:

 $\Iyy{a}{\CM}$ 

Display/Inline Mode Output

$${}^{\bar{A}}I_{yy/\!\mathrm{CM}}$$

#### 

# $\time {\tt Iyz} {\tt (frame)} {\tt (sub)}$

Inertia tensor yz-component. This command is used to express the yz-component of the inertia tensor about a specified point in a specified frame  $\langle frame \rangle$ .

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z) subSubscript; expression

## Example

Say we want to define the inertia tensor's yz-component about the center of mass in the  $\bar{A}$  frame.

This is accomplished with the  $\LaTeX$  below:

 $Iyz{a}{CM}$ 

$${}^{\bar{A}}I_{yz/\!\mathrm{CM}}$$

## $3.30 \ \text{Izx}$

# 

Inertia tensor zx-component. This command is used to express the zx-component of the inertia tensor about a specified point in a specified frame  $\langle frame \rangle$ .

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z) subSubscript; expression

#### Example

Say we want to define the inertia tensor's zx-component about the center of mass in the  $\bar{A}$  frame.

This is accomplished with the LATEX below:

 $Izx{a}{CM}$ 

Display/Inline Mode Output

 ${}^{\bar{A}}I_{zx/\!{\rm CM}}$ 

# 3.31 \Izy

# $\time {\tt lzy} {\tt drame} {\tt lxy} {\tt drame}$

Inertia tensor zy-component. This command is used to express the zy-component of the inertia tensor about a specified point in a specified frame  $\langle frame \rangle$ .

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z) subSubscript; expression

## Example

Say we want to define the inertia tensor's zy-component about the center of mass in the  $\bar{A}$  frame.

This is accomplished with the LATEX below:

 $\Izy{a}{\CM}$ 

$${}^{\bar{A}}I_{zy/\!{\rm CM}}$$

## $3.32 \setminus Izz$

# 

Inertia tensor zz-component. This command is used to express the zz-component of the inertia tensor about a specified point in a specified frame  $\langle frame \rangle$ .

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z) subSubscript; expression

## Example

Say we want to define the inertia tensor's zz-component about the center of mass in the  $\bar{A}$  frame.

This is accomplished with the LATEX below:

 $Izz{a}{CM}$ 

Display/Inline Mode Output

 ${}^{\bar{A}}I_{zz/\!\mathrm{CM}}$ 

#### $3.33 \ \text{xrel}$

## $\xrel{\langle frame \rangle} {\langle point \rangle} {\langle refpt \rangle}$

Relative x-displacement. This command is used to express the displacement in x from  $\langle point \rangle$  to  $\langle refpt \rangle$ .

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z) point Point; expression<br/>refpt Reference point; expression

#### Example

Say we want to define the displacement in x of a particle  $m_i$  relative to the center of mass in the  $\bar{A}$  body frame.

This is accomplished with the LATEX below:

 $\xrel{a}{m_{i}}{CM}$ 

Display/Inline Mode Output

 ${}^{\bar{A}}\!x_{m_i/\!\mathrm{CM}}$ 

# 3.34 \yrel

# 

Relative y-displacement. This command is used to express the displacement in y from  $\langle point \rangle$  to  $\langle refpt \rangle$ .

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z) point Point; expression<br/>refpt Reference point; expression

#### Example

Say we want to define the displacement in y of a particle  $m_i$  relative to the center of mass in the  $\bar{A}$  body frame.

This is accomplished with the LATEX below:

 $\yrel{a}{m_{i}}{CM}$ 

Display/Inline Mode Output

 ${}^{\bar{A}}y_{m_i/\!\mathrm{CM}}$ 

## 3.35 \zrel

# $\zrel{\langle frame\rangle}{\langle point\rangle}{\langle refpt\rangle}$

Relative z-displacement. This command is used to express the displacement in z from  $\langle point \rangle$  to  $\langle refpt \rangle$ .

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z) pointPoint; expressionrefptReference point; expression

#### Example

Say we want to define the displacement in z of a particle  $m_i$  relative to the center of mass in the  $\bar{A}$  body frame.

This is accomplished with the  $\LaTeX$  below:

 $zrel{a}{m_{i}}{CM}$ 

Display/Inline Mode Output

 ${}^{\bar{A}}z_{m_i/\!{\rm CM}}$ 

## Other Terms

## 3.36 \vecF

# $\vec{\mathsf{F}}{\langle frame \rangle}{\langle vector \rangle}{\langle sub \rangle}$

Vector in frame with subscript. This command is used to express a specified vector  $\langle vector \rangle$  in a frame  $\langle frame \rangle$  with a chosen subscript  $\langle sub \rangle$ .

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z) vectorVector; expressionsubSubscript; expression

## Example

Say we want to define a vector  $\vec{s}$ , which describes the position of a bus in the  $\bar{E}$  frame.

This is accomplished with the LATEX below:

\vecF{e}{s}{\mathit{bus}}

Display/Inline Mode Output

 ${}^{ar{E}} ec{s}_{bus}$ 

# $3.37 \setminus potEn$

#### \potEn

Potential energy. This command is used to express the potential energy.

No input arguments.

#### Example

Define the potential energy of a particle of mass m due to gravity near the Earth's surface while treating  $\bar{O}$  as an inertial reference frame attached to the Earth's surface.

This is accomplished with the LATEX below:

\potEn \approx mgh

Display/Inline Mode Output

 $^{\bar{o}}V \approx mgh$ 

# 3.38 \kinEn

#### \kinEn

Kinetic energy. This command is used to express the kinetic energy.

No input arguments.

# Example

Define the kinetic energy of a particle of mass m translating at speed v while treating  $\bar{O}$  as an inertial reference frame attached to the Earth's surface.

This is accomplished with the  $\LaTeX$  below:

 $\kappa = \frac{1}{2}mv^2$ 

Display Mode Output

$${}^{\bar{o}}T_o = \frac{1}{2}mv^2$$

Inline Mode Output

$$^{\bar{o}}T_{\scriptscriptstyle O}=\frac{1}{2}mv^2$$

# 4 Advanced Terms

This section includes the commands needed to typeset most of the terms from MAE 789. Note that all commands in this section should be used inside of a math environment.

## Partial Terms

#### 4.1 \Pvec

```
\Pvec{\langle frame1\rangle} \{\langle var\rangle}\}

P-vector. This command is used to express the P-vector of \langle frame2\rangle relative to \langle frame1\rangle with respect to \langle var\rangle.

\langle frame1\rangle = \text{Frame}; \text{letter (a-z, A-Z)} \langle frame2\rangle = \text{Frame}; \text{letter (a-z, A-Z)} \langle \langle var\rangle = \text{Variable}; \text{symbol}
```

#### Example

Say we want to define the P-vector of the  $\bar{B}$  frame relative to the  $\bar{A}$  with respect to  $\theta$ .

This is accomplished with the LATEX below:

 $\Pvec{a}{b}{\tilde{b}}{\tilde{a}}$ 

Display/Inline Mode Output

 ${}^{ar{A}}\!ec{P}^{ar{B}}_{\! heta}$ 

#### 4.2 \Pvecx

```
\ensuremath{\mbox{\sc Pvecx}} \langle frame1 \rangle \} \{ \langle frame2 \rangle \} \{ \langle var \rangle \}
```

P-vector x-component. This command is used to express the x-component of the P-vector of  $\langle frame2 \rangle$  relative to  $\langle frame1 \rangle$  with respect to  $\langle var \rangle$ .

```
\langle frame1 \rangle = Frame; letter (a-z, A-Z)
\langle frame2 \rangle = Frame; letter (a-z, A-Z)
\langle var \rangle = Variable; symbol
```

## Example

Say we want to define the x-component of the P-vector of the  $\bar{B}$  frame relative to the  $\bar{A}$  with respect to  $\theta$ .

This is accomplished with the LATEX below:

\Pvecx{a}{b}{\theta}

Display/Inline Mode Output

 ${}^{ar{A}}\!\vec{P}_{ heta,x}^{ar{B}}$ 

# 4.3 \Pvecy

```
\ensuremath{\mbox{\sc Pvecy}} \{\langle frame1 \rangle\} \{\langle frame2 \rangle\} \{\langle var \rangle\}
```

P-vector y-component. This command is used to express the y-component of the P-vector of  $\langle frame2 \rangle$  relative to  $\langle frame1 \rangle$  with respect to  $\langle var \rangle$ .

```
\langle frame1 \rangle = Frame; letter (a-z, A-Z)
\langle frame2 \rangle = Frame; letter (a-z, A-Z)
\langle var \rangle = Variable; symbol
```

## Example

Say we want to define the y-component of the P-vector of the  $\bar{B}$  frame relative to the  $\bar{A}$  with respect to  $\theta$ .

This is accomplished with the LATEX below:

 $\P \$ 

Display/Inline Mode Output

 ${}^{ar{A}}\!\vec{P}_{ heta,y}^{ar{B}}$ 

#### 4.4 \Pvecz

```
\Pvecz{\langle frame1\rangle}{\langle frame2\rangle}{\langle var\rangle}
```

P-vector z-component. This command is used to express the z-component of the P-vector of  $\langle frame2 \rangle$  relative to  $\langle frame1 \rangle$  with respect to  $\langle var \rangle$ .

```
\langle frame1 \rangle = Frame; letter (a-z, A-Z)
\langle frame2 \rangle = Frame; letter (a-z, A-Z)
\langle var \rangle = Variable; symbol
```

## Example

Say we want to define the z-component of the P-vector of the  $\bar{B}$  frame relative to the  $\bar{A}$  with respect to  $\theta$ .

This is accomplished with the LATEX below:

 $\Pvecz{a}{b}{\theta}$ 

Display/Inline Mode Output

 ${}^{\bar{A}}\vec{P}_{ heta,z}^{\bar{B}}$ 

## **Total Terms**

#### 4.5 \Tvec

```
\Tvec{\langle frame1 \rangle}{\langle frame2 \rangle}{\langle var \rangle}
```

T-vector. This command is used to express the T-vector of  $\langle frame2 \rangle$  relative to  $\langle frame1 \rangle$  with respect to  $\langle var \rangle$ .

```
\langle frame1 \rangle = Frame; letter (a-z, A-Z)
\langle frame2 \rangle = Frame; letter (a-z, A-Z)
\langle var \rangle = Variable; symbol
```

#### Example

Say we want to define the T-vector of the  $\bar{B}$  frame relative to the  $\bar{A}$  with respect to  $\theta$ .

This is accomplished with the LATEX below:

 $\Tvec{a}{b}{\theta}$ 

Display/Inline Mode Output

 ${}^{\bar{A}}\vec{T}_{ heta}^{\bar{B}}$ 

## 4.6 \Tvecx

```
\Tvecx{\langle frame1 \rangle}{\langle frame2 \rangle}{\langle var \rangle}
```

T-vector x-component. This command is used to express the x-component of the T-vector of  $\langle frame2 \rangle$  relative to  $\langle frame1 \rangle$  with respect to  $\langle var \rangle$ .

```
\langle frame1 \rangle = Frame; letter (a-z, A-Z)
\langle frame2 \rangle = Frame; letter (a-z, A-Z)
\langle var \rangle = Variable; symbol
```

## Example

Say we want to define the x-component of the T-vector of the  $\bar{B}$  frame relative to the  $\bar{A}$  with respect to  $\theta$ .

This is accomplished with the LATEX below:

 $\Tvecx{a}{b}{\theta}$ 

Display/Inline Mode Output

 ${}^{\bar{A}}\vec{T}_{ heta,x}^{ar{B}}$ 

# 4.7 \Tvecy

```
\verb|\Tvecy{| \langle frame1 \rangle| {\langle frame2 \rangle} {\langle var \rangle}|}
```

T-vector y-component. This command is used to express the y-component of the T-vector of  $\langle frame2 \rangle$  relative to  $\langle frame1 \rangle$  with respect to  $\langle var \rangle$ .

```
\langle frame1 \rangle = Frame; letter (a-z, A-Z)
\langle frame2 \rangle = Frame; letter (a-z, A-Z)
\langle var \rangle = Variable; symbol
```

#### Example

Say we want to define the y-component of the T-vector of the  $\bar{B}$  frame relative to the  $\bar{A}$  with respect to  $\theta$ .

This is accomplished with the LATEX below:

 $\Tvecy{a}{b}{\theta}$ 

Display/Inline Mode Output

 ${}^{ar{A}}ec{T}_{ heta,y}^{ar{B}}$ 

#### 

```
\Tvecz{\langle frame1 \rangle}{\langle frame2 \rangle}{\langle var \rangle}
```

T-vector z-component. This command is used to express the z-component of the T-vector of  $\langle frame2 \rangle$  relative to  $\langle frame1 \rangle$  with respect to  $\langle var \rangle$ .

```
\langle frame1 \rangle = Frame; letter (a-z, A-Z)
\langle frame2 \rangle = Frame; letter (a-z, A-Z)
\langle var \rangle = Variable; symbol
```

## Example

Say we want to define the z-component of the T-vector of the  $\bar{B}$  frame relative to the  $\bar{A}$  with respect to  $\theta$ .

This is accomplished with the LATEX below:

 $\Tvecz{a}{b}{\theta}$ 

Display/Inline Mode Output

 ${}^{\bar{A}}\vec{T}_{\theta,z}^{\bar{B}}$ 

# **Equations of Motion Terms**

The terms in this section are used most commonly when deriving equations of motion of systems using more advanced methods such as Lagrange's equations and Kane's equations.

# $4.9 \ \n$

```
\aggVk{\langle frame1 \rangle}{\langle frame2 \rangle}{\langle k \rangle}
```

Angular velocity k. This command is used to express the angular velocity of  $\langle frame2 \rangle$ - $\langle k \rangle$  relative to  $\langle frame1 \rangle$ .

```
\langle frame1 \rangle = First frame; letter (a-z, A-Z)
\langle frame2 \rangle = Second frame; letter (a-z, A-Z)
\langle k \rangle = Number; positive integer
```

#### Example

Say we want to define the angular velocity of the  $\bar{A}_5$  frame relative to the  $\bar{O}$  frame.

This is accomplished with the LATEX below:

 $\angVk{0}{A}{5}$ 

Display/Inline Mode Output

 $\bar{O}_{(1)} \vec{A}_5$ 

## $4.10 \setminus angAk$

Angular acceleration k. This command is used to express the angular acceleration of  $\langle frame2 \rangle - \langle k \rangle$  relative to  $\langle frame1 \rangle$ .

 $\langle frame1 \rangle$  = First frame; letter (a-z, A-Z)  $\langle frame2 \rangle$  = Second frame; letter (a-z, A-Z)

 $\langle k \rangle$  = Number; positive integer

## Example

Say we want to define the angular velocity of the  $\bar{B}_6$  frame relative to the  $\bar{O}$  frame.

This is accomplished with the LATEX below:

 $\angAk{o}{b}{6}$ 

Display/Inline Mode Output

 $\bar{A}\vec{\alpha}^{\bar{B}_6}$ 

## 4.11 \forc

## $\forc{\langle sub \rangle}$

Lowercase force. This command is used to express the forces (usually internal).

 $\langle sub \rangle = \text{Subscript}; \text{ expression}$ 

#### Example

Say we want to define the force acting on the ith body.

This is accomplished with the LATEX below:

\forc{i}

Display/Inline Mode Output

 $\vec{f_i}$ 

# $4.12 \ \text{ptraVr}$

# $\mathbf{\t Vr}(\langle frame \rangle) \{\langle point \rangle\} \{\langle r \rangle\}$

r-th partial velocity. This command is used to express rth partial velocity of  $\langle point \rangle$  in  $\langle frame \rangle$ .

 $\langle frame \rangle = Frame; letter (a-z, A-Z)$ 

 $\langle point \rangle = Point; expression$ 

 $\langle r \rangle = \text{Number}$ ; positive integer

#### Example

Say we want to define the 4th partial velocity of  $m_i$  in  $\bar{O}$ .

This is accomplished with the LATEX below:

\ptraVr{o}{m\_i}{4}

Display/Inline Mode Output

 ${}_{\scriptscriptstyle O}^{\scriptscriptstyle \bar{A}} \vec{v}_{m_i,4}$ 

# $4.13 \ \text{ptraVt}$

# $\proonup \proonup \$

time partial velocity. This command is used to express the time partial velocity of  $\langle point \rangle$  in  $\langle frame \rangle$ .

 $\langle frame \rangle = \text{Frame}; \text{ letter (a-z, A-Z)}$ 

 $\langle point \rangle = Point; expression$ 

#### Example

Say we want to define the time partial velocity of  $m_i$  in  $\bar{O}$ .

This is accomplished with the LATEX below:

\ptraVt{o}{m\_i}

Display/Inline Mode Output

 ${}^{ar{A}}_{\scriptscriptstyle O} \vec{v}_{m_i,t}$ 

# $4.14 \pangVr$

```
\propto \pro
```

r-th partial angular velocity. This command is used to express rth partial angular velocity of  $\langle frame2 \rangle$  in  $\langle frame1 \rangle$ .

```
\langle frame1 \rangle = Frame; letter (a-z, A-Z)
\langle frame2 \rangle = Frame; letter (a-z, A-Z)
\langle r \rangle = Number; positive integer
```

## Example

Say we want to define the 4th partial angular velocity of  $\bar{B}$  in  $\bar{A}$ .

This is accomplished with the LaTeX below:

 $\pary Vr{a}{b}{4}$ 

Display/Inline Mode Output

 ${}^{ar{A}} \vec{\omega}_4^{ar{B}}$ 

# $4.15 \pangVrk$

## $\parbox{$\operatorname{Vrk}(\langle frame1\rangle)$} {\langle frame2\rangle} {\langle k\rangle} {\langle r\rangle}$

r-th partial angular velocity k. This command is used to express rth partial angular velocity of  $\langle frame2 \rangle - \langle k \rangle$  in  $\langle frame1 \rangle$ .

```
\langle frame1 \rangle = Frame; letter (a-z, A-Z)
\langle frame2 \rangle = Frame; letter (a-z, A-Z)
\langle k \rangle = Number; positive integer
\langle r \rangle = Number; positive integer
```

#### Example

Say we want to define the 4th partial angular velocity of  $\bar{B}_2$  in  $\bar{A}$ .

This is accomplished with the LATEX below:

 $\partial{pangVr{a}{b}{2}{4}}$ 

Display/Inline Mode Output

 ${}^{ar{A}}ec{\omega}_4^{ar{B}_2}$ 

# $4.16 \pangVt$

```
\partial \
```

time partial angular velocity. This command is used to express the time partial angular velocity of  $\langle frame2 \rangle$  in  $\langle frame1 \rangle$ .

```
\langle frame1 \rangle = Frame; letter (a-z, A-Z) \langle frame2 \rangle = Frame; letter (a-z, A-Z)
```

#### Example

Say we want to define the time partial angular velocity of  $\bar{B}$  in  $\bar{A}$ .

This is accomplished with the LATEX below:

\pangVt{o}{b}

Display/Inline Mode Output

 ${}^{ar{A}}\!ec{\omega}_t^{ar{B}}$ 

# 4.17 \Lagr

## \Lagr

Lagrangian. Show the symbol for the Lagrangian.

No input arguments.

#### Example

Show the symbol for the Lagrangian.

This is accomplished with the LATEX below:

\Lagr

Display/Inline Mode Output

 ${}^{\bar{o}}\!\mathcal{L}_o$ 

# 4.18 \KFr

# $\KFr\{\langle r \rangle\}$

 $F_r$ . This command is used to express  $F_r$  from Kane's equations of motion.

 $\langle r \rangle$  = Number; positive integer

#### Example

Say we want to define  $F_1$ .

This is accomplished with the LATEX below:

 $\KFr{1}$ 

Display/Inline Mode Output

 $F_1$ 

## 4.19 \KFrs

# $\KFrs{\langle r \rangle}$

 $F_r^*$ . This command is used to express  $F_r^*$  from Kane's equations of motion.

 $\langle r \rangle = r$ ; positive integer

#### Example

Say we want to define  $F_1^*$ .

This is accomplished with the LATEX below:

 $\KFrs{1}$ 

Display/Inline Mode Output

 $F_1^*$ 

# 5 Differentiation

This section includes the commands needed to typeset most derivatives and their calculation. Note that all commands in this section should be used inside of a math environment.

#### Time Derivatives

## $5.1 \setminus derI$

```
\derive{derI}{\langle frame \rangle}{\langle expr \rangle}
```

1st time derivative. This command is used to take the first time derivative of  $\langle expr \rangle$  in the  $\langle frame \rangle$  frame.

```
\langle frame \rangle = Frame; letter (a-z, A-Z)
\langle expr \rangle = Expression; expression
```

#### Example

Say we want to define the first time derivative of position,  $\vec{r}$ , in the  $\bar{A}$  frame.

This is accomplished with the LATEX below:

 $\derI{a}{\vec{r}\,}$ 

Display/Inline Mode Output

$$\frac{\bar{A}_{
m d}}{{
m d}t}(ec{r})$$

#### 5.2 \derII

```
\derII{\langle frame \rangle} {\langle expr \rangle}
```

2nd time derivative. This command is used to take the second time derivative of  $\langle expr \rangle$  in the  $\langle frame \rangle$  frame.

```
\langle frame \rangle = Frame; letter (a-z, A-Z)
\langle expr \rangle = Expression; expression
```

#### Example

Say we want to define the second time derivative of position,  $\vec{r}$ , in the  $\bar{A}$  frame.

This is accomplished with the LATEX below:

 $\derII{a}{\vec{r}\,}$ 

$$\frac{\bar{A}_{d^2}}{dt^2}(\vec{r})$$

## $5.3 \ \text{derN}$

# $\der N{\langle frame \rangle} {\langle expr \rangle} {\langle n \rangle}$

nth time derivative. This command is used to take the nth time derivative of  $\langle expr \rangle$  in the  $\langle frame \rangle$  frame.

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z)  $\langle expr \rangle$  = Expression; expression  $\langle n \rangle$  = Number, positive integer

#### Example

Say we want to define the 100th time derivative of position,  $\vec{r}$ , in the  $\bar{A}$  frame.

This is accomplished with the LATEX below:

 $\derN{a}{\vec{r}\,}{100}$ 

Display/Inline Mode Output

$$rac{ar{d}^{100}}{\mathrm{d}t^{100}}(ec{r})$$

#### 5.4 \tranI

# $\tranl{dframe} {\tranl} {\dframe} {\dframe} {\dframe} {\dframe}$

First-order transport theorem. This command is used to show how to take the first  $\langle dframe \rangle$ -frame time derivative of  $\langle expr \rangle$  which is expressed in the  $\langle eframe \rangle$ -frame.

 $\langle dframe \rangle$  = Derivative frame; letter (a-z, A-Z)  $\langle eframe \rangle$  = Expressed frame; letter (a-z, A-Z)  $\langle expr \rangle$  = Expression; expression

#### Example

Say we want to find the first  $\bar{B}$  frame time derivative of the vector  $\vec{q}$ , which is expressed in the  $\bar{A}$  frame.

This is accomplished with the LaTeX below:

 $\tranI{b}{a}{\vec{q}},$ 

# 5.5 \tranII

# $\tranll{dframe} {\tranll{dframe}} {\cline{constraints}} {\cline{$

Second-order transport theorem. This command is used to show how to take the second  $\langle dframe \rangle$ -frame time derivative of  $\langle expr \rangle$  which is expressed in the  $\langle eframe \rangle$ -frame.

 $\langle dframe \rangle = Derivative frame; letter (a-z, A-Z) \langle eframe \rangle = Expressed frame; letter (a-z, A-Z)$ 

 $\langle expr \rangle = \text{Expression}; \text{ expression}$ 

## Example

Say we want to find the second  $\bar{B}$  frame time derivative of the vector  $\vec{q}$ , which is expressed in the  $\bar{A}$  frame.

This is accomplished with the LATEX below:

 $\tranII{b}{a}{\vec{q}},$ 

$$rac{ar{^B}\mathbf{d}^2}{\mathbf{d}t^2}(ec{q}) = rac{ar{^A}\mathbf{d}^2}{\mathbf{d}t^2}(ec{q}) + {^{ar{^B}}}ec{lpha}^{ar{^A}} imes ec{q} + 2^{ar{^B}}ec{\omega}^{ar{^A}} imes rac{ar{^A}\mathbf{d}}{\mathbf{d}t}(ec{q}) + {^{ar{^B}}}ec{\omega}^{ar{^A}} imes \left({^{ar{^B}}}ec{\omega}^{ar{^A}} imes ec{q}
ight)$$

## Partial Derivatives

# $5.6 \ \text{pderI}$

```
\prootemark\pderI{\langle frame \rangle}{\langle var \rangle}{\langle expr \rangle}
```

1st partial derivative. This command is used to take the first partial derivative with respect to  $\langle var \rangle$  of  $\langle expr \rangle$  in the  $\langle frame \rangle$  frame.

```
\langle frame \rangle = Frame; letter (a-z, A-Z)
\langle var \rangle = Variable; symbol
\langle expr \rangle = Expression; expression
```

#### Example

Say we want to define the first partial derivative of velocity,  $\vec{v}$ , with respect to  $\theta$  in the  $\bar{B}$  frame.

This is accomplished with the LATEX below:

Display/Inline Mode Output

$$rac{ar{ heta}\partial}{\partial heta}(ec{v}\,)$$

# 5.7 \pderII

# $\prootemark$ $\prootemark$

2nd partial derivative. This command is used to take the second partial derivative with respect to  $\langle var \rangle$  of  $\langle expr \rangle$  in the  $\langle frame \rangle$  frame.

```
\langle frame \rangle = Frame; letter (a-z, A-Z)
\langle var \rangle = Variable; symbol
\langle expr \rangle = Expression; expression
```

#### Example

Say we want to define the second partial derivative of velocity,  $\vec{v}$ , with respect to  $\theta$  in the  $\bar{B}$  frame.

This is accomplished with the LATEX below:

 $\displaystyle \left\{ \left( vec\{v\} \right), \right\} \right.$ 

$$\frac{\bar{^{B}}\partial^{2}}{\partial\theta^{2}}(\vec{v}\,)$$

# 5.8 \pderN

# 

nth partial derivative. This command is used to take the nth partial derivative with respect to  $\langle var \rangle$  of  $\langle expr \rangle$  in the  $\langle frame \rangle$  frame.

 $\langle frame \rangle = Frame; letter (a-z, A-Z)$ 

 $\langle var \rangle = \text{Variable; symbol}$ 

 $\langle expr \rangle = \text{Expression}; \text{ expression}$ 

 $\langle n \rangle = \text{Number, positive integer}$ 

## Example

Say we want to define the 30th partial derivative of velocity,  $\vec{v}$ , with respect to  $\theta$  in the  $\bar{B}$  frame.

This is accomplished with the LATEX below:

Display/Inline Mode Output

$$rac{ar{ heta}\partial^{30}}{\partial heta^{30}}(ec{v}\,)$$

# 5.9 \ptranI

# $\mathbf{\langle} dframe \} \{ \langle eframe \} \} \{ \langle var \rangle \} \{ \langle expr \rangle \}$

First-order partial derivative transport theorem. This command is used to show how to take the first partial  $\langle dframe \rangle$ -frame derivative with respect to  $\langle var \rangle$  of  $\langle expr \rangle$  which is expressed in the  $\langle eframe \rangle$ -frame.

```
\langle dframe \rangle = Derivative frame; letter (a-z, A-Z)
\langle eframe \rangle = Expressed frame; letter (a-z, A-Z)
\langle var \rangle = Variable; symbol
\langle expr \rangle = Expression; expression
```

#### Example

Say we want to find the first partial  $\bar{B}$  frame derivative with respect to  $\phi$  of the vector  $\vec{q}$ , which is expressed in the  $\bar{A}$  frame.

This is accomplished with the LATEX below:

$$rac{ar{B}}{\partial \phi}(ec{q}) = rac{ar{A}}{\partial \phi}(ec{q}) + ar{B}ec{P}_{\phi}^{ar{A}} imes ec{q}$$

## **Total Derivatives**

#### 5.10 \tderI

```
\text{ } \
```

1st total derivative. This command is used to take the first total derivative with respect to  $\langle var \rangle$  of  $\langle expr \rangle$  in the  $\langle frame \rangle$  frame.

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z)  $\langle var \rangle$  = Variable; symbol

 $\langle expr \rangle = \text{Expression}; \text{ expression}$ 

#### Example

Say we want to define the first total derivative of velocity,  $\vec{v}$ , with respect to  $\psi$  in the  $\bar{O}$  frame.

This is accomplished with the LATEX below:

 $\time I{o}{\psi}{\vec{v}\,}$ 

Display/Inline Mode Output

 $\frac{\bar{o}_{\mathrm{d}}}{\mathrm{d}\psi}(\vec{v}\,)$ 

#### 5.11 \tderII

# $\label{locality} $$ \der{II}(\langle frame \rangle) = (\langle var \rangle) = (\langle expr \rangle) $$$

2nd total derivative. This command is used to take the second total derivative with respect to  $\langle var \rangle$  of  $\langle expr \rangle$  in the  $\langle frame \rangle$  frame.

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z)  $\langle var \rangle$  = Variable; symbol  $\langle expr \rangle$  = Expression; expression

#### Example

Say we want to define the second total derivative of velocity,  $\vec{v}$ , with respect to  $\psi$  in the  $\bar{O}$  frame.

This is accomplished with the LATEX below:

 $\time {\psi}{\vec{v}\,}$ 

Display/Inline Mode Output

 $\frac{\bar{o}}{\mathrm{d}\psi^2}(\vec{v})$ 

# 5.12 \tderN

## $\t (frame)$ $\{(var)\}$ $\{(expr)\}$

nth total derivative. This command is used to take the nth total derivative with respect to  $\langle var \rangle$  of  $\langle expr \rangle$  in the  $\langle frame \rangle$  frame.

 $\langle frame \rangle = Frame; letter (a-z, A-Z)$ 

 $\langle var \rangle = \text{Variable; symbol}$ 

 $\langle expr \rangle = \text{Expression}; \text{ expression}$ 

 $\langle n \rangle = \text{Number, positive integer}$ 

#### Example

Say we want to define the 30th total derivative of velocity,  $\vec{v}$ , with respect to  $\psi$  in the  $\bar{O}$  frame.

This is accomplished with the LATEX below:

 $\t \ensuremath{\t} {\psi}{\vec{v}\,}{30}$ 

Display/Inline Mode Output

$$rac{ar{o} \mathrm{d}^{30}}{\mathrm{d} \psi^{30}} (ec{v}\,)$$

#### 5.13 \ttranI

# 

First-order total derivative transport theorem. This command is used to show how to take the first total  $\langle dframe \rangle$ -frame derivative with respect to  $\langle var \rangle$  of  $\langle expr \rangle$  which is expressed in the  $\langle eframe \rangle$ -frame.

```
\langle dframe \rangle = Derivative frame; letter (a-z, A-Z)
\langle eframe \rangle = Expressed frame; letter (a-z, A-Z)
\langle var \rangle = Variable; symbol
\langle expr \rangle = Expression; expression
```

#### Example

Say we want to find the first total  $\bar{O}$  frame derivative with respect to  $\phi$  of the vector  $\vec{q}$ , which is expressed in the  $\bar{A}$  frame.

This is accomplished with the LATEX below:

$$rac{ar{O}_{
m d}}{{
m d}\phi}(ec{q}) = rac{ar{A}_{
m d}}{{
m d}\phi}(ec{q}) + ar{O}ec{T}_{\phi}^{ec{A}} imes ec{q}$$

# 6 Vectors

This section includes the commands needed to typeset most vector quantities. Note that all commands in this section should be used inside of a math environment. Vectors should be in a display math environment.

Note that if you wish to attach a frame subscipt to any of the elementary vectors, you should also include \! or \!\! before the frame definition to remove the space between the closing bracket of the vector and the frame. This will be addressed in a future version.

## Column Vectors

#### 6.1 \vecfrc

```
\vecfrc{\langle frame \rangle}{\langle comp1 \rangle}{\langle comp2 \rangle}{\langle comp3 \rangle}
```

Column vector in frame. This command produces a column vector in  $\langle frame \rangle$  with x-component  $\langle comp1 \rangle$ , y-component  $\langle comp2 \rangle$ , and z-component  $\langle comp3 \rangle$ .

```
\langle frame \rangle = Frame; letter (a-z, A-Z)
\langle comp1 \rangle = x-component; expression
\langle comp2 \rangle = y-component; expression
\langle comp3 \rangle = z-component; expression
```

#### Example

Say we want to define a column vector in the  $\bar{F}$  frame with components a, b, and c in the  $\hat{\imath}_{\bar{r}}$ ,  $\hat{\jmath}_{\bar{r}}$ , and  $\hat{k}_{\bar{r}}$  directions, respectively.

This is accomplished with the LATEX below:

 $\vecfrc{f}{a}{b}{c}$ 

Display Mode Output

 $\begin{vmatrix} a \\ b \\ c \end{vmatrix}$ 

## 6.2 \vecfrnc

```
\vecfrnc{\langle frame \rangle}{\langle num \rangle}{\langle comp1 \rangle}{\langle comp2 \rangle}{\langle comp3 \rangle}
```

Column vector in numbered frame. This command produces a column vector in  $\langle frame \rangle$  assigned number  $\langle num \rangle$ , with x-component  $\langle comp1 \rangle$ , y-component  $\langle comp2 \rangle$ , and z-component  $\langle comp3 \rangle$ .

```
\langle frame \rangle = Frame; letter (a-z, A-Z)
\langle num \rangle = Frame; letter (a-z, A-Z)
\langle comp1 \rangle = x-component; expression
\langle comp2 \rangle = y-component; expression
\langle comp3 \rangle = z-component; expression
```

#### Example

Say we want to define a column vector in the  $\bar{F}_2$  frame with components a, b, and c in the  $\hat{\imath}_{\bar{r}}$ ,  $\hat{\jmath}_{\bar{r}}$ , and  $\hat{k}_{\bar{r}}$  directions, respectively.

This is accomplished with the LATEX below:

 $\vecfrnc{f}{2}{a}{b}{c}$ 

Display Mode Output

 $\begin{bmatrix} a \\ b \\ c \end{bmatrix}_{\bar{k}}$ 

#### 6.3 \vecAc

#### \vecAc

First elementary column vector. This command is used to express the elementary column vector along the x-direction of a frame.

No input arguments.

# Define the elementary column unit vector for the x-component. This is accomplished with the LaTeX below: \vecAc Display Mode Output [1] 0 0 0

# 6.4 \vecBc

#### \vecBc

Second elementary column vector. This command is used to express the elementary column vector along the y-direction of a frame.

No input arguments.

Example
Define the elementary column unit vector for the y-component.
This is accomplished with the LaTeX below:
\vecBc
Display Mode Output
$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

# 6.5 \vecCc

#### \vecCc

Third elementary column vector. This command is used to express the elementary column vector along the z-direction of a frame.

No input arguments.

Example
Define the elementary column unit vector for the z-component.
This is accomplished with the LaTEX below:
\vecCc
Display Mode Output
$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

# Transposed Column Vectors

## 6.6 \vecfrcT

```
\vecfrcT{\langle frame \rangle} {\langle comp1 \rangle} {\langle comp2 \rangle} {\langle comp3 \rangle}
```

Column vector in frame. This command produces a transposed column vector in  $\langle frame \rangle$  with x-component  $\langle comp1 \rangle$ , y-component  $\langle comp2 \rangle$ , and z-component  $\langle comp3 \rangle$ .

```
\langle frame \rangle = Frame; letter (a-z, A-Z)
\langle comp1 \rangle = x-component; expression
\langle comp2 \rangle = y-component; expression
\langle comp3 \rangle = z-component; expression
```

## Example

Say we want to define a transposed column vector in the  $\bar{F}$  frame with components a, b, and c in the  $\hat{\imath}_{\bar{F}}$ ,  $\hat{\jmath}_{\bar{F}}$ , and  $\hat{k}_{\bar{F}}$  directions, respectively.

This is accomplished with the LATEX below:

 $\vecfrcT{f}{a}{b}{c}$ 

Display Mode Output

$$\begin{bmatrix} a & b & c \end{bmatrix}_{\scriptscriptstyle \tilde{F}}^{\rm T}$$

#### 6.7 \vecfrncT

# $\comp1$ ${\langle comp1 \rangle} {\langle comp2 \rangle} {\langle comp3 \rangle}$

Column vector in numbered frame. This command produces a transposed column vector in  $\langle frame \rangle$  assigned number  $\langle num \rangle$ , with x-component  $\langle comp1 \rangle$ , y-component  $\langle comp2 \rangle$ , and z-component  $\langle comp3 \rangle$ .

```
\langle frame \rangle = Frame; letter (a-z, A-Z)

\langle num \rangle = Frame; letter (a-z, A-Z)

\langle comp1 \rangle = x-component; expression

\langle comp2 \rangle = y-component; expression

\langle comp3 \rangle = z-component; expression
```

## Example

Say we want to define a transposed column vector in the  $\bar{F}_2$  frame with components a, b, and c in the  $\hat{\imath}_{\bar{r}}$ ,  $\hat{\jmath}_{\bar{r}}$ , and  $\hat{k}_{\bar{r}}$  directions, respectively.

This is accomplished with the LATEX below:

 $\vecfrncT{f}{2}{a}{b}{c}$ 

Display Mode Output

$$\begin{bmatrix} a & b & c \end{bmatrix}_{\bar{\mathit{F}}_{2}}^{\mathrm{T}}$$

#### 6.8 \vecAcT

#### \vecAcT

First transposed elementary column vector. This command is used to express the transposed elementary column vector along the x-direction of a frame.

No input arguments.

#### Example

Define the transposed elementary unit vector for the x-component.

This is accomplished with the LATEX below:

\vecAcT

Display Mode Output

$$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$$

## 6.9 \vecBcT

#### \vecBcT

Second transposed elementary column vector. This command is used to express the transposed elementary column vector along the y-direction of a frame.

No input arguments.

## Example

Define the transposed elementary unit vector for the y-component.

This is accomplished with the LaTeX below:

\vecBcT

Display Mode Output

 $\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$ 

# 6.10 \vecCcT

#### \vecCcT

Third transposed elementary column vector. This command is used to express the transposed elementary column vector along the z-direction of a frame.

No input arguments.

#### Example

Define the transposed elementary unit vector for the z-component.

This is accomplished with the LATEX below:

\vecCcT

Display Mode Output

 $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$ 

## Row Vectors

## 6.11 \vecfrr

```
\vecfrr{\langle frame \rangle} {\langle comp1 \rangle} {\langle comp2 \rangle} {\langle comp3 \rangle}
```

Row vector in frame. This command produces a row vector in  $\langle frame \rangle$  with x-component  $\langle comp1 \rangle$ , y-component  $\langle comp2 \rangle$ , and z-component  $\langle comp3 \rangle$ .

```
\langle frame \rangle = Frame; letter (a-z, A-Z)
\langle comp1 \rangle = x-component; expression
\langle comp2 \rangle = y-component; expression
\langle comp3 \rangle = z-component; expression
```

## Example

Say we want to define a row vector in the  $\bar{F}$  frame with components a, b, and c in the  $\hat{i}_{\bar{F}}$ ,  $\hat{j}_{\bar{F}}$ , and  $\hat{k}_{\bar{F}}$  directions, respectively.

This is accomplished with the  $\LaTeX$  below:

 $\vecfrr{f}{a}{b}{c}$ 

Display Mode Output

$$\begin{bmatrix} a & b & c \end{bmatrix}_{\bar{F}}$$

### 6.12 \vecfrnr

```
\vecfrnr{\langle frame \rangle} {\langle num \rangle} {\langle comp1 \rangle} {\langle comp2 \rangle} {\langle comp3 \rangle}
```

Row vector in numbered frame. This command produces a row vector in  $\langle frame \rangle$  assigned number  $\langle num \rangle$ , with x-component  $\langle comp1 \rangle$ , y-component  $\langle comp2 \rangle$ , and z-component  $\langle comp3 \rangle$ .

```
\langle frame \rangle = Frame; letter (a-z, A-Z)

\langle num \rangle = Frame; letter (a-z, A-Z)

\langle comp1 \rangle = x-component; expression

\langle comp2 \rangle = y-component; expression

\langle comp3 \rangle = z-component; expression
```

#### Example

Say we want to define a row vector in the  $\bar{F}_2$  frame with components a, b, and c in the  $\hat{i}_{\bar{r}}$ ,  $\hat{j}_{\bar{r}}$ , and  $\hat{k}_{\bar{r}}$  directions, respectively.

This is accomplished with the LATEX below:

 $\operatorname{vecfrnr}_{f}_{2}_{a}_{b}_{c}$ 

Display Mode Output

 $\begin{bmatrix} a & b & c \end{bmatrix}_{\bar{F}_2}$ 

#### 6.13 \vecAr

#### \vecAr

First elementary row vector. This command is used to express the elementary row vector along the x-direction of a frame.

No input arguments.

### Example

Define the elementary row unit vector for the x-component.

This is accomplished with the LATEX below:

\vecAr

#### Display Mode Output

 $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$ 

### 6.14 \vecBr

#### \vecBr

Second elementary row vector. This command is used to express the elementary row vector along the y-direction of a frame.

No input arguments.

### Example

Define the elementary row unit vector for the y-component.

This is accomplished with the LATEX below:

\vecBr

Display Mode Output

 $\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$ 

### 6.15 \vecCr

#### \vecCr

Third elementary row vector. This command is used to express the elementary row vector along the z-direction of a frame.

No input arguments.

### Example

Define the elementary row unit vector for the z-component.

This is accomplished with the LATEX below:

\vecCr

Display Mode Output

 $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$ 

## 7 Matrices

This section includes the commands needed to typeset most matrix quantities. Note that all commands in this section should be used inside of a math environment and a display math environment should be used for the best appearance.

Note that if you wish to attach a frame subscipt to any of the matrices without one, you should also include \! or \!\! before the frame definition to remove the space between the closing bracket of the vector and the frame. This will be addressed in a future version.

### **Standard Rotation Matrices**

### 7.1 \rotx

### $\t (angle)$

Rotation matrix about x. This command writes all the terms of a rotation matrix about the x-axis using a specified angle,  $\langle angle \rangle$ .

 $\langle angle \rangle$  = Rotation angle; symbol

#### Example

Say we want to define the rotation matrix about the x-axis using the angle  $\phi$ .

This is accomplished with the LATEX below:

\rotx{\phi}

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & s\phi \\ 0 & -s\phi & c\phi \end{bmatrix}$$

#### 7.2 \roty

### $\t (angle)$

Rotation matrix about y. This command writes all the terms of a rotation matrix about the y-axis using a specified angle,  $\langle angle \rangle$ .

 $\langle angle \rangle$  = Rotation angle; symbol

### Example

Say we want to define the rotation matrix about the y-axis using the angle  $\theta$ .

This is accomplished with the LATEX below:

\roty{\theta}

Display Mode Output

$$\begin{bmatrix} c\theta & 0 & -s\theta \\ 0 & 1 & 0 \\ s\theta & 0 & c\theta \end{bmatrix}$$

#### 7.3 \rotz

### 

Rotation matrix about z. This command writes all the terms of a rotation matrix about the z-axis using a specified angle,  $\langle angle \rangle$ .

 $\langle angle \rangle$  = Rotation angle; symbol

### Example

Say we want to define the rotation matrix about the z-axis using the angle  $\psi$ .

This is accomplished with the LATEX below:

\rotz{\psi}

$$\begin{bmatrix} c\psi & s\psi & 0 \\ -s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

## 7.4 \rotxq

### $\operatorname{rotxq}\{\langle num \rangle\}$

Rotation matrix about x using q shorthand. This command writes all the terms of a rotation matrix about the x-axis using a specified  $q_n$ ,  $n = \langle num \rangle$  and shorthand notation.

 $\langle num \rangle$  = Variable number; positive integer

### Example

Say we want to define the rotation matrix about the x-axis using the variable  $q_1$ .

This is accomplished with the LATEX below:

\rotxq{1}

Display Mode Output

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_1 & s_1 \\ 0 & -s_1 & c_1 \end{bmatrix}$$

## 7.5 \rotyq

## $\operatorname{rotyq}\{\langle num \rangle\}$

Rotation matrix about y using q shorthand. This command writes all the terms of a rotation matrix about the y-axis using a specified  $q_n$ ,  $n = \langle num \rangle$  and shorthand notation.

 $\langle num \rangle$  = Variable number; positive integer

### Example

Say we want to define the rotation matrix about the y-axis using the variable  $q_2$ .

This is accomplished with the LATEX below:

\rotyq{2}

$$\begin{vmatrix} c_2 & 0 & -s_2 \\ 0 & 1 & 0 \\ s_2 & 0 & c_2 \end{vmatrix}$$

## 7.6 \rotzq

### $\operatorname{\mathsf{Notzq}}(\langle num \rangle)$

Rotation matrix about z using q shorthand. This command writes all the terms of a rotation matrix about the z-axis using a specified  $q_n$ ,  $n = \langle num \rangle$  and shorthand notation.

 $\langle num \rangle$  = Variable number; positive integer

### Example

Say we want to define the rotation matrix about the z-axis using the variable  $q_3$ .

This is accomplished with the LATEX below:

\rotzq{3}

Display Mode Output

$$\begin{bmatrix} c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

## Transposed/Inverse Rotation Matrices

### 7.7 $\rotxT$

## $\t T{\langle angle \rangle}$

Transposed rotation matrix about x. This command writes all the terms of a transposed/inverse rotation matrix about the x-axis using a specified angle,  $\langle angle \rangle$ .

 $\langle angle \rangle = \text{Rotation angle; symbol}$ 

### Example

Say we want to define the transposed/inverse rotation matrix about the x-axis using the angle  $\phi$ .

This is accomplished with the LATEX below:

\rotxT{\phi}

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix}$$

## 7.8 $\rotyT$

### $\t T{\langle angle \rangle}$

Transposed rotation matrix about y. This command writes all the terms of a transposed/inverse rotation matrix about the y-axis using a specified angle,  $\langle angle \rangle$ .

 $\langle angle \rangle$  = Rotation angle; symbol

### Example

Say we want to define the transposed/inverse rotation matrix about the y-axis using the angle  $\theta$ .

This is accomplished with the LATEX below:

\rotyT{\theta}

Display Mode Output

$$\begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix}$$

### $7.9 \ \text{rotzT}$

### 

Transposed rotation matrix about z. This command writes all the terms of a transposed/inverse rotation matrix about the z-axis using a specified angle,  $\langle angle \rangle$ .

 $\langle angle \rangle$  = Rotation angle; symbol

### Example

Say we want to define the transposed/inverse rotation matrix about the z-axis using the angle  $\psi$ .

This is accomplished with the LaTeX below:

\rotzT{\psi}

$$\begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

### $7.10 \ \text{rotxqT}$

### $\operatorname{T}(\langle num \rangle)$

Transposed rotation matrix about x using q shorthand. This command writes all the terms of a transposed/inverse rotation matrix about the x-axis using a specified  $q_n$ ,  $n = \langle num \rangle$  and shorthand notation.

 $\langle num \rangle$  = Variable number; positive integer

### Example

Say we want to define the transposed/inverse rotation matrix about the x-axis using the variable  $q_1$ .

This is accomplished with the LATEX below:

 $\rotxqT{1}$ 

Display Mode Output

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_1 & -s_1 \\ 0 & s_1 & c_1 \end{bmatrix}$$

## $7.11 \ \text{rotyqT}$

## $\t \operatorname{TotyqT}(\langle num \rangle)$

Transposed rotation matrix about y using q shorthand. This command writes all the terms of a transposed/inverse rotation matrix about the y-axis using a specified  $q_n$ ,  $n = \langle num \rangle$  and shorthand notation.

 $\langle num \rangle$  = Variable number; positive integer

### Example

Say we want to define the transposed/inverse rotation matrix about the y-axis using the variable  $q_2$ .

This is accomplished with the LATEX below:

 $\rotyqT{2}$ 

$$\begin{bmatrix} c_2 & 0 & s_2 \\ 0 & 1 & 0 \\ -s_2 & 0 & c_2 \end{bmatrix}$$

## $7.12 \ \text{rotzqT}$

### $\trotzqT{\langle num \rangle}$

Transposed rotation matrix about z using q shorthand. This command writes all the terms of a transposed/inverse rotation matrix about the z-axis using a specified  $q_n$ ,  $n = \langle num \rangle$  and shorthand notation.

 $\langle num \rangle$  = Variable number; positive integer

### Example

Say we want to define the transposed/inverse rotation matrix about the z-axis using the variable  $q_3$ .

This is accomplished with the LATEX below:

 $\rotzqT{3}$ 

Display Mode Output

$$\begin{bmatrix} c_3 & -s_3 & 0 \\ s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

### Inertia Matrices

### 7.13 \inertiaMat

## $\int {\langle frame \rangle} {\langle foint \rangle}$

Inertia matrix. This command is used to define the terms of an inertia tensor of a body about point  $\langle point \rangle$  in matrix form with body frame  $\langle frame \rangle$ .

```
\langle frame \rangle = Frame; letter (a-z, A-Z)
\langle point \rangle = Point; expression
```

#### Example

Say we want to define the inertia matrix of body with body frame  $\bar{B}$  about its center of mass.

This is accomplished with the  $\LaTeX$  below:

\inertiaMat{b}{\CM}

$$\begin{bmatrix} {}^BI_{xx/\scriptscriptstyle{\text{CM}}} & -{}^BI_{xy/\scriptscriptstyle{\text{CM}}} & -{}^BI_{xz/\scriptscriptstyle{\text{CM}}} \\ -{}^{\bar{B}}I_{yx/\scriptscriptstyle{\text{CM}}} & {}^{\bar{B}}I_{yy/\scriptscriptstyle{\text{CM}}} & -{}^{\bar{B}}I_{yz/\scriptscriptstyle{\text{CM}}} \\ -{}^{\bar{B}}I_{zx/\scriptscriptstyle{\text{CM}}} & -{}^{\bar{B}}I_{zy/\scriptscriptstyle{\text{CM}}} & {}^{\bar{B}}I_{zz/\scriptscriptstyle{\text{CM}}} \end{bmatrix}$$

### 7.14 \inertiaMatsh

#### \inertiaMatsh

Shorthand inertia matrix. This command is used to define the terms of an inertia tensor in shorthand.

No input arguments.

### Example

Define the shorthand inertia matrix.

This is accomplished with the LaTeX below:

\inertiaMatsh

Display Mode Output

$$\begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix}$$

#### 7.15 \inertiaDif

### $\inertiaDif{\langle frame \rangle} {\langle point \rangle}$

Inertia difference matrix. This command is used to define the terms of an inertia difference tensor of a body about point  $\langle point \rangle$  relative to its center of mass in matrix form with body frame  $\langle frame \rangle$ .

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z)  $\langle point \rangle$  = Point; expression

#### Example

Say we want to define the inertia difference matrix of body with body frame  $\bar{B}$  about a point A.

This is accomplished with the LATEX below:

\inertiaDif{b}{\smca{a}}}

$$\begin{bmatrix} ({}^{\bar{b}}y_{_{\mathrm{CM/A}}})^2 + ({}^{\bar{b}}z_{_{\mathrm{CM/A}}})^2 & {}^{\bar{b}}x_{_{\mathrm{CM/A}}}{}^{\bar{b}}y_{_{\mathrm{CM/A}}} & {}^{\bar{b}}x_{_{\mathrm{CM/A}}}{}^{\bar{b}}z_{_{\mathrm{CM/A}}} \\ {}^{\bar{b}}y_{_{\mathrm{CM/A}}}{}^{\bar{b}}x_{_{\mathrm{CM/A}}} & ({}^{\bar{b}}x_{_{\mathrm{CM/A}}})^2 + ({}^{\bar{b}}z_{_{\mathrm{CM/A}}})^2 & {}^{\bar{b}}y_{_{\mathrm{CM/A}}}{}^{\bar{b}}z_{_{\mathrm{CM/A}}} \\ {}^{\bar{b}}z_{_{\mathrm{CM/A}}}{}^{\bar{b}}x_{_{\mathrm{CM/A}}} & {}^{\bar{b}}z_{_{\mathrm{CM/A}}}{}^{\bar{b}}y_{_{\mathrm{CM/A}}} & ({}^{\bar{b}}x_{_{\mathrm{CM/A}}})^2 + ({}^{\bar{b}}y_{_{\mathrm{CM/A}}})^2 \end{bmatrix}$$

#### 7.16 \inertiaDifsh

#### \inertiaDifsh

Shorthand inertia difference matrix. This command is used to define the terms of an inertia difference tensor relative to its center of mass in shorthand.

No input arguments.

### Example

Define the shorthand inertia difference matrix.

This is accomplished with the LATEX below:

\inertiaDifsh

Display Mode Output

$$\begin{bmatrix} y^2 + z^2 & xy & xz \\ yx & x^2 + z^2 & yz \\ zx & zy & x^2 + y^2 \end{bmatrix}$$

#### Other Matrices

#### 7.17 \tensorMat

### $\texttt{\tensorMat}(\langle frame \rangle) \{ \langle tensor \rangle \}$

Tensor matrix. This command is used to define the terms of a matrix representation of the tensor  $\langle tensor \rangle$  relative to the frame  $\langle frame \rangle$ .

```
\langle frame \rangle = Frame; letter (a-z, A-Z) \langle tensor \rangle = Tensor; expression
```

#### Example

Say we want to define a matrix representation of an inertia tensor about a body's center of mass expressed in its body frame  $\bar{B}$ .

This is accomplished with the LATEX below:

\tensorMat{b}{\inerTen{\CM}}

$$\begin{bmatrix} \hat{\imath}_{\bar{B}} \cdot \tilde{I}_{\mathrm{CM}} \cdot \hat{\imath}_{\bar{b}} & \hat{\imath}_{\bar{b}} \cdot \tilde{I}_{\mathrm{CM}} \cdot \hat{\jmath}_{\bar{b}} & \hat{\imath}_{\bar{b}} \cdot \tilde{I}_{\mathrm{CM}} \cdot \hat{k}_{\bar{b}} \\ \hat{\jmath}_{\bar{b}} \cdot \tilde{I}_{\mathrm{CM}} \cdot \hat{\imath}_{\bar{b}} & \hat{\jmath}_{\bar{b}} \cdot \tilde{I}_{\mathrm{CM}} \cdot \hat{\jmath}_{\bar{b}} & \hat{\jmath}_{\bar{b}} \cdot \tilde{I}_{\mathrm{CM}} \cdot \hat{k}_{\bar{b}} \\ \hat{k}_{\bar{b}} \cdot \tilde{I}_{\mathrm{CM}} \cdot \hat{\imath}_{\bar{b}} & \hat{k}_{\bar{b}} \cdot \tilde{I}_{\mathrm{CM}} \cdot \hat{\jmath}_{\bar{b}} & \hat{k}_{\bar{b}} \cdot \tilde{I}_{\mathrm{CM}} \cdot \hat{k}_{\bar{b}} \end{bmatrix}$$

## 7.18 \sqMatiii

 $\label{eq:lambda_sqMatiii} $$ \left(A11\right) \left(A12\right) \left(A13\right) \left(A21\right) \left(A22\right) \left(A23\right) \left(A31\right) \left(A32\right) \left(A33\right) \right] $$$ 

3x3 Matrix. This command is used to define a 3x3 square matrix with specified terms.

 $\langle Amn \rangle = (m,n)$  term; expression

### Example

Say we want to define a 3x3 matrix with the values 1 to 9 ascending from left to right, top to bottom.

This is accomplished with the LATEX below:

\sqMatiii{1}{2}{3}{4}{5}{6}{7}{8}{9}

Display Mode Output

 $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ 

## 7.19 \eyeMatiii

### \eyeMatiii

3x3 identity matrix.

No input arguments.

### Example

Define the 3x3 identity matrix.

This is accomplished with the LATEX below:

\eyeMatiii

Display Mode Output

 $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 

## 7.20 \dcm

#### \dcm

Direction cosine matrix. This command defines the terms of a direction cosine matrix. *No input arguments.* 

### Example

Define the direction cosine matrix.

This is accomplished with the LaTeX below:

\dcm

### Display Mode Output

$$\begin{bmatrix} C_{11'} & C_{12'} & C_{13'} \\ C_{21'} & C_{22'} & C_{23'} \\ C_{31'} & C_{32'} & C_{33'} \end{bmatrix}$$

### $7.21 \ \text{crossMat}$

### $\comp$ $\comp$

Cross matrix. This command is used to define a cross matrix which when multiplied by a vector is equivalent to taking a cross product. Defined given the x, y, and z components of the first vector in the cross product.

```
\langle xcomp \rangle = x-component; expression \langle ycomp \rangle = y-component; expression \langle zcomp \rangle = z-component; expression
```

#### Example

Say we want to define a cross matrix for a vector  $\vec{r} = [x \ y \ z]$ .

This is accomplished with the LATEX below:

\crossMat{x}{y}{z}

$$\begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix}$$

# 8 Equations

This section includes the commands needed to typeset most of the fundamental equations from advanced dynamics. Note that all commands in this section should be used inside of a math environment and a display math environment should be used for the best appearance.

Additionally, it should be mentioned that the descriptions of the functions may not be enough to understand all of these definitions. This is partly intentional since it is based on a notation from a course and fully comprehending the equations would require one taking the course themself. However, the descriptions may be updated to be more detailed in the future.

#### Newton's Second Law

#### 8.1 \NewtonII

#### \NewtonII

Newton II. Shows Newton's Second Law for a system of k rigid bodies and n particles. Assumes Newton III.

No input arguments.

#### Example

Show the generic form of Newton's Second Law when Newton III applies.

This is accomplished with the LATEX below:

\NewtonII

$$\sum \vec{F}_{\rm sys, ext} = \sum_{i=1}^{k} m_{\rm T, i}{}^{\bar{o}} \vec{a}_{{\rm CM}, i/o} + \sum_{j=1}^{n} m_{j}{}^{\bar{o}} \vec{a}_{m_{j}/o}$$

## 8.2 \NewtonIIpart

#### \NewtonIIpart

Newton II for particles. Shows Newton's Second Law for a system of n particles. Assumes Newton III.

No input arguments.

### Example

Show the particle form of Newton's Second Law.

This is accomplished with the LATEX below:

\NewtonIIpart

Display Mode Output

$$\sum \vec{F}_{\text{sys,ext}} = \sum_{i=1}^{n} m_i \, \bar{\vec{o}} \vec{a}_{m_i/o}$$

## 8.3 \NewtonIIrigp

#### \NewtonIIrigp

Newton II for rigid body. Shows Newton's Second Law for a system of particles as a rigid body. Assumes Newton III.

No input arguments.

#### Example

Show the rigid body form of Newton's Second Law.

This is accomplished with the LATEX below:

\NewtonIIrigp

$$\sum ec{F}_{
m sys,ext} = m_{\scriptscriptstyle 
m T}{}^{\scriptscriptstyle ar{O}} ec{a}_{\scriptscriptstyle 
m CM/O}$$

## 8.4 \NewtonIIgen

### \NewtonIIgen

Generalized Newton II. Shows Newton's Second Law for a generic system of n particles.

No input arguments.

### Example

Show the generic form of Newton's Second Law.

This is accomplished with the LATEX below:

\NewtonIIgen

Display Mode Output

$$\sum_{i=1}^{n} ec{F_{i}} + \sum_{i=1}^{n} \sum_{j=1}^{n} ec{f_{ij}} = m_{ ext{ iny T}}{}^{ar{o}} ec{a}_{ ext{ iny CM/O}}$$

## **Full Inertia Equations**

#### 8.5 \inerTenDef

## $\iner{TenDef}{\langle point \rangle}$

Inertia tensor definition. Defines the inertia tensor about a specified point,  $\langle point \rangle$ .

 $\langle point \rangle = Point; expression$ 

#### Example

Say we want to define the inertia tensor about point A.

This is accomplished with the LATEX below:

\inerTenDef{\smca{A}}}

$$ilde{I_{\scriptscriptstyle A}} \equiv \sum_{i=1}^n m_i \left[ \left( ec{r}_{m_i/\!\scriptscriptstyle A} \cdot ec{r}_{m_i/\!\scriptscriptstyle A} 
ight) ilde{1} - ec{r}_{m_i/\!\scriptscriptstyle A} ec{r}_{m_i/\!\scriptscriptstyle A} 
ight]$$

### 8.6 \parAxisTen

### $\operatorname{\operatorname{VarAxisTen}}\{\langle point \rangle\}$

Parallel axis theorem tensor definition. Expression of the parallel axis theorem in tensor form about a specified point A.

 $\langle point \rangle = Point; expression$ 

### Example

Say we want to define the inertia tensor about point A using the parallel axis theorem.

This is accomplished with the LATEX below:

\parAxisTen{\smca{A}}}

Display Mode Output

$$\tilde{I}_{\!\scriptscriptstyle A} = \tilde{I}_{\!\scriptscriptstyle \mathrm{CM}} + m_{\!\scriptscriptstyle \mathrm{T}} \left[ \left( \vec{r}_{\!\scriptscriptstyle \mathrm{CM}/\!A} \cdot \vec{r}_{\!\scriptscriptstyle \mathrm{CM}/\!A} \right) \tilde{1} - \vec{r}_{\!\scriptscriptstyle \mathrm{CM}/\!A} \vec{r}_{\!\scriptscriptstyle \mathrm{CM}/\!A} \right]$$

#### 8.7 \rRel

### 

Relative displacement. This is the relative displacement vector for the inertia tensor of the system  $\langle sys \rangle$  relative to the point  $\langle point \rangle$  with respect to the frame  $\langle frame \rangle$ .

 $\langle frame \rangle = Frame; letter (a-z, A-Z)$ 

 $\langle sys \rangle$  = System name; expression

 $\langle point \rangle$  = Reference point; expression

### Example

Say we want to define the relative displacement of a mass element  $m_i$  relative to the point B in the body frame  $\bar{B}$ 

This is accomplished with the LATEX below:

 $\rrack{rRel{b}{m_i}{\smca{B}}}$ 

$$ec{r}_{m_i/\!\scriptscriptstyle B} \equiv \left({}^{\scriptscriptstyle ar{\scriptscriptstyle B}} x_{m_i/\!\scriptscriptstyle B}
ight) \hat{\imath}_{\scriptscriptstyle ar{\scriptscriptstyle B}} + \left({}^{\scriptscriptstyle ar{\scriptscriptstyle B}} y_{m_i/\!\scriptscriptstyle B}
ight) \hat{\jmath}_{\scriptscriptstyle ar{\scriptscriptstyle B}} + \left({}^{\scriptscriptstyle ar{\scriptscriptstyle B}} z_{m_i/\!\scriptscriptstyle B}
ight) \hat{k}_{\scriptscriptstyle ar{\scriptscriptstyle B}}$$

### 8.8 \IxxSum

### 

 $I_{xx}$  term sum definition. Defines the  $I_{xx}$  term of the inertia tensor about a point  $\langle point \rangle$  in frame  $\langle frame \rangle$ .

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z)  $\langle point \rangle$  = Point; expression

### Example

Say we want to define the  $I_{xx}$  term of the inertia tensor about point A in frame  $\bar{B}$ .

This is accomplished with the LATEX below:

\IxxSum{b}{\smca{a}}

Display Mode Output

$${}^{ar{ ilde{B}}}I_{xx/\!\scriptscriptstyle{A}} \equiv \sum_{i=1}^n m_i \left( \left( {}^{ar{ ilde{B}}}y_{m_i/\!\scriptscriptstyle{A}} 
ight)^2 + \left( {}^{ar{B}}z_{m_i/\!\scriptscriptstyle{A}} 
ight)^2 
ight)$$

## 8.9 \IxySum

## 

 $I_{xy}$  term sum definition. Defines the  $I_{xy}$  term of the inertia tensor about a point  $\langle point \rangle$  in frame  $\langle frame \rangle$ .

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z)  $\langle point \rangle$  = Point; expression

### Example

Say we want to define the  $I_{xy}$  term of the inertia tensor about point A in frame  $\bar{B}$ .

This is accomplished with the LATEX below:

\IxySum{b}{\smca{a}}

$${}^{ar{B}}I_{xy/A} \equiv \sum_{i=1}^n m_i{}^{ar{B}}x_{m_i/A}{}^{ar{B}}y_{m_i/A}$$

#### 8.10 \IxzSum

## $\IxzSum\{\langle frame \rangle\}\{\langle point \rangle\}$

 $I_{xz}$  term sum definition. Defines the  $I_{xz}$  term of the inertia tensor about a point  $\langle point \rangle$  in frame  $\langle frame \rangle$ .

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z)  $\langle point \rangle$  = Point; expression

### Example

Say we want to define the  $I_{xz}$  term of the inertia tensor about point A in frame  $\bar{B}$ .

This is accomplished with the LATEX below:

\IxzSum{b}{\smca{a}}

Display Mode Output

$${}^{\bar{B}}I_{xy/A} \equiv \sum_{i=1}^{n} m_{i}{}^{\bar{B}}x_{m_{i}/A}{}^{\bar{B}}z_{m_{i}/A}$$

### 8.11 \IyxSum

## $\label{local_sum} $$ \sup {\langle frame \rangle} {\langle point \rangle} $$$

 $I_{yx}$  term sum definition. Defines the  $I_{yx}$  term of the inertia tensor about a point  $\langle point \rangle$  in frame  $\langle frame \rangle$ .

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z)  $\langle point \rangle$  = Point; expression

### Example

Say we want to define the  $I_{yx}$  term of the inertia tensor about point A in frame  $\bar{B}$ .

This is accomplished with the LATEX below:

\IyxSum{b}{\smca{a}}

$${}^{ar{B}}I_{yx/A} \equiv \sum_{i=1}^{n} m_{i}{}^{ar{B}}y_{m_{i}/A}{}^{ar{B}}x_{m_{i}/A}$$

### $8.12 \setminus IyySum$

### $\label{local_system} $$ \sup {\langle frame \rangle} {\langle point \rangle}$$

 $I_{yy}$  term sum definition. Defines the  $I_{yy}$  term of the inertia tensor about a point  $\langle point \rangle$  in frame  $\langle frame \rangle$ .

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z)  $\langle point \rangle$  = Point; expression

### Example

Say we want to define the  $I_{yy}$  term of the inertia tensor about point A in frame  $\bar{B}$ .

This is accomplished with the LATEX below:

\IyySum{b}{\smca{a}}

Display Mode Output

$${}^{ar{B}}I_{yy/A} \equiv \sum_{i=1}^n m_i \left( \left( {}^{ar{B}}x_{m_i/A} 
ight)^2 + \left( {}^{ar{B}}z_{m_i/A} 
ight)^2 
ight)$$

## 8.13 \IyzSum

### $\IyzSum\{\langle frame \rangle\}\{\langle point \rangle\}$

 $I_{yz}$  term sum definition. Defines the  $I_{yz}$  term of the inertia tensor about a point  $\langle point \rangle$  in frame  $\langle frame \rangle$ .

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z)  $\langle point \rangle$  = Point; expression

#### Example

Say we want to define the  $I_{yz}$  term of the inertia tensor about point A in frame  $\bar{B}$ .

This is accomplished with the LATEX below:

\IyzSum{b}{\smca{a}}

$${}^{ar{B}}I_{yz/A} \equiv \sum_{i=1}^n m_i{}^{ar{B}}y_{m_i/A}{}^{ar{B}}z_{m_i/A}$$

#### 8.14 \IzxSum

### $\IzxSum\{\langle frame \rangle\}\{\langle point \rangle\}$

 $I_{zx}$  term sum definition. Defines the  $I_{zx}$  term of the inertia tensor about a point  $\langle point \rangle$  in frame  $\langle frame \rangle$ .

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z)  $\langle point \rangle$  = Point; expression

### Example

Say we want to define the  $I_{zx}$  term of the inertia tensor about point A in frame  $\bar{B}$ .

This is accomplished with the LATEX below:

\IzxSum{b}{\smca{a}}

Display Mode Output

$${}^{ar{B}}I_{zx/\!A} \equiv \sum_{i=1}^n m_i{}^{ar{B}}z_{m_i/\!A}{}^{ar{B}}x_{m_i/\!A}$$

### 8.15 \IzySum

### $\IzySum\{\langle frame \rangle\}\{\langle point \rangle\}$

 $I_{zy}$  term sum definition. Defines the  $I_{zy}$  term of the inertia tensor about a point  $\langle point \rangle$  in frame  $\langle frame \rangle$ .

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z)  $\langle point \rangle$  = Point; expression

#### Example

Say we want to define the  $I_{zy}$  term of the inertia tensor about point A in frame  $\bar{B}$ .

This is accomplished with the LATEX below:

\IzySum{b}{\smca{a}}

$${}^{ar{B}}I_{zy/A} \equiv \sum_{i=1}^n m_i{}^{ar{B}}z_{m_i/A}{}^{ar{B}}y_{m_i/A}$$

#### $8.16 \setminus IzzSum$

### $\IzzSum\{\langle frame \rangle\}\{\langle point \rangle\}$

 $I_{zz}$  term sum definition. Defines the  $I_{zz}$  term of the inertia tensor about a point  $\langle point \rangle$  in frame  $\langle frame \rangle$ .

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z)  $\langle point \rangle$  = Point; expression

### Example

Say we want to define the  $I_{zz}$  term of the inertia tensor about point A in frame  $\bar{B}$ .

This is accomplished with the LATEX below:

\IzzSum{b}{\smca{a}}

Display Mode Output

$${}^{\bar{B}}I_{zz/A} \equiv \sum_{i=1}^{n} m_i \left( \left( {}^{\bar{B}}x_{m_i/A} \right)^2 + \left( {}^{\bar{B}}y_{m_i/A} \right)^2 \right)$$

### 8.17 \parAxis

### $\operatorname{\mathtt{\baseline}} \{\langle frame \rangle\} \{\langle point \rangle\}$

Parallel axis theorem in matrix form. Defines the terms of the inertia tensor about a point  $\langle point \rangle$  in frame  $\langle frame \rangle$  using the parallel axis theorem.

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z)  $\langle point \rangle$  = Point; expression

#### Example

Say we want to define the inertia tensor in matrix form about point A in frame  $\bar{B}$  using the parallel axis theorem.

This is accomplished with the LATEX below:

\parAxis{b}{\smca{a}}

$$\begin{split} [\tilde{I}_{A}]_{\bar{B}} &= \begin{bmatrix} {}^{\bar{B}}I_{xx/A} & -{}^{\bar{B}}I_{xy/A} & -{}^{\bar{B}}I_{xz/A} \\ -{}^{\bar{B}}I_{yx/A} & {}^{\bar{B}}I_{yy/A} & -{}^{\bar{B}}I_{yz/A} \\ -{}^{\bar{B}}I_{zx/A} & -{}^{\bar{B}}I_{zy/A} & {}^{\bar{B}}I_{zz/A} \end{bmatrix} \\ &+ m_{\mathrm{T}} \begin{bmatrix} ({}^{\bar{B}}y_{\mathrm{CM/A}})^{2} + ({}^{\bar{B}}z_{\mathrm{CM/A}})^{2} & {}^{\bar{B}}x_{\mathrm{CM/A}}^{\bar{B}}y_{\mathrm{CM/A}} & {}^{\bar{B}}x_{\mathrm{CM/A}}^{\bar{B}}z_{\mathrm{CM/A}} \\ {}^{\bar{B}}y_{\mathrm{CM/A}}{}^{\bar{B}}x_{\mathrm{CM/A}} & ({}^{\bar{B}}x_{\mathrm{CM/A}})^{2} + ({}^{\bar{B}}z_{\mathrm{CM/A}})^{2} & {}^{\bar{B}}y_{\mathrm{CM/A}}^{\bar{B}}z_{\mathrm{CM/A}} \\ {}^{\bar{B}}z_{\mathrm{CM/A}}{}^{\bar{B}}x_{\mathrm{CM/A}} & {}^{\bar{B}}z_{\mathrm{CM/A}}^{\bar{B}}y_{\mathrm{CM/A}} & ({}^{\bar{B}}x_{\mathrm{CM/A}})^{2} + ({}^{\bar{B}}y_{\mathrm{CM/A}})^{2} + ({}^{\bar{B}}y_{\mathrm{CM/A}})^{2} \end{bmatrix} \end{split}$$

## **Short Inertia Equations**

#### 8.18 \rRelsh

### 

Relative displacement shorthand. This is the shorthand relative displacement vector for the inertia tensor of the system  $\langle sys \rangle$  relative to the point  $\langle point \rangle$  with respect to the frame  $\langle frame \rangle$ .

 $\langle frame \rangle$  = Frame; letter (a-z, A-Z)  $\langle sys \rangle$  = System name; expression  $\langle point \rangle$  = Reference point; expression

### Example

Say we want to define the relative displacement using shorthand summation of a mass element  $m_i$  relative to the point B in the body frame  $\bar{B}$ 

This is accomplished with the LATEX below:

 $\rRelsh\{b\}\{m_i\}\{\smca\{B\}\}\}$ 

Display Mode Output

$${}^{\bar{\scriptscriptstyle B}}\vec{r}_{m_i/\!\scriptscriptstyle B} = (x)\hat{\imath}_{\scriptscriptstyle ar{\scriptscriptstyle B}} + (y)\hat{\jmath}_{\scriptscriptstyle ar{\scriptscriptstyle B}} + (z)\hat{k}_{\scriptscriptstyle ar{\scriptscriptstyle B}}$$

### 8.19 \IxxSumsh

#### \IxxSumsh

 $I_{xx}$  sum shorthand. Defines the  $I_{xx}$  term of the inertia tensor using summation and shorthand notation.

No input arguments.

#### Example

Define the  $I_{xx}$  term of the inertia tensor using shorthand summation notation.

This is accomplished with the LATEX below:

\IxxSumsh

$$I_{xx} = \sum_{i=1}^{n} m_i \left( y_i^2 + z_i^2 \right)$$

## 8.20 \IxySumsh

### \IxySumsh

 $I_{xy}$  sum shorthand. Defines the  $I_{xy}$  term of the inertia tensor using summation and shorthand notation.

No input arguments.

### Example

Define the  $I_{xy}$  term of the inertia tensor using shorthand summation notation.

This is accomplished with the LATEX below:

\IxySumsh

Display Mode Output

$$I_{xy} = \sum_{i=1}^{n} m_i x_i y_i$$

### 8.21 \IxzSumsh

#### \IxzSumsh

 $I_{xz}$  sum shorthand. Defines the  $I_{xz}$  term of the inertia tensor using summation and shorthand notation.

No input arguments.

#### Example

Define the  $I_{xz}$  term of the inertia tensor using shorthand summation notation.

This is accomplished with the LATEX below:

\IxzSumsh

$$I_{xz} = \sum_{i=1}^{n} m_i x_i z_i$$

## $8.22 \setminus IyxSumsh$

### \IyxSumsh

 $I_{yx}$  sum shorthand. Defines the  $I_{yx}$  term of the inertia tensor using summation and shorthand notation.

No input arguments.

### Example

Define the  $I_{yx}$  term of the inertia tensor using shorthand summation notation.

This is accomplished with the LATEX below:

\IyxSumsh

### Display Mode Output

$$I_{yx} = \sum_{i=1}^{n} m_i y_i x_i$$

## 8.23 \IyySumsh

### \IyySumsh

 $I_{yy}$  sum shorthand. Defines the  $I_{yy}$  term of the inertia tensor using summation and shorthand notation.

No input arguments.

#### Example

Define the  $I_{yy}$  term of the inertia tensor using shorthand summation notation.

This is accomplished with the LATEX below:

\IyySumsh

$$I_{yy} = \sum_{i=1}^{n} m_i \left( x_i^2 + z_i^2 \right)$$

# 8.24 \IyzSumsh

### \IyzSumsh

 $I_{yz}$  sum shorthand. Defines the  $I_{yz}$  term of the inertia tensor using summation and shorthand notation.

No input arguments.

### Example

Define the  $I_{yz}$  term of the inertia tensor using shorthand summation notation.

This is accomplished with the LATEX below:

\IyzSumsh

Display Mode Output

$$I_{yz} = \sum_{i=1}^{n} m_i y_i z_i$$

### 8.25 \IzxSumsh

### \IzxSumsh

 $I_{zx}$  sum shorthand. Defines the  $I_{zx}$  term of the inertia tensor using summation and shorthand notation.

No input arguments.

#### Example

Define the  $I_{zx}$  term of the inertia tensor using shorthand summation notation.

This is accomplished with the LATEX below:

\IzxSumsh

$$I_{zx} = \sum_{i=1}^{n} m_i z_i x_i$$

## 8.26 \IzySumsh

### \IzySumsh

 $I_{zy}$  sum shorthand. Defines the  $I_{zy}$  term of the inertia tensor using summation and shorthand notation.

No input arguments.

### Example

Define the  $I_{zy}$  term of the inertia tensor using shorthand summation notation.

This is accomplished with the LATEX below:

\IzySumsh

Display Mode Output

$$I_{zy} = \sum_{i=1}^{n} m_i z_i y_i$$

### 8.27 \IzzSumsh

#### \IzzSumsh

 $I_{zz}$  sum shorthand. Defines the  $I_{zz}$  term of the inertia tensor using summation and shorthand notation.

No input arguments.

#### Example

Define the  $I_{zz}$  term of the inertia tensor using shorthand summation notation.

This is accomplished with the LATEX below:

\IzzSumsh

$$I_{zz} = \sum_{i=1}^{n} m_i \left( x_i^2 + y_i^2 \right)$$

### 8.28 \IxxInt

#### \IxxInt

 $I_{xx}$  sum shorthand. Defines the  $I_{xx}$  term of the inertia tensor using integration and shorthand notation.

No input arguments.

### Example

Define the  $I_{xx}$  term of the inertia tensor using shorthand integration notation.

This is accomplished with the LATEX below:

\IxxInt

Display Mode Output

$$I_{xx} = \int_{\text{Body}} (y^2 + z^2) dm = \iiint_V (y^2 + z^2) \rho(x, y, z) dV$$

### 8.29 \IxyInt

### \IxyInt

 $I_{xy}$  sum shorthand. Defines the  $I_{xy}$  term of the inertia tensor using integration and shorthand notation.

No input arguments.

#### Example

Define the  $I_{xy}$  term of the inertia tensor using shorthand integration notation.

This is accomplished with the LaTeX below:

\IxyInt

$$I_{xy} = \int_{\text{Body}} (xy) \, dm = \iiint_V (xy) \, \rho(x, y, z) \, dV$$

### 8.30 \IxzInt

#### \IxzInt

 $I_{xz}$  sum shorthand. Defines the  $I_{xz}$  term of the inertia tensor using integration and shorthand notation.

No input arguments.

### Example

Define the  $I_{xz}$  term of the inertia tensor using shorthand integration notation.

This is accomplished with the LATEX below:

\IxzInt

Display Mode Output

$$I_{xz} = \iint_{\text{Body}} (xz) \, dm = \iiint_V (xz) \, \rho(x, y, z) \, dV$$

## 8.31 \IyxInt

### \IyxInt

 $I_{yx}$  sum shorthand. Defines the  $I_{yx}$  term of the inertia tensor using integration and shorthand notation.

No input arguments.

#### Example

Define the  $I_{yx}$  term of the inertia tensor using shorthand integration notation.

This is accomplished with the LaTeX below:

\IyxInt

$$I_{yx} = \int_{\text{Body}} (yx) \, dm = \iiint_V (yx) \, \rho(x, y, z) \, dV$$

## 8.32 \IyyInt

### \IyyInt

 $I_{yy}$  sum shorthand. Defines the  $I_{yy}$  term of the inertia tensor using integration and shorthand notation.

No input arguments.

### Example

Define the  $I_{yy}$  term of the inertia tensor using shorthand integration notation.

This is accomplished with the LATEX below:

\IyyInt

Display Mode Output

$$I_{yy} = \int_{\text{Body}} (x^2 + z^2) \, dm = \iiint_V (x^2 + z^2) \, \rho(x, y, z) \, dV$$

### 8.33 \IyzInt

#### \IyzInt

 $I_{yz}$  sum shorthand. Defines the  $I_{yz}$  term of the inertia tensor using integration and shorthand notation.

No input arguments.

#### Example

Define the  $I_{yz}$  term of the inertia tensor using shorthand integration notation.

This is accomplished with the LATEX below:

\IyzInt

$$I_{yz} = \int_{\text{Body}} (yz) \, dm = \iiint_V (yz) \, \rho(x, y, z) \, dV$$

### 8.34 \IzxInt

#### \IzxInt

 $I_{zx}$  sum shorthand. Defines the  $I_{zx}$  term of the inertia tensor using integration and shorthand notation.

No input arguments.

### Example

Define the  $I_{zx}$  term of the inertia tensor using shorthand integration notation.

This is accomplished with the LATEX below:

\IzxInt

Display Mode Output

$$I_{zx} = \iint_{\text{Body}} (zx) \, dm = \iiint_{V} (zx) \, \rho(x, y, z) \, dV$$

### 8.35 \IzyInt

#### \IzyInt

 $I_{zy}$  sum shorthand. Defines the  $I_{zy}$  term of the inertia tensor using integration and shorthand notation.

No input arguments.

#### Example

Define the  $I_{zy}$  term of the inertia tensor using shorthand integration notation.

This is accomplished with the LaTeX below:

\IzyInt

$$I_{zy} = \int_{\text{Body}} (zy) \, dm = \iiint_V (zy) \, \rho(x, y, z) \, dV$$

### 8.36 \IzzInt

#### \IzzInt

 $I_{zz}$  sum shorthand. Defines the  $I_{zz}$  term of the inertia tensor using integration and shorthand notation.

No input arguments.

### Example

Define the  $I_{zz}$  term of the inertia tensor using shorthand integration notation.

This is accomplished with the LATEX below:

\IzzInt

Display Mode Output

$$I_{zz} = \int_{\text{Body}} (x^2 + y^2) \, dm = \iiint_V (x^2 + y^2) \, \rho(x, y, z) \, dV$$

### 8.37 \parAxissh

#### \parAxissh

Parallel axis theorem in shorthand matrix form. Defines the terms of the inertia tensor about using the parallel axis theorem and shorthand.

No input arguments.

#### Example

Say we want to define the inertia tensor in matrix form using the parallel axis theorem in shorthand notation.

This is accomplished with the LATEX below:

\parAxissh

$$\tilde{I}_{\text{CM}} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} + m_{\text{T}} \begin{bmatrix} y^2 + z^2 & xy & xz \\ yx & x^2 + z^2 & yz \\ zx & zy & x^2 + y^2 \end{bmatrix}$$

## **Angular Velocity Equations**

### $8.38 \setminus angVdef$

### $\aggle AngVdef{\langle frame1 \rangle} {\langle frame2 \rangle}$

Angular velocity definition. Defines the angular velocity of frame  $\langle frame2 \rangle$  relative to frame  $\langle frame1 \rangle$ .

```
\langle frame1 \rangle = Frame; letter (a-z, A-Z)
\langle frame2 \rangle = Frame; letter (a-z, A-Z)
```

### Example

Say we want to define the angular velocity of the  $\bar{B}$  frame relative to the  $\bar{A}$  frame.

This is accomplished with the LATEX below:

 $\agVdef{a}{b}$ 

### Display Mode Output

### $8.39 \setminus angVxdef$

### $\agVxdef{\langle frame1\rangle}{\langle frame2\rangle}$

Angular velocity x-component definition. Defines the x-component of the angular velocity of frame  $\langle frame2 \rangle$  relative to frame  $\langle frame1 \rangle$ .

```
\langle frame1 \rangle = Frame; letter (a-z, A-Z)
\langle frame2 \rangle = Frame; letter (a-z, A-Z)
```

#### Example

Say we want to define the x-component of the angular velocity of the  $\bar{B}$  frame relative to the  $\bar{A}$  frame.

This is accomplished with the LATEX below:

\angVxdef{a}{b}

$${}^{\bar{\scriptscriptstyle{A}}}\!\omega_{x\bar{\scriptscriptstyle{B}}}^{\bar{\scriptscriptstyle{B}}} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \,{}^{\bar{\scriptscriptstyle{B}}}\![C]^{\bar{\scriptscriptstyle{A}}}\,{}^{\bar{\scriptscriptstyle{A}}}\![\dot{C}]^{\bar{\scriptscriptstyle{B}}} \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^{\mathrm{T}}$$

### $8.40 \ \nydef$

### $\aggle AngVydef \{ \langle frame1 \rangle \} \{ \langle frame2 \rangle \}$

Angular velocity y-component definition. Defines the y-component of the angular velocity of frame  $\langle frame2 \rangle$  relative to frame  $\langle frame1 \rangle$ .

```
\langle frame1 \rangle = Frame; letter (a-z, A-Z) \langle frame2 \rangle = Frame; letter (a-z, A-Z)
```

### Example

Say we want to define the y-component of the angular velocity of the  $\bar{B}$  frame relative to the  $\bar{A}$  frame.

This is accomplished with the LATEX below:

\angVydef{a}{b}

Display Mode Output

$${}^{\scriptscriptstyle ar{A}}\!\omega_{\!uar{\scriptscriptstyle B}}^{\scriptscriptstyle ar{\scriptscriptstyle B}} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} {}^{\scriptscriptstyle ar{\scriptscriptstyle B}}\![C]^{\scriptscriptstyle ar{\scriptscriptstyle A}}\![\dot{C}]^{\scriptscriptstyle ar{\scriptscriptstyle B}} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^{
m T}$$

## $8.41 \ \angVzdef$

### $\aggle \Delta mgVzdef \{\langle frame1 \rangle\} \{\langle frame2 \rangle\}$

Angular velocity z-component definition. Defines the z-component of the angular velocity of frame  $\langle frame2 \rangle$  relative to frame  $\langle frame1 \rangle$ .

```
\langle frame1 \rangle = Frame; letter (a-z, A-Z)
\langle frame2 \rangle = Frame; letter (a-z, A-Z)
```

#### Example

Say we want to define the z-component of the angular velocity of the  $\bar{B}$  frame relative to the  $\bar{A}$  frame.

This is accomplished with the LaTeX below:

\angVzdef{a}{b}

$${}^{\scriptscriptstyle{\bar{A}}}\!\omega_{z\bar{\scriptscriptstyle{B}}}^{\scriptscriptstyle{\bar{B}}} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \,{}^{\scriptscriptstyle{\bar{B}}}\![C]^{\scriptscriptstyle{\bar{A}}}\,{}^{\scriptscriptstyle{\bar{A}}}\![\dot{C}]^{\scriptscriptstyle{\bar{B}}} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^{\rm T}$$

### 8.42 \Pvecdef

### $\label{eq:precdef} $$\operatorname{Cone}(\langle frame1 \rangle) {\langle frame2 \rangle} {\langle var \rangle}$$$

P-vector definition. Defines the P-vector of frame  $\langle frame2 \rangle$  relative to frame  $\langle frame1 \rangle$  with respect to  $\langle var \rangle$ .

```
\langle frame1 \rangle = Frame; letter (a-z, A-Z)
\langle frame2 \rangle = Frame; letter (a-z, A-Z)
\langle var \rangle = Variable; symbol
```

### Example

Say we want to define the P-vector of the  $\bar{B}$  frame relative to the  $\bar{A}$  frame with respect to  $\theta$ .

This is accomplished with the LATEX below:

\Pvecdef{a}{b}{\theta}

Display Mode Output

### 8.43 \Pvecxdef

### 

P-vector x-component definition. Defines the x-component of the P-vector of frame  $\langle frame2 \rangle$  relative to frame  $\langle frame1 \rangle$  with respect to  $\langle var \rangle$ .

```
\langle frame1 \rangle = Frame; letter (a-z, A-Z)
\langle frame2 \rangle = Frame; letter (a-z, A-Z)
\langle var \rangle = Variable; symbol
```

### Example

Say we want to define the x-component of the P-vector of the  $\bar{B}$  frame relative to the  $\bar{A}$  frame with respect to  $\theta$ .

This is accomplished with the LaTeX below:

\Pvecxdef{a}{b}{\theta}

$${}^{\bar{A}}\vec{P}_{\theta,x}^{\bar{B}} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} {}^{\bar{B}}[C]^{\bar{A}} \frac{\partial}{\partial \theta} {}^{\bar{A}}[C]^{\bar{B}} \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^{\mathrm{T}}$$

## 8.44 \Pvecydef

### $\ensuremath{\mbox{Pvecydef}} \langle frame1 \rangle \} \{ \langle frame2 \rangle \} \{ \langle var \rangle \}$

P-vector y-component definition. Defines the y-component of the P-vector of frame  $\langle frame2 \rangle$  relative to frame  $\langle frame1 \rangle$  with respect to  $\langle var \rangle$ .

```
\langle frame1 \rangle = Frame; letter (a-z, A-Z)
\langle frame2 \rangle = Frame; letter (a-z, A-Z)
\langle var \rangle = Variable; symbol
```

### Example

Say we want to define the y-component of the P-vector of the  $\bar{B}$  frame relative to the  $\bar{A}$  frame with respect to  $\theta$ .

This is accomplished with the LATEX below:

 $\Pvecydef{a}{b}{\theta}$ 

Display Mode Output

$${}^{\bar{A}}\!\vec{P}_{\theta,y}^{\bar{B}} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} {}^{\bar{B}}\![C]^{\bar{A}} \frac{\partial}{\partial \theta} {}^{\bar{B}}\![C]^{\bar{B}} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^{\mathrm{T}}$$

#### 8.45 \Pveczdef

## $\label{eq:precond} $$\operatorname{Pveczdef}(\langle frame1\rangle) {\langle frame2\rangle} {\langle var\rangle}$$

P-vector z-component definition. Defines the z-component of the P-vector of frame  $\langle frame2 \rangle$  relative to frame  $\langle frame1 \rangle$  with respect to  $\langle var \rangle$ .

```
\langle frame1 \rangle = Frame; letter (a-z, A-Z)
\langle frame2 \rangle = Frame; letter (a-z, A-Z)
\langle var \rangle = Variable; symbol
```

### Example

Say we want to define the z-component of the P-vector of the  $\bar{B}$  frame relative to the  $\bar{A}$  frame with respect to  $\theta$ .

This is accomplished with the LaTeX below:

\Pveczdef{a}{b}{\theta}

$${}^{\bar{A}}\!\vec{P}_{\theta,z}^{\bar{B}} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \, {}^{\bar{B}}\![C]^{\bar{A}} \frac{\partial}{\partial \theta} {}^{\bar{A}}\![C]^{\bar{B}} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$

## 8.46 \Tvecdef

# $\Tvecdef{\langle frame1\rangle}{\langle frame2\rangle}{\langle var\rangle}$

T-vector definition. Defines the T-vector of frame  $\langle frame2 \rangle$  relative to frame  $\langle frame1 \rangle$  with respect to  $\langle var \rangle$ .

```
\langle frame1 \rangle = Frame; letter (a-z, A-Z)
\langle frame2 \rangle = Frame; letter (a-z, A-Z)
\langle var \rangle = Variable; symbol
```

#### Example

Say we want to define the T-vector of the  $\bar{B}$  frame relative to the  $\bar{A}$  frame with respect to  $\beta$ .

This is accomplished with the LaTeX below:

 $\Tvecdef{a}{b}{\beta}$ 

Display Mode Output

#### 8.47 \Tvecxdef

# $\Tvecxdef{\langle frame1\rangle}{\langle frame2\rangle}{\langle var\rangle}$

T-vector x-component definition. Defines the x-component of the T-vector of frame  $\langle frame2 \rangle$  relative to frame  $\langle frame1 \rangle$  with respect to  $\langle var \rangle$ .

```
\langle frame1 \rangle = Frame; letter (a-z, A-Z)
\langle frame2 \rangle = Frame; letter (a-z, A-Z)
\langle var \rangle = Variable; symbol
```

#### Example

Say we want to define the x-component of the T-vector of the  $\bar{B}$  frame relative to the  $\bar{A}$  frame with respect to  $\beta$ .

This is accomplished with the LATEX below:

\Tvecxdef{a}{b}{\beta}

$${}^{\bar{\boldsymbol{A}}}\vec{T}_{\beta,x}^{\bar{\boldsymbol{B}}} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \, {}^{\bar{\boldsymbol{B}}}[C]^{\bar{\boldsymbol{A}}} \frac{\mathrm{d}\,\bar{\boldsymbol{A}}}{\mathrm{d}\beta} [C]^{\bar{\boldsymbol{B}}} \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^{\mathrm{T}}$$

# 8.48 \Tvecydef

#### $\Tvecydef{\langle frame1\rangle}{\langle frame2\rangle}{\langle var\rangle}$

T-vector y-component definition. Defines the y-component of the T-vector of frame  $\langle frame2 \rangle$  relative to frame  $\langle frame1 \rangle$  with respect to  $\langle var \rangle$ .

```
\langle frame1 \rangle = Frame; letter (a-z, A-Z)
\langle frame2 \rangle = Frame; letter (a-z, A-Z)
\langle var \rangle = Variable; symbol
```

#### Example

Say we want to define the y-component of the T-vector of the  $\bar{B}$  frame relative to the  $\bar{A}$  frame with respect to  $\beta$ .

This is accomplished with the LATEX below:

 $\Tvecydef{a}{b}{\beta}$ 

Display Mode Output

$${}^{\bar{A}}\vec{T}^{\bar{B}}_{\beta,y} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \, {}^{\bar{B}}[C]^{\bar{A}} \frac{\mathrm{d}\,\bar{A}}{\mathrm{d}\beta} [C]^{\bar{B}} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^{\mathrm{T}}$$

#### 8.49 \Tveczdef

# $\Tveczdef{\langle frame1\rangle}{\langle frame2\rangle}{\langle var\rangle}$

T-vector z-component definition. Defines the z-component of the T-vector of frame  $\langle frame2 \rangle$  relative to frame  $\langle frame1 \rangle$  with respect to  $\langle var \rangle$ .

```
\langle frame1 \rangle = Frame; letter (a-z, A-Z)
\langle frame2 \rangle = Frame; letter (a-z, A-Z)
\langle var \rangle = Variable; symbol
```

#### Example

Say we want to define the z-component of the T-vector of the  $\bar{B}$  frame relative to the  $\bar{A}$  frame with respect to  $\beta$ .

This is accomplished with the LaTeX below:

\Tveczdef{a}{b}{\beta}

$${}^{\scriptscriptstyle{\bar{A}}}\vec{T}_{\beta,z}^{\scriptscriptstyle{\bar{B}}} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \, {}^{\scriptscriptstyle{\bar{B}}}[C]^{\scriptscriptstyle{\bar{A}}} \frac{\mathrm{d}\,{}^{\scriptscriptstyle{\bar{A}}}}{\mathrm{d}\beta} [C]^{\scriptscriptstyle{\bar{B}}} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$

# **Angular Momentum Equations**

## 8.50 \angMomPart

```
\angMomPart{\langle frame \rangle}{\langle point1 \rangle}{\langle point2 \rangle}{\langle mass \rangle}
```

Particle angular momentum definition. This command is used to define the angular momentum of a particle with mass  $\langle mass \rangle$  with respect to  $\langle point2 \rangle$  with respect to  $\langle point1 \rangle$ , relative to the frame  $\langle frame \rangle$ .

```
\langle frame \rangle = \text{Letter (a-z, A-Z)}
\langle point1 \rangle = \text{Name of point1; expression}
\langle point2 \rangle = \text{Name of point2; expression}
\langle mass \rangle = \text{Name of mass; expression}
```

#### Example

Say we want to define the angular momentum of a particle of mass m with respect to P with respect to Q expressed in the  $\bar{F}$  frame.

This is accomplished with the LATEX below:

$$ec{r}_{\scriptscriptstyle Q}ec{h}_{m/\scriptscriptstyle P}\equivec{r}_{m/\scriptscriptstyle P} imes m^{\scriptscriptstyle ar{r}}ec{v}_{m/\scriptscriptstyle Q}$$

# 8.51 \angMomSys

# $\label{lem:lemma_sys} $$\operatorname{MomSys}(\langle frame \rangle) {\langle point1 \rangle} {\langle point2 \rangle}$$$

System angular momentum. This command is used to express the angular momentum of a system with respect to  $\langle point2 \rangle$  with respect to  $\langle point1 \rangle$ , relative to the frame  $\langle frame \rangle$ .

 $\langle frame \rangle = \text{Letter (a-z, A-Z)}$ 

 $\langle point1 \rangle$  = Name of point1; expression

 $\langle point2 \rangle$  = Name of point2; expression

#### Example

Say we want to define the angular momentum of a system with respect to P with respect to Q expressed in the  $\overline{F}$  frame.

This is accomplished with the LATEX below:

\angMomSys{f}{\smca{Q}}}{\smca{P}}

Display Mode Output

$${}_{\scriptscriptstyle Q}^{\scriptscriptstyle ar{\scriptscriptstyle F}}ec{h}_{\scriptscriptstyle P,\mathrm{sys}} \equiv \sum_{i=1}^n ec{r}_{m_i/\!\scriptscriptstyle P} imes m_i{}^{\scriptscriptstyle ar{\scriptscriptstyle F}}ec{v}_{m_i/\!\scriptscriptstyle Q}$$

# $8.52 \setminus angMomDefI$

#### \angMomDefI

Angular momentum definition I. Generic angular momentum expression.

No input arguments.

#### Example

Show the generic expression for angular momentum.

This is accomplished with the LATEX below:

\angMomDefI

$${}_{Q}^{ar{A}}ec{h}_{P,\mathrm{sys}}={}_{B}^{ar{F}}ec{h}_{B,\mathrm{sys}}+ ilde{I}_{B}\cdot{}^{ar{O}}ec{\omega}^{ar{F}}+ec{r}_{\mathrm{CM}/B} imes m_{\mathrm{T}}{}^{ar{A}}ec{v}_{B/Q}+ec{r}_{B/P} imes m_{\mathrm{T}}{}^{ar{A}}ec{v}_{\mathrm{CM}/Q}$$

# $8.53 \setminus angMomDefII$

#### \angMomDefII

Angular momentum definition II. Angular momentum of a rigid body.

No input arguments.

#### Example

Show the expression for the angular momentum of a rigid body.

This is accomplished with the LATEX below:

\angMomDefII

Display Mode Output

$${}^{\bar{o}}_{\scriptscriptstyle Q} \vec{h}_{\scriptscriptstyle 
m CM} = {}^{\bar{o}}_{\scriptscriptstyle 
m CM} \vec{h}_{\scriptscriptstyle 
m CM} = \tilde{I}_{\scriptscriptstyle 
m CM} \cdot {}^{\bar{o}} ec{\omega}^{ar{F}}$$

# $8.54 \setminus angMomDefIII$

#### \angMomDefIII

Angular momentum definition III. Angular momentum about the origin with respect to a body frame.

No input arguments.

#### Example

Show the angular momentum definition III.

This is accomplished with the LATEX below:

\angMomDefIII

$${}^{\scriptscriptstyle \bar{O}}_{\scriptscriptstyle O}\!\vec{h}_{\scriptscriptstyle O,\rm sys} = {}^{\scriptscriptstyle \bar{O}}_{\scriptscriptstyle \rm CM}\!\vec{h}_{\scriptscriptstyle \rm CM,sys} + \vec{r}_{\scriptscriptstyle \rm CM/O} \times m_{\scriptscriptstyle \rm T}{}^{\scriptscriptstyle \bar{O}}\vec{v}_{\scriptscriptstyle \rm CM/O}$$

# 8.55 \angMomDefIV

#### \angMomDefIV

Angular momentum definition IV. Angular momentum about an arbitrary point A. No input arguments.

#### Example

Show the angular momentum definition IV.

This is accomplished with the LATEX below:

\angMomDefIV

Display Mode Output

# **Torque Equations**

# 8.56 \torqueDef

#### \torqueDef

Torque definition. Torque definition for a system of n particles about point P.

No input arguments.

#### Example

Show the torque definition.

This is accomplished with the LATEX below:

\torqueDef

$$ec{ au}_{ ext{ iny p,net}} \equiv \sum_{i=1}^n ec{r}_{m_i/\! ext{ iny p}} imes ec{F}_{i, ext{net}}$$

# 8.57 \torqueDefI

#### \torqueDefI

Torque definition I. Most general case,  $\bar{O}$  is an IRF.

No input arguments.

### Example

Show torque definition I.

This is accomplished with the  $\LaTeX$  below:

\torqueDefI

Display Mode Output

$$ec{ au}_{\!\scriptscriptstyle P,\mathrm{net}} = {}^{ar{o}}_{rac{\mathrm{d}}{\mathrm{d}t}} \! \left( {}^{ar{o}}_{\!\scriptscriptstyle Q} \! ec{h}_{\!\scriptscriptstyle P,\mathrm{sys}} 
ight) + m_{\scriptscriptstyle \mathrm{T}} {}^{ar{o}} ec{v}_{\scriptscriptstyle \mathrm{CM/P}} imes {}^{ar{o}} ec{v}_{\scriptscriptstyle Q/P} + m_{\scriptscriptstyle \mathrm{T}} ec{r}_{\scriptscriptstyle \mathrm{CM/P}} imes {}^{ar{o}} ec{a}_{\scriptscriptstyle Q/O}$$

# 8.58 \torqueDefII

#### \torqueDefII

Torque definition II.  $Q=O,\,P=\mathit{CM},\,\bar{O}$  is an IRF.

No input arguments.

#### Example

Show torque definition II.

This is accomplished with the LATEX below:

\torqueDefII

$$ec{ au}_{ ext{ iny CM}, ext{net}} = rac{ar{o}_{ ext{d}}}{ ext{d}t} igg( ar{o}_{ ext{ iny CM}, ext{ iny Sys}} igg)$$

# 8.59 \torqueDefIII

#### \torqueDefIII

Torque definition III.  $Q=P=\mathit{CM},\,\bar{O}$  is an IRF.

No input arguments.

### Example

Show torque definition III.

This is accomplished with the LaTeX below:

\torqueDefIII

Display Mode Output

$$ec{ au}_{ ext{ iny CM}, ext{net}} = rac{ar{o}_{ ext{d}}}{ ext{d}t} \Big( ar{o}_{ ext{ iny CM}} ec{h}_{ ext{ iny CM}, ext{sys}} \Big)$$

# 8.60 \torqueDefIV

#### \torqueDefIV

Torque definition IV.  $Q=P=O,\,\bar{O}$  is an IRF.

No input arguments.

# Example

Show torque definition IV.

This is accomplished with the  $\LaTeX$  below:

\torqueDefIV

$$ec{ au_{\scriptscriptstyle O}} = rac{ar{\scriptscriptstyle O}_{
m d}}{{
m d}t} \Big( ar{\scriptscriptstyle O}_{\scriptscriptstyle O} ec{h}_{\scriptscriptstyle O,{
m sys}} \Big)$$

# 8.61 \torqueDefV

#### \torqueDefV

Torque definition V.  $Q=O,\,P$  is a fixed point with respect to  $O,\,\bar{O}$  is an IRF. No input arguments.

#### Example

Show torque definition V.

This is accomplished with the LATEX below:

\torqueDefV

Display Mode Output

$$ec{ au}_{ ext{ iny P,net}} = rac{ar{ iny d}}{\mathrm{d}t} \Big( ar{ iny d}_{ ext{ iny P,sys}} \Big)$$

# **Kinetic Energy Equations**

#### 8.62 \kinEnDef

#### \kinEnDef

Kinetic energy definition. Kinetic energy definition for a system of n particles.

 $No\ input\ arguments.$ 

#### Example

Show the kinetic energy definition.

This is accomplished with the LATEX below:

\kinEnDef

$${}^{\bar{o}}T_o \equiv \sum_{i=1}^n {1\over 2} m_i{}^{\bar{o}} \vec{v}_{m_i/o} \cdot {}^{\bar{o}} \vec{v}_{m_i/o}$$

#### 8.63 \kinEnDefI

#### \kinEnDefI

Kinetic energy definition I. General expression for the kinetic energy for a system of n particles and any frames  $\bar{O}$  and  $\bar{A}$ .

No input arguments.

#### Example

Show kinetic energy definition I.

This is accomplished with the LATEX below:

\kinEnDefI

Display Mode Output

$$\begin{split} {}^{\bar{o}}T_{\scriptscriptstyle O} &= \tfrac{1}{2} m_{\scriptscriptstyle T}{}^{\bar{o}} \vec{v}_{\scriptscriptstyle A/O} \cdot {}^{\bar{o}} \vec{v}_{\scriptscriptstyle A/O} + m_{\scriptscriptstyle T}{}^{\bar{o}} \vec{v}_{\scriptscriptstyle A/O} \cdot {}^{\bar{o}} \vec{v}_{\scriptscriptstyle \mathrm{CM/A}} + \sum_{i=1}^n \tfrac{1}{2} m_i{}^{\bar{A}} \vec{v}_{m_i/A} \cdot {}^{\bar{A}} \vec{v}_{m_i/A} \\ &+ \sum_{i=1}^n \tfrac{1}{2} m_i{}^{\bar{A}} \vec{v}_{m_i/A} \cdot \left( {}^{\bar{o}} \vec{\omega}^{\bar{A}} \times \vec{r}_{m_i/A} \right) + \tfrac{1}{2} {}^{\bar{o}} \vec{\omega}^{\bar{A}} \cdot \tilde{I}_{\scriptscriptstyle A} \cdot {}^{\bar{o}} \vec{\omega}^{\bar{A}} \end{split}$$

#### 8.64 \kinEnDefII

#### \kinEnDefII

Kinetic energy definition II. Expression for the kinetic energy for a rigid body with body frame  $\bar{A}$ . Valid for any frames  $\bar{O}$  and  $\bar{A}$ .

No input arguments.

### Example

Show kinetic energy definition II.

This is accomplished with the LATEX below:

\kinEnDefII

$${}^{\bar{o}}T_o = {\textstyle \frac{1}{2}} m_{\scriptscriptstyle T}{}^{\bar{o}} \vec{v}_{\scriptscriptstyle A/O} \cdot {}^{\bar{o}} \vec{v}_{\scriptscriptstyle A/O} + m_{\scriptscriptstyle T}{}^{\bar{o}} \vec{v}_{\scriptscriptstyle A/O} \cdot {}^{\bar{o}} \vec{v}_{\scriptscriptstyle \mathrm{CM/A}} + {\textstyle \frac{1}{2}} {}^{\bar{o}} \vec{\omega}^{\bar{\scriptscriptstyle A}} \cdot \tilde{I}_{\scriptscriptstyle A} \cdot {}^{\bar{o}} \vec{\omega}^{\bar{\scriptscriptstyle A}}$$

#### 8.65 \kinEnDefIII

#### \kinEnDefIII

Kinetic energy definition III. Expression for the kinetic energy for a rigid body with body frame  $\bar{A}$  and A = CM. Valid for any frames  $\bar{O}$  and  $\bar{A}$ .

No input arguments.

#### Example

Show kinetic energy definition III.

This is accomplished with the LATEX below:

\kinEnDefIII

Display Mode Output

$${}^{\bar{o}}T_{\scriptscriptstyle O} = {\textstyle \frac{1}{2}} m_{\scriptscriptstyle T}{}^{\bar{o}} \vec{v}_{\scriptscriptstyle {\rm CM/O}} \cdot {}^{\bar{o}} \vec{v}_{\scriptscriptstyle {\rm CM/O}} + {\textstyle \frac{1}{2}} {}^{\bar{o}} \vec{\omega}^{\bar{\scriptscriptstyle A}} \cdot \tilde{I}_{\scriptscriptstyle {\rm CM}} \cdot {}^{\bar{o}} \vec{\omega}^{\bar{\scriptscriptstyle A}}$$

#### 8.66 \kinEnDefIV

#### \kinEnDefIV

Kinetic energy definition IV. Expression for the kinetic energy for a rigid body with body frame  $\bar{A}$  and A is a fixed point with respect to  $O\left({}^{\bar{o}}\vec{v}_{A/o}=\vec{0}\right)$ . Valid for any frames  $\bar{O}$  and  $\bar{A}$ .

No input arguments.

#### Example

Show kinetic energy definition IV.

This is accomplished with the LATEX below:

\kinEnDefIV

$${}^{\bar{o}}T_{\scriptscriptstyle O}=rac{1}{2}{}^{\bar{o}}ec{\omega}^{\scriptscriptstyle ar{A}}\cdot \widetilde{I}_{\scriptscriptstyle A}\cdot {}^{\bar{o}}ec{\omega}^{\scriptscriptstyle ar{A}}$$

# Alternate Kinetic Energy Equations

# 8.67 \TEq

#### \TEq

Alternative kinetic energy definition. Used to evaluate Ljapunov/Liapunov stability. *No input arguments.* 

#### Example

Show the alternative kinetic energy definition.

This is accomplished with the LATEX below:

\TEq

Display Mode Output

$$^{\bar{o}}T_{o}=T_{2}+T_{1}+T_{0}$$

# 8.68 \TzeroEq

#### \TzeroEq

 $T_0$  equation. This command defines the zeroth-order term of the kinetic energy.

No input arguments.

#### Example

Show the definition of  $T_0$ .

This is accomplished with the LATEX below:

\TzeroEq

$$T_0 = \gamma(q_1, \dots, q_n, t)$$

# 8.69 \ToneEq

#### \ToneEq

 $T_1$  equation. This command defines the first-order term of the kinetic energy.

No input arguments.

#### Example

Show the definition of  $T_1$ .

This is accomplished with the LATEX below:

 $\ToneEq$ 

Display Mode Output

$$T_1 = \sum_{r=1}^n \beta_r \dot{q}_r, \quad \beta_r = \beta_r(q_1, \dots, q_n, t)$$

# 8.70 \TtwoEq

#### \TtwoEq

 $T_2$  equation. This command defines the second-order term of the kinetic energy.

No input arguments.

#### Example

Show the definition of  $T_2$ .

This is accomplished with the LATEX below:

\TtwoEq

$$T_2 = \frac{1}{2} \sum_{r=1}^{n} \sum_{s=1}^{n} \alpha_{rs} \dot{q}_r \dot{q}_s, \quad \alpha_{rs} = \alpha_{rs}(q_1, \dots, q_n, t)$$

# **Euler's Equations of Motion**

# 8.71 \EulerEqx

#### \EulerEqx

Euler's equation about the x-axis. This command defines Euler's equation of motion about the x-axis for a rigid body about its principal axes.

No input arguments.

#### Example

Show the definition of Euler's equation of motion about the x-axis.

This is accomplished with the LATEX below:

\EulerEqx

Display Mode Output

$$\tau_x = I_{xx}\dot{\omega}_x - (I_{yy} - I_{zz})\omega_y\omega_z$$

# 8.72 \EulerEqy

#### \EulerEqy

Euler's equation about the y-axis. This command defines Euler's equation of motion about the y-axis for a rigid body about its principal axes.

 $No\ input\ arguments.$ 

#### Example

Show the definition of Euler's equation of motion about the y-axis.

This is accomplished with the LATEX below:

\EulerEqy

$$\tau_y = I_{yy}\dot{\omega}_y - (I_{zz} - I_{xx})\omega_z\omega_x$$

# 8.73 \EulerEqz

#### \EulerEqz

Euler's equation about the z-axis. This command defines Euler's equation of motion about the z-axis for a rigid body about its principal axes.

No input arguments.

#### Example

Show the definition of Euler's equation of motion about the z-axis.

This is accomplished with the LaTeX below:

\EulerEqz

Display Mode Output

$$\tau_z = I_{zz}\dot{\omega}_z - (I_{xx} - I_{yy})\omega_x\omega_y$$

# Lagrange's Equations

# 8.74 \Lagrangian

#### \Lagrangian

Lagrangian. This command defines the Lagrangian. You can use  $\mathbf{L}$  to get  $\mathcal{L}$ .

No input arguments.

#### Example

Show the definition of the Lagrangian.

This is accomplished with the LATEX below:

\Lagrangian

$${}^{\bar{o}}\!\mathcal{L}_{\!o} \equiv {}^{\bar{o}}T_{\!o} - {}^{\bar{o}}V$$

# 8.75 \Lagrange

## $\Lagrange{\langle num \rangle}$

Lagrange's equation. This command defines Lagrange's equation of a specified  $q_n$  where  $n = \langle num \rangle$ .

 $\langle num \rangle = \text{Number}; \text{ positive integer}$ 

#### Example

Say we want to define Lagrange's equation for an arbitrary  $q_n$ .

This is accomplished with the LATEX below:

\Lagrange{n}

Display Mode Output

$$\frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial^{\circ} \mathcal{L}_{o}}{\partial \dot{q}_{n}} - \frac{\partial^{\circ} \mathcal{L}_{o}}{\partial q_{n}} = Q_{n,\mathrm{nc}}$$

# 8.76 \LagrangeTV

#### $\LagrangeTV{\langle num \rangle}$

Lagrange's equation energy form. This command defines Lagrange's equation of a specified  $q_n$  where  $n = \langle num \rangle$  in terms of the kinetic and potential energy.

 $\langle num \rangle = \text{Number}; \text{ positive integer}$ 

#### Example

Say we want to define Lagrange's equation in energy form for an arbitrary  $q_n$ .

This is accomplished with the  $\LaTeX$  below:

\LagrangeTV{n}

$$\frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial}{\partial \dot{q}_n} {}^{\scriptscriptstyle \bar{O}} T_{\scriptscriptstyle O} - \frac{\partial}{\partial q_n} {}^{\scriptscriptstyle \bar{O}} T_{\scriptscriptstyle O} + \frac{\partial}{\partial q_n} {}^{\scriptscriptstyle \bar{O}} V = Q_{n,\mathrm{nc}}$$

# Kane's Equations

# 8.77 \KaneEq

#### \KaneEq

Kane's equation. This command defines Kane's equation of motion.

No input arguments.

#### Example

Display Kane's equation of motion.

This is accomplished with the  $\LaTeX$  below:

\KaneEq

Display Mode Output

$$F_r + F_r^* = 0$$

# 8.78 \Kaneqdot

#### \Kaneqdot

Kane's  $\dot{q}_s$  definition. Defines the time derivative of the generalized coordinate.

No input arguments.

#### Example

Display Kane's  $\dot{q}_s$  definition.

This is accomplished with the  $\LaTeX$  below:

\Kaneqdot

$$\dot{q}_s = \sum_{i=1}^n w_{sr} u_r + x_s, \quad s = 1, \dots, n$$

# 8.79 \Kaneomegar

## \Kaneomegar

Kane's  ${}^{\bar{o}}\vec{\omega}_r^{\bar{b}}$  definition. Defines the rth partial angular velocity.

No input arguments.

#### Example

Display Kane's  ${}^{\bar{o}}\vec{\omega}_r^{\bar{B}}$  definition.

This is accomplished with the LATEX below:

\Kaneomegar

Display Mode Output

$${}^{\scriptscriptstyle{\bar{O}}}\vec{\omega}_r^{\scriptscriptstyle{\bar{B}}} = \sum_{s=1}^n w_{sr}{}^{\scriptscriptstyle{\bar{O}}}\vec{P}_{q_s}^{\scriptscriptstyle{\bar{B}}}$$

# 8.80 \Kaneomegat

#### \Kaneomegat

Kane's  ${}^{\bar{o}}\vec{\omega}_t^{\bar{b}}$  definition. Defines the time partial angular velocity.

No input arguments.

## Example

Display Kane's  ${}^{\bar{o}}\vec{\omega}_t^{\bar{B}}$  definition.

This is accomplished with the  $\LaTeX$  below:

\Kaneomegat

$${}^{\bar{o}}\vec{\omega}_t^{\bar{B}} = {}^{\bar{o}}\vec{P}_t^{\bar{B}} + \sum_{s=1}^n x_s {}^{\bar{o}}\vec{P}_{q_s}^{\bar{B}}$$

# 8.81 \Kaneomega

## \Kaneomega

Kane's  ${}^{\bar{o}}\vec{\omega}^{\bar{B}}$  definition. Defines the angular velocity.

No input arguments.

#### Example

Display Kane's  ${}^{\bar{o}}\vec{\omega}^{\bar{B}}$  definition.

This is accomplished with the LATEX below:

\Kaneomega

Display Mode Output

$${}^{{}^{_{ar{O}}}}\!ec{\omega}^{{}^{_{ar{B}}}}={}^{{}^{_{ar{O}}}\!ec{\omega}^{{}^{_{ar{B}}}}_t+\sum_{r=1}^n{}^{{}^{_{ar{O}}}\!ec{\omega}^{{}^{_{ar{B}}}}_r$$

## 8.82 \Kanevcmr

#### \Kanevcmr

Kane's  ${}^{\bar{o}}_{o}\vec{v}_{_{\mathrm{CM},r}}$  definition. Defines the rth partial velocity.

No input arguments.

#### Example

Display Kane's  ${}^{\bar{o}}_{\mathcal{O}}\vec{v}_{^{\text{CM}}\!,r}$  definition.

This is accomplished with the LATEX below:

\Kanevcmr

$${}^{ar{O}}_{O}ec{v}_{ ext{CM},r} = \sum_{s=1}^n w_{sr} {}^{ar{O}}_{\overline{\partial}q_s} ig(ec{r}_{ ext{CM}/O}ig)$$

#### 8.83 \Kanevcmt

#### \Kanevcmt

Kane's  ${}^{\bar{o}}_{\mathcal{C}_{\mathrm{M},t}}$  definition. Defines the time partial velocity.

No input arguments.

#### Example

Display Kane's  ${}^{\bar{o}}_{o}\vec{v}_{{\rm CM},t}$  definition.

This is accomplished with the LATEX below:

\Kanevcmt

Display Mode Output

$${}_{o}^{ar{o}}ec{v}_{ ext{cm},t}={}_{o}^{ar{o}}ec{d}\left(ec{r}_{ ext{cm}/o}
ight)+\sum_{s=1}^{n}x_{s}{}_{o}^{ar{o}}rac{\partial}{\partial q_{s}}\left(ec{r}_{ ext{cm}/o}
ight)$$

#### 8.84 \Kanevcm

#### \Kanevcm

Kane's  $\bar{v}_{\text{CM}/O}$  definition. Defines the velocity.

No input arguments.

#### Example

Display Kane's  ${}^{\bar{o}}\vec{v}_{_{\mathrm{CM}/\mathcal{O}}}$  definition.

This is accomplished with the LATEX below:

\Kanevcm

$${}^{ar{o}}ec{v}_{ ext{cm/o}} = {}^{ar{o}}_oec{v}_{ ext{cm,}t} + \sum_{r=1}^n {}^{ar{o}}_oec{v}_{ ext{cm,}r}u_r$$

# 8.85 \KaneFrPart

#### \KaneFrPart

Kane's  $F_r$  term for particles.

No input arguments.

# Example

Show Kane's  $\mathcal{F}_r$  term for particles.

This is accomplished with the LaTeX below:

\KaneFrPart

$$\sum_{l=1}^{N_{\scriptscriptstyle ext{P}}} ec{f_l} \cdot {}^{ar{\scriptscriptstyle O}}_{\scriptscriptstyle O} ec{v}_{m_l,r}$$

#### 8.86 \KaneFrsPart

#### \KaneFrsPart

Kane's  $F_r^*$  term for particles.

No input arguments.

#### Example

Show Kane's  $F_r^*$  term for particles.

This is accomplished with the LATEX below:

\KaneFrsPart

Display Mode Output

$$-\sum_{l=1}^{N_{\rm P}} m_l{}^{\scriptscriptstyle \bar{O}} \vec{a}_{m_l/\!\scriptscriptstyle O} \cdot {}^{\scriptscriptstyle \bar{O}}_{\scriptscriptstyle O} \vec{v}_{m_l,r}$$

#### 8.87 \KaneFrRig

#### \KaneFrRig

Kane's  $F_r$  term for rigid bodies.

No input arguments.

#### Example

Show Kane's  $F_r$  term for rigid bodies.

This is accomplished with the LaTeX below:

\KaneFrRig

$$\sum_{k=1}^{N_{ ext{R}}} \left( ec{F}_{\!\!k} \cdot {}^{ar{o}}_{o} ec{v}_{{ ext{cm}}k,r} + ec{ au}_{k,{ ext{cm}}k} \cdot {}^{ar{o}} ec{\omega}_r^{ar{b}_k} 
ight)$$

# 8.88 \KaneFrsRig

#### \KaneFrsRig

Kane's  $F_r^*$  term for rigid bodies.

No input arguments.

#### Example

Show Kane's  $F_r^*$  equation for rigid bodies.

This is accomplished with the LATEX below:

\KaneFrsRig

Display Mode Output

$$-\sum_{k=1}^{N_{\mathrm{R}}}\left(m_{k}{}^{\bar{o}}\vec{a}_{_{\mathrm{CM}}k/o}\cdot{}^{\bar{o}}\vec{v}_{_{\mathrm{CM}}k,r}+\left(\tilde{I}_{_{\mathrm{CM}}k}\cdot{}^{\bar{o}}\vec{\alpha}^{\bar{B}_{k}}+{}^{\bar{o}}\vec{\omega}^{\bar{B}_{k}}\times\tilde{I}_{_{\mathrm{CM}}k}\cdot{}^{\bar{o}}\vec{\omega}^{\bar{B}_{k}}\right)\cdot{}^{\bar{o}}\vec{\omega}^{\bar{B}_{k}}_{r}\right)$$

#### 8.89 \KaneFr

#### \KaneFr

Kane's generalized  $F_r$  equation. The command defines Kane's  $F_r$  equation for a general system of  $N_R$  rigid bodies and  $N_P$  particles.

No input arguments.

#### Example

Show Kane's generalized  $F_r$  equation.

This is accomplished with the LaTeX below:

\KaneFr

$$F_r = \sum_{k=1}^{N_{
m R}} \left( ec{F_k} \cdot {}^{ec{O}} ec{v}_{{
m CM}k,r} + ec{ au}_{k,{
m CM}k} \cdot {}^{ec{O}} ec{\omega}_r^{ar{B}_k} 
ight) + \sum_{l=1}^{N_{
m P}} ec{f_l} \cdot {}^{ec{O}} ec{v}_{m_l,r}$$

# 8.90 \KaneFrs

#### \KaneFrs

Kane's generalized  $F_r^*$  equation. The command defines Kane's  $F_r^*$  equation for a general system of  $N_R$  rigid bodies and  $N_P$  particles.

No input arguments.

#### Example

Show Kane's generalized  $F_r^*$  equation.

This is accomplished with the LATEX below:

\KaneFrs

$$\begin{split} F_r^* &= -\sum_{k=1}^{N_{\text{\tiny R}}} \left( m_k{}^{\bar{o}} \vec{a}_{\text{\tiny CM}k/o} \cdot {}^{\bar{o}}_{o} \vec{v}_{\text{\tiny CM}k,r} + \left( \tilde{I}_{\text{\tiny CM}k} \cdot {}^{\bar{o}} \vec{\alpha}^{\bar{b}_k} + {}^{\bar{o}} \vec{\omega}^{\bar{b}_k} \times \tilde{I}_{\text{\tiny CM}k} \cdot {}^{\bar{o}} \vec{\omega}^{\bar{b}_k} \right) \cdot {}^{\bar{o}} \vec{\omega}^{\bar{b}_k} \right) \cdot {}^{\bar{o}} \vec{\omega}^{\bar{b}_k} \\ &+ \left( -\sum_{l=1}^{N_{\text{\tiny P}}} m_l{}^{\bar{o}} \vec{a}_{m_l/o} \cdot {}^{\bar{o}} \vec{v}_{m_l,r} \right) \end{split}$$

# 9 Miscellaneous Commands

This section contains miscellaneous commands that do not fit into the other categories. Currently, there are only used to format characters as italicized small caps and arrange them as (pre-)super/subscripts. They are necessary to improve the appearance of capital letter superscripts and subscripts and their kerning.<sup>3</sup>

#### 9.1 \CM

#### $\CM$

Center of mass. Creates small caps of CM for scripts.

No input arguments.

### Example

Define the position vector  $\vec{r}$  relative to the center of mass.

This is accomplished with the LATEX below:

 $\vec{r} {\CM}$ 

Display/Inline Mode Output

 $ec{r}_{\scriptscriptstyle ext{CM}}$ 

#### $9.2 \setminus comma$

#### \comma

Comma. Creates a comma with adjusted kerning for scripts with small caps.

No input arguments.

#### Example

Show an example using the subscript k, mathrmCM, i.

This is accomplished with the LATEX below:

v\_{k,\CM,i} \text{ vs } v\_{k\comma\CM\comma i}

Display/Inline Mode Output

 $v_{k,\text{cm},i}$  vs  $v_{k,\text{cm},i}$ 

<sup>&</sup>lt;sup>3</sup>In future versions, other helper commands to aid in formatting may be added here. Ideally, however, commands will look at the input arguments and adjust the kerning automatically to reduce reliance on a lot of these helper commands. This requires advanced LATEX scripting which is currently beyond my level. Eventually, the switch will be made though, making the package produce results that look better for users at the expense of some additional overhead and complexity.

#### 9.3 \smca

#### $\scal{\langle string \rangle}$

Small-caps. This command is used to convert text to small caps. This is necessary when subscripts or superscripts are capital letters. All previous commands that involve points or frames use this automatically but arbitrary subscripts and superscripts need this to specified explicitly. Note that for subscripts, you may need to adjust the spacing with \! or another command that adjusts kerning such as \mkern.

 $\langle string \rangle = String of characters$ 

#### Example

Compare a superscript A and subscript B on v with and without small caps.

This is accomplished with the LATEX below:

 $v^{A}_{\sum_{b}} \text{ v}^{\sum_{a}}_{\sum_{b}}$ 

Display/Inline Mode Output

 $v^A_{\scriptscriptstyle B}$  vs  $v^{\scriptscriptstyle A}_{\scriptscriptstyle B}$ 

#### $9.4 \ \text{smn}$

# $\mbox{\sc } \{\langle expr \rangle\}$

Small number. This command is used to reduce math text to 50% of its original size after making it upright.

 $\langle expr \rangle = \text{Expression}$ ; math expression or text

#### Example

Compare a subscript of Aircraft 1 on v creating manually, using  $\mbox{\sc smca}$ , and using  $\mbox{\sc smca}$ .

This is accomplished with the LATEX below:

```
v_{\mathit{Aircraft1}}
\text{ vs }
v_{\smca{Aircraft1}}
\text{ vs }
v_{\smn{Aircraft1}}
```

#### Display/Inline Mode Output

 $v_{Aircraft1}$  vs  $v_{AIRCRAFT1}$  vs  $v_{Aircraft1}$ 

#### 9.5 \sm

### $\mbox{sm}\{\langle expr\rangle\}$

Small. This command is used to reduce any expression to 50% of its original size.

 $\langle expr \rangle = \text{Expression}; \text{ math expression or text}$ 

#### Example

Compare a subscript of Aircraft 1 on v with,  $\sm, \sm, \and \smca$ .

This is accomplished with the LATEX below:

```
v_{\sm{Aircraft1}}
\text{ vs }
v_{\smn{Aircraft1}}
\text{ vs }
v_{\smca{Aircraft1}}
```

Display/Inline Mode Output

 $v_{Aircraft1}$  vs  $v_{Aircraft1}$  vs  $v_{AIRCRAFT1}$ 

#### 9.6 \bart

### $\operatorname{\mathtt{art}}\{\langle sym \rangle\}$

Adds a slightly wider and thicker bar to the specified  $\langle sym \rangle$ .

 $\langle sym \rangle = \text{Symbol}$ ; letter or symbol

# Example

Compare \bart to \bar for the letter F

This is accomplished with the LATEX below:

\bart{0} \text{ vs } \bar{0}

Display/Inline Mode Output

 $\bar{O}$  vs  $\bar{O}$ 

#### 9.7 \Vs

```
\Vs{\langle var \rangle} {\langle sc1 \rangle} {\langle sc2 \rangle} {\langle sc3 \rangle} {\langle sc4 \rangle}
```

Variable square scripts. Adds scripts to all corners of the input  $\langle var \rangle$ . Note that unlike the following \V\* commands, this one does not adjust the kerning of any argument and is equivalent to \tensor\*[^{sc1}\_{sc2}]{var}{\_{sc3}^{sc4}}.

 $\langle var \rangle = \text{Variable}; \text{ expression}$ 

 $\langle sc1 \rangle = \text{Upper-left script; expression}$ 

 $\langle sc2 \rangle = \text{Lower-left script; expression}$ 

 $\langle sc3 \rangle$  = Lower-right script; expression

 $\langle sc4 \rangle$  = Upper-right script; expression

#### **Example**

Show what ascending letters look like as arguments.

This is accomplished with the LATEX below:

 $Vs{a}{b}{c}{d}{e}$ 

Display/Inline Mode Output

 $_{c}^{b}a_{d}^{e}$ 

#### 9.8 \Vlt

### 

Variable lower triangular scripts. Adds scripts to the lower triangular corners of the input  $\langle var \rangle$ .

 $\langle var \rangle = \text{Variable}; \text{ expression}$ 

 $\langle sc1 \rangle$  = Upper-left script; expression

 $\langle sc2 \rangle$  = Lower-left script; expression

 $\langle sc3 \rangle$  = Lower-right script; expression

#### Example

Show what ascending letters look like as arguments.

This is accomplished with the LATEX below:

 $\Vlt{a}{b}{c}{d}$ 

Display/Inline Mode Output

 $_{c}^{b}a_{d}$ 

#### 9.9 \Vut

```
\mathsf{Vut}\{\langle var \rangle\}\{\langle sc1 \rangle\}\{\langle sc2 \rangle\}\{\langle sc3 \rangle\}
```

Variable upper triangular scripts. Adds scripts to the upper triangular corners of the input  $\langle var \rangle$ .

 $\langle var \rangle = \text{Variable}; \text{ expression}$ 

 $\langle sc1 \rangle$  = Upper-left script; expression

 $\langle sc2 \rangle$  = Upper-right script; expression

 $\langle sc3 \rangle$  = Lower-right script; expression

#### Example

Show what ascending letters look like as arguments.

This is accomplished with the LATEX below:

\Vut{a}{b}{c}{d}

Display/Inline Mode Output

 $^{b}a_{d}^{c}$ 

## 9.10 \Vup

# $\operatorname{Vup}\{\langle var \rangle\}\{\langle sc1 \rangle\}\{\langle sc2 \rangle\}$

Variable upper scripts. Adds scripts to all upper corners of the input  $\langle var \rangle$ .

 $\langle var \rangle = \text{Variable}; \text{ expression}$ 

 $\langle sc1 \rangle$  = Upper-left script; expression

 $\langle sc2 \rangle$  = Upper-right script; expression

#### Example

Show what ascending letters look like as arguments.

This is accomplished with the LATEX below:

 $\Up{a}{b}{c}$ 

Display/Inline Mode Output

 $ba^c$ 

# 9.11 \Vdg

```
\Vdg{\langle var \rangle} {\langle sc1 \rangle} {\langle sc2 \rangle}
```

Variable diagonal scripts. Adds scripts to all main diagonal corners of the input  $\langle var \rangle$ .

 $\langle var \rangle = \text{Variable}; \text{ expression}$ 

 $\langle sc1 \rangle$  = Upper-left script; expression

 $\langle sc2 \rangle$  = Lower-right script; expression

### Example

Show what ascending letters look like as arguments.

This is accomplished with the LATEX below:

 $\Vdg{a}{b}{c}$ 

Display/Inline Mode Output

 $^ba_c$ 

# 9.12 \Vsup

Variable superscript. Adds superscript to the input  $\langle var \rangle$ .

 $\langle var \rangle$  = Variable; expression

 $\langle sc1 \rangle$  = Upper-right script; expression

#### Example

Show what ascending letters look like as arguments.

This is accomplished with the LATEX below:

 $\V sup{a}{b}$ 

Display/Inline Mode Output

 $a^b$ 

#### 9.13 \Vsub

```
\V sub{\langle var \rangle} {\langle sc1 \rangle}
```

Variable subscript. Adds subscript to the input  $\langle var \rangle$ .

 $\langle var \rangle = \text{Variable}; \text{ expression}$ 

 $\langle sc1 \rangle$  = Lower-right script; expression

#### Example

Show what ascending letters look like as arguments.

This is accomplished with the LATEX below:

 $\V sub{a}{b}$ 

Display/Inline Mode Output

 $a_b$ 

# 9.14 \Vpup

#### $\Vpup{\langle var \rangle}{\langle sc1 \rangle}$

Variable presuperscript. Adds presuperscript to the input  $\langle var \rangle$ .

 $\langle var \rangle = \text{Variable}; \text{ expression}$ 

 $\langle sc1 \rangle$  = Upper-left script; expression

#### Example

Show what ascending letters look like as arguments.

This is accomplished with the LATEX below:

\Vpup{a}{b}

Display/Inline Mode Output

 $^{b}a$ 

#### 9.15 \Ms

# $\Ms{\langle mat \rangle} {\langle sc1 \rangle} {\langle sc2 \rangle} {\langle sc3 \rangle} {\langle sc4 \rangle}$

Matrix square scripts. Adds scripts to all corners of the input matrix or other upright expression  $\langle mat \rangle$ .

 $\langle mat \rangle = \text{Matrix (or upright expression), expression}$ 

 $\langle sc1 \rangle$  = Upper-left script; expression

 $\langle sc2 \rangle$  = Lower-left script; expression

 $\langle sc3 \rangle$  = Lower-right script; expression

 $\langle sc4 \rangle$  = Upper-right script; expression

#### Example

Show what ascending letters look like as arguments on the 3x3 identity matrix.

This is accomplished with the LATEX below:

 $Ms{\eyeMatiii}{a}{b}{c}{d}$ 

Display Mode Output

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{c}^{d}$$

# 9.16 \Mup

# $\begin{tabular}{l} $$ \Mup{\langle mat \rangle}{\langle sc1 \rangle}{\langle sc2 \rangle}$ \end{tabular}$

Matrix upper scripts. Adds scripts to upper corners of the input matrix or other upright expression  $\langle mat \rangle$ .

 $\langle mat \rangle = \text{Matrix}$  (or upright expression), expression

 $\langle sc1 \rangle$  = Upper-left script; expression

 $\langle sc2 \rangle$  = Upper-right script; expression

#### Example

Show what ascending letters look like as arguments on the 3x3 identity matrix.

This is accomplished with the LATEX below:

\Mup{\eyeMatiii}{a}{b}

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{b}$$

## 9.17 \Msub

## $\Msub{\langle mat \rangle} {\langle sc1 \rangle}$

Matrix subscript. Adds subscript to the input matrix or other upright expression  $\langle mat \rangle$ .

 $\langle mat \rangle = \text{Matrix}$  (or upright expression), expression  $\langle sc1 \rangle = \text{Lower-right script}$ ; expression

### Example

Show what ascending letters look like as arguments on the 3x3 identity matrix.

This is accomplished with the LATEX below:

\Msub{\eyeMatiii}{a}

#### Display Mode Output

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

# 9.18 \Mpup

# $\parbox{Mpup}{\langle mat \rangle}{\langle sc1 \rangle}$

Matrix presuperscript. Adds presuperscript to the input matrix or other upright expression  $\langle mat \rangle$ .

 $\langle mat \rangle = \text{Matrix}$  (or upright expression), expression  $\langle sc1 \rangle = \text{Upper-left script}$ ; expression

#### Example

Show what ascending letters look like as arguments on the 3x3 identity matrix.

This is accomplished with the LATEX below:

\Mpup{\eyeMatiii}{a}

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

#### 9.19 \But

```
\But{\langle var \rangle}{\langle sc1 \rangle}{\langle sc2 \rangle}{\langle sc3 \rangle}
```

Bracket then upper triangular scripts. Adds scripts to the upper triangular corners of the added brackets around input  $\langle var \rangle$ .

 $\langle var \rangle = \text{Variable}; \text{ expression}$ 

 $\langle sc1 \rangle$  = Upper-left script; expression

 $\langle sc2 \rangle = \text{Upper-right script; expression}$ 

 $\langle sc3 \rangle$  = Lower-right script; expression

#### Example

Show what ascending letters look like as arguments.

This is accomplished with the LATEX below:

\But{a}{b}{c}{d}

Display/Inline Mode Output

 $^{b}[a]_{d}^{c}$ 

## 9.20 \Bup

# $\mathbb{E}_{var} \{\langle sc1 \rangle\} \{\langle sc2 \rangle\}$

Bracket then upper scripts. Adds scripts to all upper corners of the added brackets around input  $\langle var \rangle$ .

 $\langle var \rangle = \text{Variable}; \text{ expression}$ 

 $\langle sc1 \rangle$  = Upper-left script; expression

 $\langle sc2 \rangle = \text{Upper-right script; expression}$ 

#### Example

Show what ascending letters look like as arguments.

This is accomplished with the LATEX below:

 $\mathbb{B}up\{a\}\{b\}\{c\}$ 

Display/Inline Mode Output

 $^{b}[a]^{c}$ 

#### 9.21 \Bsub

#### $\Bsub{\langle var \rangle} {\langle sc1 \rangle}$

Bracket than subscript. Adds subscript to the added brackets around input  $\langle var \rangle$ .

 $\langle var \rangle = \text{Variable}; \text{ expression}$ 

 $\langle sc1 \rangle$  = Lower-right script; expression

#### Example

Show what ascending letters look like as arguments.

This is accomplished with the LATEX below:

\Bsub{a}{b}

Display/Inline Mode Output

 $[a]_b$ 

#### 9.22 \Bsubv

#### $\Bsubv{\langle var \rangle} {\langle sc1 \rangle}$

Bracket then subscript for vectors. Adds subscript to the added brackets around input  $\langle var \rangle$ . Note that this is needed to adjust the second bracket so that it is further from the vector arrow. Equivalent to  $\Bsubt\{var\mbox{mkern+2mu}\}$ . Additionally, some symbols may appear fine without this additional kerning, which is why it was added in a separate command rather than by default in the previous command.

 $\langle var \rangle = \text{Variable}; \text{ expression}$ 

 $\langle sc1 \rangle$  = Lower-right script; expression

#### Example

Show what ascending letters look like as arguments and compare to \Bsub

This is accomplished with the LATEX below:

Display/Inline Mode Output

 $[\vec{a}]_b$  vs  $[\vec{a}]_b$ 

# 10 Additional Information

This section contains additional information about the package including installation instructions, packages used for documentation and a brief description of what they are used for, and planned features for future versions.

#### 10.1 Installation Instructions

There are currently two main ways to use this package:

- 1. The first is to download the advanceddynamics.sty file from GitHub and simply place it in the same folder as your document.
- 2. The second is to copy the package to your local distribution so that you can use it from any document. We will examine the first method here.

First download your preferred release from GitHub and then extract it.

Next, locate the install location for local packages. Default locations for the most common distributions are listed below:

- MacTeX (Mac Only)
  - \$HOME\Library\texmf<sup>4</sup>
- MiKTeX (Windows/Linux/Mac)
  - User Specified: https://miktex.org/kb/texmf-roots#:~:text=Your%20own% 20TEXMF%20root%20directories
- TeXLive (Windows/Linux/Mac)
  - Linux/Mac: \usr\local\texlive\texmf-local
  - Windows: C:\Users\<user>\texlive\texmf-local

Once you have located the folder, copy the tex and doc folders from the extracted package into this folder. The package should now be accessible to your preferred LATEX distribution and IDE for all documents.

In the future, I hope that this package will be included with the distributions and then this setup will be unnecessary.

# 10.2 Additional Packages

In addition to the five packages used for typesetting equations in the main package, several additional packages were used to create this documentation. Note that for convenience, all of the packages used (including the ones mentioned in the Introduction, Section 1) and a brief description of what they are used for are listed on the following page:

<sup>&</sup>lt;sup>4</sup>Note that the Library directory is hidden by default on Mac

1. advanceddynamics.sty: Provides package macros and commands

```
accents for defining custom bar accent \bart [1] amsmath for math notation [4] amssymb for math symbols [6] graphicx for scaling subscripts and superscripts [2] mathtools for additional math functionality [3] tensor for prescripts [5]
```

2. advdyndoc.sty: Provides documentation macros and commands

```
amsmath for math align* environment [4] fontenc for ASCII/monospace characters [Standard LATEX 2\varepsilon package] tcolorbox for titled example boxes and documentation commands [16] xcolor for custom text colors [13] xparse for \NewDocumentCommand command [15]
```

3. advanceddynamicsmanual.tex: Creates documentation PDF

```
advanceddynamics for typesetting examples [See above] advdyndoc for documentation and formatting [See above] biblatex for generating the references section [9] datetime2 for getting and formatting the current date [17] enumitem for formatting lists like these [8] fancyhdr for setting the document header [12] geometry for setting page layout [18] hyperref for linking to sections/labels and urls [14] imakeidx for generating the index section [7] lastpage for getting the last page number [11] parskip for removing paragraph indents [10]
```

#### 10.3 Todos

This section details a list of planned future features to the package. Current plans include:

- 1. Use xparse to migrate commands from \newcommand\* to \DeclareDocumentCommand and \DeclareExpandableDocumentCommand
  - Keep backwards compatibility, only modify/add commands
  - Unify frame and numbered frame notation by adding an optional input argument
  - Update commands that commonly have added subscripts/superscripts with an optional input argument
- 2. Update package to support importing macros/commmands by section
- 3. Format package to work with normal LATEX installation methods
- 4. Allow for automatic splitting of long equations rather than predefined splits without breaking other math packages (difficult)

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