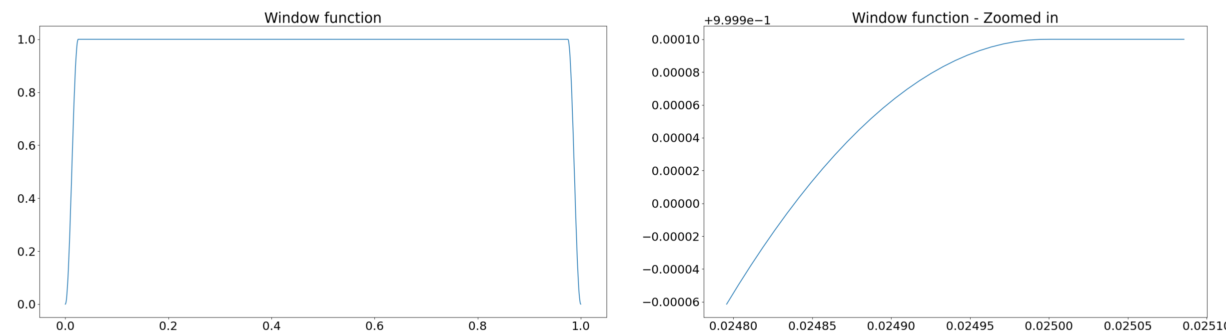
Problem 1

a)

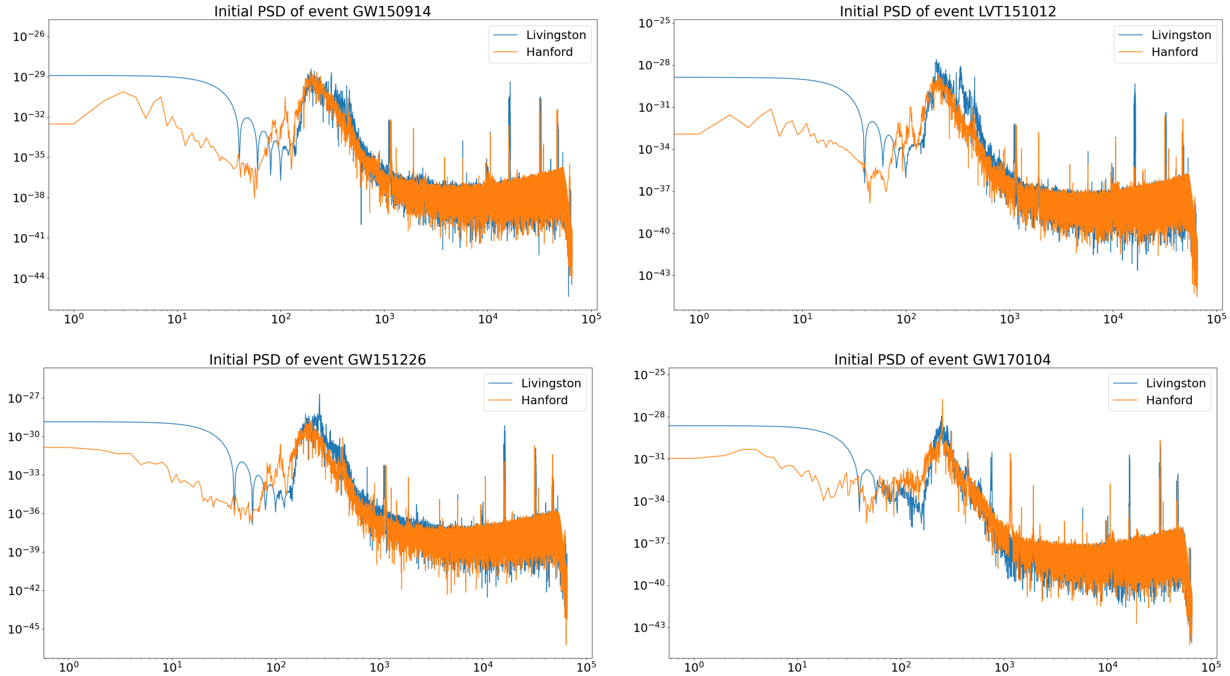
Each set of measurements has 131072 points - 32 seconds at a 4096Hz sampling rate. The first step is to window the data. The window I used is a flattened version of the Hanning window, where 95% of the window is flat - this is setup to be continuous.



To estimate the noise, first look at the Power Spectral Density. If it's averaged over the 4 events, some of the white noise cancels/flattens out while the frequency-dependant noise remains.

Taking the average over **n**-neighbours in the PSD, further flattens out the white noise in the PSD. Repeating this process **t**-times, eventually gets rid of everything except the frequency dependent noise our data is convolved with.

The operation looks like this:



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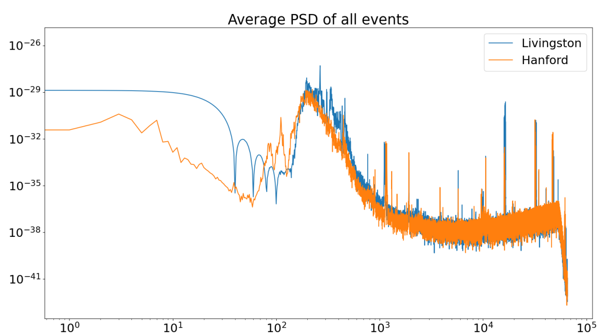

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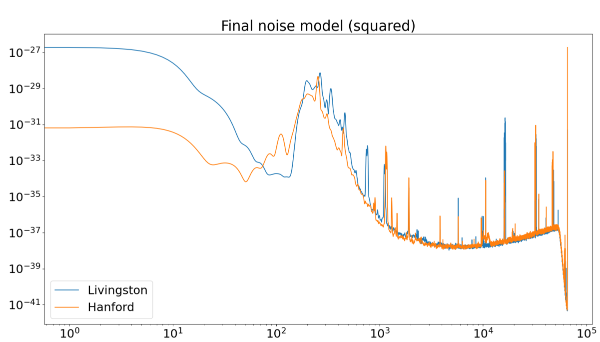
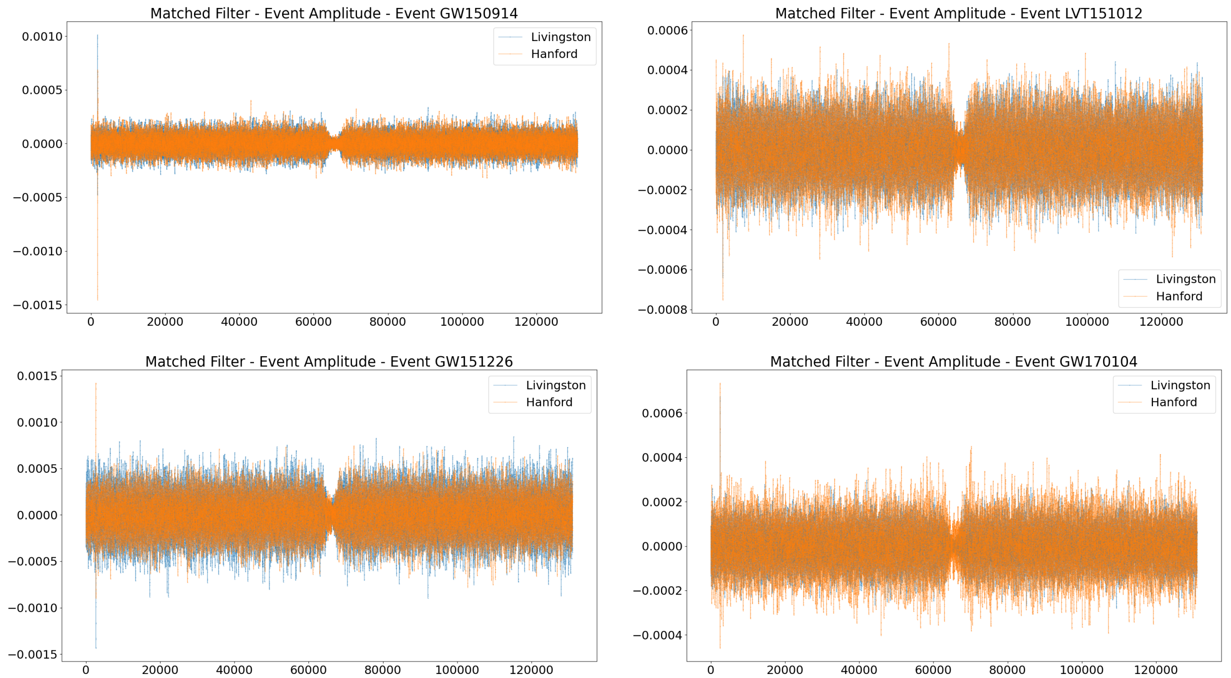


The lines could be eliminated (through a median filter), but they seem to carry information about the noise rather than the signal and getting rid of the peaks only seems to worsen the matched filter.

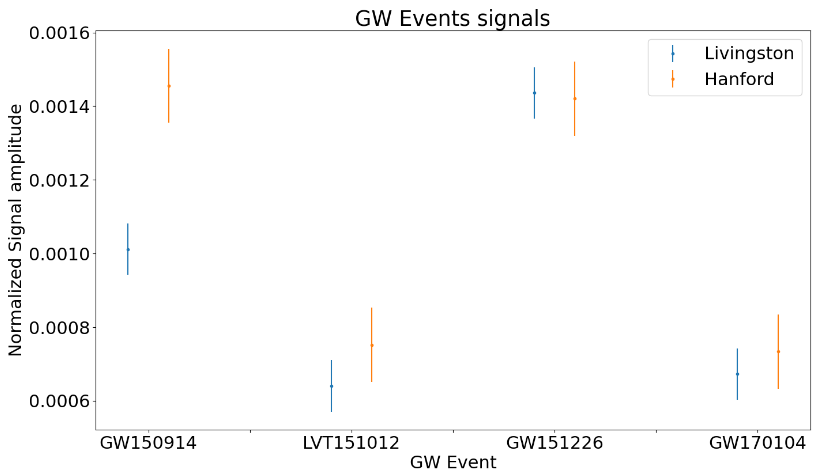
b)

The matched filter output looks like:

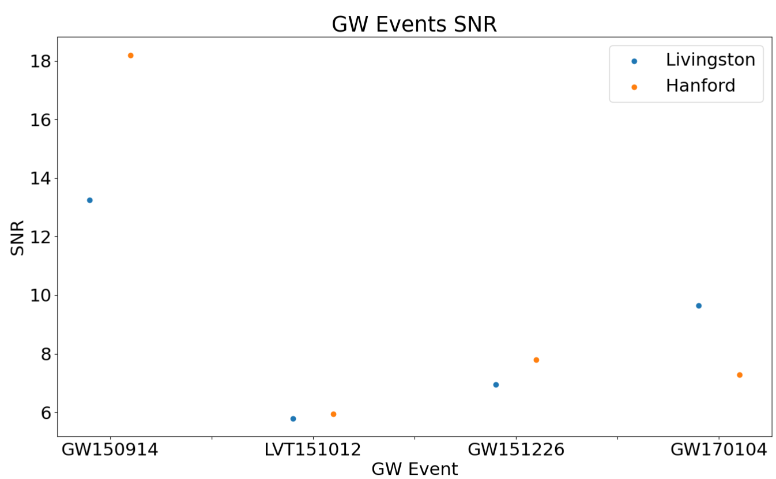


c)

Which allows us to obtain the signal amplitude (using the last 80000 data points to get σ):



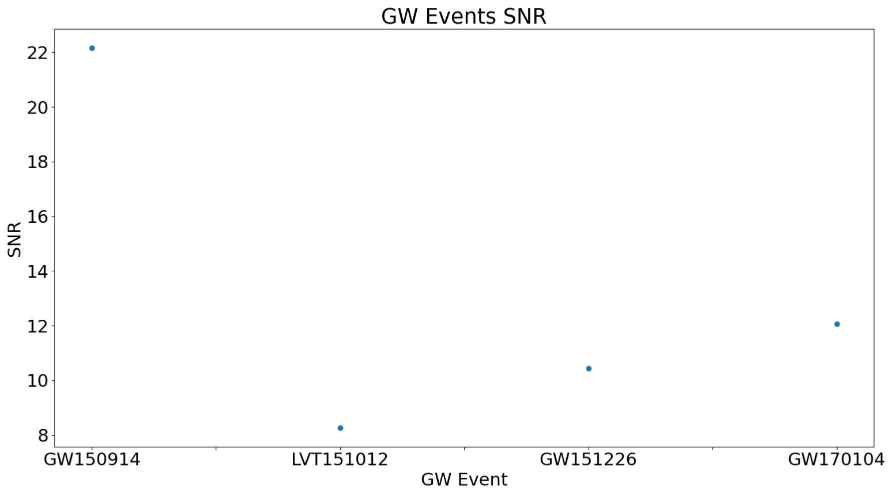
Then, we can get the Signal to Noise Ratio (SNR):



We can use a weighted average - over the uncertainty - to combine the signal to noise ratio from both detectors.

$$X_C = \frac{\sum_i \omega_i X_i}{\sum_k \omega_k}$$
$$\omega_i = \frac{1}{\sigma_i^2}$$

Then, the combined SNR is:



d)

As we saw in class,

$$m = (A^T N^{-1} A)^{-1} A^T N^{-1} d \tag{1}$$

$$A_w = N^{-\frac{1}{2}} A \tag{2}$$

$$d_w = N^{-\frac{1}{2}} d \tag{3}$$

$$\implies m = (A_w^T A_w)^{-1} A_w^T d_w \tag{4}$$

Then,

$$m \cdot m^T = (A_w^T A_w)^{-1} A_w^T d_w d_w^T A_w (A_w^T A_w)^{-1} \tag{5}$$

Again, recall (I also verified this computationally) $d_w^T d_w = 1$. Then,

$$m \cdot m^T = (A_w^T A_w)^{-1} A_w^T A_w (A_w^T A_w)^{-1} \tag{6}$$

$$\therefore$$

$$\boxed{m \cdot m^T = (A_w^T A_w)^{-1}} \tag{7}$$

It's quite trivial then to obtain $< mm^T >$ from this - and hence the theoretical σ_m from the noise model. Below a table comparing the SNRs obtained from the two methods.

Event	SNR Scatter		SNR Theoretical	
	Livingston	Hanford	Livingston	Hanford
GW150914	13.2	18.2	13.9	21.9
LVT151012	5.8	5.9	5.5	7.2
GW151226	6.9	7.8	8.0	8.8
GW170104	9.6	7.3	7.5	9.0

The results are on average relatively close, the scattered SNR values are in general lower than theoretical ones, but they tend to stay within ± 1 of each other.

e)

From the template and noise model, we can obtain the whitened template and apply a DFT to obtain an estimate for the frequency of the signal.

Event	Livingston Frequency (Hz)	Hanford Frequency (Hz)
GW150914	47.9	42.2
LVT151012	47.86	47.4
GW151226	51.8	42.2
GW170104	52.6	49.1

Note that depending on whether you want angular frequency or not, the above changes by a 2π factor.

f)

These waves move at the speed of light so an estimate on the uncertainty in the Δt of detection between both detectors should give us an estimate on the positional uncertainty.

To get the time uncertainty, I look at the number of points in the matched filter that are within σ of the maximum signal. Then use this to estimate the σ_{time} (which seems like a decent approximation).

Event	Livingston Time Uncertainty (s/4096)	Hanford Time Uncertainty (s/4096)
GW150914	1	1
LVT151012	3	3
GW151226	2	3
GW170104	2	2.5

Then, using uncertainty propagation we can obtain for each of the events that $\sigma_{\Delta t} = \{2, 6, 5, 4.5\}$ s/4096

We can now get a positional uncertainty; $\sigma_x = c\sigma_{\Delta t} = \{146, 439, 366, 330\}$ km is then the expected uncertainty in the position.