Problem 2

a) Which solver should you use?

Due to the large differences in half-lives between the various decay products, this is considered a stiff equation. Hence I'll use the Radau method - which comes with Scipy's IVP solver.

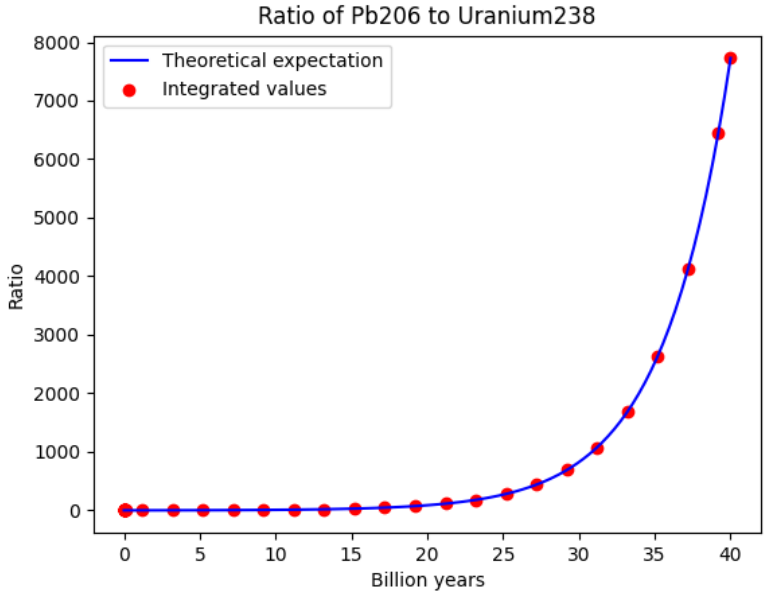
b) Does the ratio of Pb206 to U238 make sense?

The ratio does indeed make sense. Since all the half-lives in the decay chain are short compared to Uranium 238, you can approximate the decay to be direct:

$$\frac{\partial Y_{U238}}{\partial t} = -\lambda Y_{U238}$$
$$\frac{\partial Y_{Pb206}}{\partial t} = \lambda Y_{U238}.$$

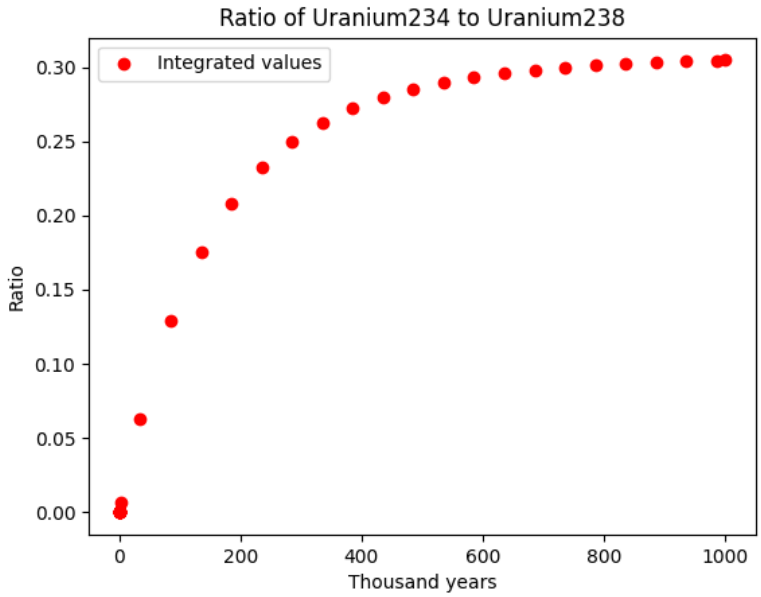
Then we can write:

$$\frac{Y_{Pb206}}{Y_{U238}} = \frac{1 - e^{-\lambda t}}{e^{-\lambda t}}. \tag{1}$$



Equation 1 is the blue line shown in the plot above. Hence the computed values - in red - make sense analytically.

As for the ratio of Thorium 230 to Uranium 234, you can see it below:



The timescale is indeed much smaller and would allow you to determine the age in the order of thousands of years, of a rock containing uranium.