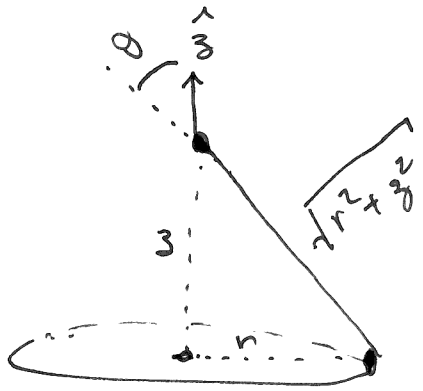
Problem 1



$$\vec{E} = \int \frac{dq \cos\theta \, \hat{z}}{4\pi\varepsilon_0 (r^2+z^2)}$$

This is because $E_{x,y}$ terms cancel.

Then, $dq \to \lambda r \, d\phi$, $\cos\theta \to \frac{z}{\sqrt{r^2+z^2}}$

$$E_z = \int_0^{2\pi} \frac{\lambda r z \, d\phi}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}}$$

$$\boxed{\vec{E} = \frac{\lambda r z}{2\varepsilon_0 (r^2+z^2)^{3/2}} \hat{z}}$$

Then, for a sphere

$$\lambda \to \sigma R \, d\varphi$$

$$r \to R\sin\vartheta$$

$$z \to z - R\cos\vartheta$$

$$\left(\sigma = \frac{Q}{4\pi R^2}\right)$$

Then, for a sphere

$$\vec{E} = \frac{\sigma R^2}{2\varepsilon_0}\int_0^{\pi} \frac{\sin\vartheta\,(z - R\cos\vartheta)\,d\vartheta}{(R^2 + z^2 - 2Rz\cos\vartheta)^{3/2}}\,\hat{z}$$