Problem 1

$$\frac{f(t+dt,x)-f(t-dt,x)}{2dt} = -v\frac{f(t,x+dx)-f(t,x-dx)}{2dx} \tag{1}$$

$$f(t+dt,x)-f(t-dt,x) = -\alpha\Big[f(t,x+dx)-f(t,x-dx)\Big] \tag{2}$$

$$\tag{3}$$

for $\alpha = k/\frac{dx}{dt}$,

$$\Big[\xi^{2dt}-1\Big]\xi^{t-dt}e^{ikx} = -\alpha\xi^t e^{ikx}\Big[e^{ikdx}-e^{-ikdx}\Big] \tag{4}$$

$$\xi^{2dt}-1 = -2i\alpha\xi^{dt}\sin(kdx) \tag{5}$$

for $\tilde{\xi} = \xi^{dt}$

$$\tilde{\xi}^2 + 2i\alpha\tilde{\xi}\sin(kdx) - 1 = 0 \tag{6}$$

$$\boxed{\tilde{\xi} = -i\alpha\sin(kdx) \pm \sqrt{1-\alpha^2\sin^2(kdx)}.} \tag{7}$$

We can then take the magnitude, $\left|\tilde{\xi}\right|^2 = \tilde{\xi}\tilde{\xi}^*$

$$\left|\tilde{\xi}\right|^2 = \alpha^2\sin^2(kdx) + 1 - \alpha^2\sin^2(kdx) \tag{8}$$

$$\left|\tilde{\xi}\right|^2 = 1 \tag{9}$$

$$\implies \boxed{|\xi| = 1.} \tag{10}$$

Then since the amplitude of our function (f) stays constant w.r.t. time and as the energy depends on our amplitude only (like in the kinetic case), then energy is conserved. □