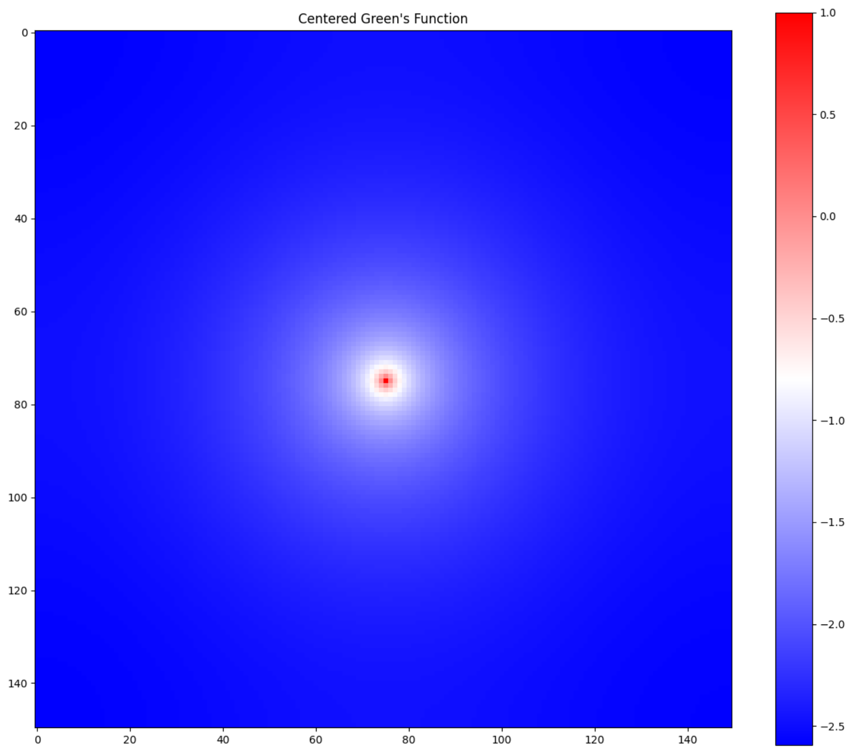
Problem 2

a)

The resulting Green's function leads to the following values.

V[1, 0]	$6.5729 \cdot 10^{-5}$
V[2, 0]	-0.4533
V[5, 0]	-1.0502

The two dimensional plot of the potential is included below.



b)

The crucial part of this problem is setting up the convolution as a matrix multiplication. This is relatively straight forward.

Let ρ be an N^2 flattened vector (instead of working with two dimensions) s.t.:

$$\vec{\rho} = (\rho[0,0], \rho[1,0], ..., \rho[0,1], \rho[1,1], ...). \tag{1}$$

Then, let $\boldsymbol{G}$ be an N^2 by N^2 matrix, s.t. each column contains a shifted version of the green's function g.

$$\boldsymbol{G} = \begin{pmatrix} \rho[0,0] & \rho[1,0] & ... & \rho[0,1] & \rho[1,1] & ... \\ \rho[N-1,0] & \rho[0,0] & ... & \rho[0,1] & \rho[1,1] & ... \\ & & ... & & & \\ \rho[N-1,0] & \rho[0,0] & ... & \rho[0,1] & \rho[1,1] & ... \\ \rho[N-2,0] & \rho[N-1,0] & ... & \rho[0,1] & \rho[1,1] & ... \\ & & ... & & & \\ \rho[0,N-1] & \rho[0,N-1] & ... & \rho[0,0] & \rho[1,0] & ... \\ & & ... & & & \end{pmatrix}. \tag{2}$$

Which is more clear written as:

$$\boldsymbol{G_{i,j}} = \rho[j_1 - i_1][j_2 - i_2], \tag{3}$$

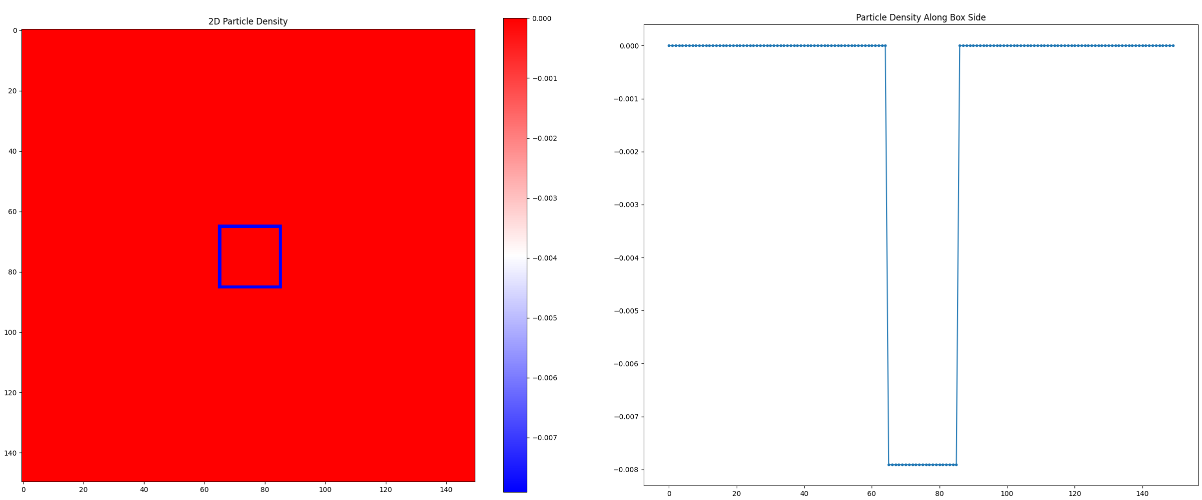
where $i = i_1 + Ni_2$ and $j = j_1 + Nj_2$.

This naturally translates into code (which I put here, because it is much easier to understand) as:

$$
\boldsymbol{G} = \begin{pmatrix} \textbf{np.roll}(\rho,\ \textbf{shift=}(0,0),\ \textbf{axis=}(0,1))\textbf{.ravel()} \\ \textbf{np.roll}(\rho,\ \textbf{shift=}(1,0),\ \textbf{axis=}(0,1))\textbf{.ravel()} \\ \ldots \\ \textbf{np.roll}(\rho,\ \textbf{shift=}(0,1),\ \textbf{axis=}(0,1))\textbf{.ravel()} \\ \ldots \\ \textbf{np.roll}(\rho,\ \textbf{shift=}(N-1,N-1),\ \textbf{axis=}(0,1))\textbf{.ravel()} \end{pmatrix}, \tag{4}
$$

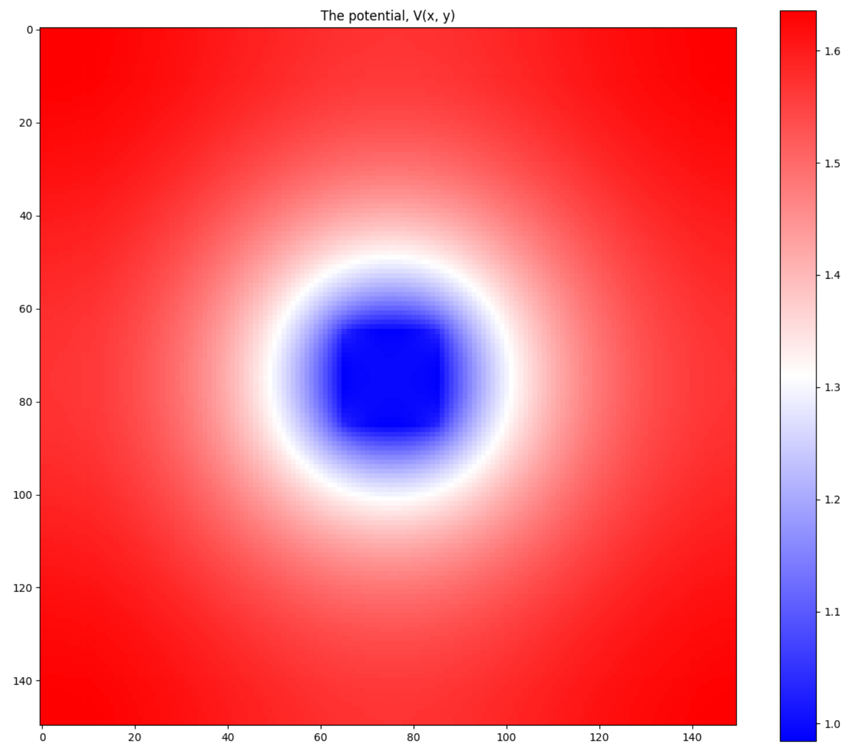
which corresponds to a convolution.

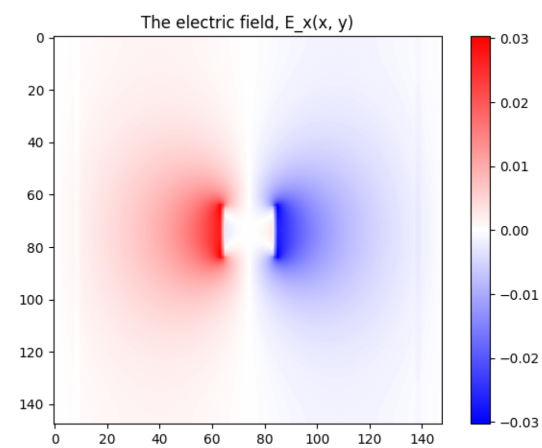
The resulting charge density is plotted below.

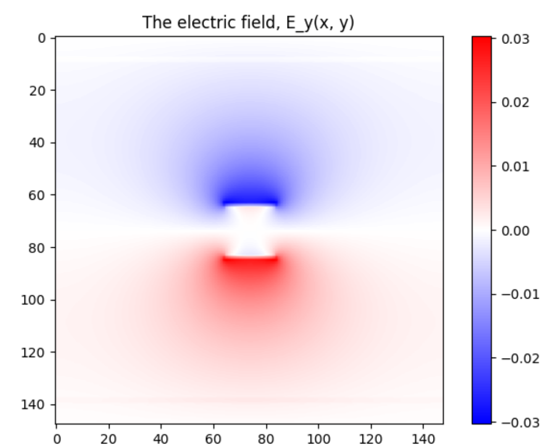


c)

Now it's quite trivial to obtain the potential everywhere in space, as well as the electric field. These are shown below and do indeed agree with I'd expect..







You can see below that the potential is roughly constant in the interior of the box (or close to it) and the electric field points perpendicular to the equipotential lines.