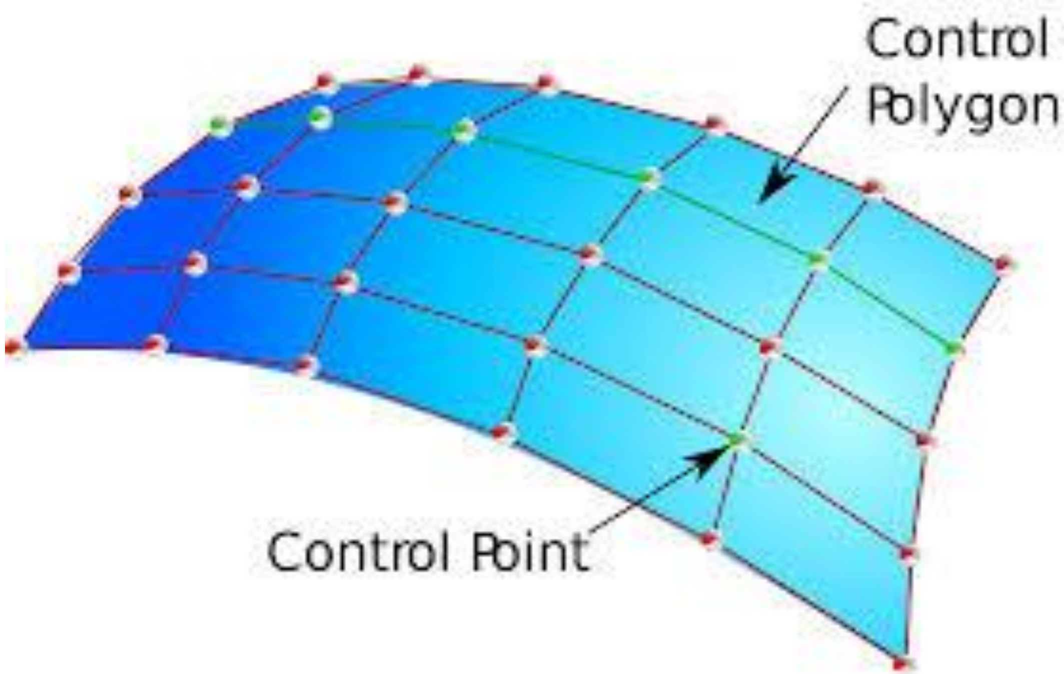
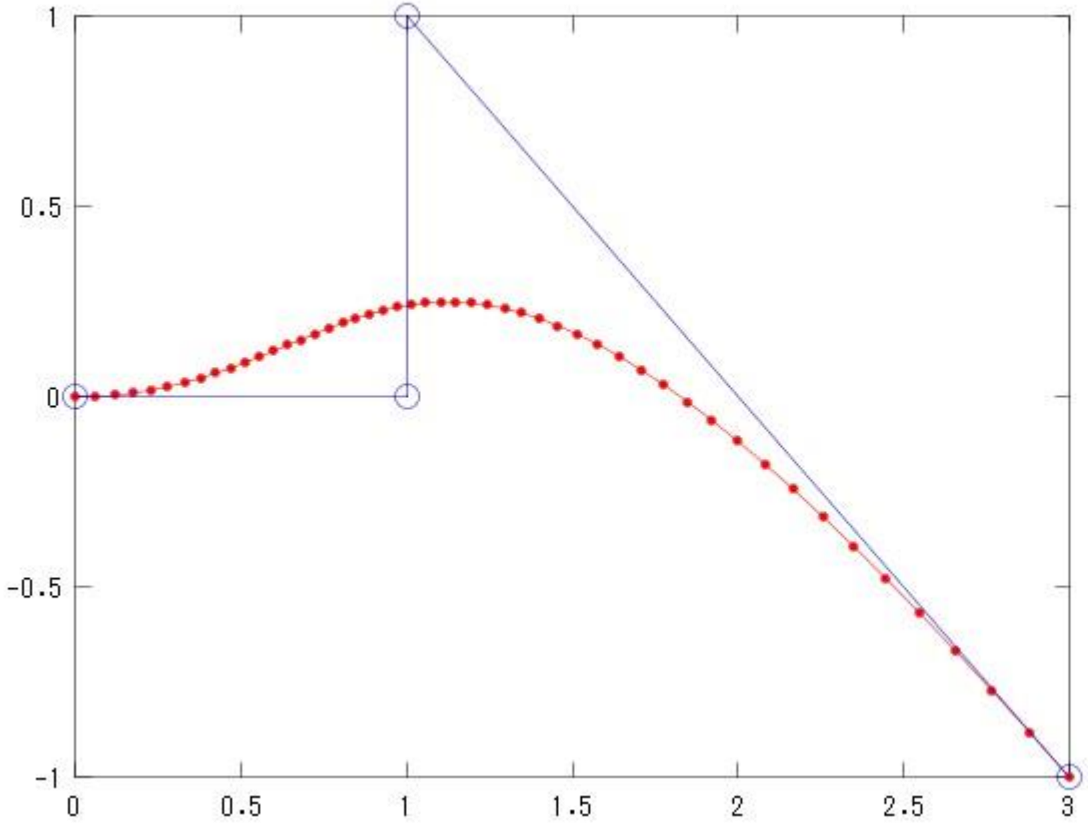
Curvas de Bézier

Introducción



Polígono de control y base

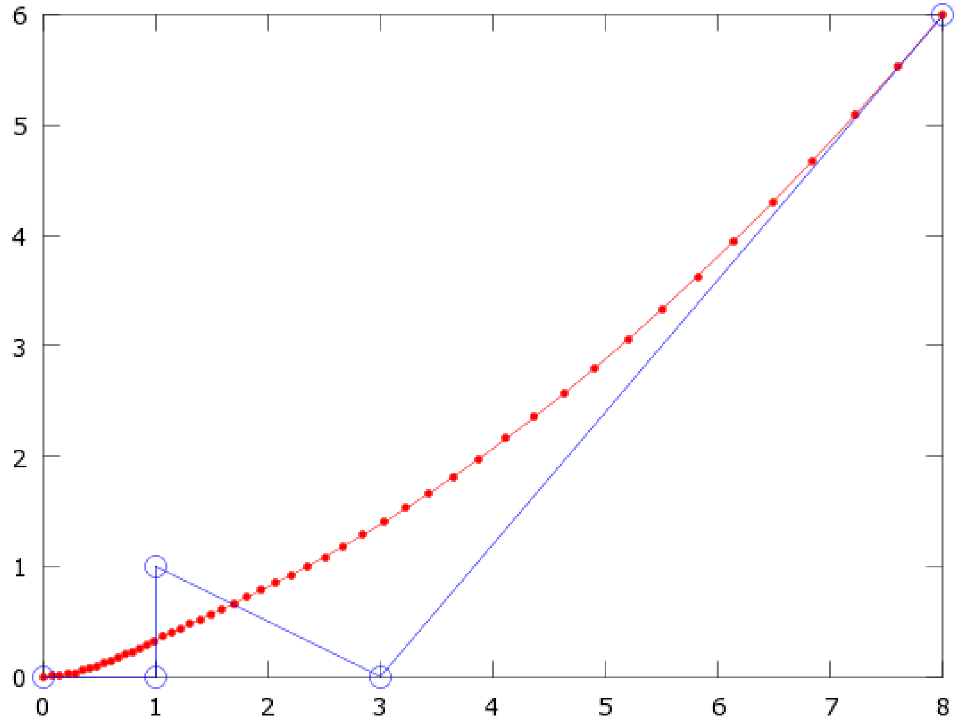


$$B_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i}$$

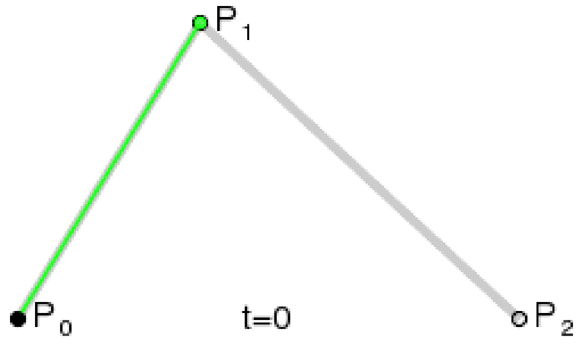
$$\{\mathrm{B}_0^{\mathrm{n}}(\mathrm{t}), \dots, \mathrm{B}_{\mathrm{n}}^{\mathrm{n}}(\mathrm{t})\}$$

Expresión de la curva

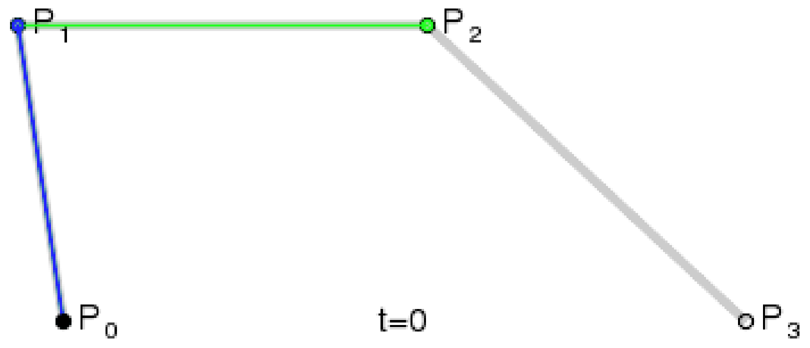
$$c(t) = \sum_{i=0}^{n} c_i \, B_i^n(t) \qquad t \in [0,1]$$



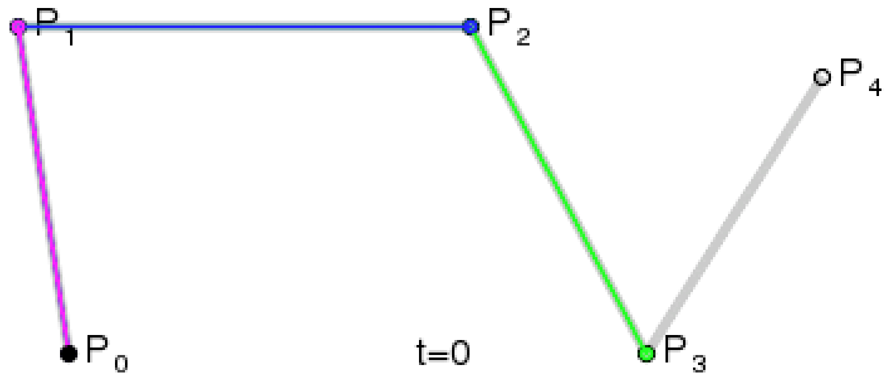
Curva cuadrática



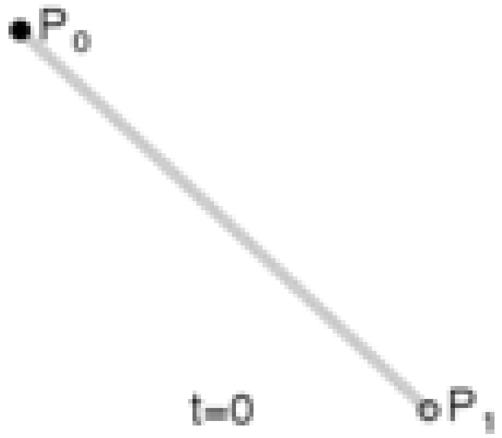
Curva cúbica



Curva de grado 4



Curva lineal

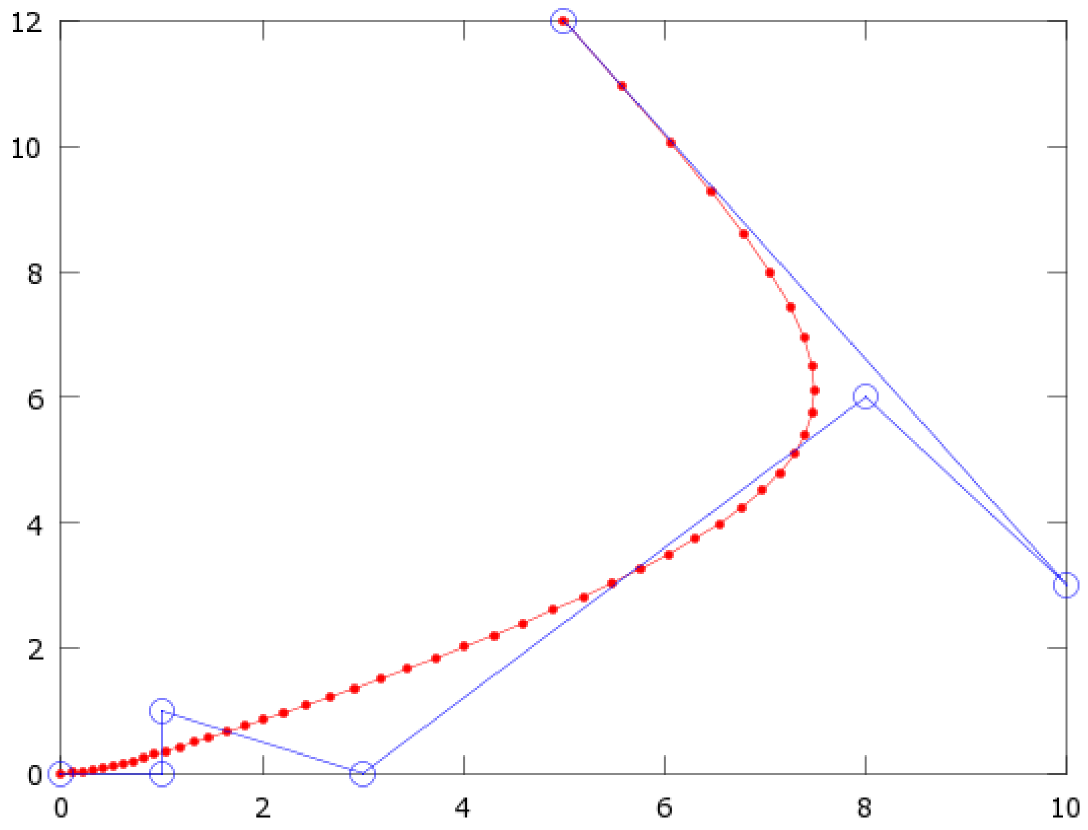


Propiedades

Interpolación en los extremos

$$c(0) = \sum_{i=0}^{n} c_i \, B_i^n(0) = c_0$$

$$c(1) = \sum_{i=0}^{n} c_i \, B_i^n(1) = c_n$$



Propiedades

Simetría

{c_0 , … , cn} {cn , cn_{-1} , … , c_0}

$$\binom{n}{i} = \frac{n!}{i!\,(n-i)!} = \binom{n}{n-i}$$

$$B_i^n(1-t) = B_{n-i}^n(t)$$

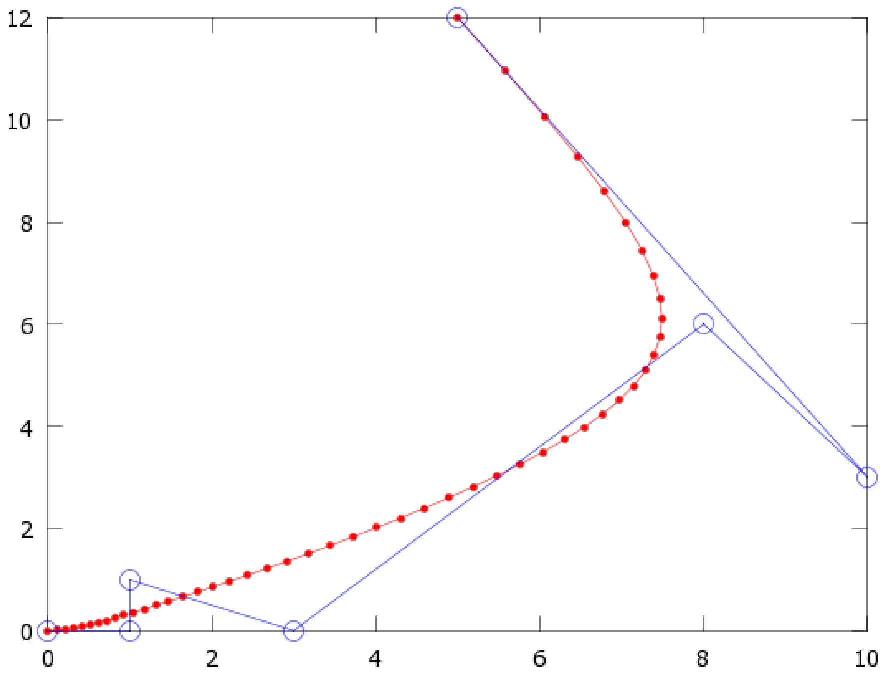
$$c(1-t) = \sum_{i=0}^{n} c_i\, B_i^n(1-t)$$

$$= \sum_{i=0}^{n} c_i B_{n-i}^n\,(t) = \sum_{j=0}^{n} c_{n-j} B_j^n(t)$$

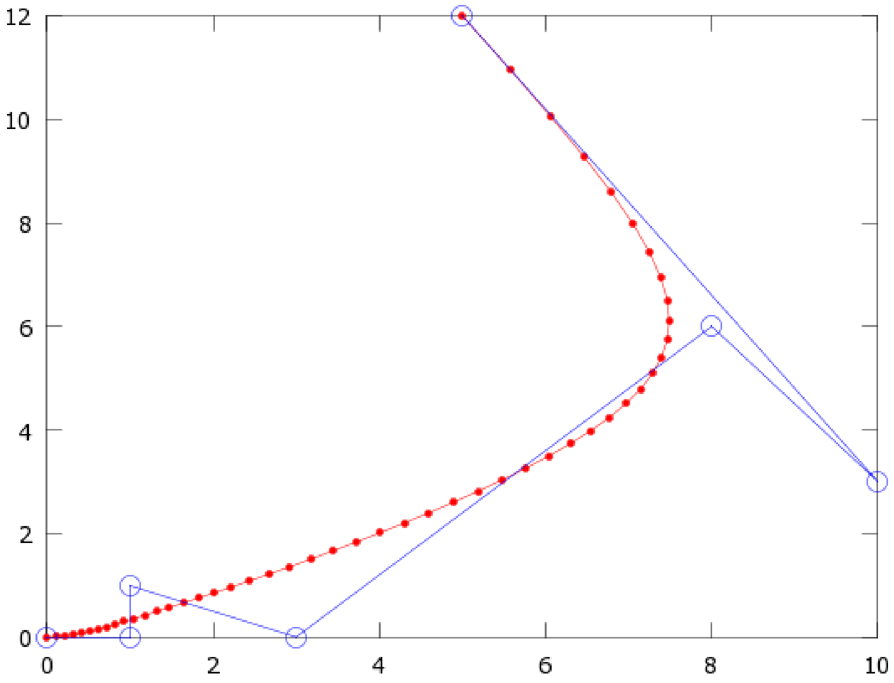
Propiedades

Simetría

$y = [0\ 0\ 1\ 0\ 6\ 3\ 12];$
$x = [0\ 1\ 1\ 3\ 8\ 10\ 5];$



$y = [12\ 3\ 6\ 0\ 1\ 0\ 0];$
$x = [5\ 10\ 8\ 3\ 1\ 1\ 0];$



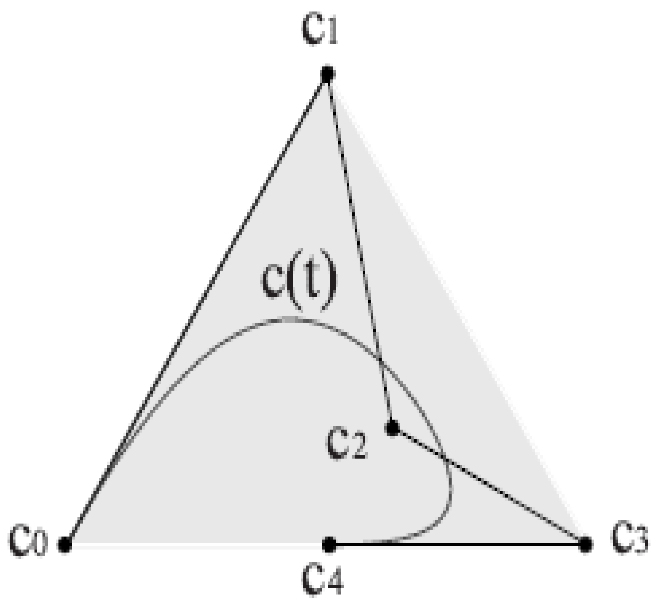
Propiedades

Invarianza afín

$$t(u) = \frac{u-a}{b-a} \qquad u \in [a,b] \qquad\qquad c^*(u) = c\big(t(u)\big) = c\left(\frac{u-a}{b-a}\right) \qquad u \in [a,b]$$

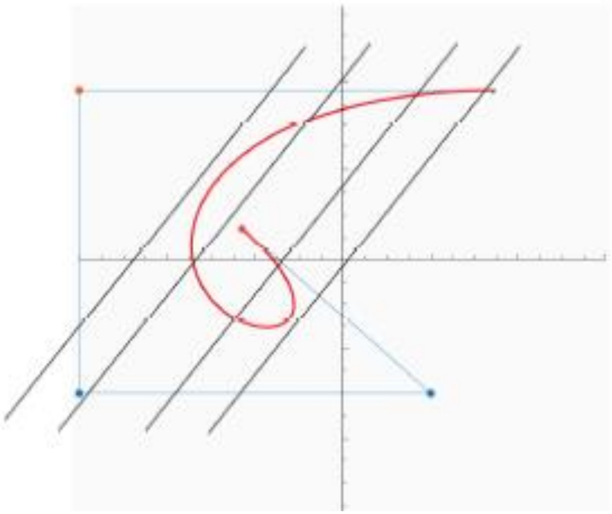
Propiedades

Envolvente convexa



Propiedades

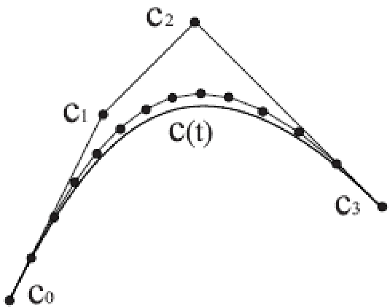
Disminución de la variación



$$c(t) = \sum_{i=0}^{n} c_i B_i^n(t)$$

$$1 = (1 - t + t)$$

$$\{c_0^1, \dots, c_{n+1}^1\}$$



Propiedades

Precisión lineal

$t = t(1-t+t)^{n-1}$ = $\sum_{i=1}^{n} c_i^n B_i^n(t)$

$c_i^n = \frac{i}{n}$

Propiedades

Control pseudo-local