1. Basic Statistics

1.1 What is Statistics?

Statistics is the study of data. It involves:

- **Collecting data** (e.g., survey results).
- **Analyzing data** (e.g., finding averages).
- **Summarizing data** (e.g., creating graphs).

1.2 Measures of Central Tendency

These measures show where the "center" of the data lies.

1. **Mean (Average)**:
   - Formula:

$$\text{Mean} = \frac{\text{Sum of all data points}}{\text{Total number of points}}$$

   - Example:
     - Data: 5, 10, 15.
     - Mean = $\frac{5+10+15}{3} = 10$.

2. **Median**:
   - Steps:
     1. Arrange data in ascending order.
     2. If the number of values is:
        - Odd: Take the middle value.
        - Even: Take the average of the two middle values.
   - Example:
     - Data: 3, 1, 4 → Arrange: 1, 3, 4.

- Median = 3.
- Data: 6, 2, 8, 4 → Arrange: 2, 4, 6, 8 → Median = $\frac{4+6}{2} = 5$.

3. **Mode**:

- Most frequently occurring value(s) in the data.
- Example:
  - Data: 2, 3, 3, 5 → Mode = 3.
  - Data: 1, 2, 2, 3, 3 → Modes = 2 and 3 (bimodal).

1.3 Measures of Spread

1. **Range**:

- Formula:

$$\text{Range} = \text{Maximum value} - \text{Minimum value}$$

- Example:
  - Data: 4, 8, 15, 16 → Range = $16 - 4 = 12$.

2. **Variance (σ^2)**:

- Measures how far data points are from the mean.
- Formula:

$$\sigma^2 = \frac{\sum(x - \mu)^2}{n}$$

- Steps:
  1. Find the mean (μ).
  2. Subtract the mean from each value and square the result.
  3. Take the average of these squared differences.
- Example:
  - Data: 2, 4, 6.

- Mean = 4.
- Differences: $(2-4)^2 = 4, (4-4)^2 = 0, (6-4)^2 = 4$.
- Variance = $\frac{4+0+4}{3} = 2.67$.

3. **Standard Deviation (σ)**:

   - Formula:

$$\sigma = \sqrt{\sigma^2}$$

   - Example:

     - Variance = 2.67 → Standard deviation = $\sqrt{2.67} \approx 1.63$.

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2. Probability of Events and Axioms

2.1 What is Probability?

Probability is the likelihood that an event will happen.

- Formula:

$$P(E) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}}$$

2.2 Key Terms

1. **Experiment**: An activity with uncertain outcomes (e.g., tossing a coin).

2. **Sample Space (S)**: All possible outcomes.

   - Example: Tossing a coin → $S = \{H, T\}$.

3. **Event**: A subset of the sample space.

   - Example: Getting heads → $E = \{H\}$.

2.3 Probability Rules

1. **Non-Negativity**:

   - Probability is always ≥ 0.

2. **Normalization**:

   - Probability of all outcomes = 1.

   - Example: Tossing a coin $\rightarrow P(H) + P(T) = 1$.

3. **Addition Rule**:

   - For mutually exclusive events:

$$P(A \cup B) = P(A) + P(B)$$

2.4 Example

- Roll a die. Find the probability of rolling:

  - A 4 $\rightarrow P(4) = \frac{1}{6}$.

  - An even number $\rightarrow P(2, 4, 6) = \frac{3}{6} = 0.5$.

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3. Conditional Probability and Bayes' Theorem

3.1 Conditional Probability

- Probability of A, given B has occurred.

- Formula:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

3.2 Example

- A bag contains 3 red and 2 green balls. One ball is drawn. What is $P(\text{red}|\text{ball is drawn})$?

  - Total outcomes = 5.
  - $P(\text{red}) = \frac{3}{5}$.

3.3 Bayes' Theorem

- Formula:

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

- Example:
  - Disease test:
    - $P(\text{disease}) = 0.01$,
    - $P(\text{positive}|\text{disease}) = 0.99$,
    - $P(\text{positive}|\text{no disease}) = 0.05$,
    - $P(\text{no disease}) = 0.99$.
  - $P(\text{disease}|\text{positive}) = \frac{0.99 \cdot 0.01}{(0.99 \cdot 0.01) + (0.05 \cdot 0.99)} \approx 0.167$.

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4. Random Variables (Single)

4.1 What is a Random Variable?

- A random variable assigns a numerical value to each outcome in a sample space.

4.2 Types

1. **Discrete**:

- Takes specific values (e.g., 0, 1, 2).

- Example: Number of heads in 3 coin tosses.

2. **Continuous**:

- Takes any value within a range (e.g., height, time).

4.3 Probability Distribution

1. **Discrete**:

- Probability Mass Function (PMF): $P(X = x)$.

- Example: Rolling a die $\rightarrow P(X = 3) = \frac{1}{6}$.

2. **Continuous**:

- Probability Density Function (PDF): $f(x)$.

- Total area under the curve = 1.

4.4 Expected Value

- Formula:

$$E[X] = \sum x \cdot P(x) \quad \text{(Discrete)}$$

- Example: Tossing a coin 3 times:

  - $P(X = 0) = 0.125, P(X = 1) = 0.375$, etc.

  - $E[X] = 0 \cdot 0.125 + 1 \cdot 0.375 + \cdots = 1.5$.

4.5 Variance

- Formula:

$$\text{Var}(X) = E[X^2] - (E[X])^2$$

- Example: Die roll:

  - $E[X] = 3.5, E[X^2] = 15.17$.

  - $\text{Var}(X) = 15.17 - (3.5)^2 = 2.92$.

Summary Table

Topic	Key Concept	Example
Basic Statistics	Mean, Median, Mode	Data: [5, 10, 15] → Mean = 10
Probability	Events, Rules, Addition/Multiplication	Rolling a die, $P(4) = 1/6$
Conditional Probability	(P(A	B) = P(A \cap B)/P(B))
Random Variables	PMF, Expected Value, Variance	Tossing 3 coins: $E[X] = 1.5$