# Comprehensive Guide: ZC416, Lecture 0 (46 Slides)

This detailed article breaks down the content from Lecture 0 (46 slides) into easy-to-understand steps, with examples, solutions, and explanations for all key concepts. The goal is to ensure that even a beginner can grasp the fundamental mathematical principles required for machine learning.

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## 1. Introduction to Matrices and Linear Systems

A matrix is a rectangular array of numbers or elements arranged in rows and columns. It is a fundamental tool in linear algebra, widely used in machine learning for data representation and transformations.

**Example**:

Matrix $A$:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

- Rows: $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}, \begin{bmatrix} 4 & 5 & 6 \end{bmatrix}$
- Columns: $\begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \end{bmatrix}$

Matrices represent linear transformations in machine learning models.

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## 2. Types of Matrices

1. **Square Matrix**: Rows = Columns. Example: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
2. **Diagonal Matrix**: Non-diagonal elements are zero. Example: $\begin{bmatrix} 5 & 0 \\ 0 & 8 \end{bmatrix}$
3. **Identity Matrix**: Diagonal elements are 1. Example: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

**Application**: Identity matrices act as the "multiplicative identity" in transformations.

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## 3. Determinants

The determinant helps identify properties of a matrix, such as invertibility. For a $2 \times 2$ matrix:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad \det(A) = ad - bc$$

**Example**:

$$A = \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix}, \quad \det(A) = (3)(5) - (4)(2) = 15 - 8 = 7$$

Since $\det(A) \neq 0$, $A$ is invertible.