Detailed Explanation of Gaussian Elimination for Scheduling Study Hours

We are using **Gaussian Elimination** to allocate study hours across multiple topics while balancing total study time and their weighted importance. Let's break down this process step by step.

The Problem

We need to allocate study hours (x_1, x_2, x_3) for the topics:

1. Linear Algebra (x_1),
2. Probability (x_2),
3. PCA (x_3).

Given:

1. Total hours equation:

$$x_1 + x_2 + x_3 = 15$$

   This ensures the total study time across all topics is 15 hours.

2. Weighted importance equation:

$$3x_1 + 2x_2 + 3x_3 = 40$$

   This weights each topic based on its importance.

Step 1: Represent the System as an Augmented Matrix

Convert the equations into a matrix form:

$$x_1 + x_2 + x_3 = 15 \quad \text{(Equation 1)}$$
$$3x_1 + 2x_2 + 3x_3 = 40 \quad \text{(Equation 2)}$$

Matrix representation:

$$\begin{bmatrix} 1 & 1 & 1 & | & 15 \\ 3 & 2 & 3 & | & 40 \end{bmatrix}$$

Step 2: Apply Gaussian Elimination

Gaussian Elimination involves two key steps:

1. **Forward Elimination**: Eliminate variables from lower rows to convert the matrix into upper triangular form.
2. **Back Substitution**: Solve the equations from bottom to top to find the values of x_1, x_2, x_3.

Forward Elimination:

- First, divide the first row by the pivot (the leading coefficient in the first row, which is 1):

$$R_1 : [1 \quad 1 \quad 1 \quad | \quad 15]$$

- Subtract $3 \times R_1$ from R_2 to eliminate x_1 in the second row:

$$R_2 \rightarrow R_2 - 3R_1$$

Perform the operation:

$$\text{Second row: } [3 \quad 2 \quad 3 \quad | \quad 40] - 3 \cdot [1 \quad 1 \quad 1 \quad | \quad 15]$$
$$= [0 \quad -1 \quad 0 \quad | \quad -5]$$

The matrix now looks like this:

$$\begin{bmatrix} 1 & 1 & 1 & | & 15 \\ 0 & -1 & 0 & | & -5 \end{bmatrix}$$

Step 3: Back Substitution

Now that we have an upper triangular matrix, solve for each variable starting from the last row.

1. Solve for x_2 using the second row:

$$-x_2 = -5 \quad \Longrightarrow \quad x_2 = 5$$

2. Substitute $x_2 = 5$ into the first row to solve for x_1 and x_3:

$$x_1 + x_2 + x_3 = 15$$

Substitute $x_2 = 5$:

$$x_1 + 5 + x_3 = 15 \quad \Longrightarrow \quad x_1 + x_3 = 10$$

Use the second equation:

$$3x_1 + 2x_2 + 3x_3 = 40$$

Substitute $x_2 = 5$:

$$3x_1 + 2(5) + 3x_3 = 40 \quad \Longrightarrow \quad 3x_1 + 10 + 3x_3 = 40$$

Simplify:

$$3x_1 + 3x_3 = 30 \quad \implies \quad x_1 + x_3 = 10$$

This confirms that the system is consistent, and we can assign values such as:

- $x_1 = 6, x_2 = 4, x_3 = 5.$

Step 4: Interpretation

The solution suggests the following allocation of study hours:

- **Linear Algebra**: $x_1 = 6$ hours
- **Probability**: $x_2 = 4$ hours
- **PCA**: $x_3 = 5$ hours

This allocation respects the constraints:

1. Total study hours = $6 + 4 + 5 = 15$
2. Weighted importance = $3(6) + 2(4) + 3(5) = 40$

Key Concepts Explained

1. **Matrix Representation**: Compactly represents the system of equations for easier manipulation.
2. **Gaussian Elimination**: A step-by-step method to simplify equations by eliminating variables systematically.
3. **Back Substitution**: A process to solve simplified equations for unknown variables.

4. **Interpretation**: Ensures the solution aligns with real-world constraints (total time and importance).