Task 2: Is Rectangle?

Given four positive integers A, B, C, D, determine if there's a rectangle such that the lengths of its sides are A, B, C and D (in any order). If any such rectangle exist return 1 else return 0. def is_rectangle (int:A, int:B, int:C, int:D): A : integer value that is one side of the rectangle. B : integer value that is one side of the rectangle.C : integer value that is one side of the rectangle. D : integer value that is one side of the rectangle. Return if is a rectangle return 1 else 0.

Approach 1

The simple approach is generate all possible combination of 2 pairs of sides and check if each pair contains numbers that are equal to each other. Now let's consider that we have a quantum circuit to compare 2 numbers, then we'll follow following steps: 1) Generate all possible combinations of sides. 2) For each pairs of sides: i) Convert both numbers in pair to binary representation ii) Prefix number with less number of bits with 0 to make both numbers of same lengths in binary. iii) Intialize qbits states for both numbers according to binary representation. iv) Run the quantum circuit to generate result if 2 numbers are equal. 3) If both pairs have equal length sides, the combination can possibly generate a rectangle

Binary Representation

Let's start with generating binary representation of the number.

```
def convertToBinStr(num: int) -> str:
    """
    Generate binary string representation for decimal number.
    Note:- Not handling the negative numbers as the length of
    side can't be negative.
    """
    ans = ''
    if num == 0:
        return '0'
    while num:
        ans = str(num % 2) + ans
        num //= 2
    return ans

# Let's try out 3-4 numbers
# bin() converts decimal to binary in python
assert "0b"+format(convertToBinStr(8)) == bin(8), "Both
representations are not equal"
assert "0b"+format(convertToBinStr(15)) == bin(15), "Both
representations are not equal"
assert "0b"+format(convertToBinStr(894)) == bin(894), "Both
```

```
representations are not equal"
assert "0b"+format(convertToBinStr(0)) == bin(0), "Both
representations are not equal"
```

Make both binary representations of same length

Binary representations of both numbers might not be of the same length. So , we'll prefix the smaller string with zeros

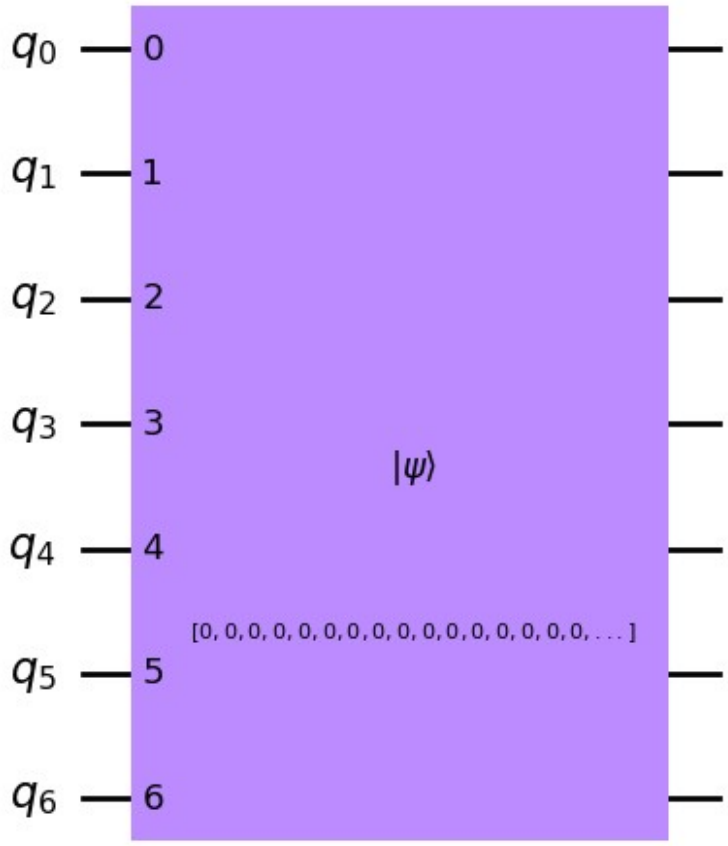
```
from typing import Tuple

def equalizeLength(str1: str, str2: str) -> Tuple[str, str]:
    if len(str2) > len(str1):
        for _ in range(len(str1), len(str2)):
            str1 = '0' + str1
    elif len(str1) > len(str2):
        for _ in range(len(str2), len(str1)):
            str2 = '0' + str2
    return (str1, str2)

# Lets try out few examples here.
str1, str2 = equalizeLength(convertToBinStr(8), convertToBinStr(80))
print(str1, str2, len(str1) == len(str2))

str1, str2 = equalizeLength(convertToBinStr(15), convertToBinStr(13))
print(str1, str2, len(str1) == len(str2))

str1, str2 = equalizeLength(convertToBinStr(0), convertToBinStr(1387))
print(str1, str2, len(str1) == len(str2))

str1, str2 = equalizeLength(convertToBinStr(235),
convertToBinStr(111003))
print(str1, str2, len(str1) == len(str2))
```

```
0001000 1010000 True
1111 1101 True
00000000000 10101101011 True
00000000011101011 11011000110011011 True
```

Initialize Qbits according to binary representation

We'll use qiskit's Statevector.from_label to initilize qbits from string.

```
from qiskit import QuantumCircuit, execute, BasicAer
from qiskit.quantum_info import Statevector

str1 = '1010000'
n = len(str1)
qc = QuantumCircuit(n)
qc.initialize(Statevector.from_label(str1), range(n))
qc.draw('mpl')
```

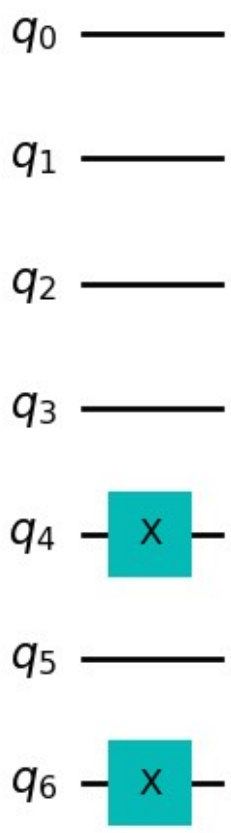



```
qc.measure_all()
execute(qc,
backend=BasicAer.get_backend('qasm_simulator')).result().get_counts()
```

```
{'1010000': 1024}
```

We can achieve the same result using below function

```
def init(bit_str: str) -> QuantumCircuit:
    n_qbits = len(bit_str)
    qc = QuantumCircuit(n_qbits)
    for i, chr in enumerate(bit_str):
        if chr == '1':
            qc.x(n_qbits-i-1)
    return qc

str1 = '1010000'
qc = init(str1)
qc.draw('mpl')
```




```
qc.measure_all()
execute(qc,
backend=BasicAer.get_backend('qasm_simulator')).result().get_counts()
```

```
{'1010000': 1024}
```

Quantum circuit

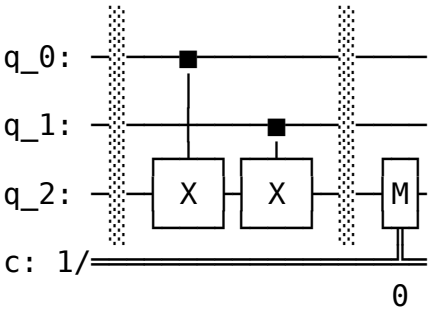
We want to check if 2 bits are same. In classical realm, we have XOR gate for this purpose. The truth table for XOR gate is as follows:

Bit1	Bit2	Result
0	0	0
0	1	1
1	0	1
1	1	0

In Quantum world, we can generate same results using 2 Qbits and 1 ancilla Qbit to read the results. And using 2 CNOT gates. Let's create this circuit.

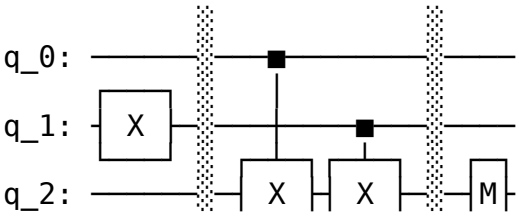
```
def xor(bit1, bit2):
    qc = QuantumCircuit(3, 1)
    if bit1 == 1:
        qc.x(0)
    if bit2 == 1:
        qc.x(1)
    qc.barrier()
    qc.cx(0, 2)
    qc.cx(1, 2)
    qc.barrier()
    qc.measure(2, 0)
    return qc

bits = [[0, 0], [0, 1], [1, 0], [1, 1]]

for pair in bits:
    circ = xor(*pair)
    print("Circuit:\n")
    print(circ)
    counts = execute(circ,
backend=BasicAer.get_backend('qasm_simulator')).result().get_counts()
    print("Bit 1: {}, Bit 2: {}, Result: {}".format(pair[0], pair[1],
counts))
    print("\n\n\n\n")
```

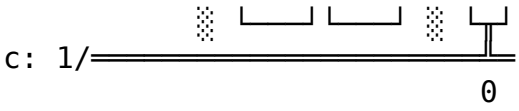
```
Circuit:
```




```
Bit 1: 0, Bit 2: 0, Result: {'0': 1024}
```

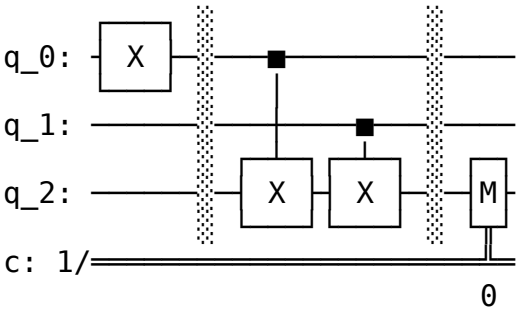
```
Circuit:
```





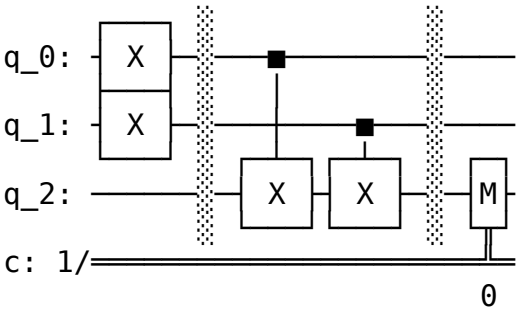

```
Bit 1: 0, Bit 2: 1, Result: {'1': 1024}
```

```
Circuit:
```




```
Bit 1: 1, Bit 2: 0, Result: {'1': 1024}
```

```
Circuit:
```




```
Bit 1: 1, Bit 2: 1, Result: {'0': 1024}
```

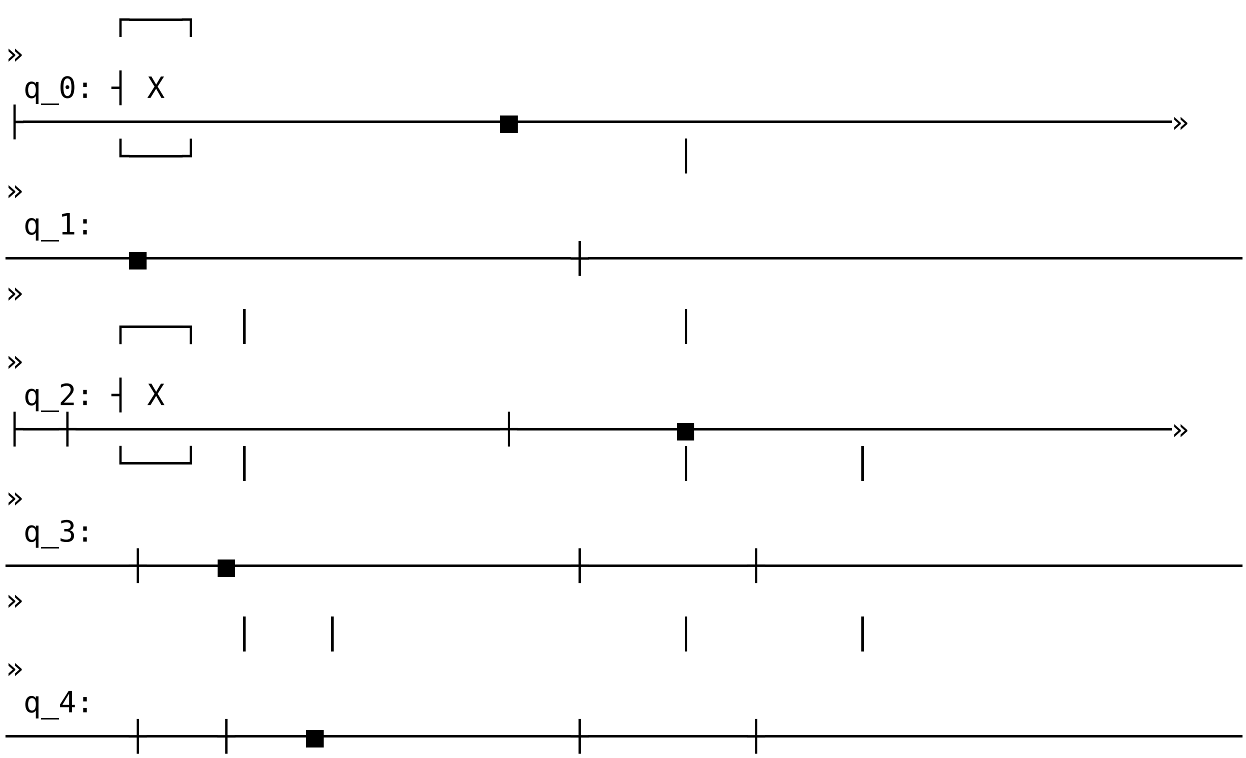
Extending circuit for n-bit number

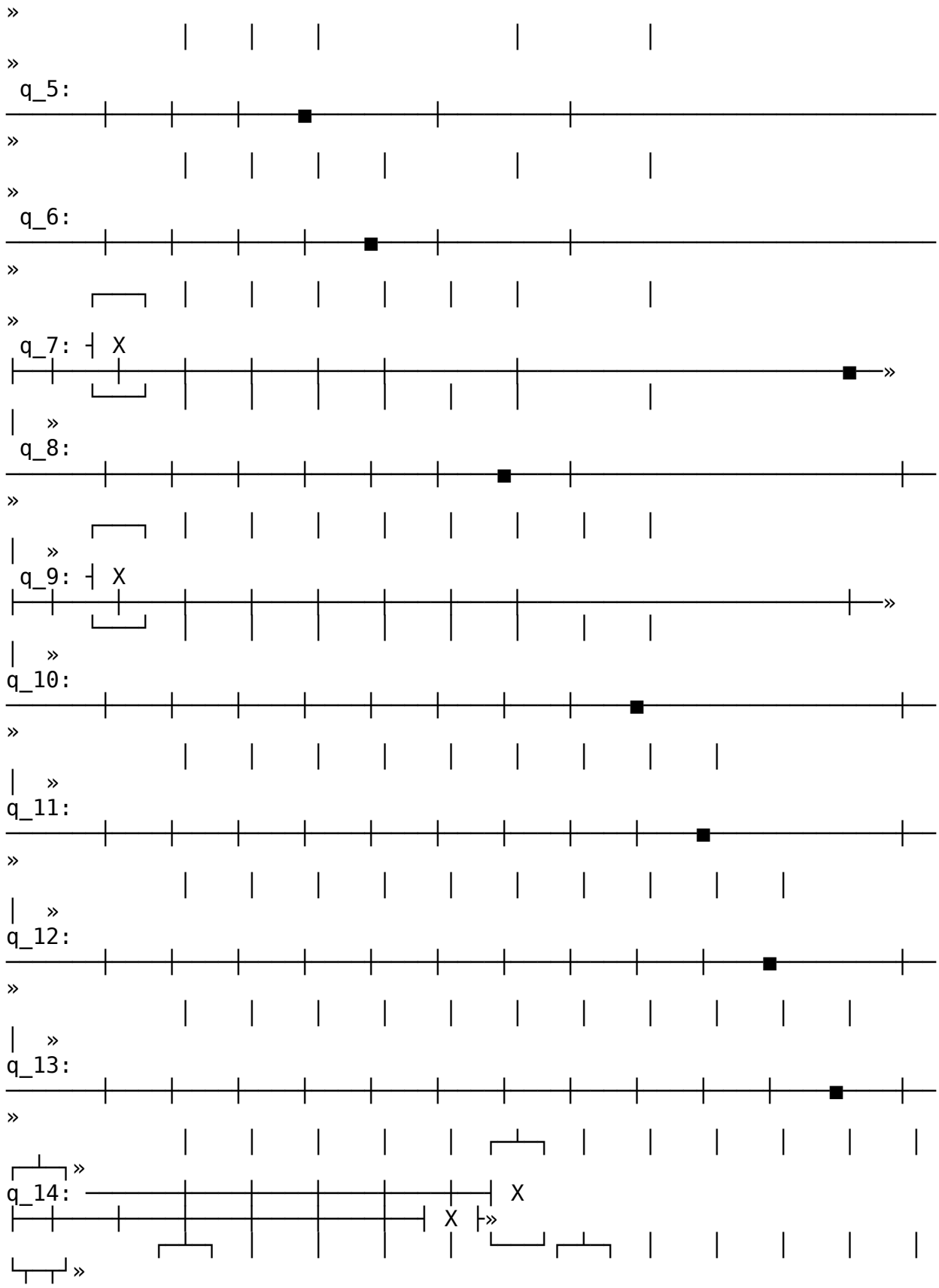
We'll repeat the above circuit for n-bits. For 2 n-bit numbers, we'll require n qbits each , so total of 2n qbits to represent 2 bits. And we'll require n ancilla qbits to read the results. So

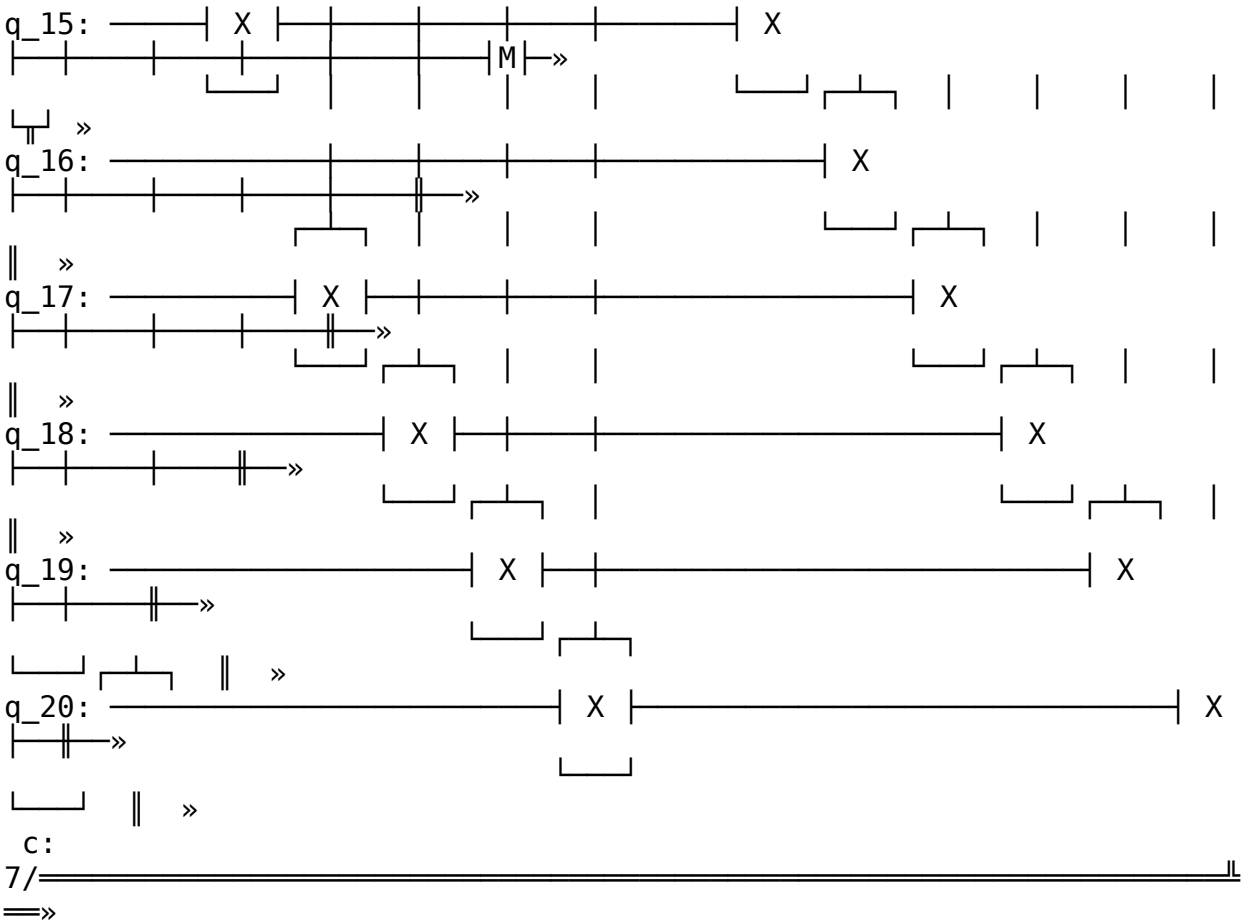
we'll need 3n qbits and n classical registers to read the results We'll check if the result is 000..(n times) so that both the numbers match

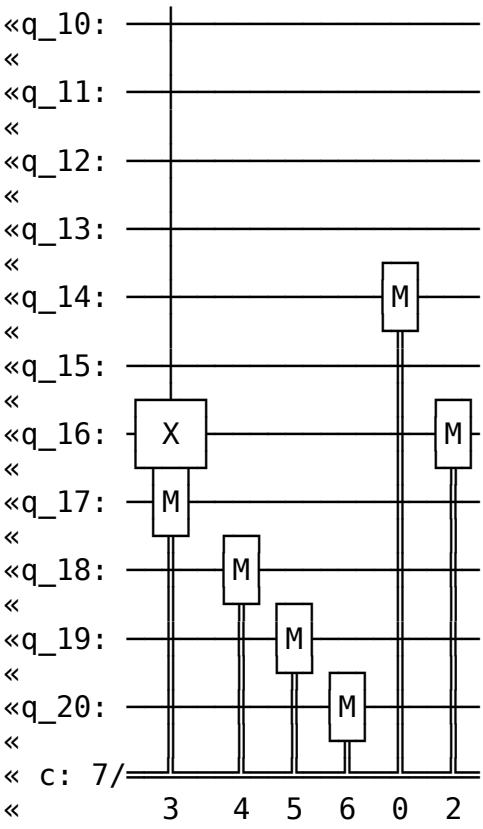
```python
def compareNumbers(str1: str, str2: str) -> QuantumCircuit:
    length = len(str1)
    qc = QuantumCircuit(3 * length, length)
    for i, (bit1, bit2) in enumerate(zip(str1, str2)):
        if bit1 == '1':
            qc.x(i)
        if bit2 == '1':
            qc.x(length + i)
        qc.cx(i, 2*length+i)
        qc.cx(length+i, 2*length+i)
        qc.measure(2*length+i, i)
    return qc

# Let's try out this circuit
str1 = '1010000'
str2 = '1010000'
qc = compareNumbers(str1, str2)
print("Circuit: \n")
print(qc)
print(f"\n Let's run the circuit, {str1} and {str2} should be equal")
counts = execute(qc,
backend=BasicAer.get_backend('qasm_simulator')).result().get_counts()
print(counts)
```

```
Circuit:
```




q_5:
q_6:
q_7:
X
q_8:
q_9:
X
q_10:
q_11:
q_12:
q_13:
q_14:
X
X

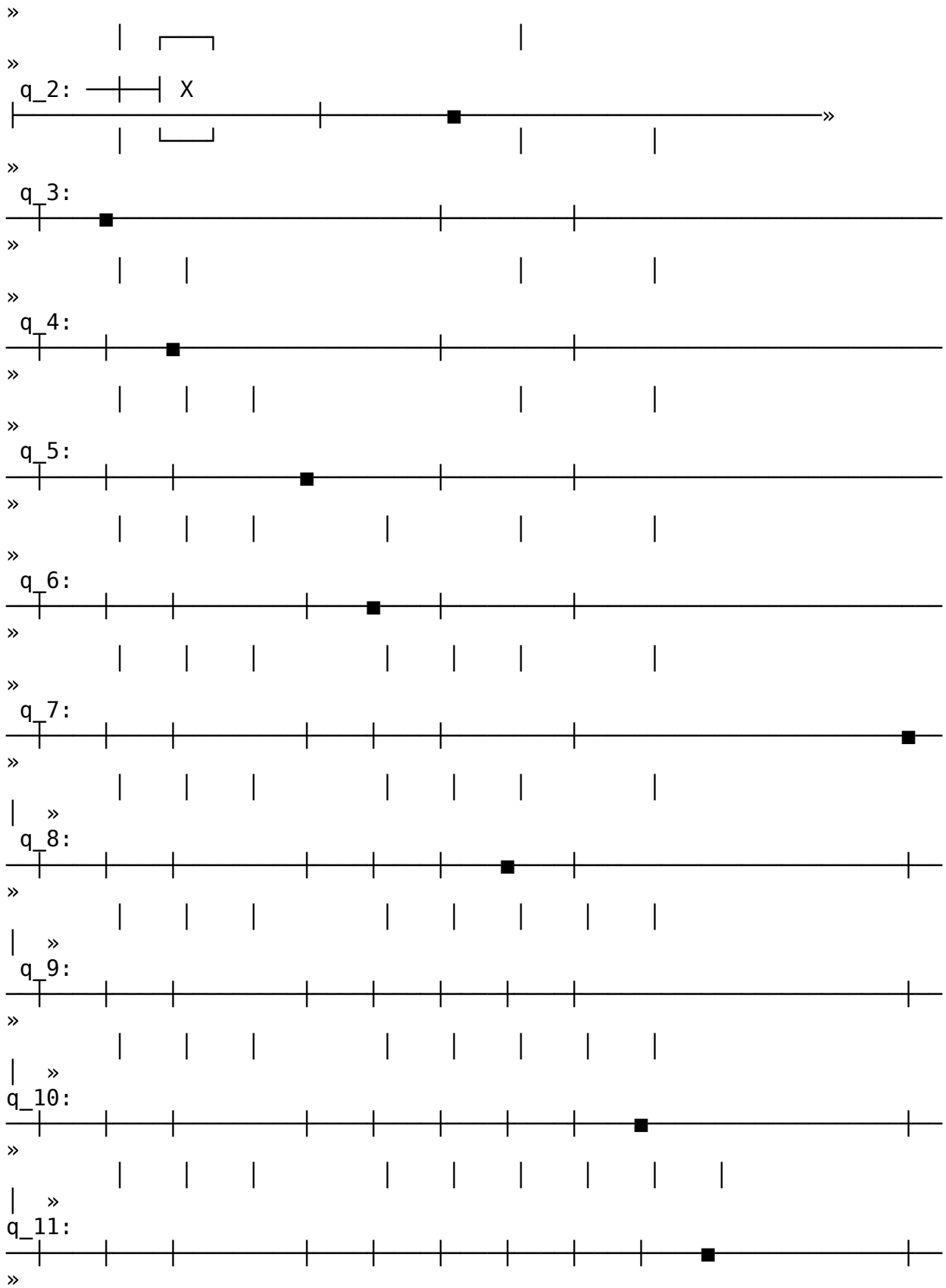

q_15:
M
q_16:
q_17:
q_18:
q_19:
q_20:
c:
7/
1
q_0:
q_1:
q_2:
q_3:
q_4:
q_5:
q_6:
q_7:
q_8:
q_9:

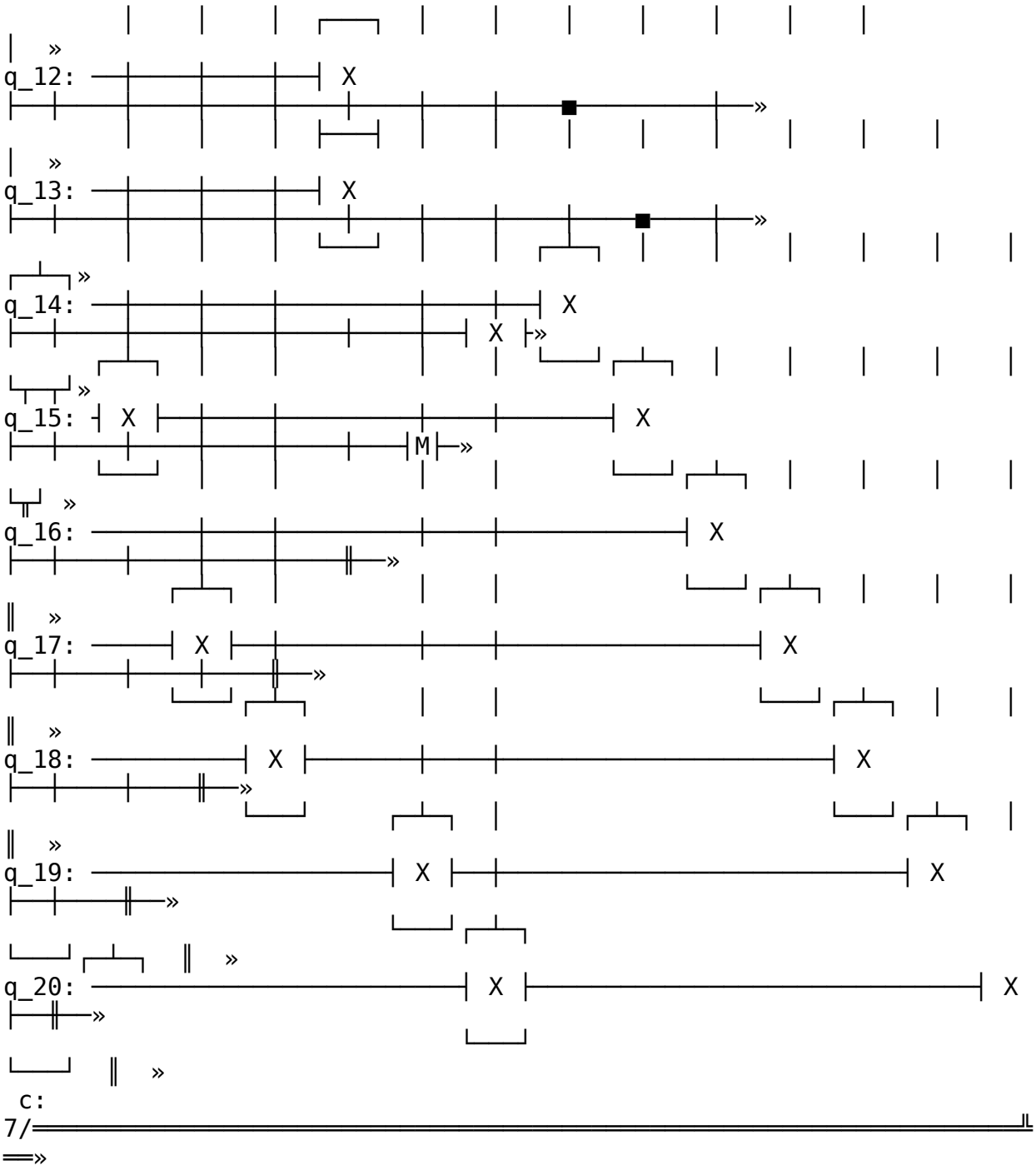


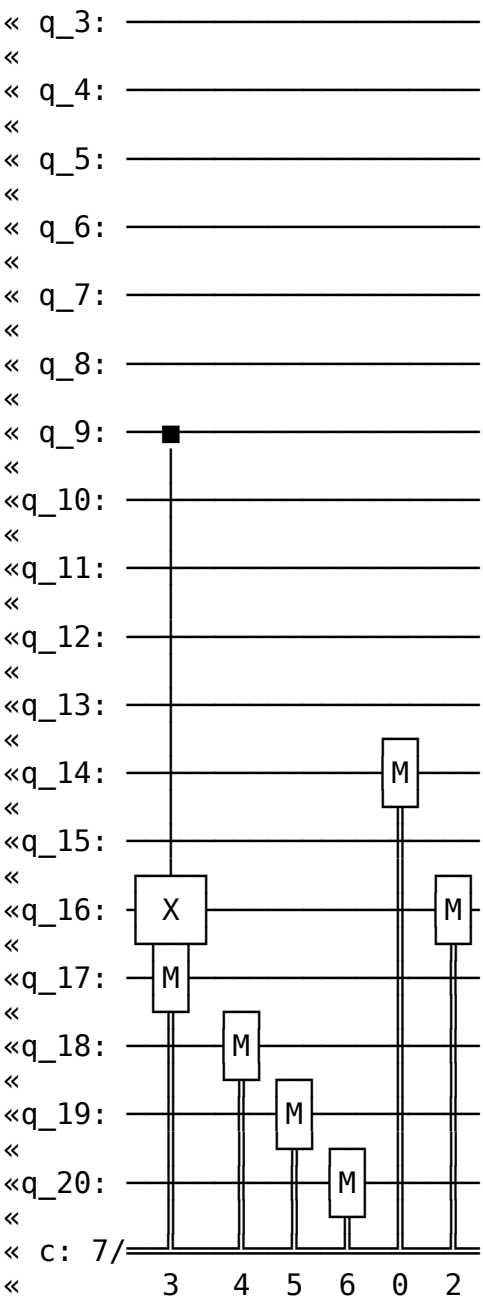

```
 Let's run the circuit, 1010000 and 1010000 should be equal
{'0000000': 1024}
```

```
# Let's try out this circuit
str1 = '1010000'
str2 = '0000011'
qc = compareNumbers(str1, str2)
print("Circuit: \n")
print(qc)
print(f"\n Let's run the circuit, {str1} and {str2} should not be
equal")
counts = execute(qc,
backend=BasicAer.get_backend('qasm_simulator')).result().get_counts()
print(counts)
```

```
Circuit:

      ┌───┐
»
 q_0: ┤ X
      └───┘
»
 q_1:
```


q_2:
X
q_3:
q_4:
q_5:
q_6:
q_7:
q_8:
q_9:
q_10:
q_11:


q_12:
q_13:
q_14:
q_15:
q_16:
q_17:
q_18:
q_19:
q_20:
X
M
c:
7/
1
q_0:
q_1:
q_2:




```
 Let's run the circuit, 1010000 and 0000011 should not be equal
{'1100101': 1024}
```

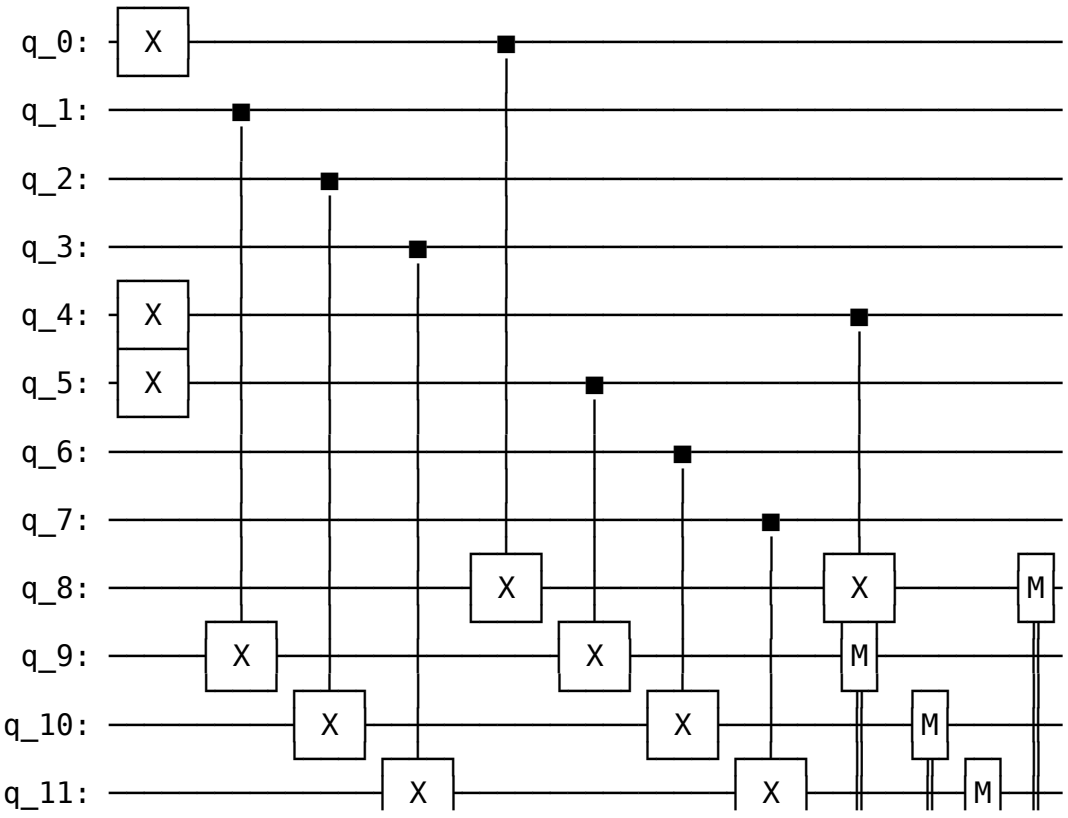
Putting everything together

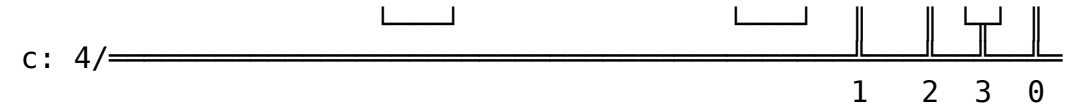
Now let's put everything together: With lengths of 4 sides A, B, C, D we have 3 combinations to check: 1) A-B and c-D 2) A-C and B-D 3) A-D and B-C For each combination, we'll run the quantum circuit above and check if both pairs are equal. If its true for even for one combination, we say that this combination of lengths can possibly form a rectangle

```
from typing import List

from itertools import combinations

def yieldAllPairs(lst: List[int]) -> List[List[int]]:
    if lst == []:
        yield []
        return
    ll = lst[1:]
    for j in range(len(ll)):
        for end in yieldAllPairs(ll[:j] + ll[j+1:]):
            yield [(lst[0], ll[j])] + end

def getCircuit(str1: str, str2: str) -> QuantumCircuit:
    length = len(str1)
    print(str1, str2)
    qc = QuantumCircuit(3 * length, length)
    for i, (bit1, bit2) in enumerate(zip(str1, str2)):
        if bit1 == '1':
            qc.x(i)
        if bit2 == '1':
            qc.x(length + i)
        qc.cx(i, 2*length+i)
        qc.cx(length+i, 2*length+i)
        qc.measure(2*length+i, i)
    return qc

def compareNumbers(num1: int, num2: int) -> bool:
    # Make each number of same length
    str1, str2 = equalizeLength(num1, num2)

    # Generate circuit to compare numbers
    circ = getCircuit(str1, str2)
    print(circ)

    # Answer should be 000..(n times)
    # Lets get expected answer ready to
    # compare with circuit output.
    actual_ans = '0' * len(str1)

    # Run the circuit and generate counts.
    generated_ans = execute(circ,
backend=BasicAer.get_backend('qasm_simulator')).result().get_counts()
    print(generated_ans)

    # Generated answer should have only 1 output with 100% probably
    # i.e. all 1024 shots should have same results.
    # And that result should be 000...(n times)
    if len(list(generated_ans.keys())) == 1 and
generated_ans[list(generated_ans.keys())[0]] == 1024:
        if list(generated_ans.keys())[0] == actual_ans:
```

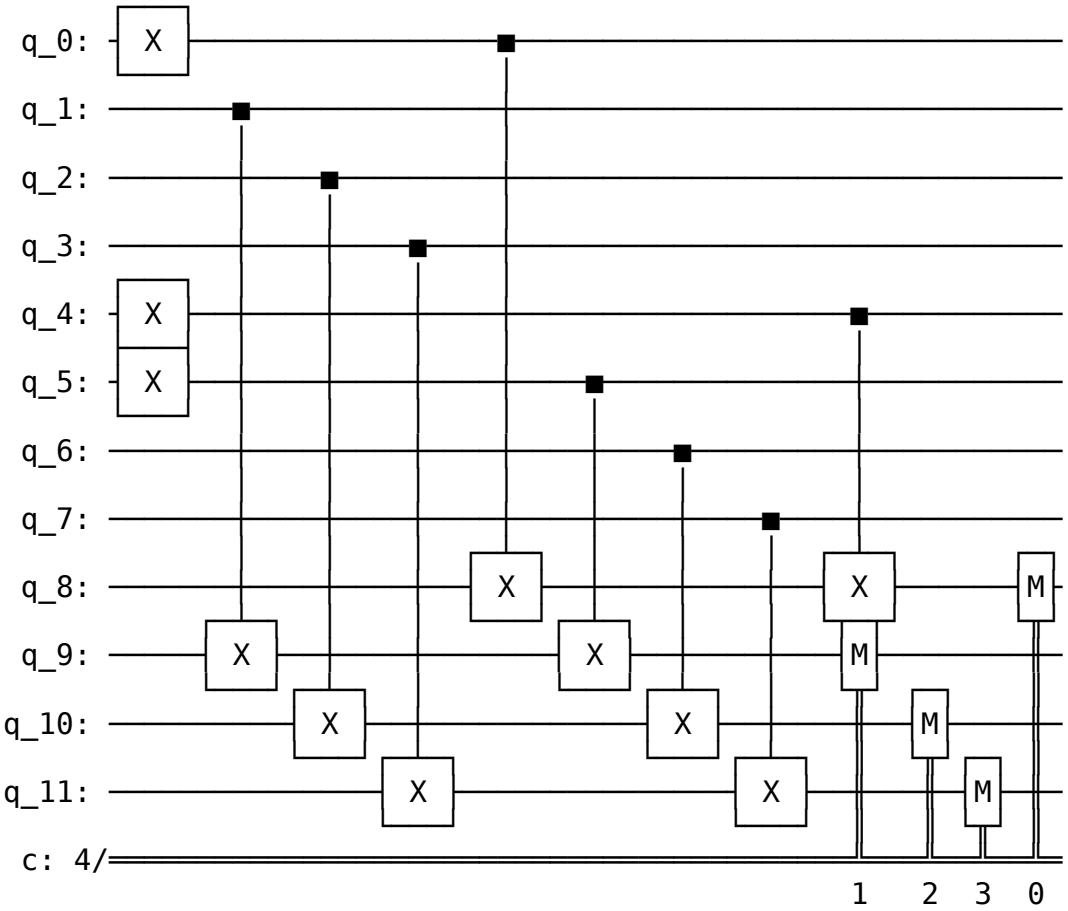
```
                return True

        return False

def isRectangle(lengths: List[int]) -> bool:
    assert all(i > 0 for i in lengths), "Length of side must be > 0."

    # Now convert all the numbers to their binary representation.
    bin_lengths = [convertToBinStr(num) for num in lengths]

    # Let's generate all possible combination of pairs
    pairs_gen = yieldAllPairs(bin_lengths)

    # For each combination, check if both pairs have
    # equal numbers.
    for pairs in pairs_gen:
        if compareNumbers(*pairs[0]) and compareNumbers(*pairs[1]):
            return True

    return False

# Let's test the code with few examples
lengths = [8, 12, 12, 8]

isRectangle(lengths)
```

```
1000 1100
```

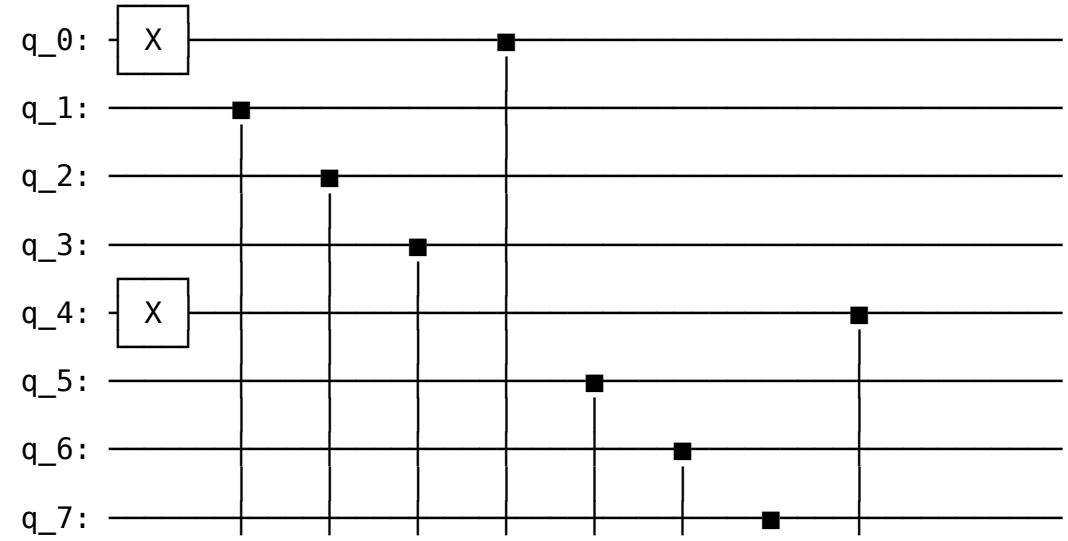


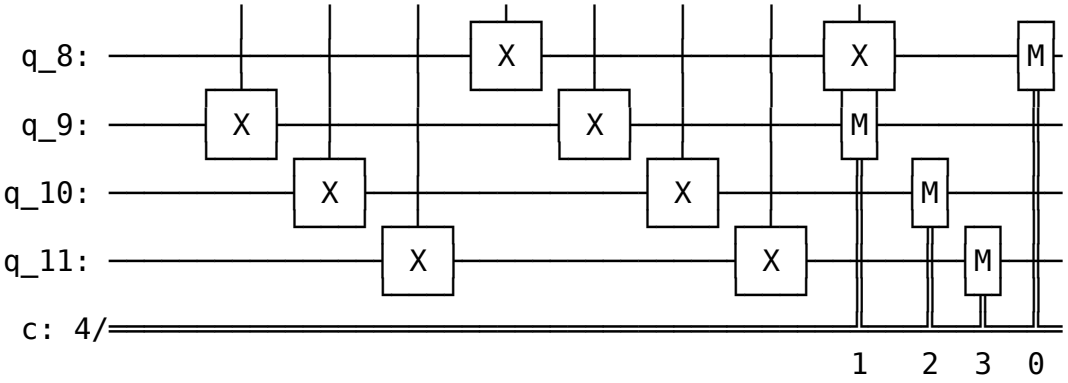


{'0010': 1024}
1000 1100

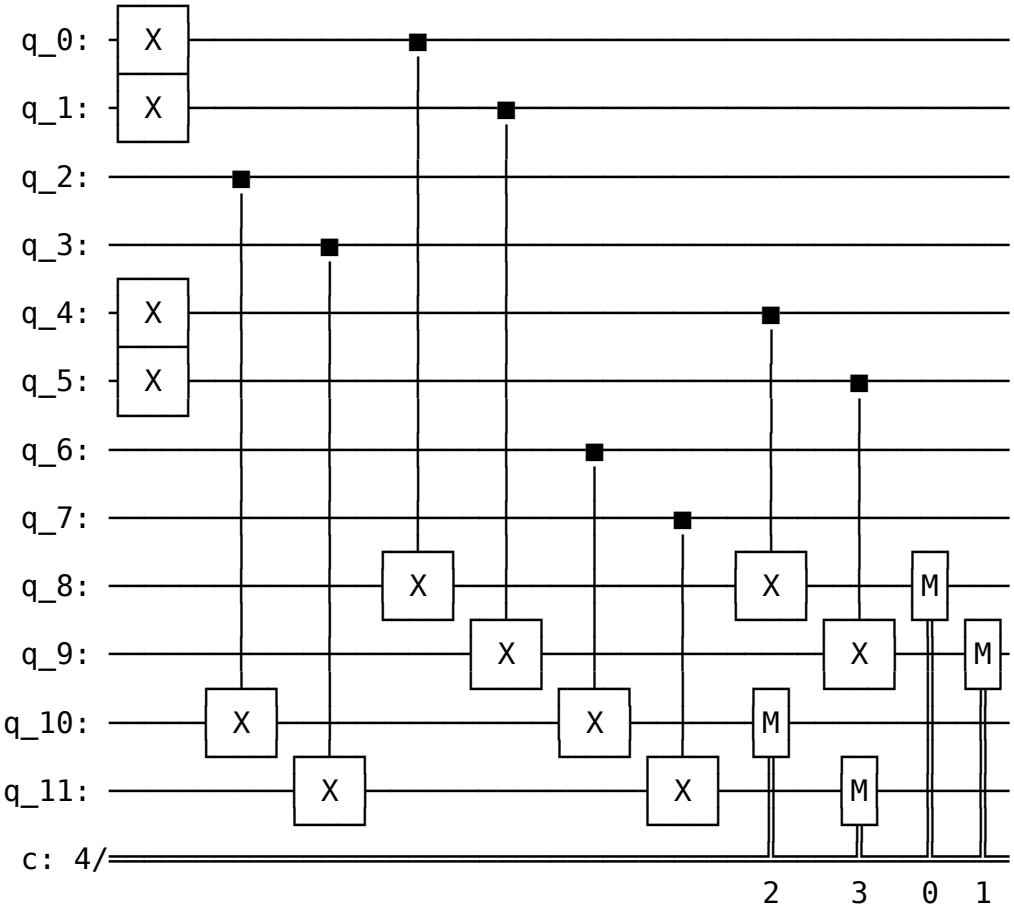


{'0010': 1024}
1000 1000





{'0000': 1024}
1100 1100



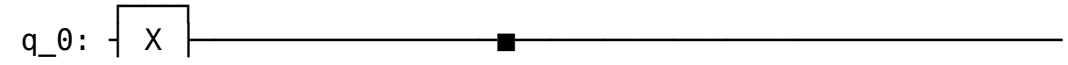
{'0000': 1024}

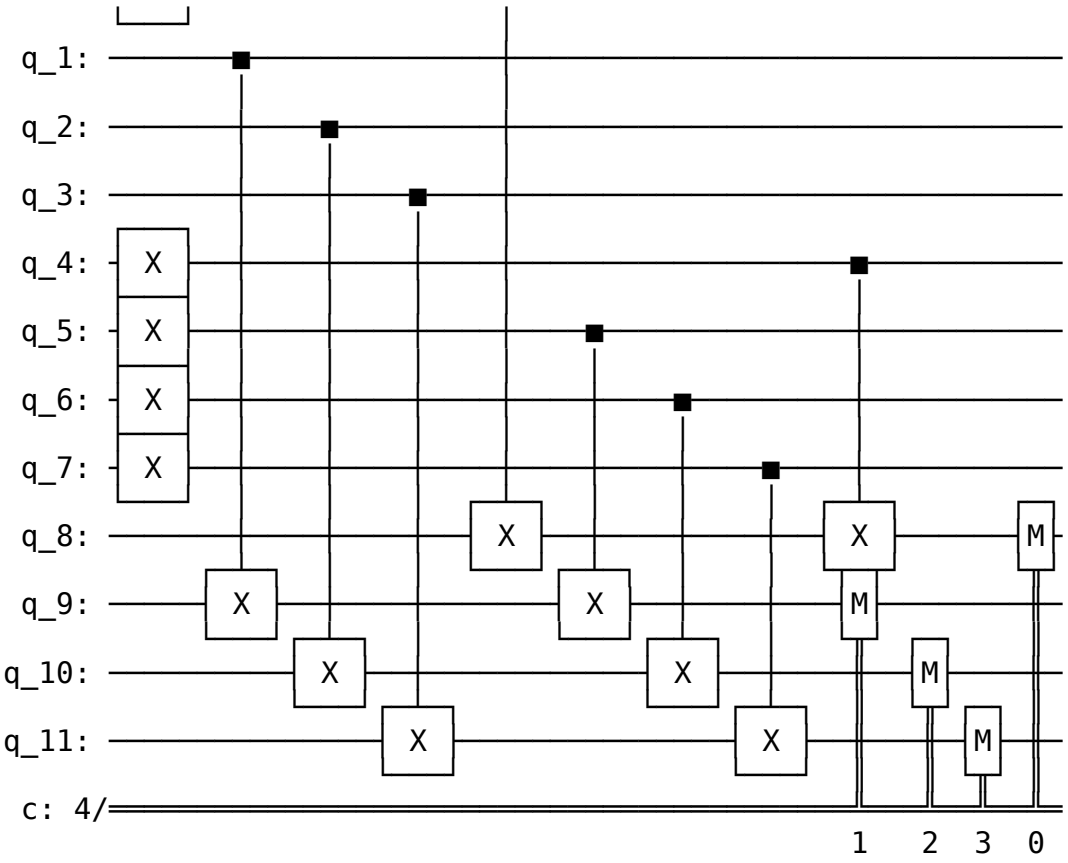
True

```
lengths = [8, 15, 12, 8]
```

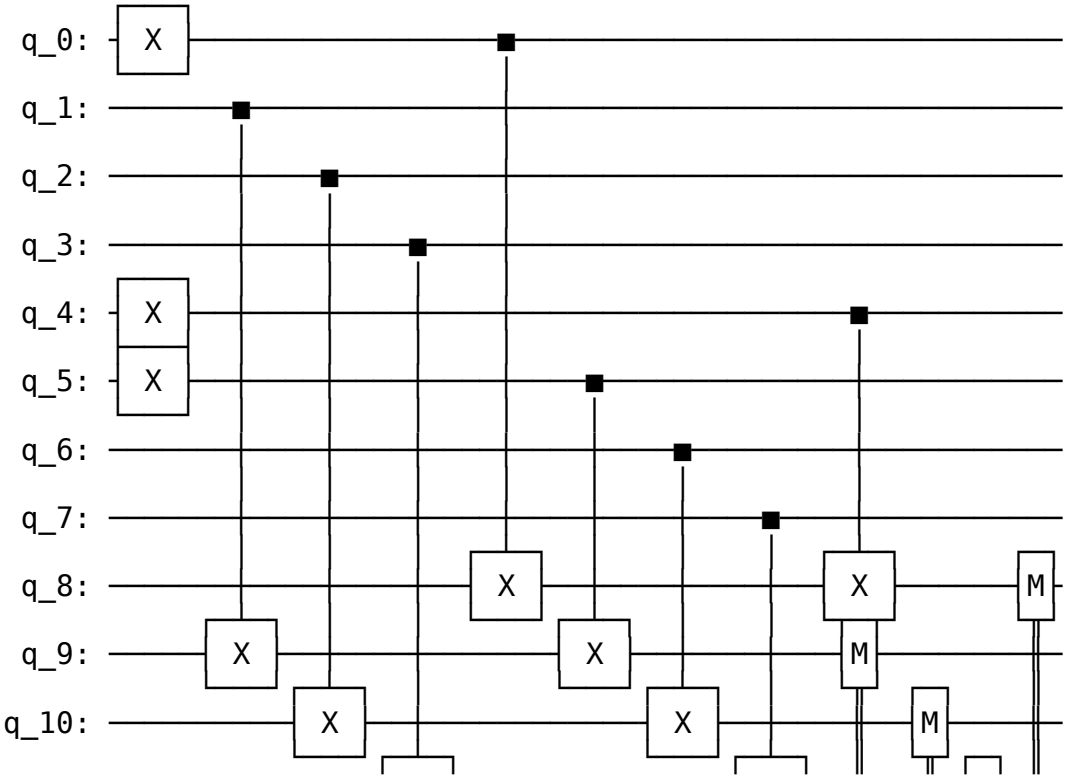
```
isRectangle(lengths)
```

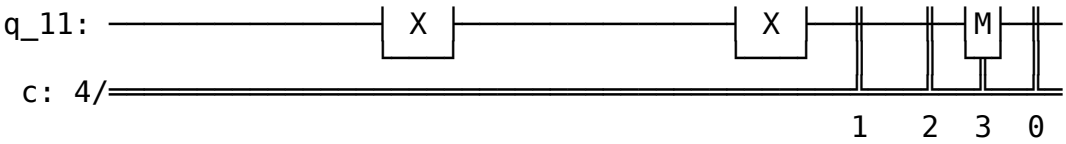
1000 1111



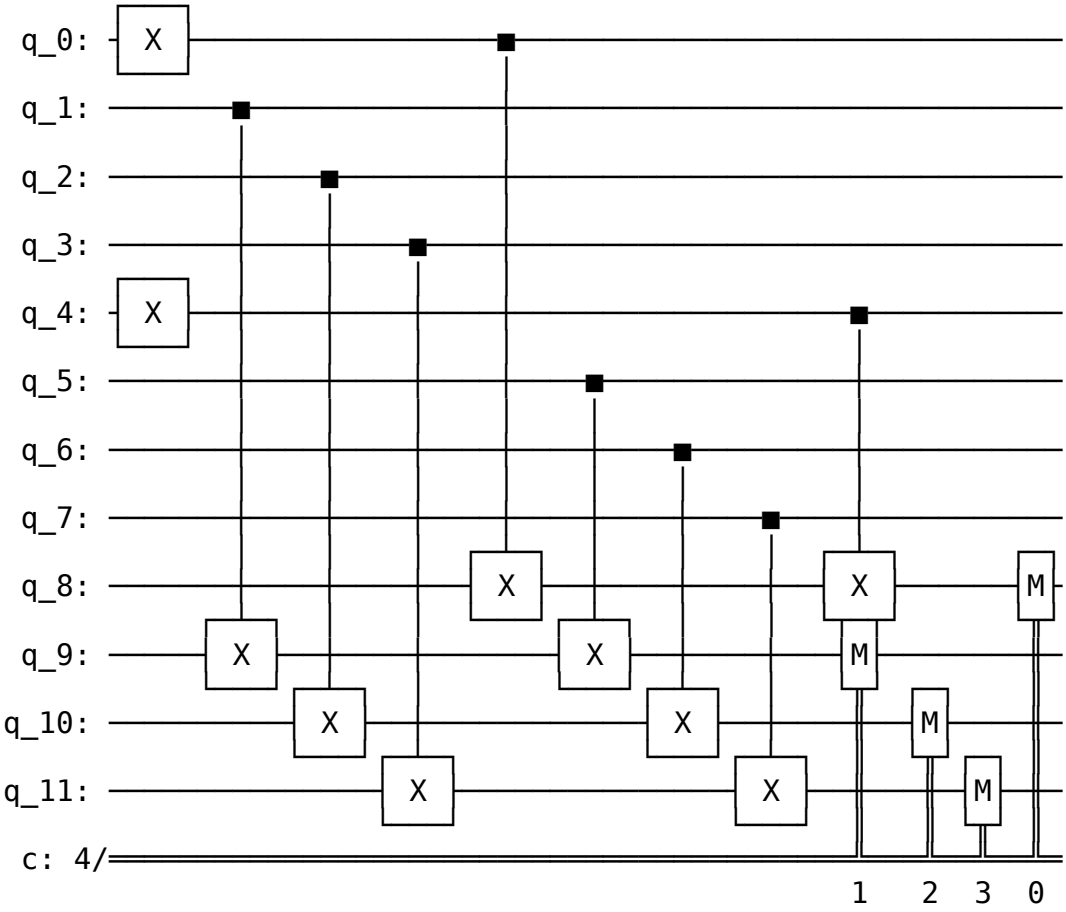



```
{'1110': 1024}
1000 1100
```

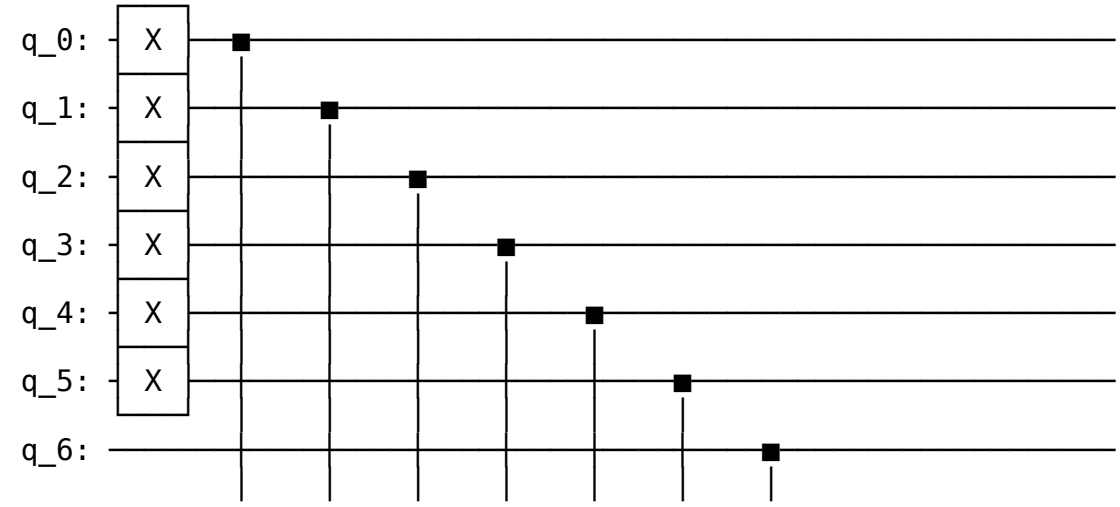


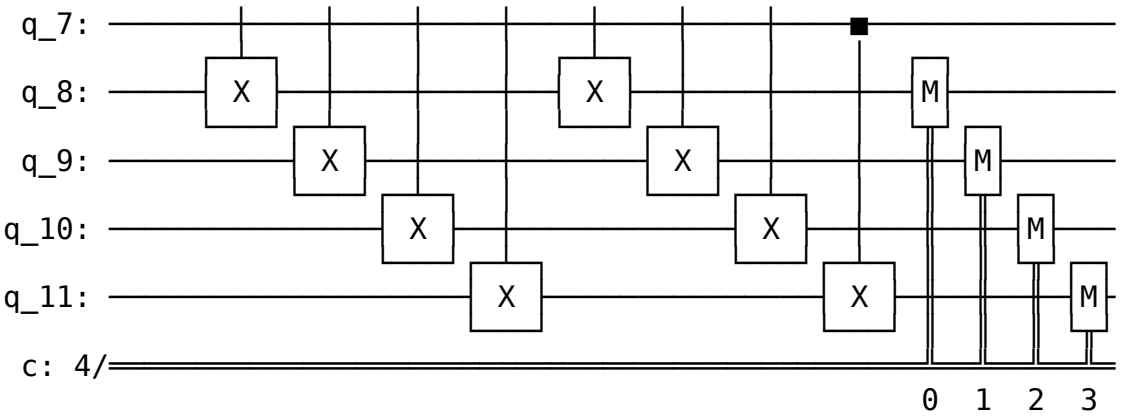


{'0010': 1024}
1000 1000



{'0000': 1024}
1111 1100






```
{'1100': 1024}

False

lengths = [8, 12, -12, 8]

isRectangle(lengths)

---------------------------------------------------------------------------
-----
AssertionError                            Traceback (most recent call
last)
Cell In[26], line 3
      1 lengths = [8, 12, -12, 8]
----> 3 isRectangle(lengths)

Cell In[23], line 2, in isRectangle(lengths)
      1 def isRectangle(lengths: List[int]) -> bool:
----> 2     assert all(i > 0 for i in lengths), "Length of side must
be > 0."
      4     # Now convert all the numbers to their binary
representation.
      5     bin_lengths = [convertToBinStr(num) for num in lengths]

AssertionError: Length of side must be > 0.
```

Approach 2

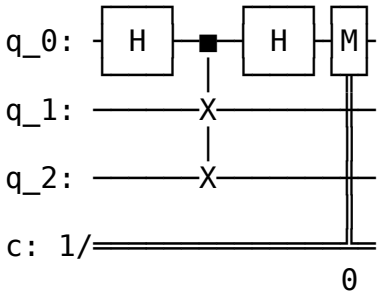
One of the problem with above approach is number of qbits and number of classical bits required. For numbers with N bit representation, this requires 3N Qbits and N classical bits Also, we need to compare 2 pairs of numbers separately, so for 4 side lengths, we end up doing 6 comparisons So, we can think of better approach. Two states are equal if inner product of two states is 0 And we can find out the inner product between 2 states using swap test.

Swap Test

The swap test uses 1 ancilla qbit which is put into superposition with a Hadamard gate. Then the controlled swap gate is applied to 2 qbits representing the states contorlled on ancilla qbit The Hadamard gate is applied again to ancilla qbit and this ancilla qbit state is measured If both qbits have equal states then ancilla qbit will be |0> However if the states of both qbits are orthogonal ancilla qbit will be |0⟩ or |1⟩ with equal probability.

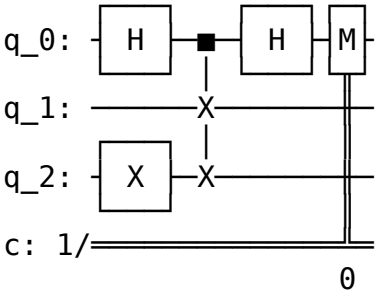
```
from qiskit import QuantumRegister, ClassicalRegister

# Let's code the swap test gate
def swapTest(num1: int, num2: int)-> QuantumCircuit:
    q = QuantumRegister(3,'q')
    c = ClassicalRegister(1,'c')
    circuit = QuantumCircuit(q,c)
    circuit.h(q[0])
    if num1 == 1:
        circuit.x(q[1])
    if num2 == 1:
        circuit.x(q[2])
    circuit.cswap(q[0],q[1],q[2]) # Contolled SWAP gate
    circuit.h(q[0])
    circuit.measure(q[0],c[0])
    return circuit

nums = [[0, 0], [0, 1], [1, 0], [1, 1]]

for num in nums:
    qc = swapTest(*num)

    print("Circuit: \n")
    print(qc)

    # run the circuit and measure the result.
    counts = execute(qc,
backend=BasicAer.get_backend('qasm_simulator')).result().get_counts()

    print(f"Num1: {num[0]}, Num2: {num[1]}, Ans: {counts}")
    print("\n\n\n")
```

```
Circuit:
```



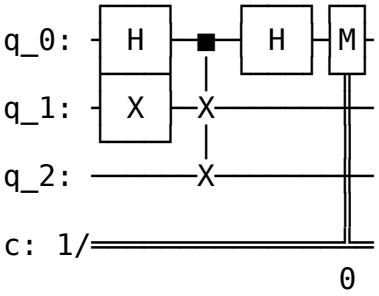
Num1: 0, Num2: 0, Ans: {'0': 1024}

Circuit:



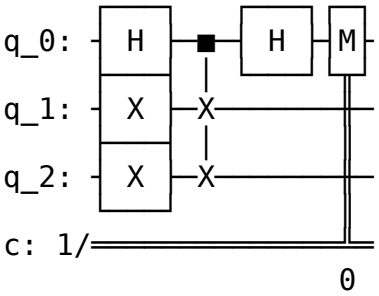
Num1: 0, Num2: 1, Ans: {'0': 500, '1': 524}

Circuit:



Num1: 1, Num2: 0, Ans: {'0': 539, '1': 485}

Circuit:



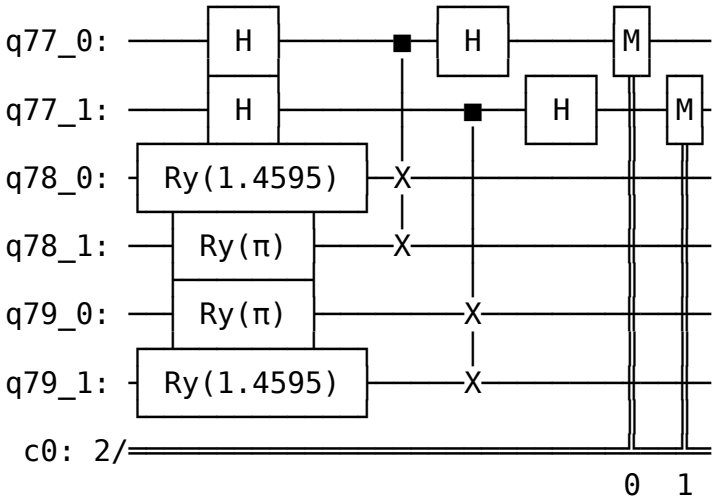
Num1: 1, Num2: 1, Ans: {'0': 1024}

Now for each combination of pair of lengths (A-B, C-D), we'll follow below steps: 1) We'll initialize 3 QuantumRegisters each with 2 qbits and 1 ClassicalRegister with 2 bits to read result. 2) We'll encode the numbers into qbit using rotation gate and angle equivalent to the arcsin of numbers. 3) We'll apply the swap test gate 4) We'll check the measurement result, if the result is |00> with 100% probabaility then both the states in each pair are equal.

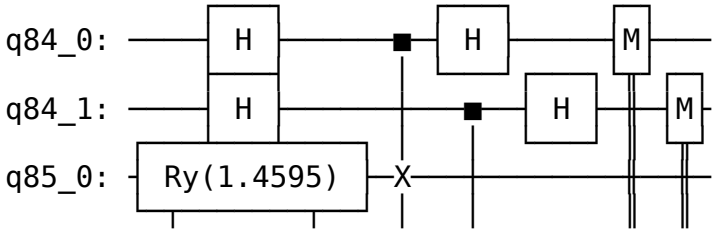
```
import numpy as np

def getCircuit2(pair1: Tuple[int, int], pair2: Tuple[int, int])->
QuantumCircuit:
    qrs = [QuantumRegister(2) for _ in range(3)]
    cr = ClassicalRegister(2)

    # Create the quantum circuit
    qc = QuantumCircuit(*qrs, cr)

    qc.ry(2*np.arcsin(pair1[0]), qrs[1][0])
    qc.ry(2*np.arcsin(pair1[1]), qrs[1][1])
    qc.ry(2*np.arcsin(pair2[0]), qrs[2][0])
    qc.ry(2*np.arcsin(pair2[1]), qrs[2][1])

    # Apply the swap circuit
    qc.h(qrs[0])
    qc.cswap(qrs[0][0], qrs[1][0], qrs[1][1])
    qc.cswap(qrs[0][1], qrs[2][0], qrs[2][1])
    qc.h(qrs[0])

    # Measure ancilla qubits without collapsing the wavefunction
    qc.measure(qrs[0][0], cr[0])
    qc.measure(qrs[0][1], cr[1])

    return qc

# Now we'll just check if the result is |00>
def isRectangle(lengths: List[int]) -> bool:
    assert all(i > 0 for i in lengths), "Length of side must be > 0."

    # normalize the lengths
    lengths = [length/max(lengths) for length in lengths]

    # Let's generate all possible combination of pairs
    pairs_gen = yieldAllPairs(lengths)

    # For each combination, check if both pairs have
```

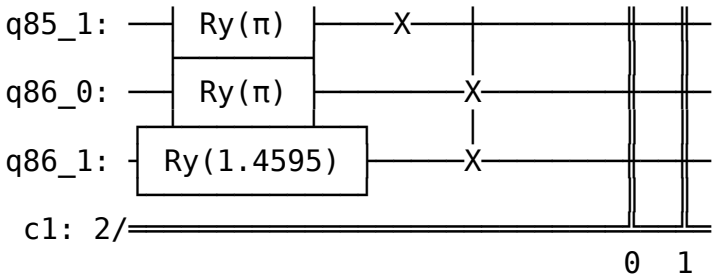
```
    # equal numbers.
    for pairs in pairs_gen:
        qc = getCircuit2(pairs[0], pairs[1])
        print("Circuit:\n")
        print(qc)

        counts = execute(qc,
backend=BasicAer.get_backend('qasm_simulator')).result().get_counts()
        print(counts)

        if len(list(counts.keys())) == 1 and
counts[list(counts.keys())[0]] == 1024:
            print(counts[list(counts.keys())[0]])
            if list(counts.keys())[0] == '00':
                return True

    return False

# Now let's again run few examples
lengths = [8, 12, 12, 8]

isRectangle(lengths)
```

```
Circuit:
```

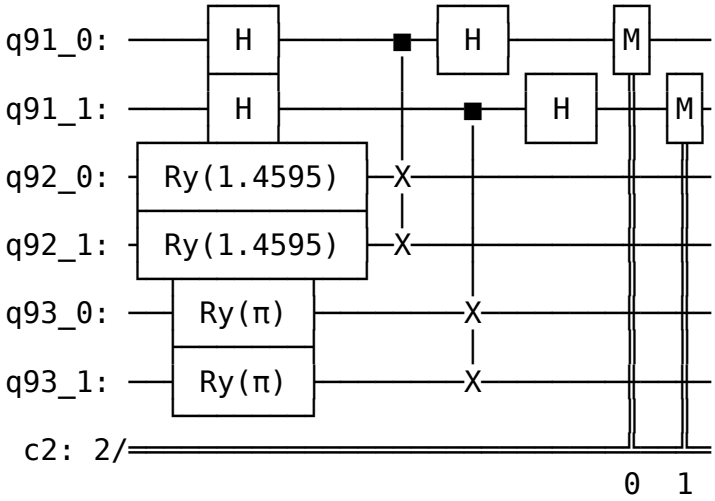



```
{'00': 523, '01': 212, '10': 214, '11': 75}
Circuit:
```





{'00': 548, '01': 200, '10': 197, '11': 79}
Circuit:

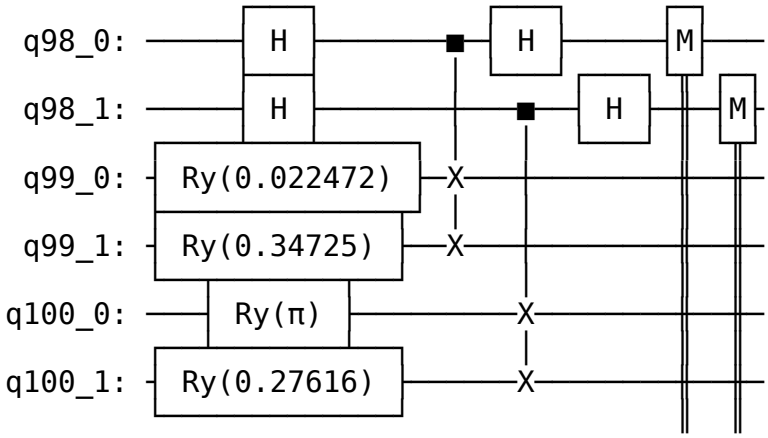


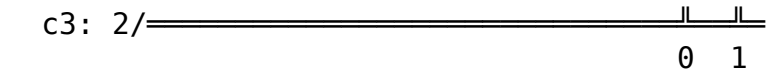
{'00': 1024}
1024

True

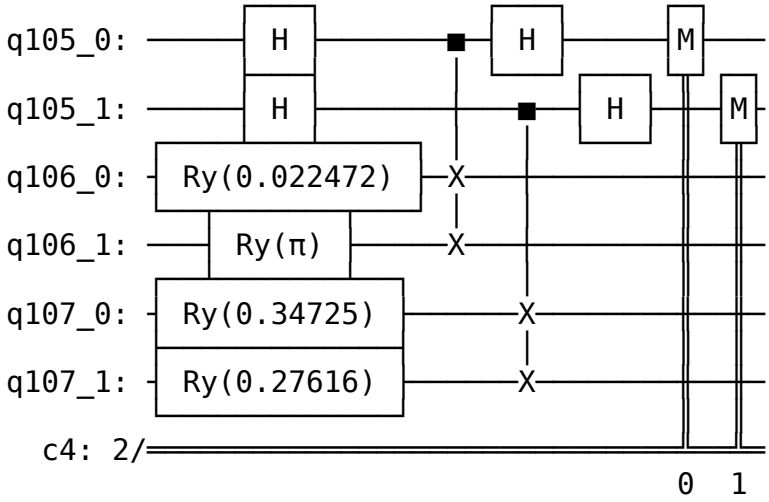
```
lengths = [8, 123, 712, 98]

isRectangle(lengths)
```

Circuit:

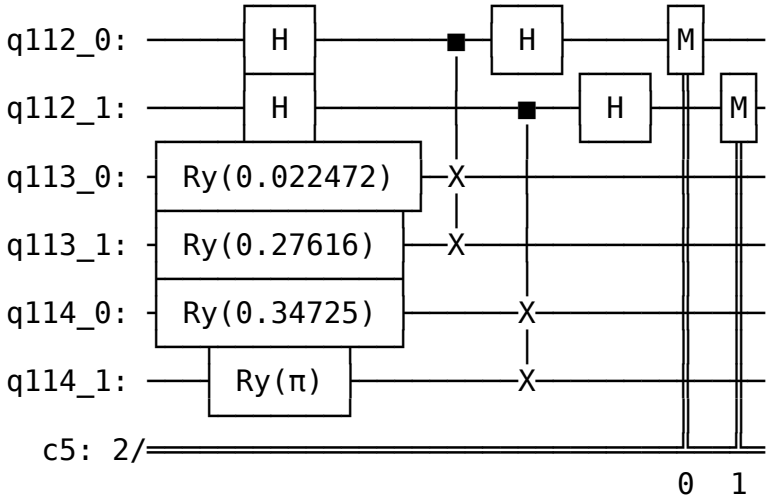





```
{'10': 506, '00': 510, '01': 5, '11': 3}
Circuit:
```




```
{'01': 523, '00': 499, '10': 2}
Circuit:
```




```
{'10': 516, '00': 500, '01': 5, '11': 3}

False
```