# Matrix Project

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# Geometry Question:

Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at $(-1, -2)$, then which one of the following is a vertex of this rhombus. Find its vertices. (Q 13 JEE MAINS CODE-G)

# Question in Matrix form:

Two sides of a rhombus are along the lines

$\begin{bmatrix} 1 & -1 \end{bmatrix} \mathbf{x} + 1 = 0$

$\begin{bmatrix} 7 & -1 \end{bmatrix} \mathbf{x}$ - 5 $= 0$

If its diagonals intersect at $\begin{bmatrix} -1 \\ -2 \end{bmatrix}$ .Find its vertices.

( Q26 in jee-linalg-2d)

## Solution Approach:

Since we are given 2 sides of rhombus **L1**, **L2** we will get a vertex **A** of rhombus from intersection of the given lines.
We also know the point of intersection of diagonals, **O**, thus we now know the equation of diagonal **D1** through **O** and **A**. Since we know the equation of the diagonal, we know the vertex **B** opposite to **A** because **O** is the midpoint of **A** and **B**.
As we know the equation of the diagonal **D1** passing through **AB** we can find the equation of other diagonal **D2** which is perpendicular to the former and passing through **O** (by the property of Rhombus). Now we can find another vertices **C** and **D** which are the intersections of the diagonal **D2** with each of the given lines or we can find one point using this method and the other using the same method as used for finding **B**.

# Method to find intersection of two lines:

Let the respective equations be
$\mathbf{n}_1^T\mathbf{x} = p_1$
$\mathbf{n}_2^T\mathbf{x} = p_2$
This can be written as the matrix equation $\begin{bmatrix} \mathbf{n}_1^T \\ \mathbf{n}_2^T \end{bmatrix} \mathbf{x} = \mathbf{p}$

$\mathrm{N}^T\mathbf{x} = \mathbf{p}$
where $\mathrm{N} = \begin{bmatrix} \mathbf{n}_1 & \mathbf{n}_2 \end{bmatrix}$
The point of intersection is then obtained as $\mathbf{x} = (\mathrm{N}^T)^{-1}\mathbf{p}$
$\mathbf{x} = \mathrm{N}^{-T}\mathbf{p}$

# Method to find intersection of two lines(calculations):

We use this method to find **A** from intersection of **L1** and **L2**

$\mathbf{n}_1^T = \begin{bmatrix} 1 & -1 \end{bmatrix}$ $\mathbf{n}_2^T = \begin{bmatrix} 7 & -1 \end{bmatrix}$

$\mathrm{N}^T = \begin{bmatrix} 1 & -1 \\ 7 & -1 \end{bmatrix}$ $N^{-T} = \begin{bmatrix} -1/6 & 1/6 \\ -7/6 & 1/6 \end{bmatrix}$

$\mathbf{p} = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$

$\mathbf{A} = \mathrm{N}^{-T}\mathbf{p} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

And **C** and **D** from intersection of **L1** and **L2** with **D2**

# Method to find intersection of two lines(calculations):

We use this method to find **C** from intersection of **L1** or **L2** with **D2**

for **L1**

$\mathbf{n}_1^T = \begin{bmatrix} 1 & -1 \end{bmatrix}$

p1=-1

for **D2**

**n**d is directional vector of **D1**, which is equal to **O-A**

$\mathbf{n}d^T = \begin{bmatrix} -2 & -4 \end{bmatrix}$

pd=$\mathbf{n}d^T\mathbf{O} = 10$

$\mathrm{N}^T = \begin{bmatrix} 1 & -1 \\ -2 & -4 \end{bmatrix}$ $N^{-T} = \begin{bmatrix} 2/3 & -1/6 \\ -1/3 & -1/6 \end{bmatrix}$

$\mathbf{p}= \begin{bmatrix} -1 \\ 10 \end{bmatrix}$

$\mathbf{C}= \mathrm{N}^{-T}\mathbf{p} = \begin{bmatrix} -2.33333333 \\ -1.33333333 \end{bmatrix}$

# Method using midpoints:

If **O** is the midpoint of **A** and **B**
**O**=(**A**+**B**)/2
**B**=2**O**-**A**
**B**=**A**+2(**O**-**A**) We know that the diagonals in a Rhombus bisect each other
**O** is the midpoint of opposite vertices of the Rhombus
Thus given **O** and **A** we can find **B**

$$\mathbf{B}=\mathbf{A}+2(\mathbf{O}\text{-}\mathbf{A})=\begin{bmatrix} -3 \\ -6 \end{bmatrix}$$

And given **O** and either of **C** or **D** we can find the other.

$$\mathbf{D}=\mathbf{C}+2(\mathbf{O}\text{-}\mathbf{C})=\begin{bmatrix} 0.33333333 \\ -2.66666667 \end{bmatrix}$$

# Graph