Matrix Project

EE17BTECH11023 EE17BTECH11007

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Geometry Question:

Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at $(-1, -2)$, then which one of the following is a vertex of this rhombus. Find its vertices.
(Q 13 JEE MAINS CODE-G)

Question in Matrix form:

Two sides of a rhombus are along the lines

$$\begin{bmatrix} 1 & -1 \end{bmatrix} \mathbf{x} + 1 = 0$$
$$\begin{bmatrix} 7 & -1 \end{bmatrix} \mathbf{x} - 5 = 0$$

If its diagonals intersect at $\begin{bmatrix} -1 \\ -2 \end{bmatrix}$ .Find its vertices.

(Q26 in jee-linalg-2d)

Solution Approach:

Since we are given 2 sides of rhombus **L1**, **L2** we will get a vertex **A** of rhombus from intersection of the given lines.
We also know the point of intersection of diagonals, **O**, thus we now know the equation of diagonal **D1** through **O** and **A**. Since we know the equation of the diagonal, we know the vertex **B** opposite to **A** because **O** is the midpoint of **A** and **B**.
As we know the equation of the diagonal **D1** passing through **AB** we can find the equation of other diagonal **D2** which is perpendicular to the former and passing through **O** (by the property of Rhombus). Now we can find another vertices **C** and **D** which are the intersections of the diagonal **D2** with each of the given lines or we can find one point using this method and the other using the same method as used for finding **B**.

Method to find intersection of two lines:

Let the respective equations be

$\mathbf{n}_1^T \mathbf{x} = p_1$

$\mathbf{n}_2^T \mathbf{x} = p_2$

This can be written as the matrix equation

$$\begin{bmatrix} \mathbf{n}_1^T \\ \mathbf{n}_2^T \end{bmatrix} \mathbf{x} = \mathbf{p}$$

$N^T \mathbf{x} = \mathbf{p}$

where N $= \begin{bmatrix} \mathbf{n}_1 & \mathbf{n}_2 \end{bmatrix}$

The point of intersection is then obtained as $\mathbf{x} = (N^T)^{-1}\mathbf{p}$

$\mathbf{x} = N^{-T}\mathbf{p}$

Method to find intersection of two lines(calculations):

We use this method to find **A** from intersection of **L1** and **L2**

$\mathbf{n}_1^T = \begin{bmatrix} 1 & -1 \end{bmatrix}$

$\mathbf{n}_2^T = \begin{bmatrix} 7 & -1 \end{bmatrix}$

$N^T = \begin{bmatrix} 1 & -1 \\ 7 & -1 \end{bmatrix}$

$N^{-T} = \begin{bmatrix} -1/6 & 1/6 \\ -7/6 & 1/6 \end{bmatrix}$

$\mathbf{p} = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$

$\mathbf{A} = N^{-T}\ \mathbf{p} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

And **C** and **D** from intersection of **L1** and **L2** with **D2**

Method to find intersection of two lines(calculations):

We use this method to find **C** from intersection of **L1** or **L2** with **D2**

for **L1**

$\mathbf{n}_1^T = \begin{bmatrix} 1 & -1 \end{bmatrix}$

$p1 = -1$

for **D2**

nd is directional vector of **D1**, which is equal to **O-A**

$\mathbf{n}d^T = \begin{bmatrix} -2 & -4 \end{bmatrix}$

pd$=\mathbf{n}d^T\ \mathbf{O} = 10$

$N^T = \begin{bmatrix} 1 & -1 \\ -2 & -4 \end{bmatrix}$

$N^{-T} = \begin{bmatrix} 2/3 & -1/6 \\ -1/3 & -1/6 \end{bmatrix}$

$\mathbf{p} = \begin{bmatrix} -1 \\ 10 \end{bmatrix}$

$\mathbf{C} = N^{-T}\ \mathbf{p} = \begin{bmatrix} -2.33333333 \\ -1.33333333 \end{bmatrix}$

Method using midpoints:

If **O** is the midpoint of **A** and **B**
$\mathbf{O}=(\mathbf{A}+\mathbf{B})/2$
$\mathbf{B}=2\mathbf{O}\text{-}\mathbf{A}$
$\mathbf{B}=\mathbf{A}+2(\mathbf{O}\text{-}\mathbf{A})$ We know that the diagonals in a Rhombus bisect each other
O is the midpoint of opposite vertices of the Rhombus
Thus given **O** and **A** we can find **B**

$$\mathbf{B}=\mathbf{A}+2(\mathbf{O}\text{-}\mathbf{A})=\begin{bmatrix}-3\\-6\end{bmatrix}$$

And given **O** and either of **C** or **D** we can find the other.

$$\mathbf{D}=\mathbf{C}+2(\mathbf{O}\text{-}\mathbf{C})=\begin{bmatrix}0.33333333\\-2.66666667\end{bmatrix}$$

Graph