EE465: Fall 2011 Simulation Project

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1 Part 1

1.1 Description

The aim of this part is to simulate queues with exponential arrival and service times under various conditions given. To this end a generic queue class has been implemented that simulates an $M/M/K/N$ queue. The implementation of this class (named *MMKNQ*) may be found in the source file *part1.h*. The file *part1.cpp* contains the front end that accepts user inputs and simulates the queue under the appropriate conditions specified initially.

For the cases I, II and III the number of arrivals given is not sufficient to ensure convergence of the values to what is expected theoretically. Hence I have added new cases as given below:

1. **Case VI** : Same as Case I but with 1000000 arrivals.
2. **Case VII** : Same as Case II but with 5000000 arrivals.
3. **Case VIII** : Same as Case III but with 1000000 arrivals.

Note that these cases will take around a minute to run. You can run the above in the same way as you would run the other cases. For example, to run case VII do:

```
$./part1 7
```

1.2 Questions

1.2.1 Finding the theoretical values expected at each given case.

Given below is a table that takes gives the theoretical values expected for each given case.

Case	Utilization	L	W	P_b	$P(0)$	Stationary Distribution
I	0.7	2.333..	3.333..	0	0.3	$0 \rightarrow 0.3, 1 \rightarrow 0.21, 2 \rightarrow 0.147$
II	0.95	19	20	0	0.05	$0 \rightarrow 0.05, 1 \rightarrow 0.0475, 2 \rightarrow 0.045125$
III	0.2499	0.7636	1.019	5.1708×10^{-4}	0.4707	$0 \rightarrow 0.4707, 1 \rightarrow 0.353, 2 \rightarrow 0.1324$
IV	0.84211	5.333..	6.666..	0	0.15789	$0 \rightarrow 0.15789, 1 \rightarrow 0.13296, 2 \rightarrow 0.11197$
V	0.5	1	3.33..	0	0.5	$0 \rightarrow 0.5, 1 \rightarrow 0.25, 2 \rightarrow 0.125$

1.2.2 The simulated results

Given below is a table that takes gives the simulated values for each given case.

Case	Utilization	L	W	P_b	$P(0)$	Stationary Distribution
I	0.698	2.308	3.304	0	0.302	$0 \to 0.302, 1 \to 0.212, 2 \to 0.147$
II	0.95	18.91	19.91	0	0.0501	$0 \to 0.0501, 1 \to 0.0477, 2 \to 0.0452$
III	0.2488	0.76	1.018	4.67×10^{-4}	0.4719	$0 \to 0.4719, 1 \to 0.3532, 2 \to 0.1316$
IV	0.8416	5.267	6.589	0	0.1584	$0 \to 0.1584, 1 \to 0.133, 2 \to 0.112$
V	0.4988	0.993	3.23	0	0.501	$0 \to 0.501, 1 \to 0.25, 2 \to 0.1251$

Note that the above values were obtained for the case when the number of arrivals were large enough for the values to converge. In all the above cases the values have converged to the theoretically expected values and are within the margin of error. The simulated and the expected values can be compared easily using the two tables above.

1.2.3 Theory vs Simulation

In all cases the values converge to that expected theoretically. Although the number or arrivals required for each case to converge is different. As was mentioned in the description case II and case III did not converge with the given number of arrivals. The number of arrivals needed to be significantly increased in these cases. The exact values are given in the description section.

1.2.4 P-K formula

The simulation data for Case I with the required modifications is given below:

```
Utilization: 0.700953
Average number of packets in the system: 2.3309
Average delay in the system: 3.32223
Blocking ratio: 0
Idle period of the server: 0.299047
Average service time: 0.997535
Average second moment of service time: 2.0027
Average wait time in queue: 2.3247
Stationary distribution: 0->0.299047, 1->0.208858, 2->0.147489, 3->0.101869, 4->0.072979 .....
```

The P-K formula is given by

$$W_Q = \frac{\lambda E\left[S^2\right]}{2\left(1 - \lambda E\left[S\right]\right)}$$

Using the formula given above and taking the required data from the simulator output, we have:

$$\begin{aligned} W_Q &= \frac{0.7 \times 2.0027}{2\left(1 - 0.7 \times 0.997535\right)} \\ &= 2.3231 \end{aligned}$$

The value obtained using the P-K formula (2.3231) compares very well with the value obtained from the simulation (2.3247) thereby verifying the P-K formula.

2 Part 2

2.1 Description

The aim of this part is to analyze the given M/G/1 system both theoretically and using simulations. As previously the implementation of the M/G/1 queue is in the form of a class *MG1Q* that can be found in the file *part2.h*. The file *part1.h* contains the front end that accepts the seed and the number of arrivals from the user. Note that the first argument passed to the executable is the number of arrivals followed by the seed. It is recommended that both values be lesser than or equal to 10^6.

2.2 Questions

2.2.1 Theoretical Values

The theoretical values of the bounded pareto distribution for the given parameters are listed below:

$$\begin{aligned} E[S] &= 3.41209 \\ E\left[S^2\right] &= 595.4159 \end{aligned}$$

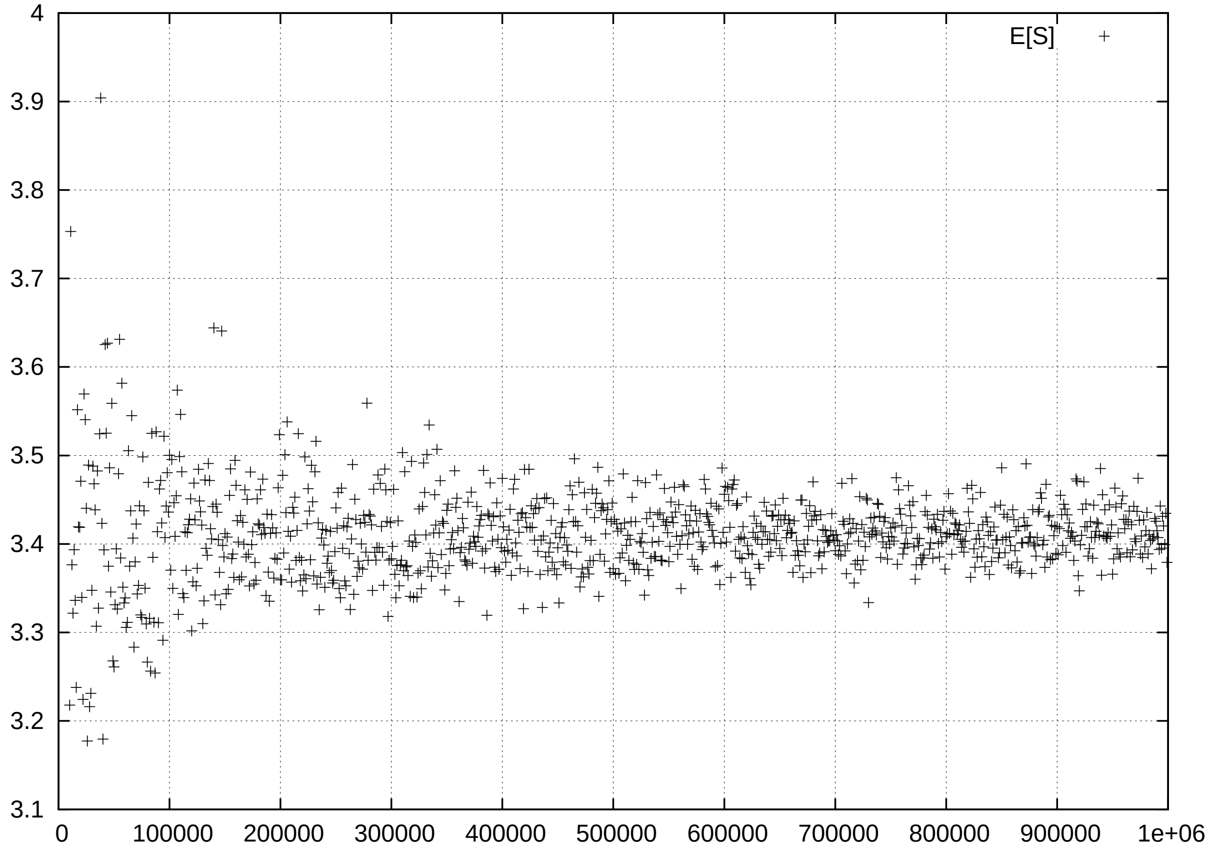
2.2.2 Average with sample size 10^4

The below table gives the average value obtained with sample size 10^4 with various seeds.

Seed	$E[S]$
7992020	3.29004
1439027	3.32156
9338211	3.24692
1323674001	3.11164
1323674031	3.5055

The mean values obtained are not close enough to the actual mean. As can be seen above there is significant fluctuation in values of the expected service time about the mean.

2.2.3 Plot



The sample size was varied from 10^4 to 10^6 in steps of 10^3. Initially the number of samples have not converged to the sample mean and the behavior progressively improves as the sample size is increased. At sample size around 10^6 the variation about the mean is much smaller compared to the behavior initially.

2.2.4 Optimal sample size

$$0.9 \times E[S] = 3.070881$$

$$1.1 \times E[S] = 3.753299$$

From the plot $N = 5 \times 10^5$ will ensure that the sample mean is well within the specified range.

2.2.5 Theoretical delay

From the P-K formula we find

$$W_Q = \frac{0.2 \times 595.4159}{2 \times (1 - 0.2 \times 3.41209)} = 187.4841458$$

$$E\left[S\right] = 3.41209$$

$$\therefore W = W_Q + E[S] = 190.8962358$$

The above is the theoretical average delay in the system for the queue.

2.2.6 Simulation run

Seed	Delay (W)	Difference from expected delay
1323684707	211.035	20.139
1323684798	127.363	-63.533
1323684852	195.704	4.808
1323684880	220.018	29.122
1323684919	191.754	0.858
1323684943	179.581	-11.315
1323685003	206.473	15.577
1323685026	208.194	17.298
1323685050	192.018	1.122
1323685070	143.137	-47.759

Each of the above values were obtained using the sample size (L) of 10^6. The results are not entirely satisfactory compared to the theoretical value. There is rather wide variation in the values obtained, indicating that it has not converged. The average of the 10 values above is 187.5277 which is quite close to the expected value.

2.2.7 Variance

The variance of the simulated average delay as is obtained from the table given above is 826.6039. The variance is very high, indicating that the values that we have obtained through the simulation have not converged.