Project

HALL Thruster Design

Aerospace Engineering
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Electric force a = acceleration

$F = q\vec{E}$ $a = \dfrac{q\vec{E}}{m}$

Magnetic Force:

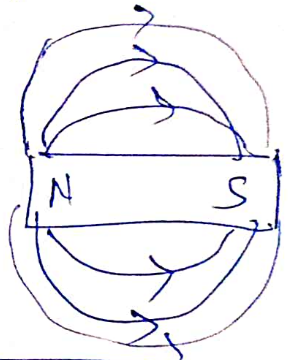
$F = q\vec{V}\vec{B}$ $[V \times B]$ cross product

$= q\vec{V}\vec{B}\sin\theta$ θ = angle b/w Velocity and magnetic field.

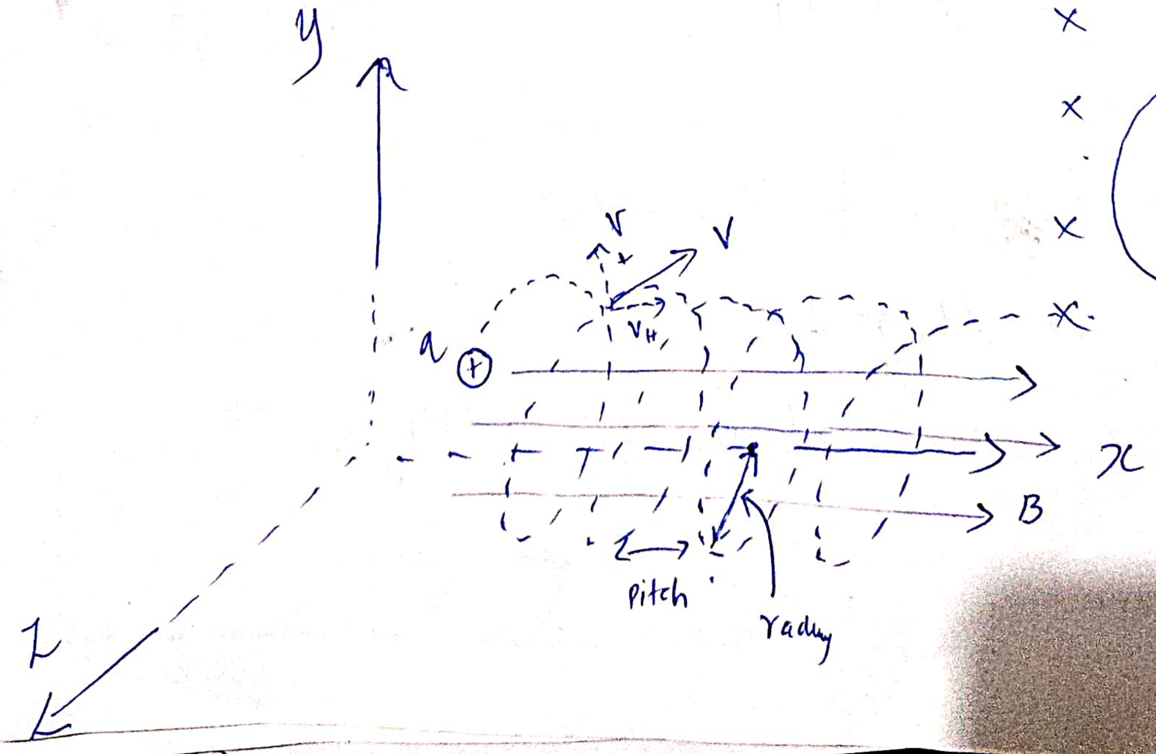
Magnetic force on current carrying conductor:

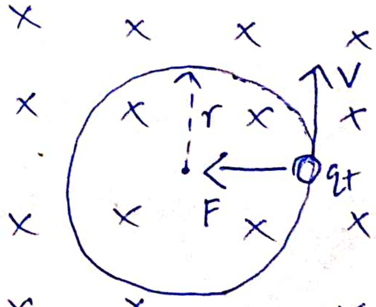
$F = \int I\, dL \times B$ ← magnetic field.

↑ Current ↑ length of conductor



Motion of Charge in a Magnetic field.



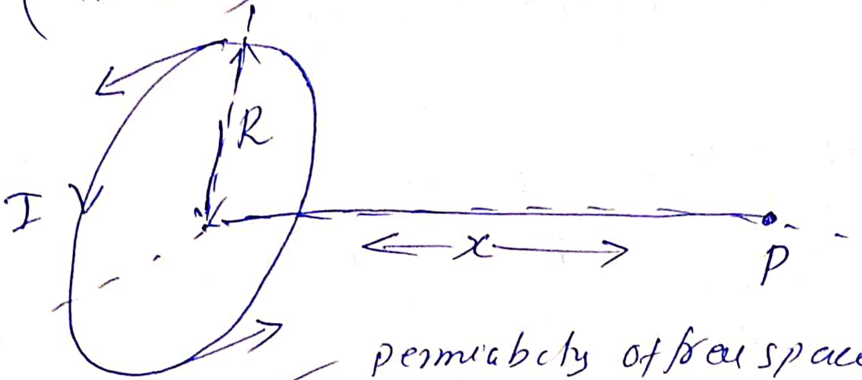


Centripetal force during rotation of charge.

$$F_c = \frac{mv^2}{r} \quad \text{but } F = qvB$$

$$\frac{mv^2}{r} = qvB \qquad \boxed{r = \frac{mv}{qB}}$$

Magnetic field due to circular current loop:
(in axis)



permeability of free space.

$$B = \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}$$

$\mu_0 = 4\pi \times 10^{-7}$

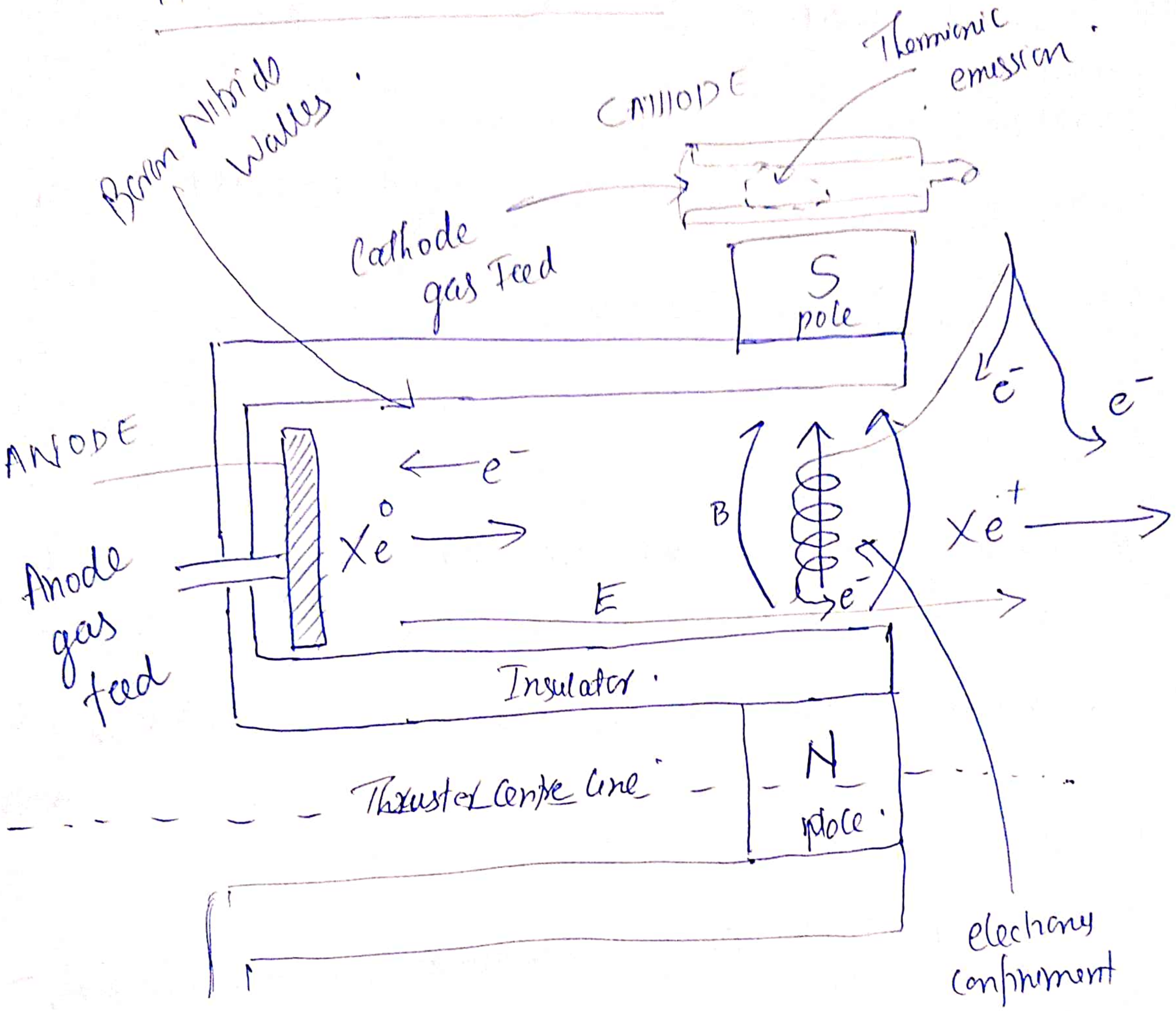
Magnetic field due to solenoid:

$B = \mu_0 n I$

number of turns per unit length.

Magnetic field inside solenoid

Hall Thruster Schematic :



Operation principle :

uses electrostatic potential to accelerate ions to high speed .

* Radial Magnetic field
generated by,
Central coil and outer coil   G - Guass

Radial magnetic field of 100 - 300 G (0.01 - 0.03 T)

Nominal Input Voltage - 300V

* Axial Electric Field   (350)

generated by Electrical Potential in range 150-800V
applied b/w anode and cathode (20V)

This creats an Electricfield in The range 2-40 kV/m

* Anode:

contains positively charged Electrode, and gas distributor.
to introduce xenon gas in The chamber.

mass flow rate Xe can typically be 5-15mg/s

Cathode:

The Cathode has a dedicated heater, gas distributer.

The heater in the cathode, causes Thermoionic emission of electrons

and the gases distributer introduces Xe.

The amount of gas distributed by ~~the cathode can typically be~~
5-10% of the gas introduced by anode.

Principle: Lorentz force: generated on a particle of charge q by mutually perpendicular electric and magnetic fields.

: $F = q(V \times B)$

* This force, in turn causes electron confinement in the hall thruster as opposed to the ringcusp magnetic field in the ion thruster.

* The Electrons move in circular motion, giving rise to the Hall current.

→ Radial magnetic field deflects the electrons but not the ions

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Larmor Radius, or Cyclotron Radius

$$R = \frac{mV}{qB}$$

Component of Velocity ⊥cler to the magnetic field.

magnetic field

q charge of particle.

mass of particle.

* Electron Larmor radius:

at Temperature of 25eV, for radial magnetic field of 150G

$r = 0.13cm$.

* Ion Larmor radius:

for 300eV of energy. for radial magnetic field of 150G

$= 180cm$.

→ Cyclotron frequency

Time period for one revolution - $T = 2\pi R/V$

= Substitute R.

We get $T = \frac{2\pi m}{qB}$

frequency $f = 1/T = qB/2\pi m$.

cyclotron frequency $= \omega = 2\pi f = qB/m$.

For Boundary Condition:

External pressure: for space near Geostationary orbit

Can be approximate to be least 5e-5 Torr or lower (till ($1.322 e^{-11}$ Pa))

Two type of Hall thrusters:

* Stationary Plasma Thruster:

SPT'S have Dielectric insulating wall

Dielectric materials have low sputtering yield.

Electron collisions with wall keep the electron temperature low (due to emission of secondary electrons)

* Thruster with Anode layer:

TALs have metallic conducting wall instead of dielectric wall.

SPT's more efficient than TAL'S

Sputterings Effect:
If ion kinetic energy exceeds the materials lattice displacement energy, molecules in the wall may be pushed into new position.

Primarily occurs where the striking energy is in the range 60–500 eV.

When high energy electrons strike it releases the secondary electrons from wall boundary.

Materials:
Anode: Stainless Steel.

Cathode: Porous Tungsten impregnated with Barium; Barium oxide, lanthanum Hexaboride (LaB_6)

Walls of plasma chamber: Boron Nitride (h-BN), Borosil ($BN-SiO_2$)
(High termal Conductivity, low Thermal expansion, High Electrical Resistivity, Non Toxic)

propellent: Xe (High molecular Weight, non toxic, non corrosive low ionization potential)

Design:

Writing Temperature in eV

T in eV

$E = \frac{3}{2} K_B T$

Thruster requirments:

Thrust: 123 mN → T

Dischage voltage: 350 V = V_d

Specific impulse: 1765 sec - I_{sp}

Its known as that;:

for $150V < V_d < 350V \rightarrow 800K < T_a < 1000K$

For $150V < V_d < 900V \rightarrow 10eV < T_{ev} < 20ev$

T_a - atom temperatuer.

T_{ev} - electron temperature.

Calculate T_a:

$$\frac{1000-800}{350-150} = \frac{T_a - 800}{V_d - 150}$$

$V_d = 350$

$$\therefore \frac{1000-800}{350-150} = \frac{T_a - 800}{350-150} \Rightarrow T_a = 1000K$$

Calculate T_{ev}.

$$\frac{20-10}{900-150} = \frac{T_{ev} - 10}{V_d - 150} = \frac{T_{ev}-10}{350-150} = T_{ev} = 12.667 \text{ eV}$$

For maxwellian electrons.

$\sigma_i V_e \rightarrow$ ionization Reaction Rate.

For:

$T_{eV} < 5eV$

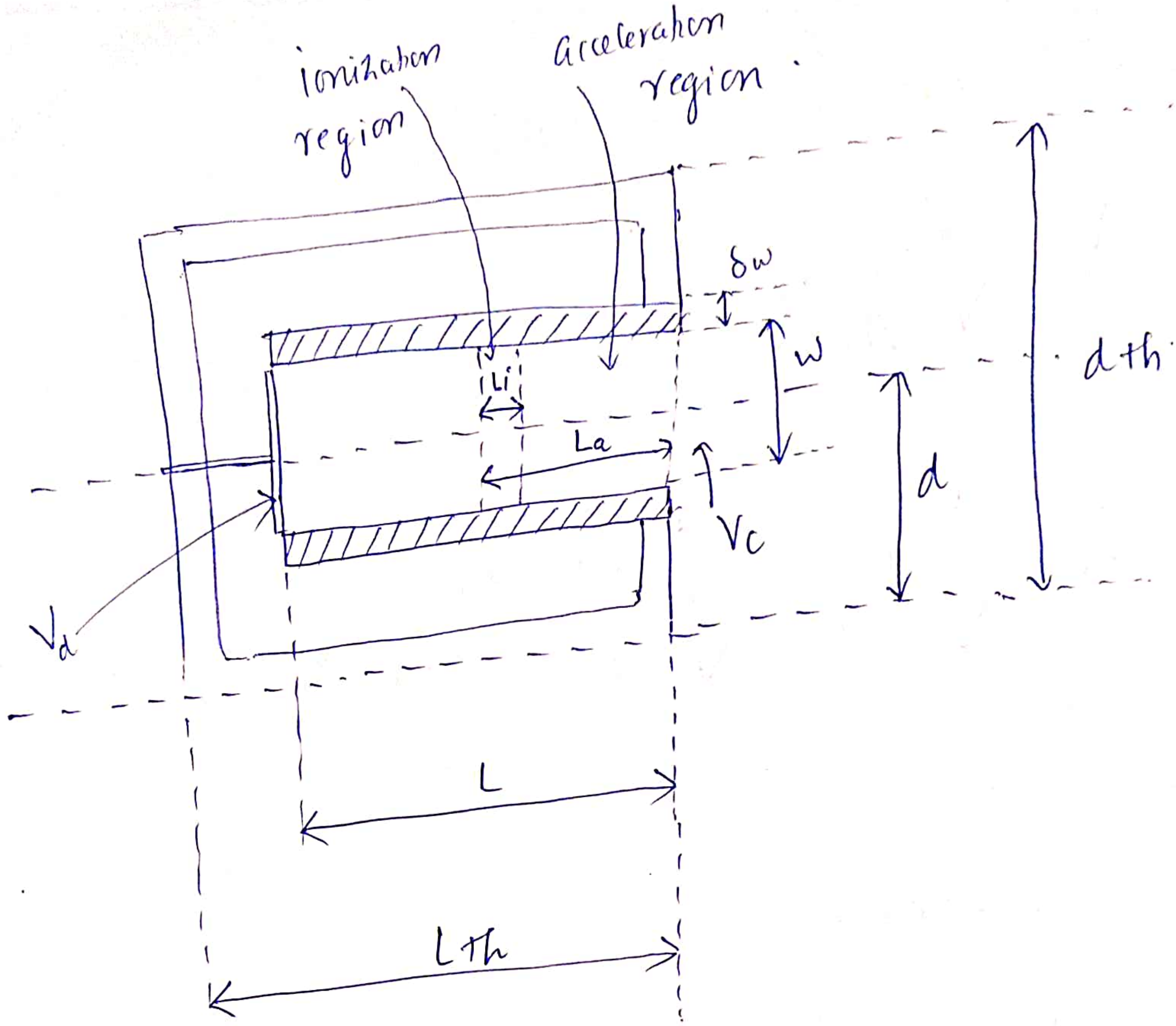
$$\sigma_i V_e = 10^{-20}\left[\left(3.97 + 0.643 T_{eV} - 0.0368 T_{eV}^2\right) e^{-\frac{12.127}{T_{eV}}}\right]\sqrt{\frac{8eT_{eV}}{\pi m}}$$

for:

$T_{eV} > 5eV$

$$\sigma_i V_e = 10^{-20}\left[-(1.031\times10^{-4}) T_{eV}^2 + 6.386\, e^{-12.125/T_{eV}}\right]\sqrt{\frac{8eT_{eV}}{\pi m}}.$$

$$\sigma_i V_e = \underline{\underline{5.801\times10^{-14}\ m^3/s}}.$$



3. Acceleration potential of ion in Acceleration layer.

1st ionization potential of Xenon which is (12.1)

$$V_d = \Delta V_{acl} + (3+1)\,\psi_{xe} + V_c$$

12.1299 eV

$V_d = 330$

$V_c = 20$

$$\Delta V_{acl} = 350 - 4(2.1) - 20$$

$$\Delta V_{acl} = 281.6\ V$$

Ion and Atom Velocity:

$$V_i = \sqrt{\frac{2qV_d}{M}} = \sqrt{\frac{2\times 1.6\times 10^{-19}\times 350}{2.18\times 10^{-25}}} = 22666.31 \text{ m/s}$$

$$V_a = \sqrt{\frac{8kT_a}{\pi M}} = \sqrt{\frac{8\times 1.38\times 10^{-23}\times 1000}{\pi \times 2.18\times 10^{-25}}} = 401.496 \text{ m/s}$$

$\dot{m}$

Electron Velocity:

$$V_e = \sqrt{\frac{8kT_{ev}}{\pi m_e}\times\frac{1}{8.617\times 10^{-5}}} = 3\times 10^{5} \text{ m/s}$$

Total mass flow:

$$\dot{m} = \dot{m}_a + \dot{m}_c$$

amount of mass flow rate at cathode is 5-10% of anode:

$$\dot{m} = \dot{m}_a + 10\%(\dot{m}_a)$$

$\cancel{\dot{m}_a}$

Thrust is given by:

$$T = \dot{m} V_{ex}$$

$$= \dot{m} (I_{sp}\, g)$$

$$123 \times 10^{-3} = \dot{m} (1756 \times 9.81)$$

$$\dot{m} = 7.104 \times 10^{-6}\ kg/s.$$

$$\dot{m} = 7.104\ mg/s.$$

$$\dot{m} = \dot{m}_a + \dot{m}_c$$

$$\therefore \dot{m} = \dot{m}_a + 10\%\, \dot{m}_a$$

$$\dot{m}_a = 6.458\ mg/s. \qquad \dot{m}_c = 0.646\ mg/s$$

Mean diameter.

Scaling index variable.   mass of Xenon atom.

$$\frac{\dot{m}_a}{d} \geq \xi \frac{\pi M V_i V_a}{\sigma_i V_e}$$

$M = 2.18017714 \times 10^{-25}$ kg

$$\frac{5.458 \times 10^{-6}}{d} \geq \frac{0.5 \times \pi \times 2.18 \times 10^{-25} \times}{5.801 \times 10^{-1}}$$

$$\therefore d \leq \frac{\sigma_i V_e \times \dot{m}_a}{\xi \pi M V_i V_a}$$

$d \leq 0.1202$ m. $= 12$cm

dimensions :

$W = 0.25 \times d$ ⟶ 3cm – Channel width.

$\delta w = 0.1 \times d$ ⟶ 1.2cm – Wall thickness.

$L = W + (2 \times \delta w)$ ⟶ 5.4 cm – Channel length.

$d_{th} = 2 \times d$ ⟶ 24 cm – Diameter of thruster.

$L_{th} = d$ ⟶ 12 cm – Length of thruster.

Current to Anode: = (Current comming out of Anode)

$$I_{\dot{m}} = \frac{e\,\dot{m}_a}{M}$$

e ← charge of electrone

$\dot{m}_a$ ← anode mass flow rate.

M ↑ mass of Xenon Atom.

$$= \frac{1.6\times10^{-19}\times 6.458\times10^{-6}}{2.18\times10^{-25}} = \underline{\underline{4.7398\ A}}$$

Discharge Current $\cong 1.3\times I_{\dot{m}}$ {1.2 - 1.4 time $I_{\dot{m}}$

$$\underline{\underline{I_d = 6.16\ A}}$$

Checking out with the market product! $V_d \times I_d = P_{input}$

$$I_d = \frac{P_{input}}{V_d} = \frac{2200\,W}{350\,V}$$

$$\underline{\underline{I_d = 6.3A}}.$$

which satisfies.

Current getting into Anode = Current of Ions getting stacked + Current of Ions giving Thrust.

Cathode efficiency :

$$\eta_c = \frac{\dot{m}_a}{\dot{m}} = \frac{\dot{m}a}{1.1\,\dot{m}.a} = 1/1.1$$

20 - 5
100 - ?
$\frac{5 \times 100}{20}$
1 - 100
10 - 100

Jet Power :

$$P_{jet} = \frac{T^2}{2\dot{m}_a}\eta_c = \frac{(123\times10^{-3})^2 \times (1/1.1)}{2\times 6.452\times10^{-6}}$$

$T = m.v$

$KE = \frac{1}{2}mv^2$. which is power of Ions

$$= \underline{\underline{1064.8\ W}}$$

$$\text{Acceleration power} = \frac{P_{jet}}{\gamma} = \frac{P_{jet}}{0.9} = \underline{\underline{1183.11\ W}}.$$

$\gamma =$

faster $x \to xe^{t}$
$x \to xe^{2t}$

$$\text{Acceleration power} = \Delta V_{acc} \times (I_{in} - I_{iw})$$

I_{in}: Entire Anode current

I_{iw}: Part of anode current that hits the wall.

$$I_{iw} = I_{in} - \frac{P_{acc}}{\Delta V_{acc}} \Rightarrow I_{iw} = \underline{\underline{0.538\ A}}$$

Length of acceleration layer = L_{ac}

Emperical Relation:

$$I_{iw} = 0.5\, K_{iw}\, I_{\dot{m}}$$

$$K_{iw} = 1.5\, L_{ac}/d$$

$L_{ac} = 0.01816\ m$

$$\text{Electric field} = \frac{\Delta V_{accl}}{L_{ac}} = \frac{281.6}{0.01816}\ V/m$$

$$\vec{E} = 15.506\ kV/m$$

$$\text{Magnetic field} := \frac{\sqrt{V_d}}{W B_{max}} = 5\sqrt{\frac{e}{M}}\,\frac{1}{0.03(e^{1.3}-1)}$$

$$\vec{B}_{max} = 0.011658\ T$$
$$= 11.658\ mT$$
$$= 116.58\ G$$

Larmor Radius Verification:

Ion Larmor Radius $= \frac{M}{eB}\left(\frac{E}{B}\right) = 155.45\ m \gg w\ (3cm)$

Electron Larmor Radius $= \frac{m}{eB}\left(E/B\right) = 0.649\ mm \ll w\ (3cm)$

Power balance and efficiency:

$$P_d = P_b + P_w + P_a + P_R + P_{ion}$$

P_d: discharge power

P_b: beam power — useful power comming out.

P_w: Power due to channel walls (due to ion/electron loss) — highest among all the power.

P_a: power to anode. (due to electron collection)

P_R: Radiative power loss — not much sometime ignored.

P_{ion}: Power to produce ions

using → Current ion efficiency

Beam power (P_b)

$$P_b = I_b V_b \qquad 1386.59\,W$$

electron temp. in kelvin

energy

sheath potential

Power to channel walls (P_w)

$$P_w = I_{iw}\left[\left(\frac{M}{2\pi m}\right)^{1/2} e^{e\phi_s/kT_e}\left(\frac{2kT_e}{e}\right) + (\mathcal{E} - \phi_s)\right]$$

$$\{\phi_s = -1.02\,T_{eV}, \quad \mathcal{E} = 0.58\,T_{eV}\}$$

$$P_w = 473.55\,W$$

ion impact is very less than electron impact.

Power to Anode (P_a)

$$P_a = 2\,T_{eV}\,I_m \cong 2\,T_{eV}\,I_a = 754.6\,W$$

current to anode ~ Discharge Current

$V_b = 330 V$ $\qquad$ $V_b = V_d - V_c$

$I_b = 4.2018 A$ $\qquad$ $I_d = I_{ib} + I_{ec}$

ideal range

voltage efficiency

$\eta_v = 330/350 = 0.942 \leftarrow (0.95)$

$\eta_v = V_b / V_d$

current efficiency.

$\eta_b = I_b / I_d = 0.682 \leftarrow (0.6 - 0.8)$

· Radiactive power loss (P_R)

$P_R = n_o n_e < \sigma_* V_e > V$

Power to produces Ions. P_{ion}

$P_{ion} = (I_b + I_{iw}) U^+ \leftarrow$ ionization potential of propellent gas.

$(4.2018 + 0.538) \times 5/100 \times V_b$

$P_{ion} = \underline{78.21 \; W}$

ted

$P_d = P_b + P_w + P_a + P_R + P_{ion}$

$2156.7 = 1386.59 + 473.55 + 154.6 + \underline{\quad} + 78.21.$

Secondary electron yield Scaling

$$\gamma = \Gamma(2+b)\, a\, T_{ev}^{b}$$

For boron nitride (BN)

$a = 0.150$

$b = 0.549$

$\Gamma(2+b) = 1.38$

For ion and Hall thruster, ions are accelerated to high exhaust velocity using electrical power source; The velocity of ions greatly exceeds that of any unionized propellant that may escape from thruster, so thrust can be described as

$$T = \frac{dm_p}{dt} v_{ex} \approx \underbrace{\dot{m}_i}_{\text{ion mass flow rate}} \underbrace{v_i}_{\text{ion velocity}}$$

$$v_i = \sqrt{\frac{2q V_b}{M}}$$

V_b ← net voltage through which ion was accelerated

M ← ion Mass

$$\dot{m}_i = \frac{I_b M}{q}$$

Substituting above EQ. ; The Thrust for singly charged propellent

$q = e$

$$\therefore T = \sqrt{\frac{2m}{e}} \times I_b \times \sqrt{V_b} \quad \text{(Newtons)}$$

here : If propellent is Xenon

$$\sqrt{2m/e} = 1.65 \times 10^{-3}$$

$$\therefore T = 1.65\, I_b \sqrt{V_b} \; [mN]$$

In amperes volts.

Total hall thruster efficieny :

$$\eta_T = \frac{T^2}{2 \dot{m}_p P_{in}}$$

$$\eta_T = \gamma^2 \eta_b \times \eta_v \times \eta_m \times \eta_o .$$

secondary emission scaling (gamma function)

discharge current efficiency

discharge voltage efficiency.

mass utilization efficiency

electrical utilization efficiency.

$$\eta_m = \frac{\dot{m}_i}{\dot{m}_p} = \frac{\dot{m}_i}{\dot{m}_a + \dot{m}_c}$$

$$\dot{m}_i = \frac{M}{e} I_d \eta_b$$

$$\eta_o = \frac{P_d}{P_T} = \frac{P_d}{P_d + P_k + P_{mag}}.$$

discharge power — keeper power — Power to generate Magnetic field.

V_0 = Neutral velocity, initial ion velocity.

n_0 = Neutral density; plasma density

n_0 = Loschmidt's number (gas density at STP) - $2.6868 \times 10^{25}\ m^{-3}$

A = Cross section Area πr^2 $r = w$

$$P_w = \frac{1}{4}\left(\frac{8kT_e}{\pi m}\right)^{1/2} e n_0 A e^{(e\phi_s/kT_e)}\left(\frac{2kT_e}{e}\right) + n_0 e V_0 A(\varepsilon - \phi_s)$$

k = Wave number = $2\pi / \lambda$

λ = mean free Path

Neutral gas atom Velocity.

$$\lambda = \frac{v_n}{n_e \langle \sigma_i v_e \rangle}$$

In Hall Thruster electron temperature is about $1/10$th the beam Voltage.

$$T_{eV} = 1/10 \times V_b$$

Calculating efficiency:

electrical efficiency

$$\eta_T = (\gamma)^2 \times \eta_e \times \eta_m$$

mass utilization efficiency.

$$= \gamma^2 \times \frac{I_b V_b}{P_d} \times \frac{\dot{m}_i}{\dot{m}_p}$$

$P_a = 2 T_{eV} I_d$

$P_{ion} = (I_b + I_{iw}) U^+$

$= (1+\eta_b) I_b U^+$

$$P_d = P_b + P_w + P_a + P_{ion}.$$

$$= I_b V_b + 0.49 I_b V_b + 2\eta_b I_b (0.01 V_b) +$$

ionization potential is roughly 5% of beam Volage.

Innavatiewe areas:

* Channel Walls.

* rather than Xe^{+}, going for Xe^{2+}

Hallow Cathode placement

Materall Selection for channel walls.