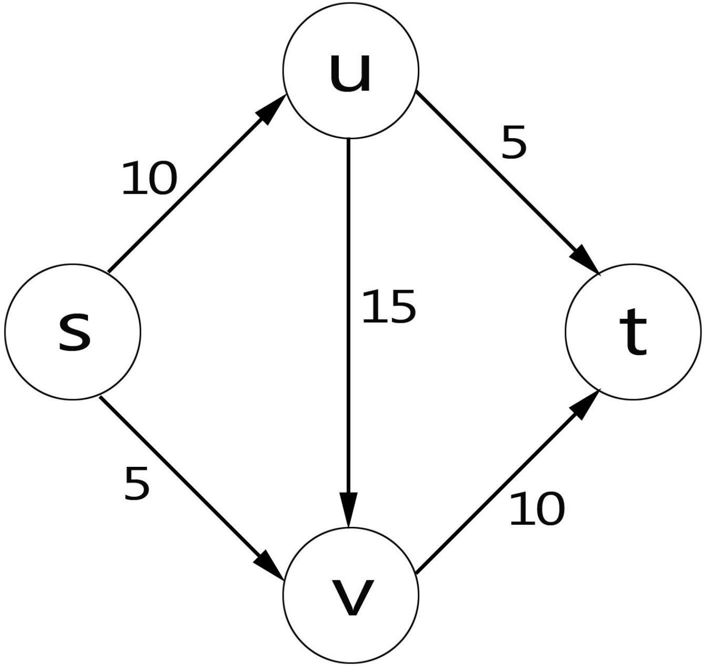
Ford Fulkerson (Max Flow)

CSCI 5454

By: Nachiket Bhagwat

Introduction

- What are Network Flow Problems
- What is Max Flow Problem
- Where is this useful



Problem Setting

Capacity constraints: $$\forall (u,v) \in E \ f(u,v) \leq c(u,v)$$

Skew symmetry: $$\forall (u,v) \in E \ f(u,v) = -f(v,u)$$

Flow conservation: $$\forall u \in V : u \neq s \text{ and } u \neq t \Rightarrow \sum_{w \in V} f(u,w) = 0$$

Value(f): $$\sum_{(s,u) \in E} f(s,u) = \sum_{(v,t) \in E} f(v,t)$$

Solutions

- Ford Fulkerson Algorithm
- Edmond Karp Algorithm

Residual Graph

Given a flow network G, and a flow f on G, we define the residual graph G_f of G with respect to f as follows.

1. The node set of G_f is the same as that of G.
2. Each edge e = (u, v) of G_f is with a capacity of c_e - f(e).
3. Each edge e' = (v, u) of G_f is with a capacity of f(e).

Residual Graph Example

Ford Fulkerson Algorithm - 1956

1. $f(u,v) \leftarrow 0$ for all edges (u,v)
2. While there is a path p from s to t in G_f, such that $c_f(u,v) > 0$ for all edges $(u,v) \in p$
   :
   1. Find $c_f(p) = \min\{c_f(u,v) : (u,v) \in p\}$
   2. For each edge $(u,v) \in p$
      1. $f(u,v) \leftarrow f(u,v) + c_f(p)$ (*Send flow along the path*)
      2. $f(v,u) \leftarrow f(v,u) - c_f(p)$ (*The flow might be "returned" later*)

Augmenting path. Find an undirected path from s to t such that:

- Can increase flow on forward edges (not full).
- Can decrease flow on backward edge (not empty).

Ford Fulkerson Algorithm Example

Running Time Analysis for Integral Case

Running Time = Time Taken for 1 iteration * #Iterations

Non Terminating Case

Edmond Karp Algorithm

```
EDMONDS-KARP ALGORITHM(G,s,t)

begin
   initialise flow f to 0
   while there exists a shortest augmenting path p in
   the residual network Gf do
      augment flow f along p
   end
end
```

Edmonds-Karp Algorithm Example

Proof Of Termination in Polynomial Time

Running Time = Time Taken for 1 iteration * #Iterations

Citations

http://cms.math.ca/openaccess/cjm/v8/cjm1956v08.0399-0404.pdf

https://en.wikipedia.org/wiki/Maximum_flow_problem

https://en.wikipedia.org/wiki/Ford%E2%80%93Fulkerson_algorithm

https://en.wikipedia.org/wiki/Edmonds%E2%80%93Karp_algorithm