GRASP

Exercise Given the following instance of the *Parallel Machine Scheduling Problem* (*PMSP*) with 3 machines:

t	a	b	c	d	e	f
d_t	27	12	15	25	11	16

a) apply a basic greedy heuristic in which the selection criterium is split into two steps:

1. select the task with the maximum time length: $i^* := \arg\max_{t \in T} d_t$;
2. select the machine that minimises the objective function: $m^* := \arg\min_{m \in M} f(x \cup \{t, m\})$.

In case of ties, in both steps choose the item of minimum index.

b) Apply a *GRASP* metaheuristic with a *Restricted Candidate List* (*RCL*) of two elements, assuming the following sequence of pseudorandom numbers: $r = (0.2, 0.2, 0.3, 0.8, 0.6, 0.1)$.

Solution **Part a)** The basic greedy heuristic performs the following operations:

1. start with $x := \emptyset$;
2. select task a, compute the processing time on the three machines for each possible choice (respectively, $(27, 0, 0)$ for m_1, $(0, 27, 0)$ for m_2 and $(0, 0, 27)$ for m_3) and select the one implying the minimum completion time, that is m_1, (based on the minimum index rule for ties);
3. select task d, compute the processing time on the three machines for each possible choice (respectively,$(52, 0, 0)$ for m_1, $(27, 25, 0)$ for m_2 and $(27, 0, 25)$ for m_3) and select the one implying the minimum completion time, that is m_2;
4. select task f, compute the processing time on the three machines for each possible choice (respectively $(43, 25, 0)$, $(27, 41, 0)$, $(27, 25, 16)$) and choose the minimum one, that is m_3;
5. select task c, compute the processing time on the three machines for each possible choice (respectively $(42, 25, 16)$, $(27, 40, 16)$, $(27, 25, 31)$) and choose the minimum one, that is m_3;
6. select task b, compute the processing time on the three machines for each possible choice (respectively $(39, 25, 31)$, $(27, 37, 31)$, $(27, 25, 43)$) and choose the minimum one, that is m_2;
7. select task e, compute the processing time on the three machines for each possible choice (respectively $(38, 37, 31)$, $(27, 48, 31)$, $(27, 37, 43)$) and choose the minimum one, that is m_1;
8. terminate, because there is no possible augmentation.

The final solution assigns tasks a and e to m_1, tasks b and d to m_2 tasks c and f to m_3, with total processing times equal to $(38, 37, 31)$ and a completion time $f(x) = 38$.

Part b) The *GRASP* metaheuristic finds at each step the two best alternatives and selects the first when $r \leq 0.5$, the second otherwise:

1. start with $x := \emptyset$;
2. select task a, compute the processing time on the three machines for each possible choice (respectively, $(27, 0, 0)$ for m_1, $(0, 27, 0)$ for m_2 and $(0, 0, 27)$ for m_3), put m_1 and m_2 in the RCL (based on the minimum index rule for ties) and choose m_1 because $r = 0.2$;
3. select task d, compute the processing time on the three machines for each possible choice (respectively,$(52, 0, 0)$ for m_1, $(27, 25, 0)$ for m_2 and $(27, 0, 25)$ for m_3), put m_2 and m_3 in the RCL (both with a completion time of 27) and choose m_2 because $r = 0.2$;
4. select task f, compute the processing time on the three machines for each possible choice (respectively $(43, 25, 0)$, $(27, 41, 0)$, $(27, 25, 16)$), put m_3 (competion time 27) and m_2 (completion time 41) in the RCL and choose m_3 because $r = 0.3$;
5. select task c, compute the processing time on the three machines for each possible choice (respectively $(42, 25, 16)$, $(27, 40, 16)$, $(27, 25, 31)$) put m_3 (competion time 31) and m_2 (completion time 40) in the RCL and choose m_2 because $r = 0.8$;
6. select task b, compute the processing time on the three machines for each possible choice (respectively $(39, 40, 16)$, $(27, 52, 16)$, $(27, 40, 28)$), put m_1 (competion time 40) and m_3 (completion time 40) in the RCL and choose m_3 because $r = 0.6$;
7. select task e, compute the processing time on the three machines for each possible choice (respectively $(38, 40, 28)$, $(27, 51, 28)$, $(27, 40, 39)$), put m_1 (competion time 40) and m_3 (completion time 40) in the RCL and choose m_1 because $r = 0.1$;
8. terminate, because there is no possible augmentation.

The final solution assigns tasks a and e to m_1, tasks c and d to m_2 tasks b and f to m_3, with total processing times equal to $(38, 40, 28)$ and a completion time $f(x) = 40$.