Results from the Sequential Auction Model of CPVR (2006)

Ignacio Cigliutti

New York University

October 25, 2021

1 Sequential Auction Model

For completeness, I replicate Katka's notes about the model of Cahuc, Postel–Vinay & Robin (2006) here.

1.1 The Model

We consider an economy populated by heterogeneous workers who differ in their productive type x. Each worker has a constant birth/death rate μ (or lives for 35 years, it probably does not matter). A newborn worker draws her type from the distribution $F(x)$, with the probability density function given by $f(x)$.

Firms differ in technologies they operate, which we denote by y. There is a distribution of types, $G(y)$. Unemployed workers receive offers from this distribution with probability λ_0, employed with probability λ_1.

The production function has constant returns to scale. A match between a worker with type x and a firm with type y produces output xy. An employed worker receive wage which is determined by bargaining as in Cahuc, Postel-Vinay, Robin, with worker's bargaining power given by α. A job separates for exogenous reasons at the rate δ. An unemployed worker receives value b (or bx).

Let U, W, J be the values of being unemployed and employed, and the value of the job, respectively. Let S be the surplus of a job. The wages are determined as in Cahuc, Postel-Vinay, Robin, and they will be a function of the worker's type x, firm's type y and also firm type of worker's employer, y_{-1}, $w(x, y, y_{-1})$. If a worker comes from unemployment, we denote her last employer as $y_{-1} = u$.

When an unemployed worker of type x and firm of type y meet, they split the surplus according to

$$W(x, y, u) - U(x) = \alpha S(x, y).$$

If a worker x employed at y with negotiation benchmark given by y_{-1} receives an offer from y', then three things can happen. If $S(x,y') > S(x,y)$, the worker accepts the offer and becomes employed at the new firm. Her wage will be such that she receives her previous surplus and a α-share of the extra surplus generated by the move,

$$W(x,y',y) - U(x) = S(x,y) + \alpha\left(S(x,y') - S(x,y)\right).$$

We denote the set of firm types that a worker would accept as $M_1(x,y)$. That is, a worker moves iff $y' \in M_1(x,y)$.

Second, if $S(x,y') < S(x,y)$, the worker stays with the current firm x but might be able to negotiate a wage increase if the contacting firm's type y' is better than her current negotiation benchmark y_{-1}: $S(x,y) > S(x,y') > S(x,y_{-1})$. Then

$$W(x,y,y') - U(x) = S(x,y') + \alpha\left(S(x,y) - S(x,y')\right).$$

We denote the set of y' for which this happens as $M_2(x,y,y_{-1})$.

Finally, if the contacting firm is worse than the negotiation benchmark, $S(x,y') < S(x,y_{-1})$, then the worker does not leave and does not re-negotiate her wage.

With this, we can formulate the value functions. Let $U(x)$ be the value of being unemployed:

$$\begin{aligned} U(x) = bx + \beta &\left(\lambda_0 \int_{y' \in M_1(u)} W(x,y,u)dG(y)\right) \\ &+ \beta\left(1 - \lambda_0 \int_{y' \in M_1(u)} dG(y)\right) U(x). \end{aligned}$$

The value of being employed at firm y with negotiation benchmark y_{-1} is

$$\begin{aligned} W(x,y,y_{-1}) = w(x,y,y_{-1}) &+ \beta(1-\delta)\left(\lambda_1 \int_{y' \in M_1(y)} W(x,y',y)dG(y')\right) \\ &+ \beta(1-\delta)\left(\lambda_1 \int_{y' \in M_2(x,y,y_{-1})} W(x,y,y')dG(y')\right) \\ &+ \beta(1-\delta)\left(1 - \lambda_1 \int_{y' \in M_1(x,y) \cup M_2(x,y,y_{-1})} dG(y')\right) W(x,y,y_{-1}) \\ &+ \beta\delta U(x). \end{aligned}$$

The value of the job is

$$\begin{aligned} J(x,y,y_{-1}) = xy - w(x,y,y_{-1}) &+ \beta(1-\delta)\lambda_1 \int_{y' \in M_2(x,y,y_{-1})} J(x,y,y')dG(y') \\ &+ \beta(1-\delta)\left(1 - \lambda_1 \int_{y' \in M_1(x,y) \cup M_2(x,y,y_{-1})} dG(y')\right) J(x,y,y_{-1}). \end{aligned}$$

Finally, the surplus satisfies

$$S(x,y) = xy - bx + \beta(1-\delta)\left(S(x,y) + \alpha\lambda_1 \int_{y' \in M_1(x,y)} (S(x,y') - S(x,y))dG(y')\right)$$
$$- \beta\alpha\lambda_0 \int_{y' \in M_1(x,u)} S(x,y')dG(y')$$

As usual, the surplus does not depend on wage.

We can solve the model for each type x separately since there are not interactions between the types. We can start with 5 types of x, and all other values common.

1.2 Solution

We can solve the model separately for each x, so I am going to simplify by dropping x from the arguments. When we introduce x back, the only thing that changes is production function from y to xy, and unemployment benefits from b to bx.

We can easily solve for the surplus $S(y)$. Once we have $S(y)$, then we have

$$W(y,u) - U = \alpha S(y),$$

which we substitute into value function for U. We observe that $M_1(u)$ is given by $S(y) \geq 0$:

$$U = b + \beta\left(\lambda_0 \int_{y:S(y)\geq 0} (\alpha S(y) + U)\, dG(y)\right)$$
$$+ \beta\left(1 - \lambda_0 \int_{y:S(y)\geq 0} dG(y)\right) U,$$

which after some manipulation leads to

$$U = \frac{b}{1-\beta} + \frac{\beta}{1-\beta}\lambda_0\alpha \int_{y:S(y)\geq 0} S(y)dG(y).$$

Knowing $S(y)$, we use this equation to get U.

Now we have U and $S(y)$ for all y. We can use the surplus splitting rule to compute $W(y, y_{-1})$ for any y, y_{-1} such that $S(y) \geq S(y_{-1})$:

$$W(y, y_{-1}) = U + S(y_{-1}) + \alpha\left(S(y) - S(y_{-1})\right).$$

Once we have this $W(y, y_{-1})$, we finally compute wages $w(y, y_{-1})$ from the value function directly:

$$
\begin{aligned}
W(y, y_{-1}) = w(y, y_{-1}) &+ \beta(1-\delta)\left(\lambda_1 \int_{y' \in M_1(y)} W(y', y) dG(y')\right) \\
&+ \beta(1-\delta)\left(\lambda_1 \int_{y' \in M_2(y, y_{-1})} W(y, y') dG(y')\right) \\
&+ \beta(1-\delta)\left(1 - \lambda_1 \int_{y' \in M_1(y) \cup M_2(y, y_{-1})} dG(y')\right) W(y, y_{-1}) \\
&+ \beta\delta U.
\end{aligned}
$$

In the above equation, we know all terms besides $w(y, y_{-1})$.

2 Calibration and Notes

Parameter	Value
z	0
β	0.98
α	0.13
δ	0.024
λ_0	0.450
λ_1	0.020

Table 1: Calibration is done in Quarterly frequency

Additionally, firm's productivity distribution, $G(y)$, is parametrized with a Log–Normal distribution with parameters $(\mu, \sigma^2) = (\log(200), 0.4)$, while worker's heterogeneity is given by $\mathcal{U}[0, 4]$. Note that stationary unemployment rate is given by $u = \frac{\delta}{\delta + \lambda_0} \simeq 0.05$

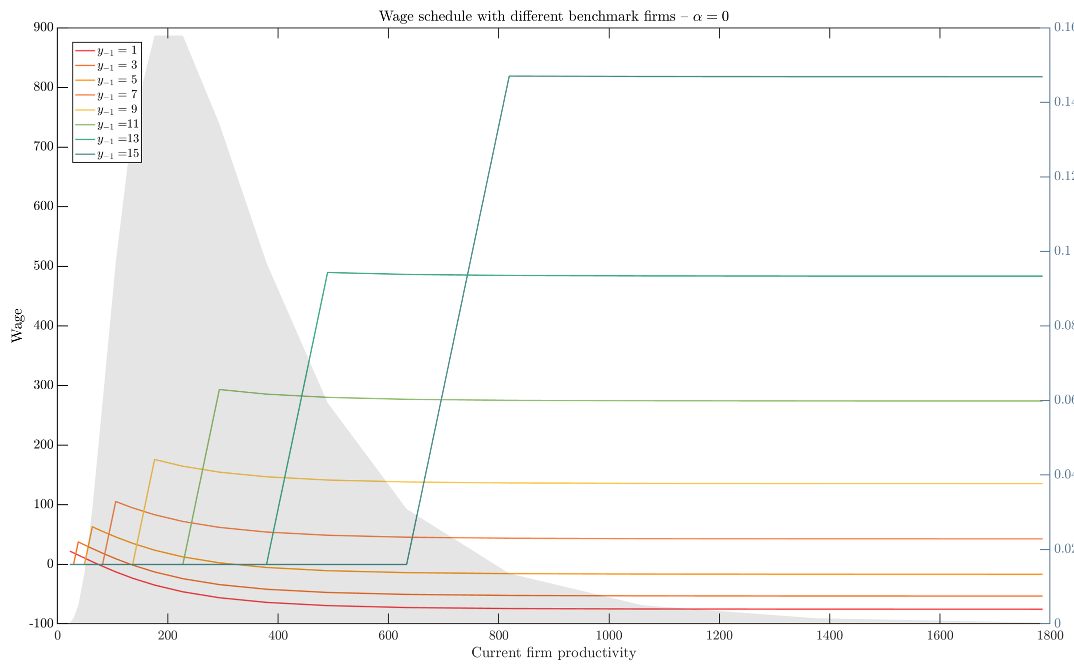
- Shaded area in the wage schedule is the density $g(y)$
- The lowest value for y_{-1} represent unemployment, so this wage schedule is the one the workers face when they are coming out of unemployment.
- Shaded area in the evolution of wages are the potential wages for UE transitions (out of unemployment)
- Contribution for each component of wage schedule helps to understand the patterns for mean productivity at the beginning of career.
- Results using the median are at the end.

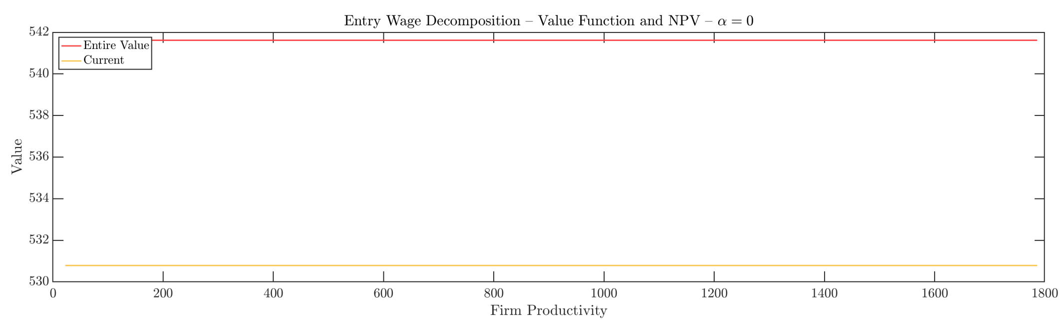
- I've run the model with monthly and yearly frequency. esults changing frequency to monthly or yearly. Results (in particular for # firms) are qualitatively and quantitatively similar.

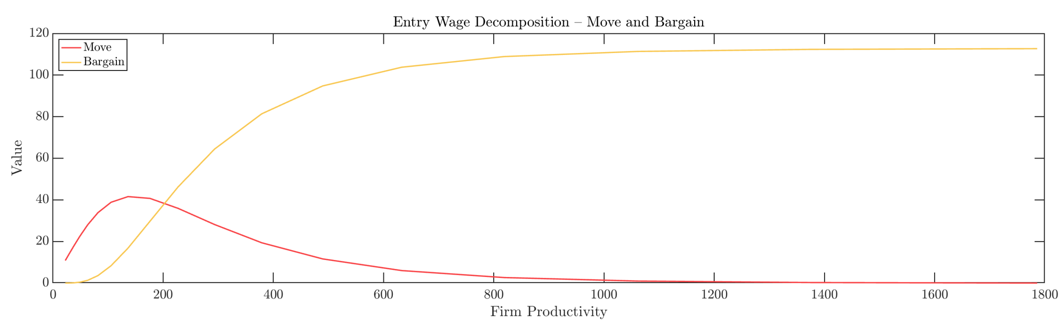
- For the calibrated values of contact and destruction rates $(\delta, \lambda_0, \lambda_1)$ the model starts running into trouble as I increase the bargaining parameter α. In particular, for $\alpha > 0.15$ it fails to converge.

3 Results

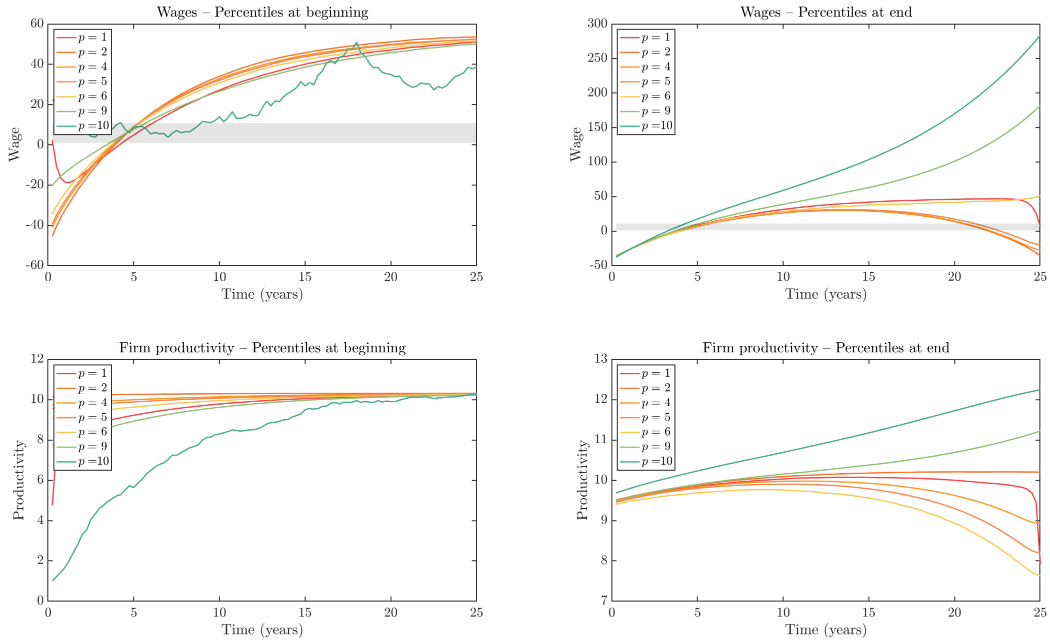
3.1 $\alpha = 0$



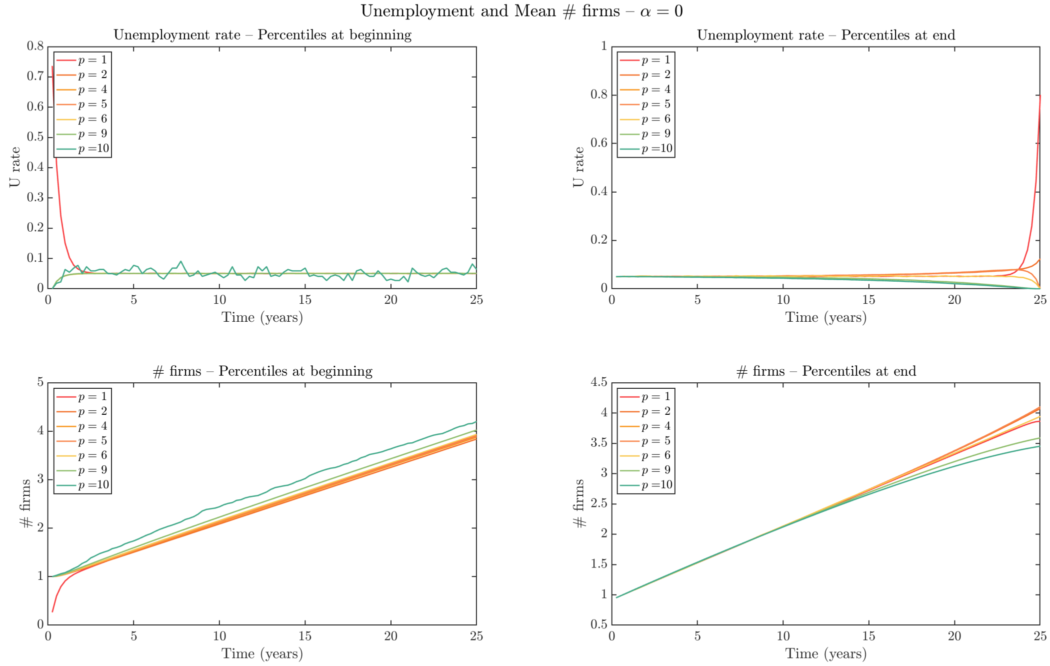




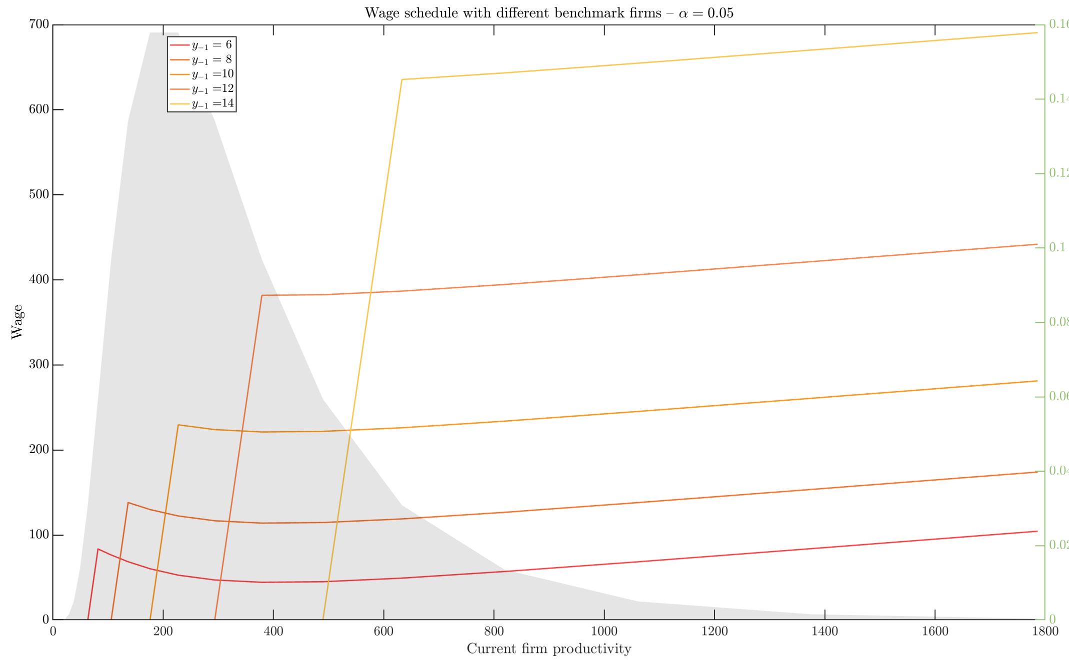
Mean Wages and Productivity – $\alpha = 0$

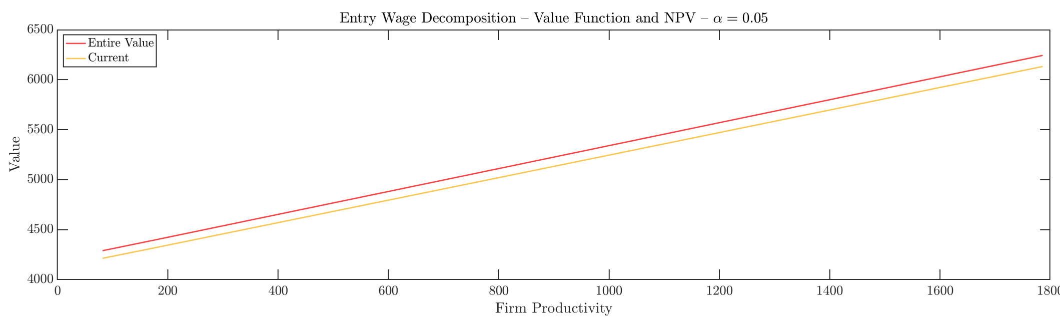


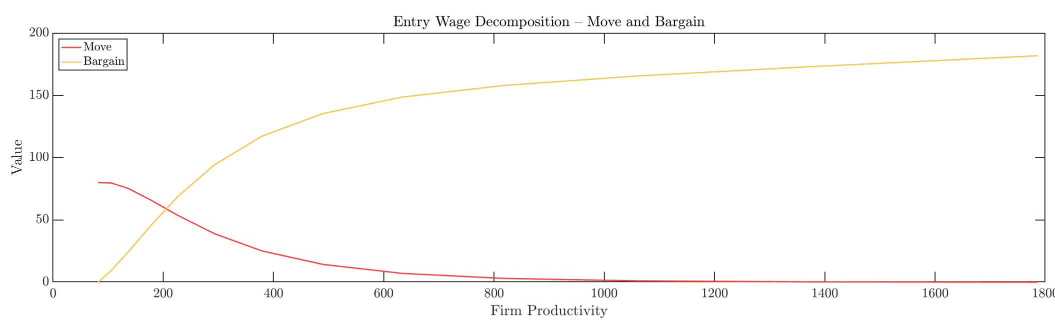
Unemployment and Mean # firms – $\alpha = 0$



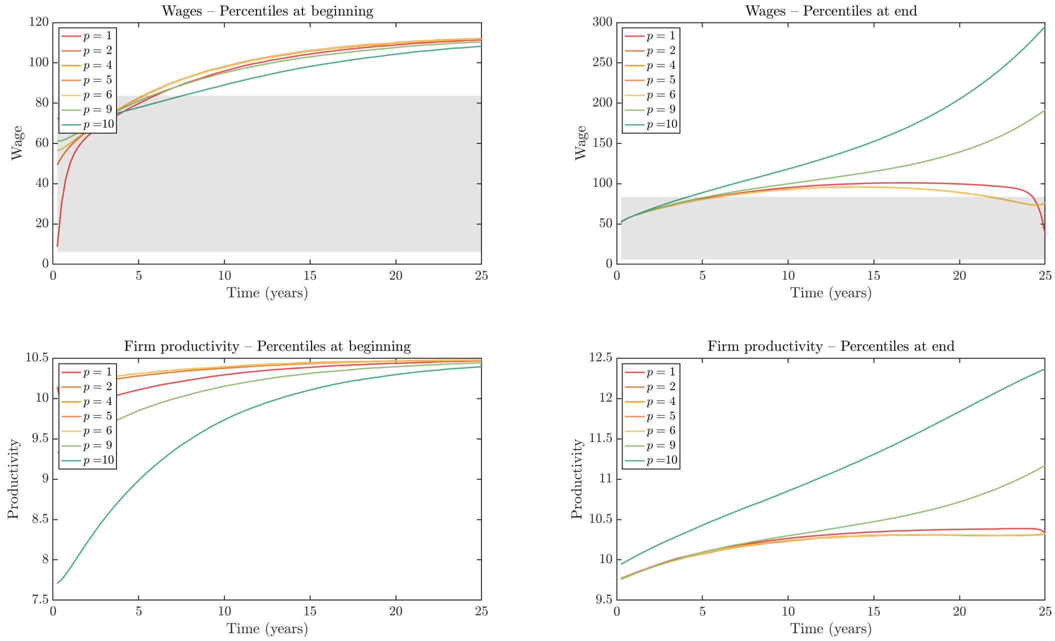
3.2 $\alpha = 0.05$


Wage schedule with different benchmark firms – α = 0.05
y−1 = 6
y−1 = 8
y−1 =10
y−1 =12
y−1 =14
Wage
Current firm productivity

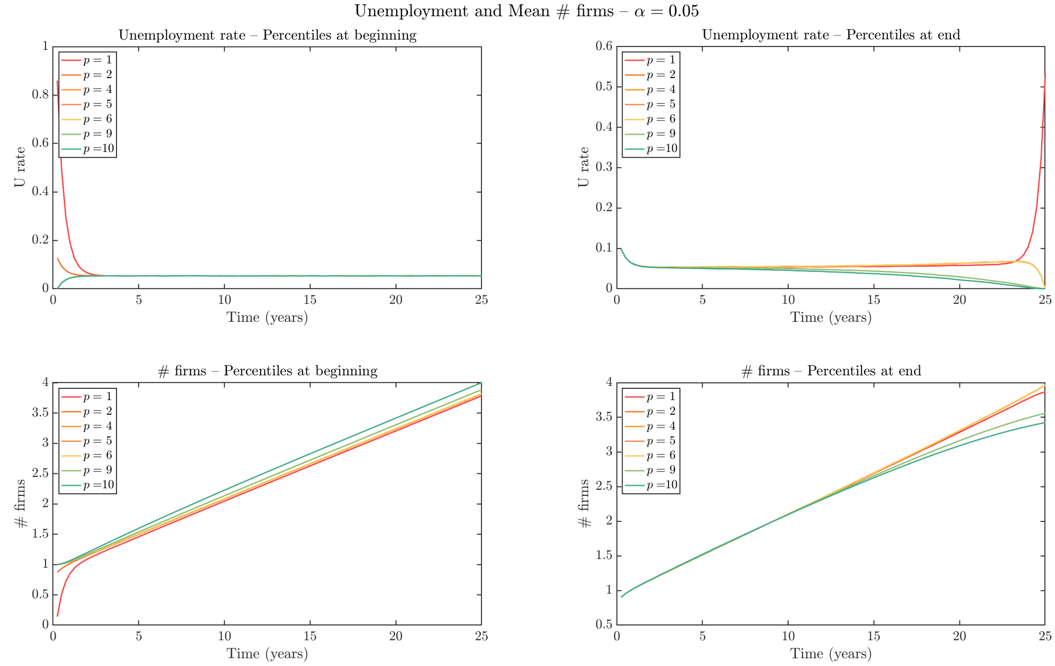

Entry Wage Decomposition – Value Function and NPV – α = 0.05
Entire Value
Current
Value
Firm Productivity


Entry Wage Decomposition – Move and Bargain
Move
Bargain
Value
Firm Productivity

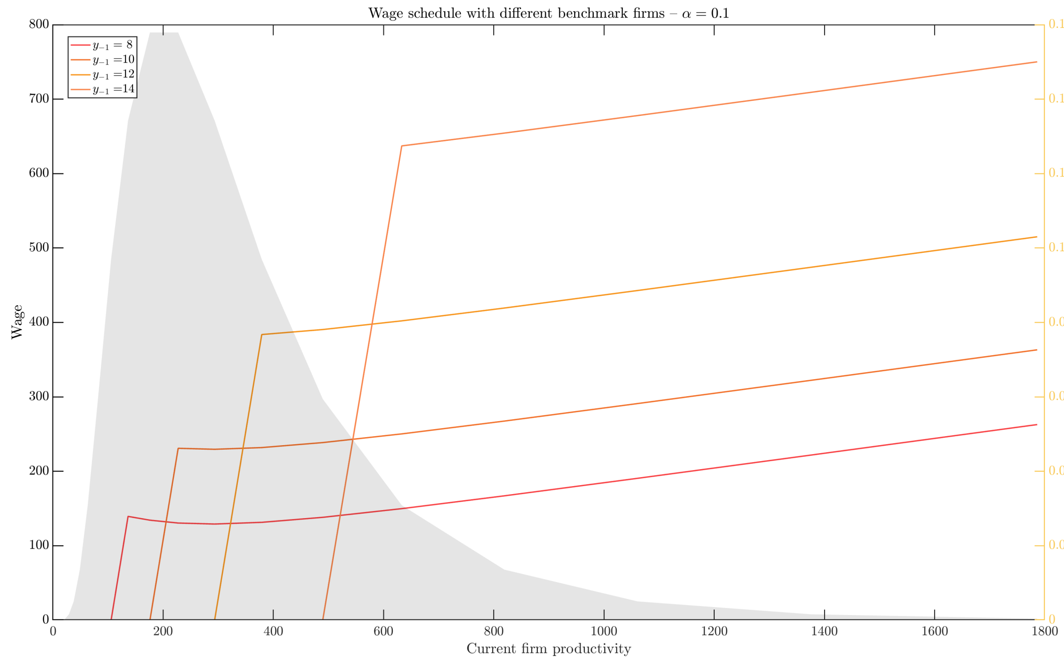
Mean Wages and Productivity – $\alpha = 0.05$

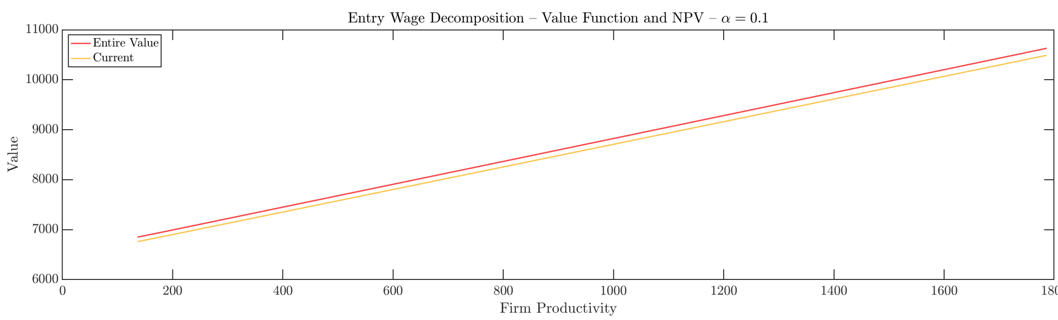


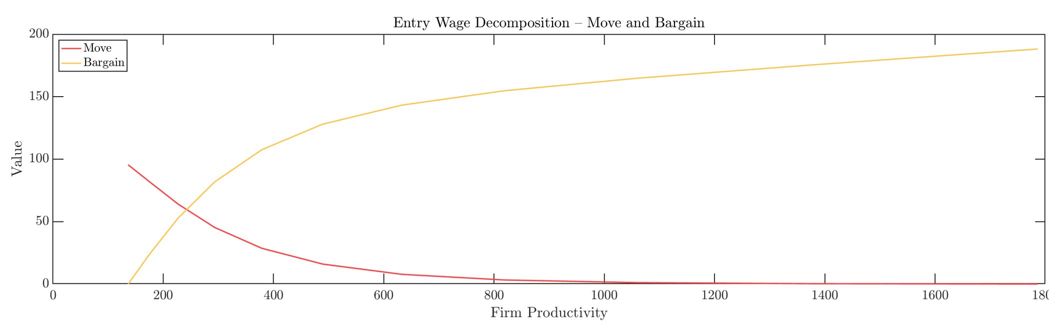
Unemployment and Mean # firms – $\alpha = 0.05$

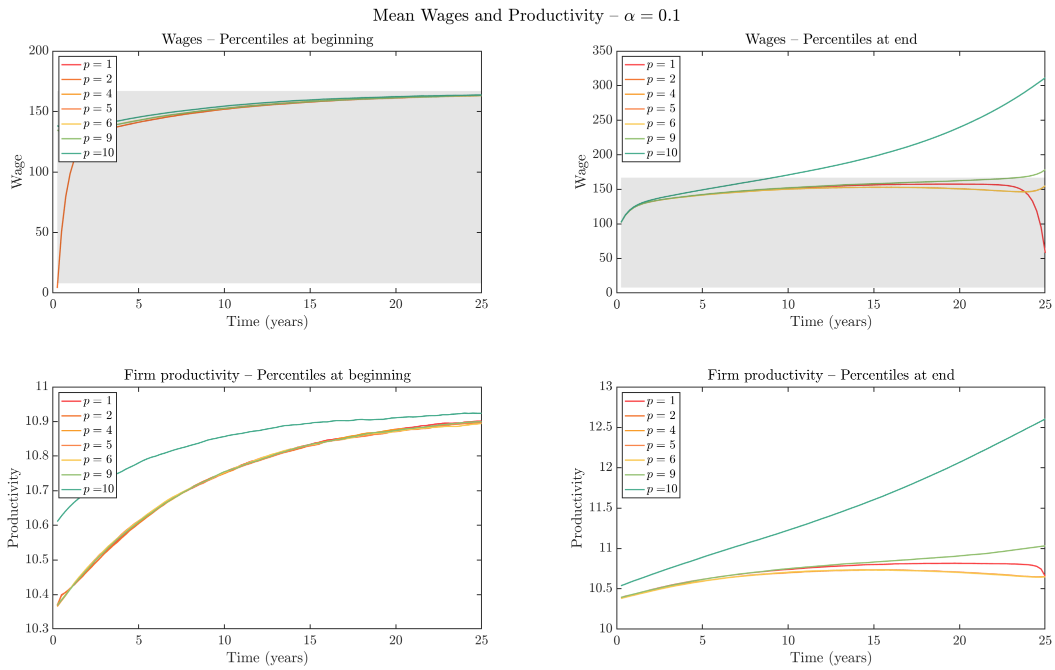


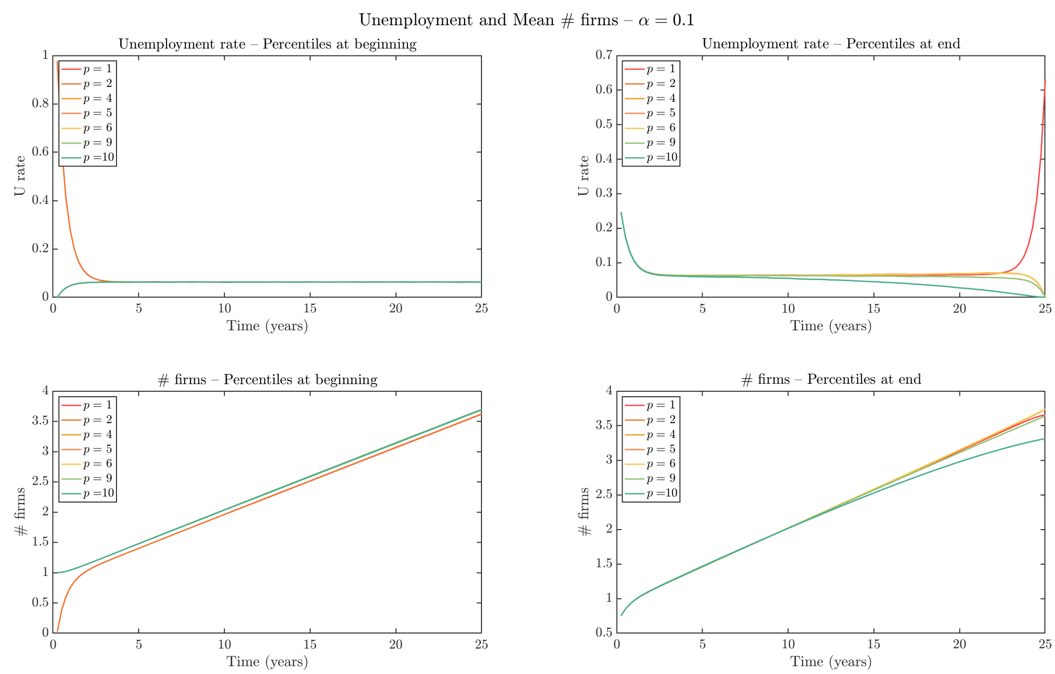
3.3 $\alpha = 0.10$


Wage schedule with different benchmark firms – $\alpha = 0.1$
$y_{-1} = 8$
$y_{-1} = 10$
$y_{-1} = 12$
$y_{-1} = 14$
Wage
Current firm productivity

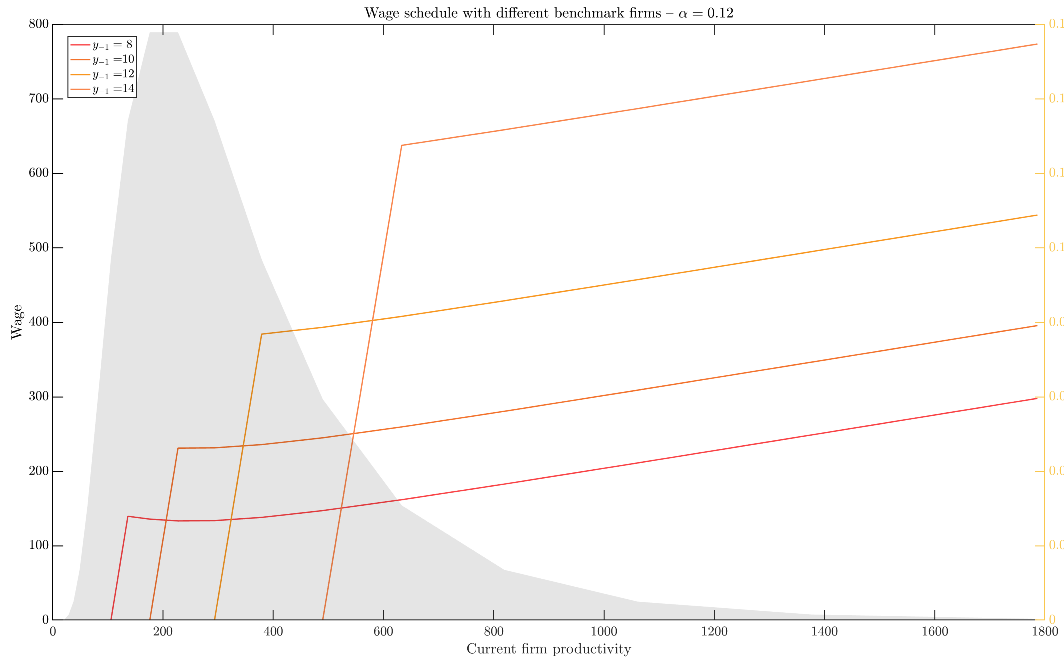

Entry Wage Decomposition – Value Function and NPV – $\alpha = 0.1$
Entire Value
Current
Value
Firm Productivity

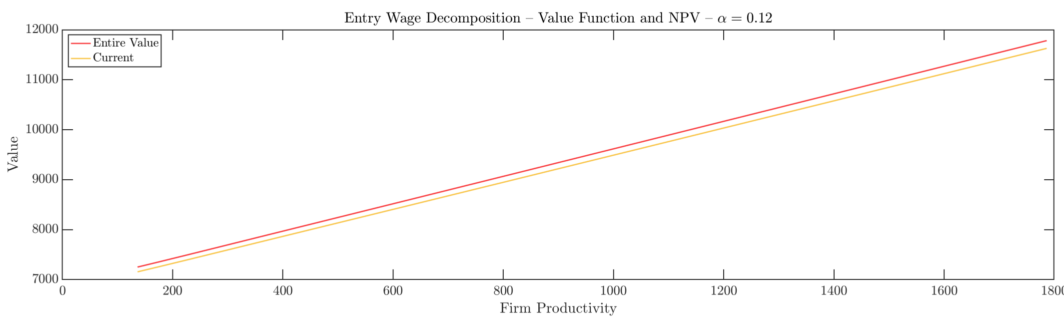

Entry Wage Decomposition – Move and Bargain
Move
Bargain
Value
Firm Productivity

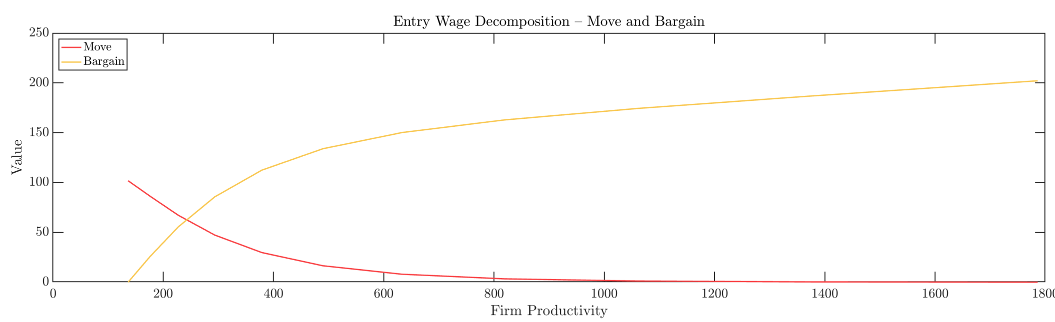

Mean Wages and Productivity – $\alpha = 0.1$
Wages – Percentiles at beginning
Wages – Percentiles at end
Firm productivity – Percentiles at beginning
Firm productivity – Percentiles at end
Wage
Productivity
Time (years)
$p = 1$
$p = 2$
$p = 4$
$p = 5$
$p = 6$
$p = 9$
$p = 10$


Unemployment and Mean # firms – $\alpha = 0.1$
Unemployment rate – Percentiles at beginning
Unemployment rate – Percentiles at end
firms – Percentiles at beginning
firms – Percentiles at end
U rate
firms
Time (years)
$p = 1$
$p = 2$
$p = 4$
$p = 5$
$p = 6$
$p = 9$
$p = 10$

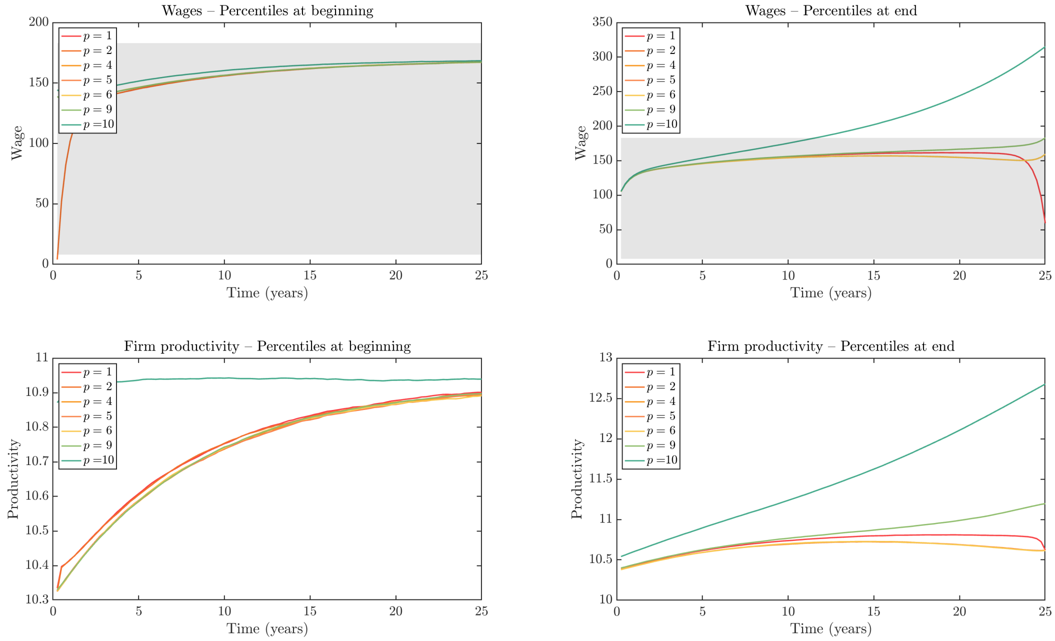
3.4 $\alpha = 0.12$


Wage schedule with different benchmark firms – α = 0.12
y−1 = 8
y−1 =10
y−1 =12
y−1 =14
Wage
Current firm productivity

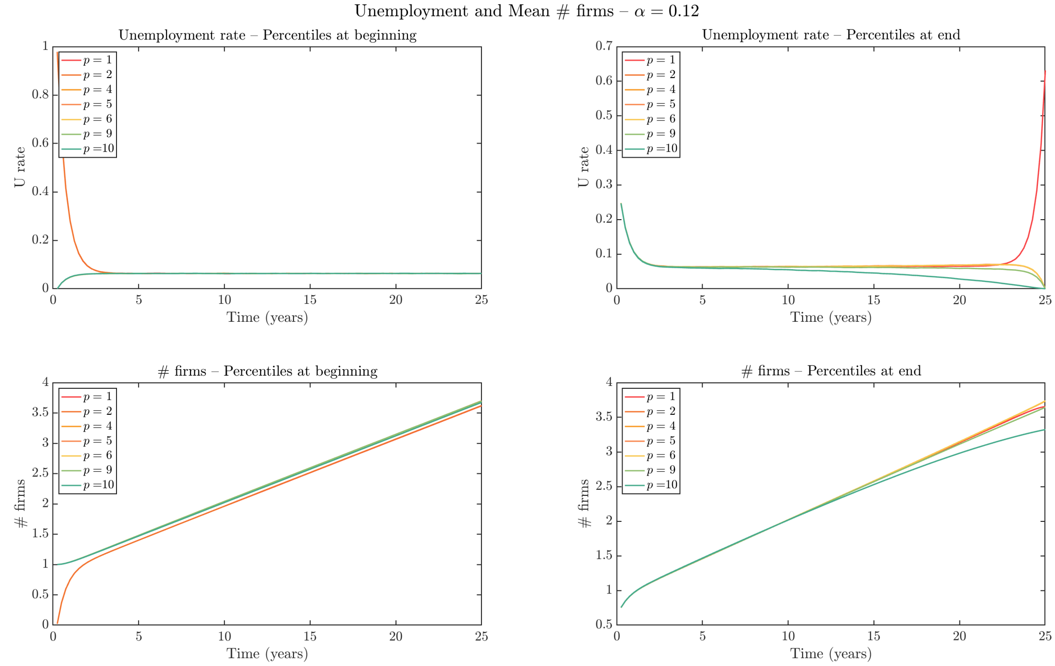

Entry Wage Decomposition – Value Function and NPV – α = 0.12
Entire Value
Current
Value
Firm Productivity


Entry Wage Decomposition – Move and Bargain
Move
Bargain
Value
Firm Productivity

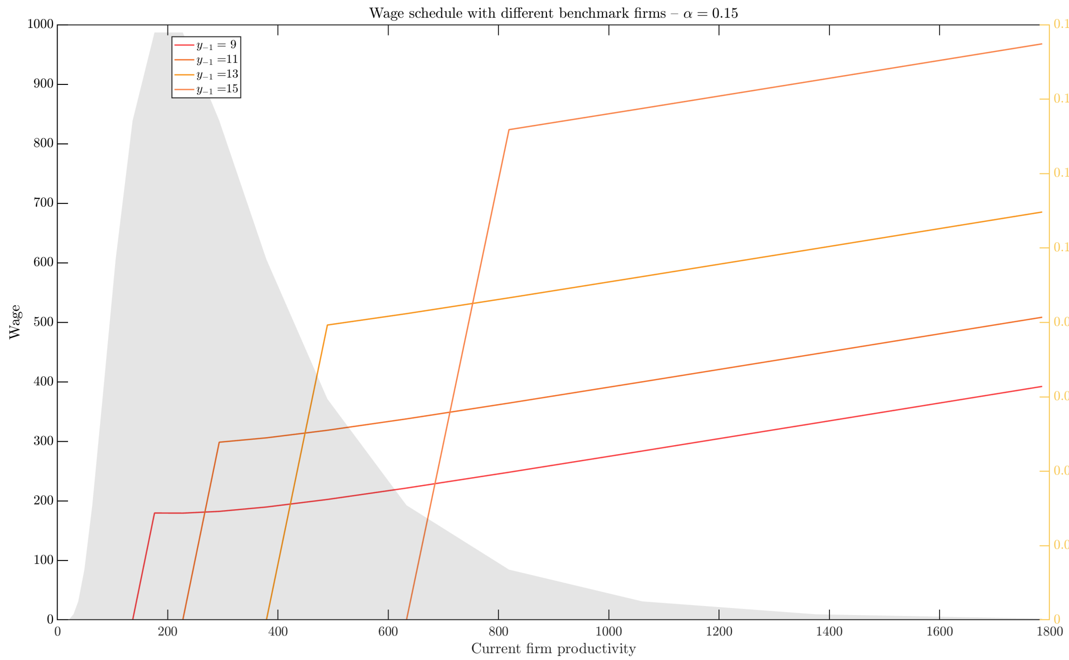
Mean Wages and Productivity – $\alpha = 0.12$



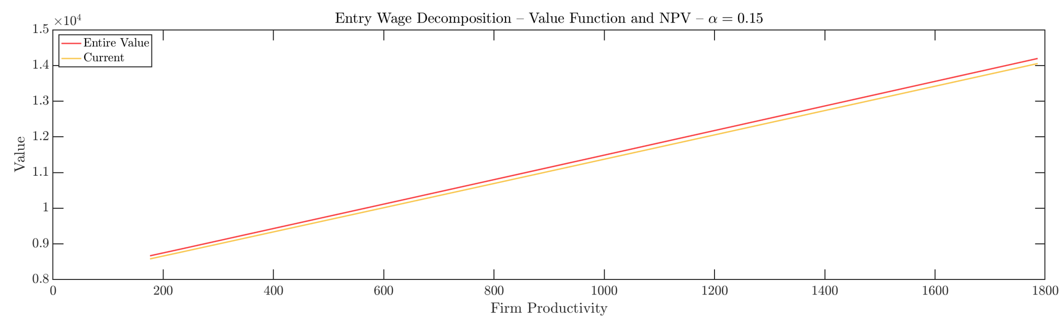
Unemployment and Mean # firms – $\alpha = 0.12$



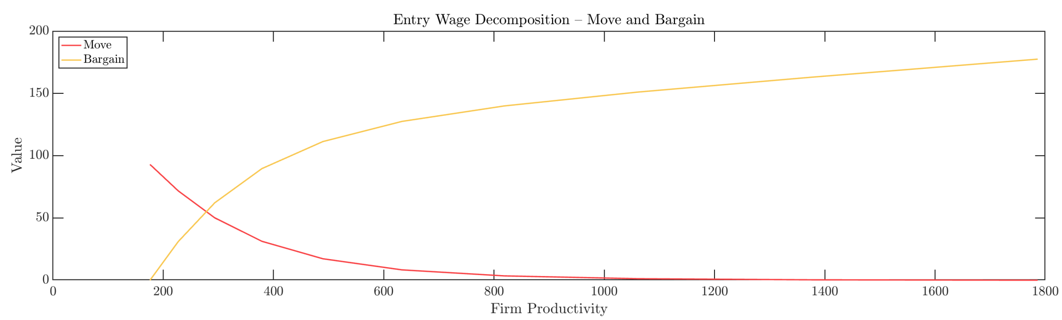
3.5 $\alpha = 0.15$



Wage schedule with different benchmark firms – $\alpha = 0.15$
$y_{-1} = 9$
$y_{-1} = 11$
$y_{-1} = 13$
$y_{-1} = 15$
Wage
Current firm productivity

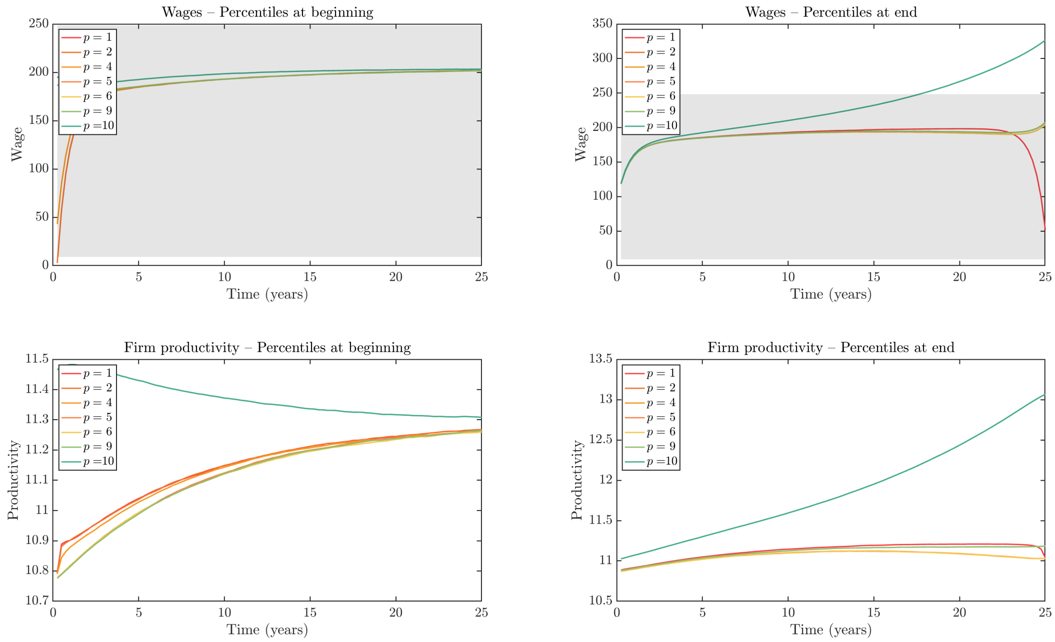


Entry Wage Decomposition – Value Function and NPV – $\alpha = 0.15$
$\times 10^4$
Entire Value
Current
Value
Firm Productivity



Entry Wage Decomposition – Move and Bargain
Move
Bargain
Value
Firm Productivity

Mean Wages and Productivity – $\alpha = 0.15$



Unemployment and Mean # firms – $\alpha = 0.15$