

# Collective risk

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# Introduction

- Goal:
  - Compute the probability that the capital of an insurance company remains positive during a given time period
- Data:
  - Premium:  $a$
  - Claims rate:  $Poisson(\lambda)$
  - Premium amount:  $Pareto(2.5, 100)$
  - Enrollment rate:  $Poisson(\nu)$
  - Departure rate:  $Exp(\mu)$
  - Initial capital:  $c_0$

# Introduction

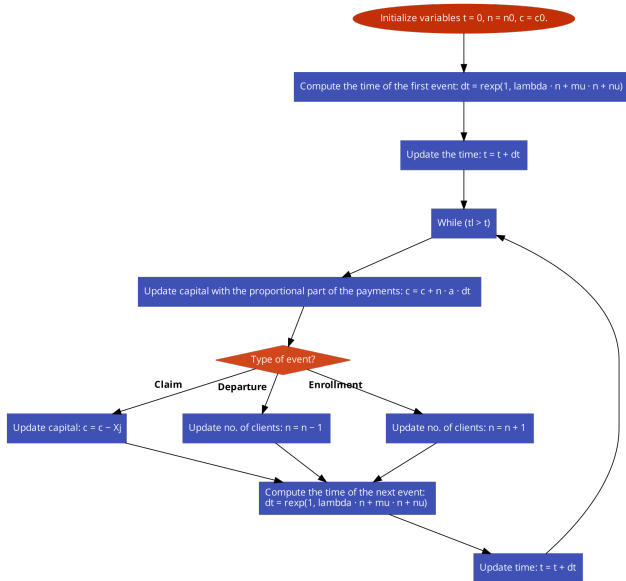
So in general, the capital of the company at any time  $t$  will be:

$$C(t) = c_0 + at(n_0 + N_A(t) - N_D(t)) - \sum_{j=1}^{N_C(t)} X_j$$

where:

- $N_A(t)$  is the number of clients that arrive by time  $t$
- $N_D(t)$  is the number of clients that leave by time  $t$
- $N_C(t)$  is the number of claims that arrive by time  $t$
- $X_j$  is the amount of the  $j$ -th claim
- $n(t)$  is the number of clients at time  $t$ .

# The project



# Results

We used 2 different approaches for this problem

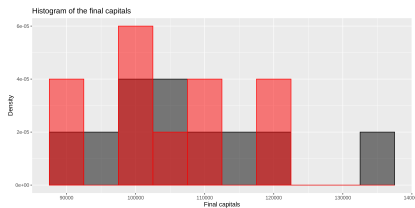
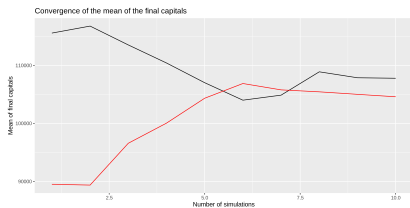
- A discrete event simulation algorithm we developed
- An improved version of our algorithm using antithetic variables to reduce the variance

These were the results for  $c_0 = 1000$ ,  $n_0 = 100$ ,  $a = 100$ ,  $t_l = 100$ ,  $\lambda = 0.1$ ,  $\mu = 0.1$ ,  $\nu = 0.1$  and  $MC = 10000$ :

<code>fraction</code>	<code>mean_final_capital</code>	<code>sd_final_capital</code>
0.9967	106475.971675903	10516.1123920357
1	106666.637870849	10361.4480989

# Results

These graphs show both the convergence of the mean of the final capitals in terms of the number of simulations and the density of the final capitals. Note that the color red indicates the antithetic variables approach



# Conclusions

- The antithetic approach does not significantly reduce the variance but it does improve the computing time, cutting it in half
- We also encountered some difficulties while developing the simulations
  - Updating the proportional part of payments after simulating the type of event
  - Trying to obtain the total number of events beforehand
  - Antithetic variable approach showing the same results as our own approach
- The project was a fine representation of how simulations can be used in the insurance policies business



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