Collective risk

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Simulation in Prob and Stats BSc AMC at UC3M

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Contents

Introduction

- Goal:
  - Compute the probability that the capital of an insurance company remains positive during a given time period
- Data:
  - Premium: a
  - Claims rate: $Poisson(\lambda)$
  - Premium amount: $Pareto(2.5, 100)$
  - Enrollment rate: $Poisson(\nu)$
  - Departure rate: $Exp(\mu)$
  - Initial capital: c_0

Introduction

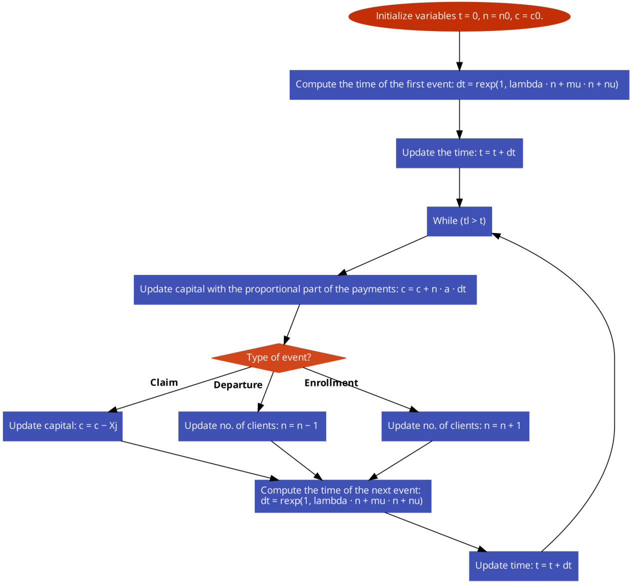
So in general, the capital of the company at any time t will be:

$$C(t) = c_0 + at(n_0 + N_A(t) - N_D(t)) - \sum_{j=1}^{N_C(t)} X_j$$

where:

- $N_A(t)$ is the number of clients that arrive by time t
- $N_D(t)$ is the number of clients that leave by time t
- $N_C(t)$ is the number of claims that arrive by time t
- X_j is the amount of the j-th claim
- $n(t)$ is the number of clients at time t.

The project


Initialize variables t = 0, n = n0, c = c0.
Compute the time of the first event: dt = rexp(1, lambda · n + mu · n + nu)
Update the time: t = t + dt
While (tl > t)
Update capital with the proportional part of the payments: c = c + n · a · dt
Type of event?
Claim
Departure
Enrollment
Update capital: c = c − Xj
Update no. of clients: n = n − 1
Update no. of clients: n = n + 1
Compute the time of the next event:
dt = rexp(1, lambda · n + mu · n + nu)
Update time: t = t + dt

Results

We used 2 different approaches for this problem

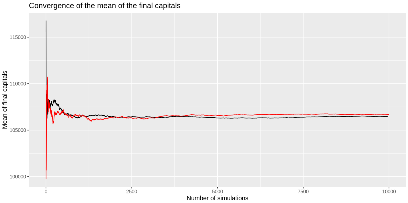
- A discrete event simulation algorithm we developed
- An improved version of our algorithm using antithetic variables to reduce the variance

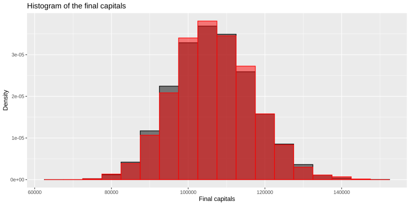
These were the results for $c_0 = 1000$, $n_0 = 100$, $a = 100$, $tl = 100$, $\lambda = 0.1$, $\mu = 0.1$, $\nu = 0.1$ and $MC = 10000$:

fraction	mean_final_capital	sd_final_capital
0.9967	106475.971675903	10516.1123920357
1	106666.637870849	10361.4480989

Results

1 Simulating with $c_0 = 1000$, $n_0 = 100$, $a = 100$, $t_l = 100$, $\lambda = 0.1$, $\mu = 0.1$, $\nu = 0.3$ and $M_C = 10000$.





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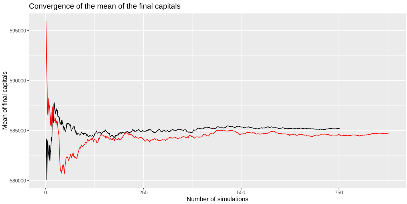
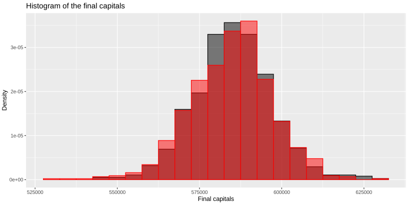
Results

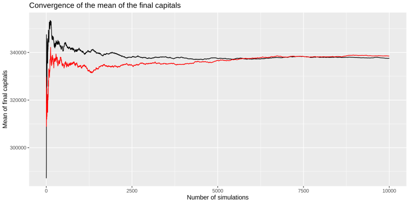
2 Simulating with $c_0 = 100$, $n_0 = 10000$, $a = 20$, $t_l = 50$, $\lambda = 0.05$, $\mu = 0.2$, $\nu = 0.6$ and $M_C = 1000$.

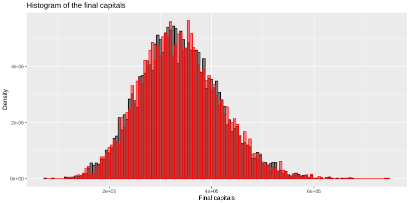




Results

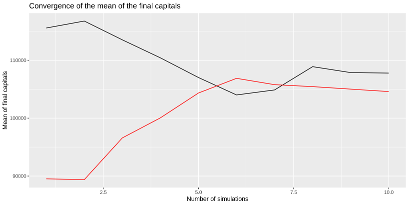
3 Simulating with $c_0 = 10000$, $n_0 = 5$, $a = 2000$, $t_l = 25$, $\lambda = 0.5$, $\mu = 0.1$, $\nu = 0.8$ and $M_C = 10000$.

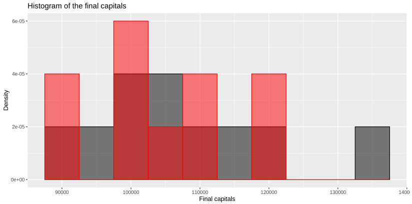




Results

These graphs show both the convergence of the mean of the final capitals in terms of the number of simulations and the density of the final capitals. Note that the color red indicates the antithetic variables approach





Conclusions

- The antithetic approach does not significantly reduce the variance but it does improve the computing time, cutting it in half
- We also encountered some difficulties while developing the simulations
  - Updating the proportional part of payments after simulating the type of event
  - Trying to obtain the total number of events beforehand
  - Antithetic variable approach showing the same results as our own approach
- The project was a fine representation of how simulations can be used in the insurance policies business

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