Collective risk

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Simulation in Prob and Stats BSc AMC at UC3M

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Contents

Introduction

- Goal:
  - Compute the probability that the capital of an insurance company remains positive during a given time period
- Data:
  - Yearly payment: a
  - Claims rate: $Poisson(\lambda)$
  - Claim amount: $Pareto(2.5, 100)$
  - Enrollment rate: $Poisson(\nu)$
  - Departure rate: $Exp(\mu)$
  - Initial capital: c_0

Introduction

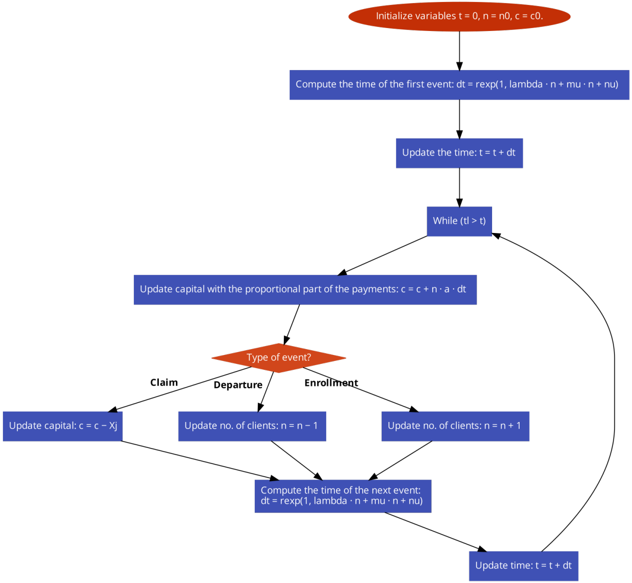
So in general, the capital of the company at any time t will be:

$$C(t) = c_0 + at(n_0 + N_A(t) - N_D(t)) - \sum_{j=1}^{N_C(t)} X_j$$

where:

- $N_A(t)$ is the number of clients that arrive by time t
- $N_D(t)$ is the number of clients that leave by time t
- $N_C(t)$ is the number of claims that arrive by time t
- X_j is the amount of the j-th claim
- $n(t)$ is the number of clients at time t.

The project



Initialize variables t = 0, n = n0, c = c0.
Compute the time of the first event: dt = rexp(1, lambda · n + mu · n + nu)
Update the time: t = t + dt
While (tl > t)
Update capital with the proportional part of the payments: c = c + n · a · dt
Type of event?
Claim
Departure
Enrollment
Update capital: c = c − Xj
Update no. of clients: n = n − 1
Update no. of clients: n = n + 1
Compute the time of the next event:
dt = rexp(1, lambda · n + mu · n + nu)
Update time: t = t + dt

Results

We used 2 different approaches for this problem

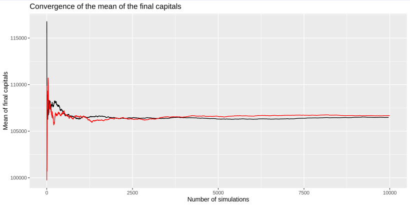
- A discrete event simulation algorithm we developed
- An improved version of our algorithm using antithetic variables to reduce the variance

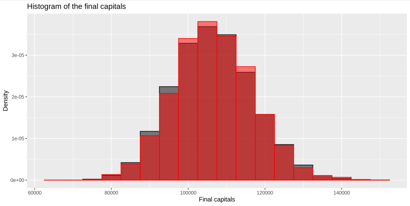
Results

Simulating with $c_0 = 1000$, $n_0 = 100$, $a = 100$, $t_l = 100$, $\lambda = 0.1$, $\mu = 0.1$, $\nu = 0.3$ and $M_C = 10000$.

Table 1: Results of the simulation with and without antithetic variates

Approach	Fraction	Mean	SD	Time (s)
Normal	0.997	106475.972	10516.112	17.86
Antithetic	1	106666.638	10361.448	10.923



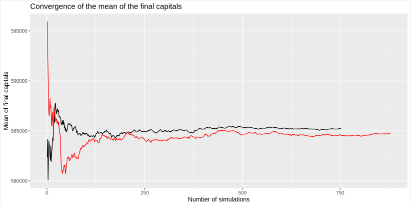


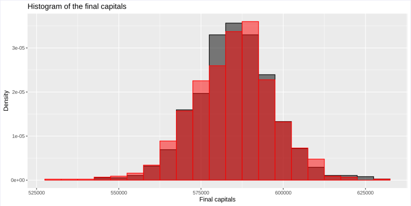
Results

Simulating with $c_0 = 100$, $n_0 = 10000$, $a = 20$, $t_l = 50$, $\lambda = 0.05$, $\mu = 0.2$, $\nu = 0.6$ and $M_C = 1000$.

Table 2: Results of the simulation with and without antithetic variates

Approach	Fraction	Mean	SD	Time (s)
Normal	0.752	585228.003	11936.203	50.856
Antithetic	0.878	584761.761	12552.333	28.775



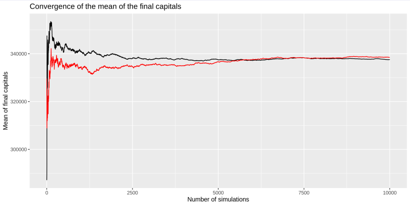


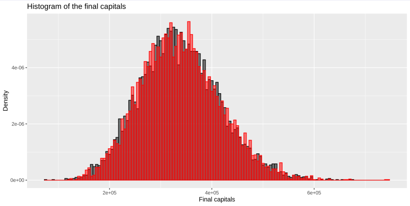
Results

Simulating with $c_0 = 10000$, $n_0 = 5$, $a = 2000$, $t_l = 25$, $\lambda = 0.5$, $\mu = 0.1$, $\nu = 0.8$ and $M_C = 10000$.

Table 3: Results of the simulation with and without antithetic variates

Approach	Fraction	Mean	SD	Time (s)
Normal	1	337549.707	78783.99	7.77
Antithetic	1	338472.373	78277.099	3.988





Conclusions

- The antithetic approach does not significantly reduce the variance but it does improve the computing time, cutting it in half
- We also encountered some difficulties while developing the simulations
  - Updating the proportional part of payments after simulating the type of event
  - Trying to obtain the total number of events beforehand
  - Antithetic variable approach showing the same results as our own approach
- The project was a fine representation of how simulations can be used in the insurance policies business

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