Collective risk

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Contents

Introduction

- Goal:
  - Compute the probability that the capital of an insurance company remains positive during a given time period
- Data:
  - Premium: a
  - Claims rate: $Poisson(\lambda)$
  - Premium amount: $Pareto(2.5, 100)$
  - Enrollment rate: $Poisson(\nu)$
  - Departure rate: $Exp(\mu)$
  - Initial capital: c_0

Introduction

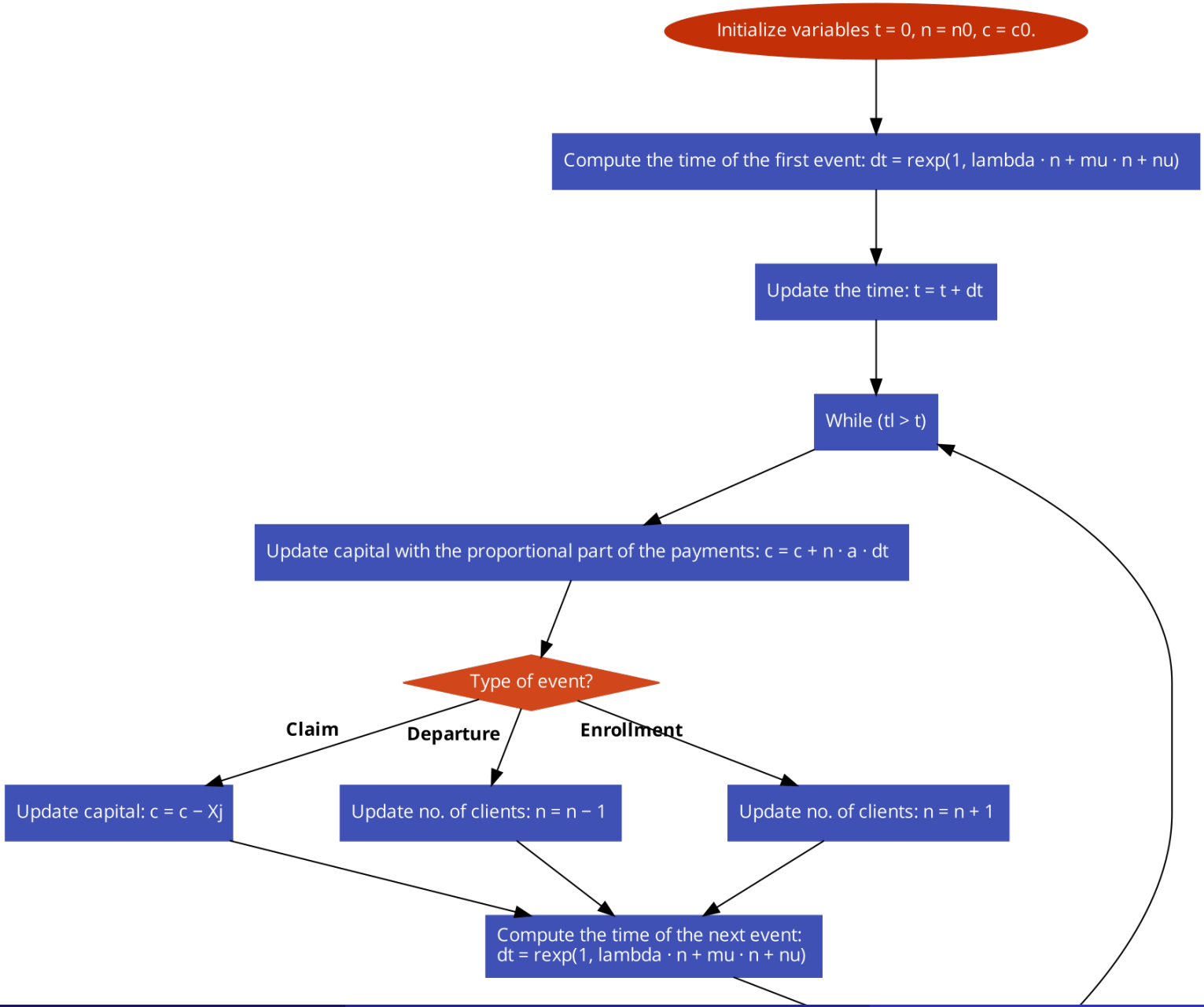
So in general, the capital of the company at any time t will be:

$$C(t) = c_0 + at(n_0 + N_A(t) - N_D(t)) - \sum_{j=1}^{N_C(t)} X_j$$

where:

- $N_A(t)$ is the number of clients that arrive by time t
- $N_D(t)$ is the number of clients that leave by time t
- $N_C(t)$ is the number of claims that arrive by time t
- X_j is the amount of the j-th claim
- $n(t)$ is the number of clients at time t.

The project


Initialize variables t = 0, n = n0, c = c0.
Compute the time of the first event: dt = rexp(1, lambda · n + mu · n + nu)
Update the time: t = t + dt
While (tl > t)
Update capital with the proportional part of the payments: c = c + n · a · dt
Type of event?
Claim
Departure
Enrollment
Update capital: c = c − Xj
Update no. of clients: n = n − 1
Update no. of clients: n = n + 1
Compute the time of the next event:
dt = rexp(1, lambda · n + mu · n + nu)

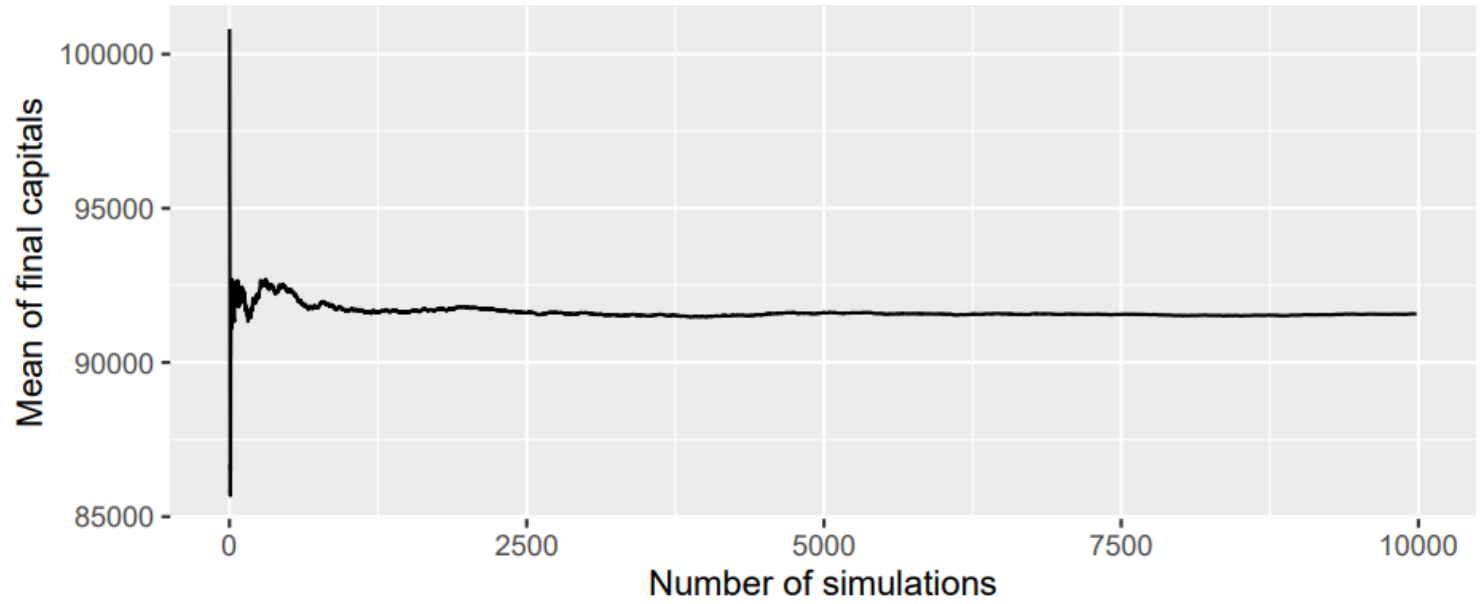
Results

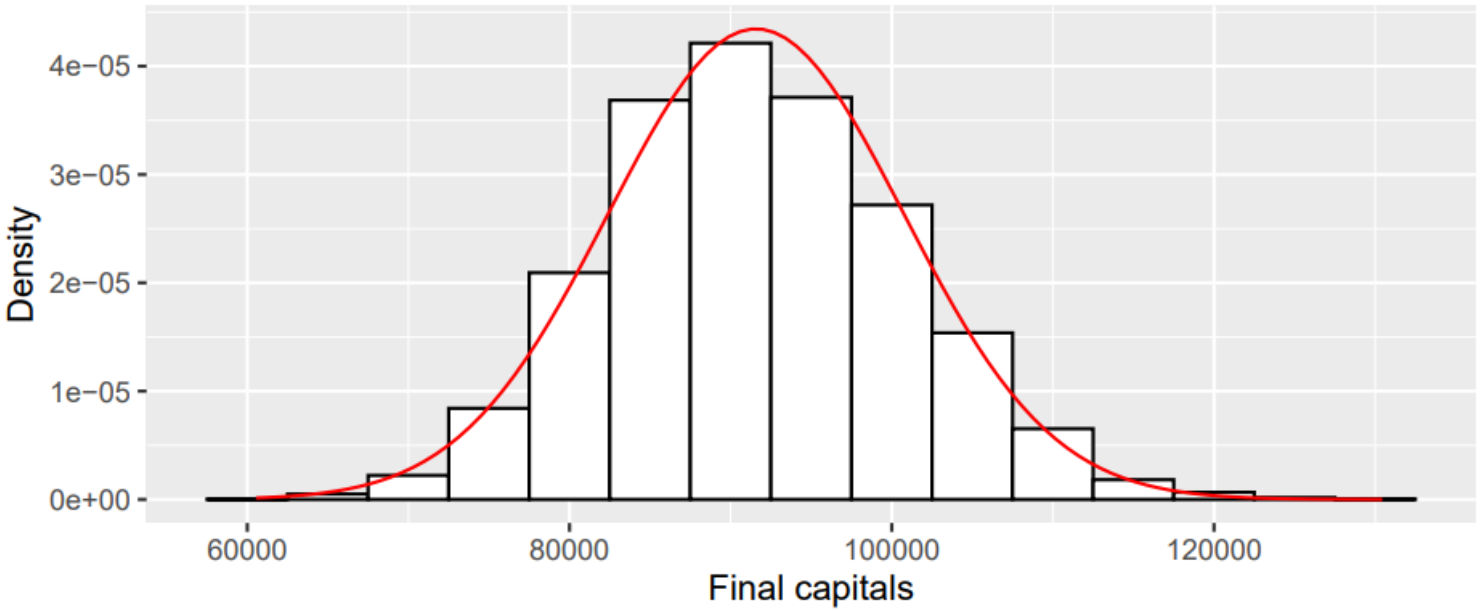
Both the discrete event simulation algorithm we developed and the one using antithetic variables gave the same results, meaning that the variance was not reduced from the original algorithm. These were the results:

- Fraction of paths (simulations) that remained positive throughout their respective lengths: 0.9984
- Mean of the final capital for those positive paths: 91573.45
- Standard deviation of the final capital for those positive paths: 9182.815

These were the results for $c_0 = 1000$, $n_0 = 100$, $a = 100$, $tl = 100$, $\lambda = 0.1$, $\mu = 0.1$, $\nu = 0.1$ and $MC = 10000$:

Results


100000
95000
90000
85000
Mean of final capitals
0
2500
5000
7500
10000
Number of simulations


4e−05
3e−05
2e−05
1e−05
0e+00
Density
60000
80000
100000
120000
Final capitals

Conclusions

About the results, how the difficulties were solved, and possible alternative approaches. Keep the focus, the conclusions must be as brief as possible.

References

Including textbooks, webpages, and class notes.