# Collective risk

Ignacio Gómez, Gonzalo Pato, Gonzalo Prats

Simulation in Prob and Stats BSc AMC at UC3M

April 2023

# Contents

# Introduction

- Goal:
  - Compute the probability that the capital of an insurance company remains positive during a given time period
- Data:
  - Premium: $a$
  - Claims rate: $Poisson(\lambda)$
  - Premium amount: $Pareto(2.5, 100)$
  - Enrollment rate: $Poisson(\nu)$
  - Departure rate: $Exp(\mu)$
  - Initial capital: $c_0$

# Introduction

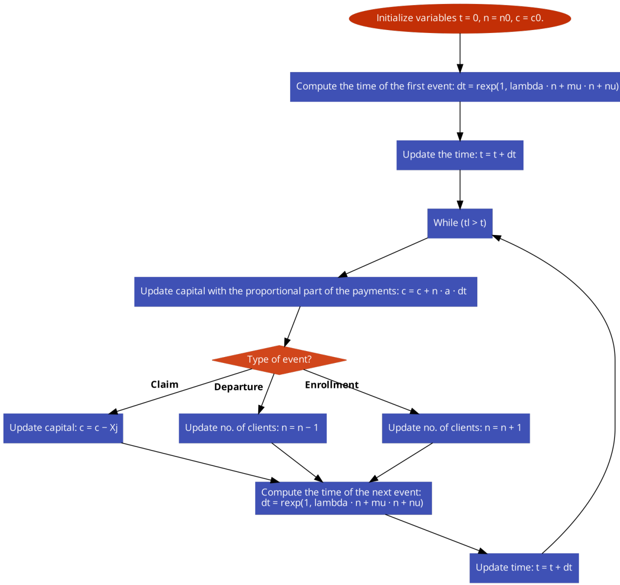
So in general, the capital of the company at any time $t$ will be:

$$C(t) = c_0 + at(n_0 + N_A(t) - N_D(t)) - \sum_{j=1}^{N_C(t)} X_j$$

where:

- $N_A(t)$ is the number of clients that arrive by time $t$
- $N_D(t)$ is the number of clients that leave by time $t$
- $N_C(t)$ is the number of claims that arrive by time $t$
- $X_j$ is the amount of the $j$-th claim
- $n(t)$ is the number of clients at time $t$.

# The project


Initialize variables t = 0, n = n0, c = c0.
Compute the time of the first event: dt = rexp(1, lambda · n + mu · n + nu)
Update the time: t = t + dt
While (tl > t)
Update capital with the proportional part of the payments: c = c + n · a · dt
Type of event?
Claim
Departure
Enrollment
Update capital: c = c − Xj
Update no. of clients: n = n − 1
Update no. of clients: n = n + 1
Compute the time of the next event:
dt = rexp(1, lambda · n + mu · n + nu)
Update time: t = t + dt

# Results

We used 2 different approaches for this problem

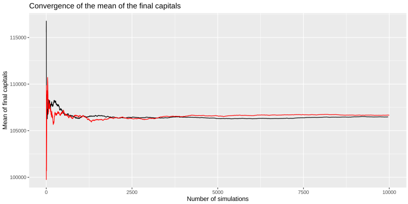
- A discrete event simulation algorithm we developed
- An improved version of our algorithm using antithetic variables to reduce the variance

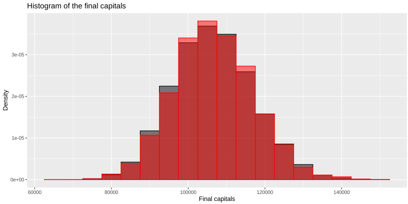
These were the results for $c_0 = 1000$, $n_0 = 100$, $a = 100$, $tl = 100$, $\lambda = 0.1$, $\mu = 0.1$, $\nu = 0.1$ and $MC = 10000$:

| fraction | mean_final_capital | sd_final_capital |
|---|---|---|
| 0.9967 | 106475.971675903 | 10516.1123920357 |
| 1 | 106666.637870849 | 10361.4480989 |

# Results

These graphs show both the convergence of the mean of the final capitals in terms of the number of simulations and the density of the final capitals. Note that the color red indicates the antithetic variables approach



# Conclusions

About the results, how the difficulties were solved, and possible alternative approaches. Keep the focus, the conclusions must be as brief as possible.

# References

Including textbooks, webpages, and class notes.