Collective risk

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Contents

Introduction

- Goal:
    - Compute the probability that the capital of an insurance company remains positive during a given time period
- Data:
    - Premium: a
    - Claims rate: $Poisson(\lambda)$
    - Premium amount: $Pareto(2.5, 100)$
    - Enrollment rate: $Poisson(\nu)$
    - Departure rate: $Exp(\mu)$
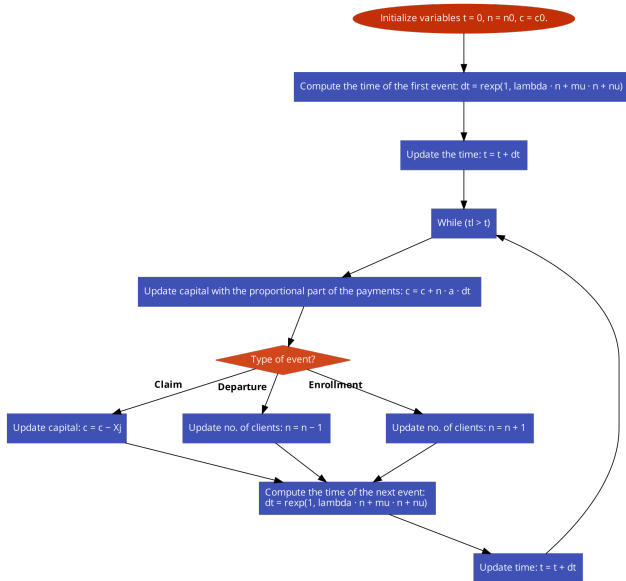   - Initial capital: c_0

Introduction

So in general, the capital of the company at any time t will be:

$$C(t) = c_0 + at(n_0 + N_A(t) - N_D(t)) - \sum_{j=1}^{N_C(t)} X_j$$

where:

- $N_A(t)$ is the number of clients that arrive by time t
- $N_D(t)$ is the number of clients that leave by time t
- $N_C(t)$ is the number of claims that arrive by time t
- X_j is the amount of the j-th claim
- $n(t)$ is the number of clients at time t.

Results

We used 2 different approaches for this problem

- A discrete event simulation algorithm we developed

- An improved version of our algorithm using antithetic variables to reduce the variance

These were the results for $c_0 = 1000$, $n_0 = 100$, $a = 100$, $tl = 100$, $\lambda = 0.1$, $\mu = 0.1$, $\nu = 0.1$ and $MC = 10000$:

fraction	mean_final_capital	sd_final_capital
0.9967	106475.971675903	10516.1123920357
1	106666.637870849	10361.4480989

Results

These graphs show both the convergence of the mean of the final capitals in terms of the number of simulations and the density of the final capitals. Note that the color red indicates the antithetic variables approach
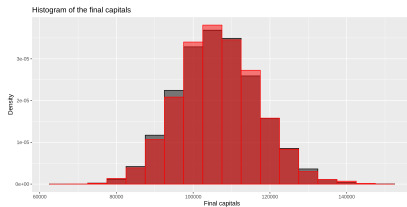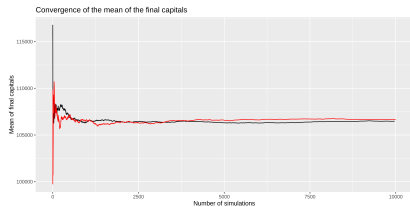
Conclusions

About the results, how the difficulties were solved, and possible alternative approaches. Keep the focus, the conclusions must be as brief as possible.

References

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