(9) Si A es una matriz cuadrada $n \times n$, se define la *traza* de A como $\mathrm{Tr}(A) = \sum_{i=1}^{n} a_{ii}$.

a) Calcular la traza de las matrices del ejercicio (10).

b) ⓐ Probar que si $A, B \in \mathbb{R}^{n\times n}$ y $c \in \mathbb{R}$ entonces

$$\mathrm{Tr}(A + cB) = \mathrm{Tr}(A) + c\,\mathrm{Tr}(B) \quad \text{y} \quad \mathrm{Tr}(AB) = \mathrm{Tr}(BA).$$

a) $Tr\left(\begin{bmatrix} 3 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 0 \end{bmatrix}\right) = 3+1+0 = 4$

$Tr\left(\begin{bmatrix} -1 & -1 & 4 \\ 1 & 3 & 8 \\ 1 & 2 & 5 \end{bmatrix}\right) = (-1)+3+5 = 7$

$Tr\left(\begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & -3 & 3 & -8 \\ -2 & 1 & 2 & -2 \\ 1 & 2 & 1 & 4 \end{bmatrix}\right) = 1+(-3)+2+4 = 4$

$Tr\left(\begin{bmatrix} 1 & -3 & 5 \\ 2 & -3 & 1 \\ 0 & -1 & 3 \end{bmatrix}\right) = 1+(-3)+3 = 1$

b) $$\begin{aligned} Tr(A+cB) &= \sum_{i=1}^{n} a_{ii} + cb_{ii} \\ &= \sum_{i=1}^{n} a_{ii} + \sum_{i=1}^{n} cb_{ii} \\ &= \sum_{i=1}^{n} a_{ii} + c\sum_{i=1}^{n} b_{ii} \\ &= Tr(A) + c.Tr(B) \end{aligned}$$

$$\begin{aligned} Tr(AB) &= \sum_{i=1}^{n} a_{ii} b_{ii} \\ &= \sum_{i=1}^{n} a_{ii} \cdot \sum_{i=1}^{n} b_{ii} \\ &= \sum_{i=1}^{n} b_{ii} \cdot \sum_{i=1}^{n} a_{ii} \\ &= \sum_{i=1}^{n} b_{ii} a_{ii} \\ &= Tr(BA) \end{aligned}$$