

(10) Para cada una de las siguientes matrices, usar operaciones elementales por fila para decidir si son invertibles y hallar la matriz inversa cuando sea posible.

$$\begin{bmatrix} 3 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 0 \end{bmatrix}, \quad \begin{bmatrix} -1 & -1 & 4 \\ 1 & 3 & 8 \\ 1 & 2 & 5 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & -3 & 3 & -8 \\ -2 & 1 & 2 & -2 \\ 1 & 2 & 1 & 4 \end{bmatrix}, \quad \begin{bmatrix} 1 & -3 & 5 \\ 2 & -3 & 1 \\ 0 & -1 & 3 \end{bmatrix}.$$

$$\begin{array}{c} \left[ \begin{array}{ccc|ccc} 3 & -1 & 2 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & -3 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\substack{f_1 - 3f_3 \\ f_2 - 2f_3}} \left[ \begin{array}{ccc|ccc} 0 & 8 & 2 & 1 & 0 & -3 \\ 0 & 7 & 1 & 0 & 1 & -2 \\ 1 & -3 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{f_1 \cdot 1/8} \left[ \begin{array}{ccc|ccc} 0 & 1 & 1/4 & 1/8 & 0 & -3/8 \\ 0 & 7 & 1 & 0 & 1 & -2 \\ 1 & -3 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\substack{f_2 - 7f_1 \\ f_3 + 3f_1}} \left[ \begin{array}{ccc|ccc} 0 & 1 & 1/4 & 1/8 & 0 & -3/8 \\ 0 & 0 & -3/4 & -7/8 & 1 & 5/8 \\ 1 & 0 & 3/4 & 3/8 & 0 & -1/8 \end{array} \right] \\ \\ \xrightarrow{f_2 \cdot -4/3} \left[ \begin{array}{ccc|ccc} 0 & 1 & 1/4 & 1/8 & 0 & -3/8 \\ 0 & 0 & 1 & 7/6 & -4/3 & -5/6 \\ 1 & 0 & 3/4 & 3/8 & 0 & -1/8 \end{array} \right] \xrightarrow{\substack{f_1 - \frac{1}{4}f_2 \\ f_3 - \frac{3}{4}f_2}} \left[ \begin{array}{ccc|ccc} 0 & 1 & 0 & -1/6 & 1/3 & -1/6 \\ 0 & 0 & 1 & 7/6 & -4/3 & -5/6 \\ 1 & 0 & 0 & -1/2 & 1 & 1/2 \end{array} \right] \xrightarrow{} \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & -1/2 & 1 & 1/2 \\ 0 & 1 & 0 & -1/6 & 1/3 & -1/6 \\ 0 & 0 & 1 & 7/6 & -4/3 & -5/6 \end{array} \right] \end{array}$$

Por lo tanto,  $\begin{bmatrix} 3 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} -1/2 & 1 & 1/2 \\ -1/6 & 1/3 & -1/6 \\ 7/6 & -4/3 & -5/6 \end{bmatrix}$

$$\begin{array}{c} \left[ \begin{array}{ccc|ccc} -1 & -1 & 4 & 1 & 0 & 0 \\ 1 & 3 & 8 & 0 & 1 & 0 \\ 1 & 2 & 5 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\substack{f_2 + f_1 \\ f_3 + f_1}} \left[ \begin{array}{ccc|ccc} -1 & -1 & 4 & 1 & 0 & 0 \\ 0 & 2 & 12 & 1 & 1 & 0 \\ 0 & 1 & 9 & 1 & 0 & 1 \end{array} \right] \xrightarrow{\substack{f_1(-1) \\ f_2 \cdot 1/2}} \left[ \begin{array}{ccc|ccc} 1 & 1 & -4 & -1 & 0 & 0 \\ 0 & 1 & 6 & 1/2 & 1/2 & 0 \\ 0 & 1 & 9 & 1 & 0 & 1 \end{array} \right] \xrightarrow{\substack{f_1 - f_2 \\ f_3 - f_2}} \left[ \begin{array}{ccc|ccc} 1 & 0 & -10 & -3/2 & -1/2 & 0 \\ 0 & 1 & 6 & 1/2 & 1/2 & 0 \\ 0 & 0 & 3 & 1/2 & -1/2 & 1 \end{array} \right] \\ \\ \xrightarrow{f_3 \cdot 1/3} \left[ \begin{array}{ccc|ccc} 1 & 0 & -10 & -3/2 & -1/2 & 0 \\ 0 & 1 & 6 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1 & 1/6 & -1/6 & 1/3 \end{array} \right] \xrightarrow{\substack{f_1 + 10f_3 \\ f_2 - 6f_3}} \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 1/6 & -13/6 & 10/3 \\ 0 & 1 & 0 & -1/2 & 2/3 & -2 \\ 0 & 0 & 1 & 1/6 & -1/6 & 1/3 \end{array} \right] \end{array}$$

Por lo tanto,  $\begin{bmatrix} -1 & -1 & 4 \\ 1 & 3 & 8 \\ 1 & 2 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} 1/6 & -13/6 & 10/3 \\ -1/2 & 2/3 & -2 \\ 1/6 & -1/6 & 1/3 \end{bmatrix}$

$$\begin{array}{c} \left[ \begin{array}{cccc|cccc} 1 & 1 & 1 & 2 & 1 & 0 & 0 & 0 \\ 1 & -3 & 3 & -8 & 0 & 1 & 0 & 0 \\ -2 & 1 & 2 & -2 & 0 & 0 & 1 & 0 \\ 1 & 2 & 1 & 4 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\substack{f_2 - f_1 \\ f_3 + 2f_1 \\ f_4 - f_1}} \left[ \begin{array}{cccc|cccc} 1 & 1 & 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & -4 & 2 & -10 & -1 & 1 & 0 & 0 \\ 0 & 3 & 4 & 2 & 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\substack{f_1 - f_4 \\ f_2 + 4f_4 \\ f_3 - 3f_4}} \left[ \begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 2 & 0 & 0 & -1 \\ 0 & 0 & 2 & -2 & -5 & 1 & 0 & 4 \\ 0 & 0 & 4 & -4 & 5 & 0 & 1 & -3 \\ 0 & 1 & 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{f_2 \cdot 1/2} \left[ \begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 2 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 & -5/2 & 1/2 & 0 & 2 \\ 0 & 0 & 4 & -4 & 5 & 0 & 1 & -3 \\ 0 & 1 & 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \\ \\ \xrightarrow{\substack{f_1 - f_2 \\ f_3 - 4f_2}} \left[ \begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 9/2 & -1/2 & 0 & -3 \\ 0 & 0 & 1 & -1 & -5/2 & 1/2 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & -2 & 1 & -11 \\ 0 & 1 & 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \end{array}$$

La matriz no tiene inversa.

$$\begin{array}{c} \left[ \begin{array}{ccc|ccc} 1 & -3 & 5 & 1 & 0 & 0 \\ 2 & -3 & 1 & 0 & 1 & 0 \\ 0 & -1 & 3 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\substack{f_2 - 2f_1 \\ f_3(-1)}} \left[ \begin{array}{ccc|ccc} 1 & -3 & 5 & 1 & 0 & 0 \\ 0 & 3 & -9 & -2 & 1 & 0 \\ 0 & 1 & -3 & 0 & 0 & -1 \end{array} \right] \xrightarrow{f_2 - 3f_3} \left[ \begin{array}{ccc|ccc} 1 & -3 & 5 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2 & 1 & 3 \\ 0 & 1 & -3 & 0 & 0 & -1 \end{array} \right] \end{array}$$

La matriz no tiene inversa.