(6) Demostrar que dados z, z_1, z_2 en $\mathbb{C}$ se cumple:

$$|\bar{z}| = |z|, \qquad |z_1\, z_2| = |z_1|\,|z_2|.$$

Sea $z = a+bi$, $|\bar{z}| = |a-bi| = \sqrt{a^2+(-b)^2} = \sqrt{a^2+b^2} = |a+bi| = |z|$

$(-k)^2 = k^2$

Sean $z_1 = a+bi$, $z_2 = c+di$,

$$|z_1 z_2| = |(a+bi)(c+di)| = |(ac-bd)+i(ad+bc)| = \sqrt{(ac-bd)^2+(ad+bc)^2} \overset{(*)}{=} \sqrt{(ac)^2+(bd)^2+(ad)^2+(bc)^2}$$

$$|z_1||z_2| = |a+bi||c+di| = \sqrt{a^2+b^2}\sqrt{c^2+d^2} = \sqrt{(a^2+b^2)(c^2+d^2)} = \sqrt{(ac)^2+(ad)^2+(bc)^2+(bd)^2}$$

(*) $(ac-bd)^2+(ad+bc)^2 = (ac)^2-2acbd+(bd)^2+(ad)^2+2adbc+(bc)^2$