(5) Calcular las derivadas parciales de las siguientes funciones y evaluarlas en el punto dado.

(a) $f(x,y) = x - y$, $(3,2)$

$f_x(x,y) = 1 \Rightarrow f_x(3,2) = 1$

$f_y(x,y) = -1 \Rightarrow f_y(3,2) = -1$

(b) $f(x,y,z) = \dfrac{xz}{y+z}$, $(1,1,1)$

$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

$f_x(x,y,z) = \dfrac{z(y+z) - xz.0}{(y+z)^2} = \dfrac{z}{y+z} \Rightarrow f_x(1,1,1) = \dfrac{1}{2}$

$f_y(x,y,z) = \dfrac{0.(y+z) - xz.1}{(y+z)^2} = \dfrac{-xz}{(y+z)^2} \Rightarrow f_y(1,1,1) = \dfrac{-1}{4}$

$f_z(x,y,z) = \dfrac{x(y+z) - xz.1}{(y+z)^2} = \dfrac{xy + xz - xz}{(y+z)^2} = \dfrac{xy}{(y+z)^2} \Rightarrow f_z(1,1,1) = \dfrac{1}{4}$

(c) $f(x,y) = xy + x^2$, $(2,0)$

$f_x(x,y) = y + 2x \Rightarrow f_x(2,0) = 4$

$f_y(x,y) = x \Rightarrow f_y(2,0) = 2$

(d) $w = \mathrm{e}^{y \ln z}$, $(\mathrm{e}, 2, \mathrm{e})$

$f_x(x,y,z) = e^{y \ln z} \Rightarrow f_x(e,2,e) = e^{2\ln e} = e^2$

$f_y(x,y,z) = e^{y\ln z}.(1 \ln z + y.0) = \ln(z) e^{y \ln z} \Rightarrow f_y(e,2,e) = \ln(e) e^{2\ln e} = e^2$

$f_z(x,y,z) = e^{y\ln z}.(0 + y.\frac{1}{z}) = \frac{y}{z} e^{y\ln z} \Rightarrow f_z(e,2,e) = \frac{2}{e} e^{2 \ln e} = 2e$

(e) $f(x,y,z) = x^3 y^4 z^5$, $(0,-1,-1)$

$f_x(x,y,z) = 3x^2 y^4 z^5 \Rightarrow f_x(0,-1,-1) = 0$

$f_y(x,y,z) = x^3 4 y^3 z^5 \Rightarrow f_y(0,-1,-1) = 0$

$f_z(x,y,z) = x^3 y^4 5 z^4 \Rightarrow f_z(0,-1,-1) = 0$

(f) $w = \ln(1 + e^{xyz})$, $(2,0,-1)$

$f_x(x,y,z) = \dfrac{yz e^{xyz}}{1+e^{xyz}} \Rightarrow f_x(2,0,-1) = 0$

$f_y(x,y,z) = \dfrac{xz e^{xyz}}{1+e^{xyz}} \Rightarrow f_y(2,0,-1) = \dfrac{2(-1)\, e^0}{1+e^0} = -1$

$f_z(x,y,z) = \dfrac{xy e^{xyz}}{1+e^{xyz}} \Rightarrow f_z(2,0,-1) = 0$