(12) Aplique la regla de la cadena para hallar dz/dt

$$\frac{df}{dt} = \frac{\partial f}{\partial x}(x(t), y(t)) \cdot x'(t) + \frac{\partial f}{\partial y}(x(t), y(t)) \cdot y'(t)$$

(a) $z = x^2 + y^2 + xy$, $x = \sin t$, $y = e^t$

$f_x(x,y) = 2x + y \quad \Rightarrow \quad f_x(\sin t, e^t) = 2\sin t + e^t$

$f_y(x,y) = 2y + x \quad \Rightarrow \quad f_y(\sin t, e^t) = 2e^t + \sin t$

$x'(t) = \cos t \qquad y'(t) = e^t$

Finalmente, $\frac{dz}{dt} = (2\sin t + e^t)\cos t + (2e^t + \operatorname{sen} t)e^t$

(b) $z = \cos(x + 4y)$, $x = 5t^4$, $y = 1/t$

$f_x(x,y) = -\operatorname{sen}(x+4y) \quad \Rightarrow \quad f_x(5t^4, 1/t) = -\operatorname{sen}(5t^4 + 4/t)$

$f_y(x,y) = -\operatorname{sen}(x+4y).4 \quad \Rightarrow \quad f_y(5t^4, 1/t) = -4\operatorname{sen}(5t^4 + 4/t)$

$x'(t) = 20t^3 \qquad y'(t) = -1/t^2$

Finalmente, $\frac{dz}{dt} = -20t^3\operatorname{sen}(5t^4 + 4/t) + \frac{4}{t^2}\operatorname{sen}(5t^4 + 4/t)$

(c) $z = \sqrt{1 + x^2 + y^2}$, $x = \ln t$, $y = \cos t$

$f_x(x,y) = \frac{1}{\cancel{2}}(1+x^2+y^2)^{-1/2} \cdot \cancel{2}x = x(1+x^2+y^2)^{-1/2} \quad \Rightarrow \quad f_x(\ln t, \cos t) = \ln(t)(1 + (\ln t)^2 + \cos^2 t)^{-1/2}$

$f_y(x,y) = \frac{1}{\cancel{2}}(1+x^2+y^2)^{-1/2} \cdot \cancel{2}y = y(1+x^2+y^2)^{-1/2} \quad \Rightarrow \quad f_y(\ln t, \cos t) = \cos t(1 + (\ln t)^2 + \cos^2 t)^{-1/2}$

$x'(t) = 1/t \qquad y'(t) = -\operatorname{sen}(t)$

Finalmente $\frac{dz}{dt} = \ln(t)(1 + (\ln t)^2 + \cos^2 t)^{-1/2} \cdot 1/t + \cos t(1 + (\ln t)^2 + \cos^2 t)^{-1/2}(-\operatorname{sen}(t))$