(11) Obtener el coseno del ángulo comprendido entre los planos S_1 y S_2, donde:

(a) $S_1 : x + y + z = 0,\ S_2 : x + 2y + 3z = 1.$

Tenemos que $N_1 = (1,1,1)$ y $N_2 = (1,2,3)$

Luego, $\langle N_1, N_2 \rangle = \langle (1,1,1), (1,2,3) \rangle = 1+2+3 = 6 = \|N_1\| \|N_2\| \cos(N_1, N_2)$

$\|N_1\| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$ $\qquad$ $\|N_2\| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$

Finalmente, $\sqrt{3}\sqrt{14} \cos(N_1, N_2) = 6 \Rightarrow \cos(N_1 N_2) = \dfrac{6}{\sqrt{3}\sqrt{14}}$

(b) $S_1 : 3x+2y-z = 0,\ S_2 : 6x-3y+2z = 5.$

Tenemos que $N_1 = (3,2,-1)$ y $N_2 = (6,-3,2)$

Luego, $\langle N_1, N_2 \rangle = \langle (3,2,-1), (6,-3,2) \rangle = 18-6-2 = 10 = \|N_1\| \|N_2\| \cos(N_1, N_2)$

$\|N_1\| = \sqrt{3^2+2^2+(-1)^2} = \sqrt{9+4+1} = \sqrt{14}$ $\qquad$ $\|N_2\| = \sqrt{6^2+(-3)^2+2^2} = \sqrt{36+9+4} = \sqrt{49} = 7$

Finalmente, $\sqrt{14}\, 7 \cos(N_1, N_2) = 10 \Rightarrow \cos(N_1, N_2) = \dfrac{10}{7\sqrt{14}}$

(c) $S_1 : x + z = 1,\ S_2 : y + z = 1.$

Tenemos que $N_1 = (1,0,1)$ y $N_2 = (0,1,0)$

Luego, $\langle N_1, N_2 \rangle = \langle (1,0,1), (0,1,1) \rangle = 1 = \|N_1\| \|N_2\| \cos(N_1, N_2)$

$\|N_1\| = \sqrt{1^2+0^2+1^2} = \sqrt{2}$ $\qquad$ $\|N_2\| = \sqrt{0^2+1^2+1^2} = \sqrt{2}$

Finalmente, $\sqrt{2}\sqrt{2} \cdot \cos(N_1, N_2) = 1 \Rightarrow \cos(N_1, N_2) = \dfrac{1}{2\sqrt{2}}$