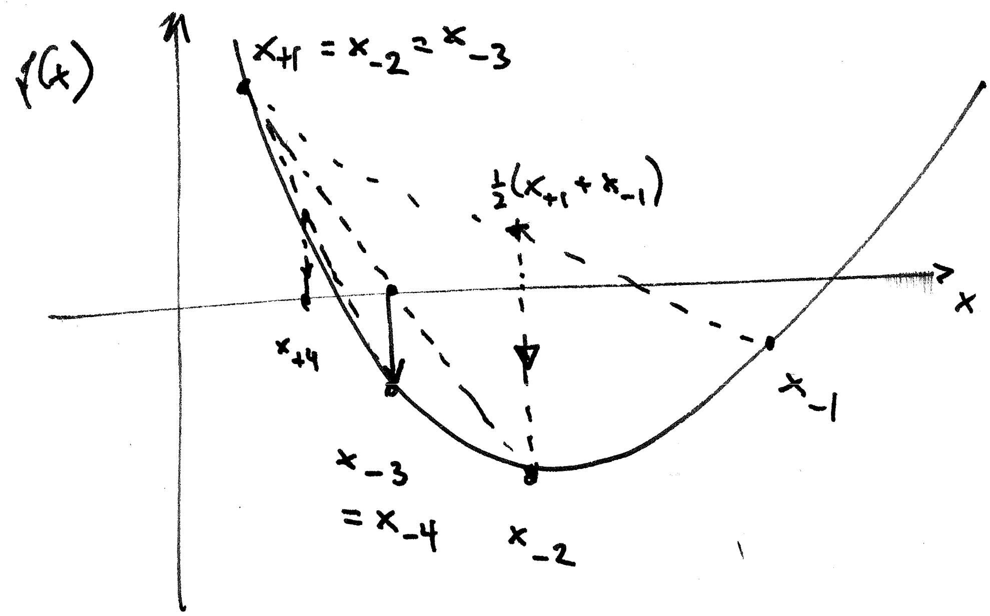
Bisection



Trial & error:

$f(x) = 0$

1. guess x_i (trial)
2. $f(x_i) \stackrel{?}{=} 0$ (error)
3. improve x_i

until error acceptable or iterations exceeded

Bisection:

$f(x_-) < 0$ $\quad f(x_+) > 0$ $\quad$ (here: $x_- > x_+$)

1. bisect
2. pick half with sign change

$x = \frac{1}{2}(x_+ + x_-)$

if $f(x) \cdot f(x_+) > 0$:

\# sign change in x_-, x:

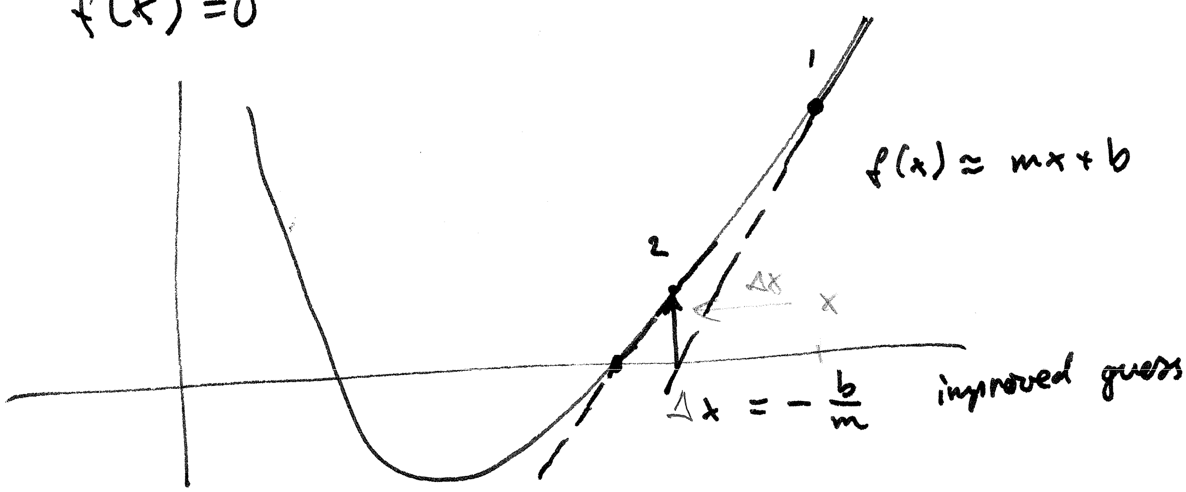
$x_+ = x$

else:

$x_- = x$

Newton – Raphson

$f(x) = 0$



Algorithm: Derivation

x_0 = old guess $\quad \Delta x$: unknown correction

$x = x_0 + \Delta x$ = (unknown) new guess

Expand $f(x)$:

$$f(x = x_0 + \Delta x) \approx f(x_0) + \Delta x \left.\frac{df}{dx}\right|_{x_0}$$

Determine correction: intercept of lin approx w/ x-axis

$f(x_0 + \Delta x) = 0$

$$f(x_0) + f'(x_0)\,\Delta x = 0$$

$$\Delta x = -\frac{f(x_0)}{f'(x_0)}$$

Repeat!

1) can use ~~analytical~~ analytical derivative
2) or numerical forward difference

$$\frac{df}{dx} \approx \frac{f(x + h) - f(x)}{h}$$

or central $$\frac{df}{dx} = \frac{f(x + \frac{h}{2}) - f(x - \frac{h}{2})}{h}$$

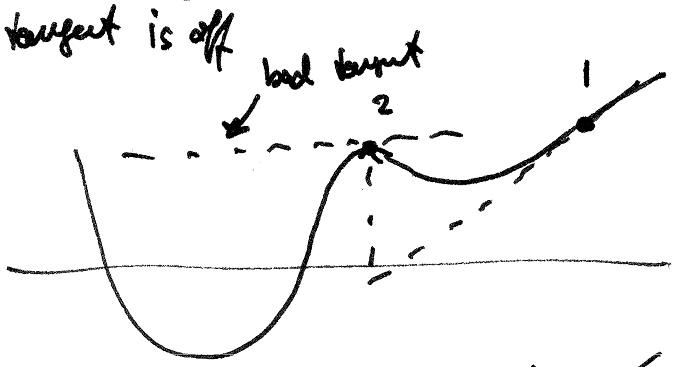
while $|f(x) > \epsilon|$:

$\quad \Delta x = -\frac{f(x)}{f'(x)}$

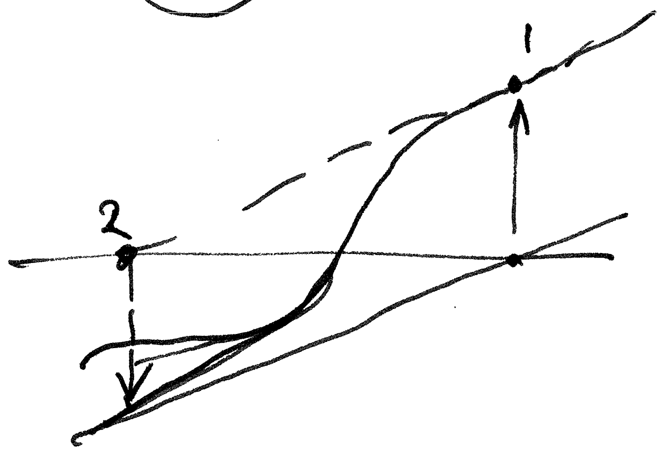
$\quad x \leftarrow x + \Delta x$

Possible issues

- Initial guess must be close
- tangent is off



local min/max



inf. loop

$\Rightarrow$ Solutions:

1) start with bisection
2) implement backtracking

If new guess increases magnitude (i.e. error increases)

$$|f(x+\Delta x)|^2 > |f(x)|^2$$

then go back to x and try smaller guess

$$x \rightarrow x + \frac{\Delta x}{2}$$

Reduce Δx more if necessary.

Advantages

- quadratical convergence
- fast