

Heat equation

heat flow $\underline{H} = -K \underline{\nabla} T(\underline{x}, t)$ (Fourier's law)

Change in internal energy (without work) must come from heat Q:

$$\Delta Q = C \rho \Delta T$$

or $Q(t) = \int d^3x \, C\rho \, T(\underline{x}, t)$

Energy conservation: $\boxed{\text{heat flow} = \text{change of energy inside}}$

$$\frac{\partial Q}{\partial t} = -\oint d\underline{A} \cdot \underline{H}$$ (heat flowing through boundary per unit time)

$$\frac{\partial}{\partial t} \int d^3x \, C\rho T(\underline{x}, t) \underset{\substack{\uparrow \\ \text{divergence theorem}}}{=} -\int d^3x \, \underline{\nabla} \cdot (-K \underline{\nabla} T)$$

$$\left(\int_V d^3x \, \underline{\nabla} \cdot \underline{\phi} = \oint_{\partial V} \underline{n} \cdot \underline{\phi} \, dA \right)$$

$$\frac{\partial}{\partial t} C\rho T(\underline{x}, t) = -\left(-K \underline{\nabla}^2 T(\underline{x}, t)\right)$$

$$\boxed{\frac{\partial T}{\partial t} = + \frac{K}{C\rho} \underline{\nabla}^2 T(\underline{x}, t)}$$

Diffusion equations are wide-spread:

- heat
- particle diffusion & Brownian motion
- finance (Black-Scholes)
- Schrödinger equation