

# PDEs

$$\phi(x, t), \quad \phi(x, y, z, t)$$

$$\phi(x, y)$$

General:

$$A \frac{\partial^2 \phi}{\partial x^2} + 2B \frac{\partial^2 \phi}{\partial x \partial y} + C \frac{\partial^2 \phi}{\partial y^2} + D \frac{\partial \phi}{\partial x} + E \frac{\partial \phi}{\partial y} = F$$

$$A \neq A(x, y) \dots$$

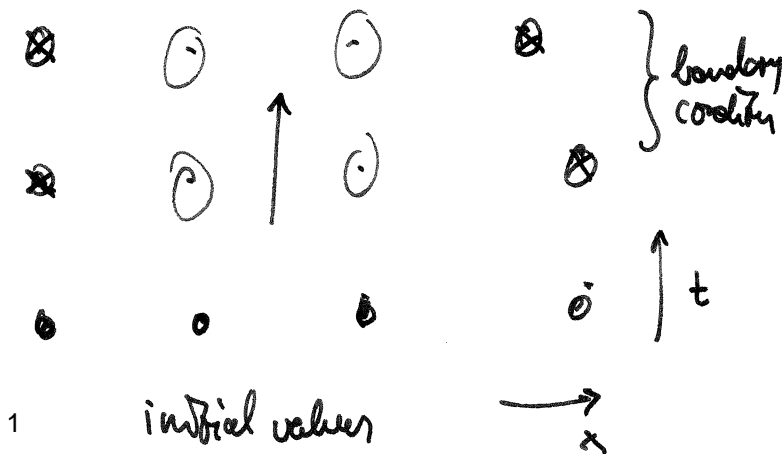
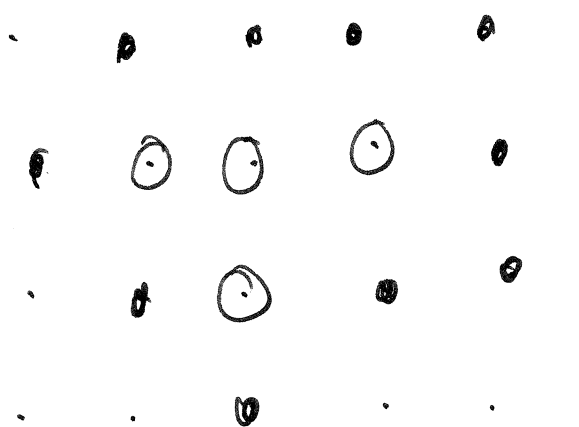
Mathematicians:

$$\text{discriminant } d = AC - B^2$$

	$d$	name	example	
static boundary value	$> 0$	elliptic	$\nabla^2 \phi(x) = -4\pi \rho(x)$ $(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})\phi = -4\pi \rho(x, y)$	Poisson
	$= 0$	parabolic	$D \nabla^2 \phi(x) = \frac{\partial \phi}{\partial t}$	diffusion, heat
time evolution initial value	$< 0$	hyperbolic	$\nabla^2 \phi(x) = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2}$	wave

boundary value problem

initial value problem



Boundary conditions:

values of  $\phi$  on boundary: Dirichlet

values of  $\frac{\partial \phi}{\partial x}$  normal to bond: Neumann

+ others

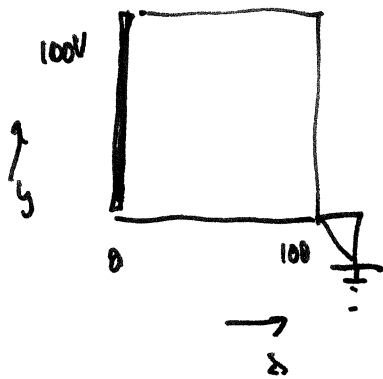
Different types of equation require different boundary conditions for unique solutions.

Solving PDEs:

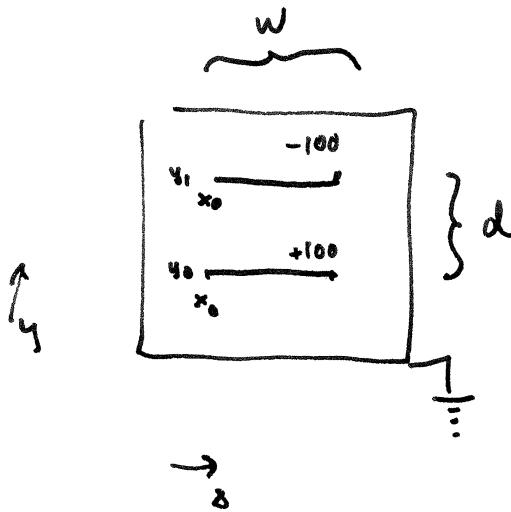
- harder than ODEs : all ODEs:

$$\frac{d\underline{y}(t)}{dt} = \underline{f}(\underline{y}, t) \quad \leftarrow \text{RK4} \dots$$

- specific to problem and boundary conditions



wire on a box



capacitor in a box