PHY 494 Project #2
TRAPPIST-1 Model

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Background

Problem Description

In very recent months, an announcement was made in regards to a discovery of a solar system which contained seven Earth-sized planets orbiting a dwarf star. This star is called TRAPPIST-1, and hence the system is called the TRAPPIST system. Each of these planets orbit close to the host star, which results in relatively short orbits. Six of the seven planets have data recorded on them and are named TRAPPIST-1b to TRAPPIST-1g. The last of which, TRAPPIST-1h, does not have adequate data for this project's purposes.

This project aims to analyze the system in many ways, including planetary and solar trajectories, energy and momentum conservation, etc. Part of this project is to also determine which planet in the system offers the best view on nearby other planets.

Equations and Definitions

A python module titled `parameters.py` was given, which contained data on each of the planets and the host star. These data included mass, orbital period, radius, semi-major axis, and eccentricity. The gravitational constant, G, was given to be

$$G = 4\pi^2 \times 0.0802 \times \frac{(10^3)^3}{364.25^2}$$

The initial conditions of each planet can be determined by Keplarian equations for aphelion (r_{max}) and perihelion (r_{min}), where a is defined as their major axes.

$$r_{max} = a(1+e)$$

$$r_{min} = a(1-e)$$

Velocity is also a component needed for the initial conditions of each body in the system. The velocity at aphelion is most relevant here. v_{ap} is defined below, with M as the mass of the host star.

$$v_{ap} = \sqrt{\frac{GM(1-e)}{a(1+e)}}$$

For our analysis, we treated the host star as another body in the simulation as opposed to keeping it stationary. This required that the initial conditions be set such that the total momentum of the system, $\mathbf{P}$, is conserved ($\mathbf{P} = 0$). Total momentum is defined below as

$$\mathbf{P} = \mathbf{M_{star}v_{star}} + \sum_{i=1}^{6} \mathbf{m_i v_i}$$

A check was also made for energy conservation ($dE/dt = 0$). The total energy of the system was calculated with kinetic energy, T, and potential energy, U.

$$E = T + U$$

$$T = \frac{1}{2}\sum_{i=1}^{N} m_i \mathbf{v_i}^2$$

$$U = \frac{1}{2}\sum_{i=1}^{N}\sum_{\substack{j=1 \\ j\neq i}}^{N} U_{ij}$$

The accuracy of the integration algorithm was estimated using the base-10 logarithm of relative error in the total energy

$$\epsilon(t) = \log_{10} \frac{E(t) - E(t=0)}{E(t=0)}$$

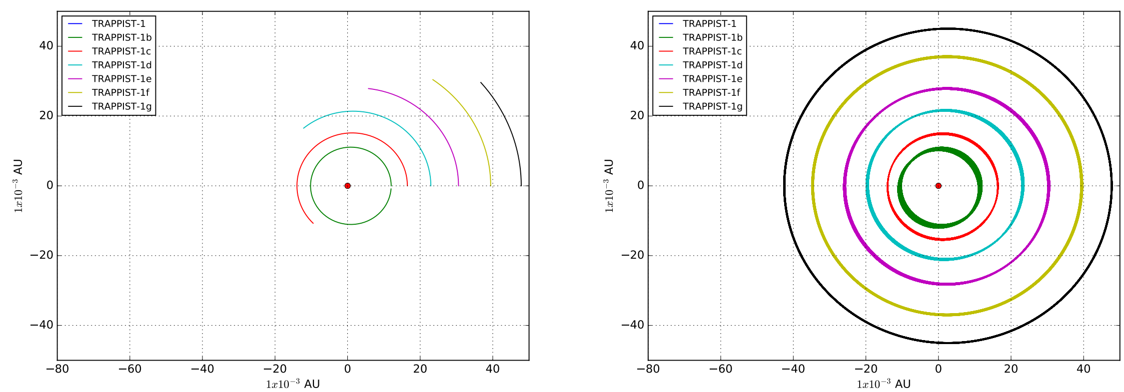
As far as determining which planets would be visible from each other, we define a planet that is nearby enough to resolve an object of 1000 km or less with the naked eye as a planet that is close enough for human terrestrial viewing. This allows us to determine which planets will be most ideal for tourism.

Results and Discussion

Orbits of the Planets

After setting the initial conditions of each body in the system, a plot was made of planetary orbits over a time period of 1.5 days. This confirmed that the orbit for TRAPPIST-1b is consistent with the known period of 1.51087081 days (as given in `parameters.py`). After confirmation of each planet's period, the orbits were extended for a time period of 1000 days. Figure 1 displays the data for both orbital period lengths.

We decided to use the Velocity Verlet algorithm with a time step 'h' of 0.01. While it is not as accurate as the RK4, it is much more stable. This stability is necessary for maintaining accuracy for the large amount of time steps required to simulate 1,000 days at 100 steps per day.



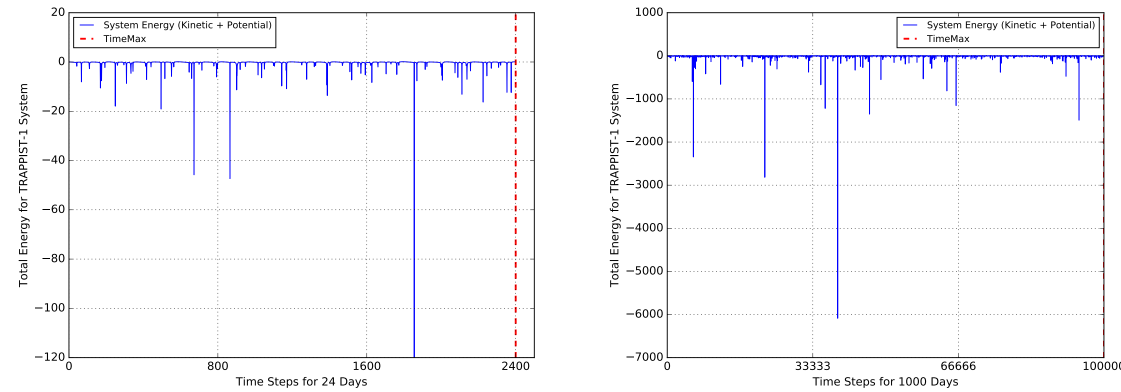
(a) Time period of 1.5 days

(b) Time period of 1000 days

Figure 1: Orbits of each planet in TRAPPIST-1 system

Energy and Momentum Conservation

The integration algorithm used for this simulation was the velocity verlet, which is known for its accuracy for long term simulations. It is an important algorithm to use for simulations which require energy and momentum conservation. Figure 2 displays the total energy ($E(t) = T + U$) as a function of time. Plots were created for time periods of 24 days and 1000 days respectively.



(a) $E(t) = T + U$ (b) Time period of 1000 days

Figure 2: Total energy of the TRAPPIST-1 system

A plot of the total change in energy ($\partial E/\partial t$) is shown, which results in 0, conserving the energy over the particular time period. Since momentum needs to be conserved as well, a plot of the change in momentum was also created. Figure 3 displays both plots, confirming that energy and momentum were both conserved in our simulation.

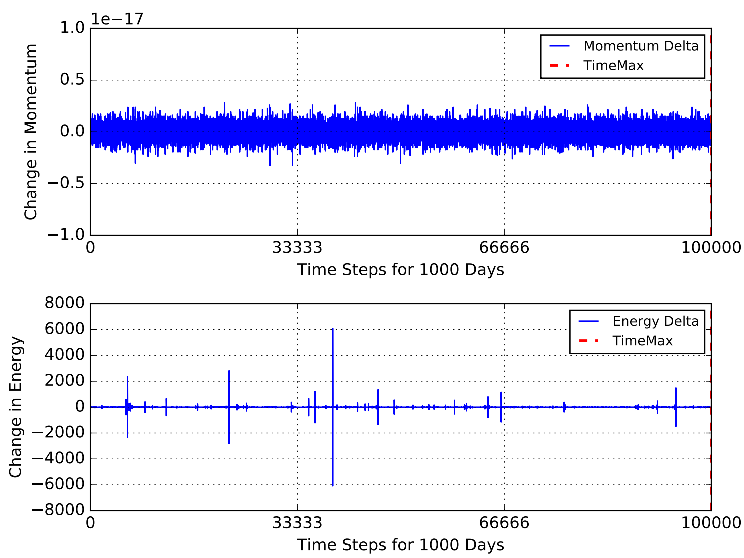
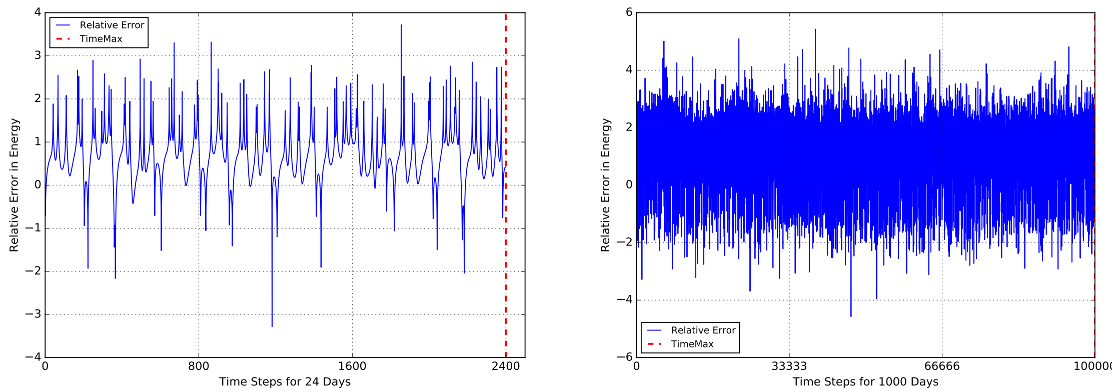


Figure 3: Conservation of Momentum and Energy in TRAPPIST-1 system

The accuracy of our integration algorithm was measured by using the base-10 logarithm of relative error in the total energy as explained above. Figure 4 displays a plot of the relative error in the algorithm for periods of 24 days and 1000 days, respectively.



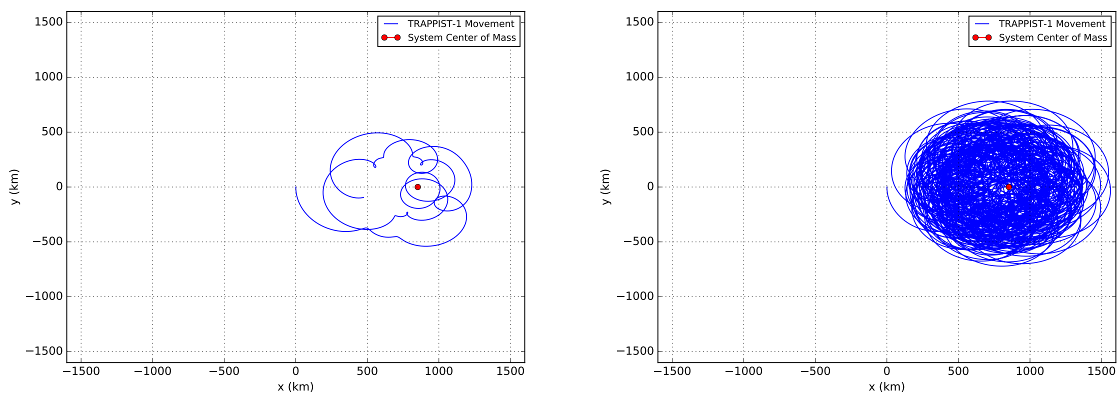
(a) $\log_{10}$(relative error) for 24 days

(b) $\log_{10}$(relative error) for 1000 days

Figure 4: Relative error in energy calculations

Adding the Star as a Body in the System

In all of the above calculations, the star was treated as another body in the system as opposed to being stationary. The motion of the star was plotted over two time periods, 24 and 1000 days. This was to show the movement of the star more precisely, then more globally. Figure 5 displays the 24 day and 1000 day plots of the stars motion, respectively.



(a) TRAPPIST-1 motion over 24 days

(b) TRAPPIST-1 motion over 1000 days

Figure 5: TRAPPIST-1 motion when including it in all calculations as a body in the system

For detection of the wobble of the star via a doppler spectrometer viewing the star on the $y - axis$, it was calculated that the star could easily be seen. Figure 6 displays the velocity of the star in the y direction in the reference frame of a laboratory observer and the detection limit of the doppler spectrometer.

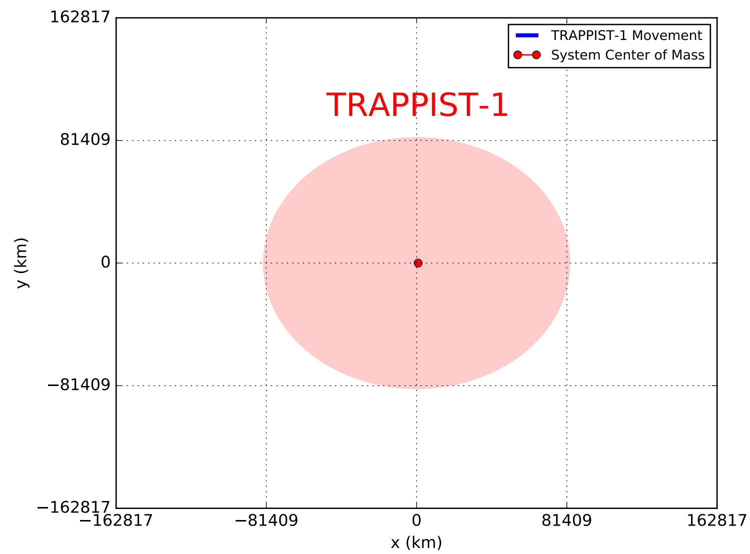


Figure 6: Center of mass in TRAPPIST-1

It was determined that the center of mass for TRAPPIST-1 is, in fact, inside the star. Figure 7 displays the star in a 2-D plot as well as the center of mass. The marker for the center of mass is eclipsing the movement of the star itself, demonstrating the relative smallness of the star's movements compared to its overall size.

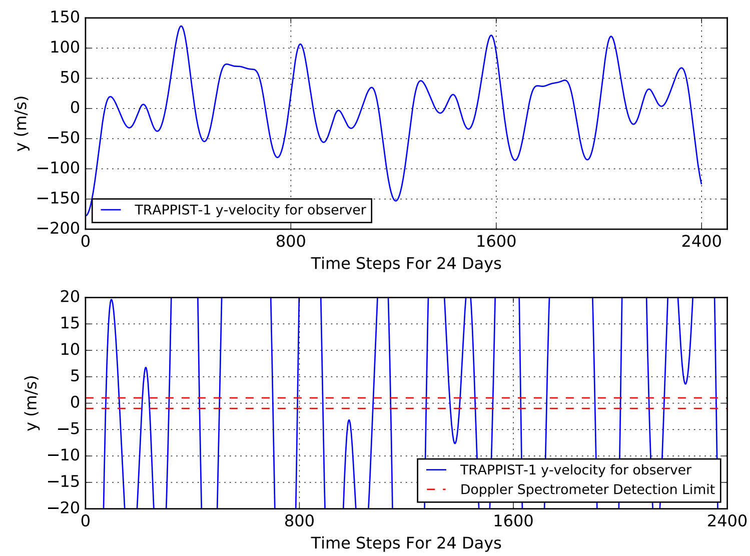


Figure 7: Doppler spectrometer analysis

Planet Watching Probability

Using the resolution of the human naked eye, it was calculated that the maximum distance to be able to resolve a 1,000 kilometer object clearly is 22.977 AU^{-3}. So, we answer the question of the probability (over 1000 days) that an observer could see any of the planets in the TRAPPIST system. Figure 8 is a histogram of the probabilities from each planet. This gives a complete interpretation of the probabilities of viewing any number of planets from any particular planet over the time period of 1000 days.

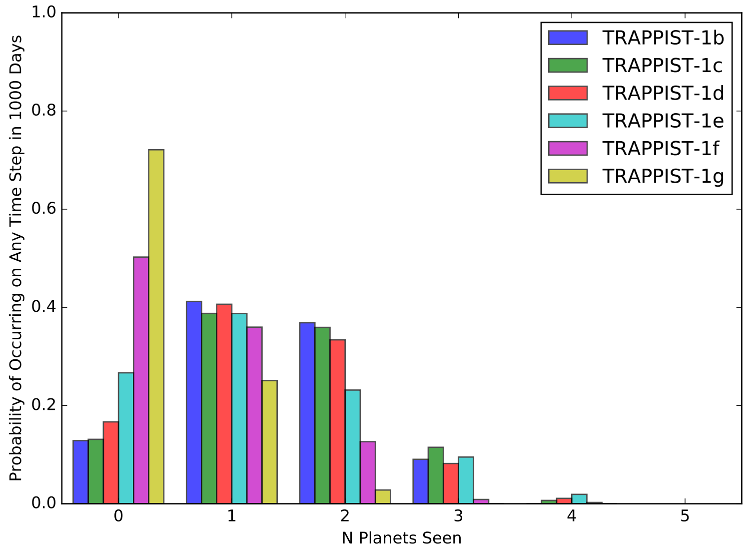


Figure 8: Planet Watching Probabilities

The values are too small to see, but TRAPPIST-1e does have times in which it can see five planets, and this only occurs when all of the planets are at or very near a rare conjunction. For a more detailed view of TRAPPIST-1e to see how the probability changes over a shorter period of 80 days, the appendix has a separate, more detailed plot of the behavior.

Summary

As the error tests and plots have shown throughout the report, the code performed well in regards to accurately calculating the system's movements. While TRAPPIST-1e was able to "see" five planets at once during a conjunction, common sense tells us that the poster is still inaccurate. The poster describes all five planets at a narrow viewing angle in the sky, while a conjunction would have three planets on one side of TRAPPIST-1e and two viewable only on the other. That being said, TRAPPIST-1e and TRAPPIST-1d would still make for views known only in science fiction and desktop wallpapers.

A final note is that an algorithm inefficiency is present in the code. Redundant gravity calculations for velocity verlet were discovered in the current design. This could be resolved by storing the calculated values in a global GBody() array for system-wide use and reuse each time step, but there was not enough time to implement this.

Contributions

The majority of the time spent on this project was with BP as the driver, KC and NC as the navigators. Many of the concepts of OOP were discussed among us all, which led to the decision of using OOP to make the code more efficient and modular. KC implemented the error calculations in regards to the velocity verlet algorithm. NC implemented the code for energy calculations. BP implemented the class `GBody` and its internals. BP, NC, and KC all helped implement all of the calculations in general. NC was responsible for constructing the final report in LaTeX.

Appendix

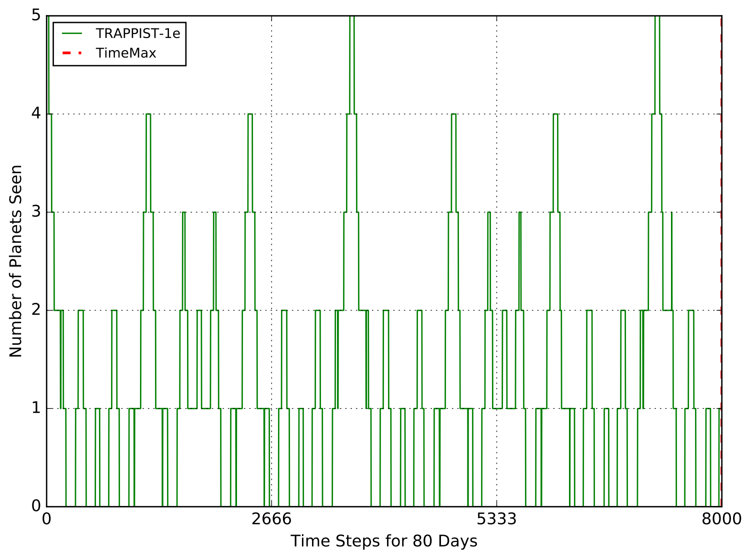


Figure 9: TRAPPIST-1e Planet Watching Probability for $t = 80$ days