

World In A Word 2013 Competition: Evolving Parameters for a Cayley Graph Visualiser Using 64 bits

Abstract

A 64-bit string is mapped using the Grammatical Evolution system, to evolve parameters for a visualiser of Cayley graphs. The resulting visualisations, albeit restricted, still exhibit a large degree of complexity and evolvability, and are representative of the domain.

1 Introduction

The “World in a Word” 64-Bit Design Challenge [1] is a competition run for the IEEE Congress on Evolutionary Computation. The objective is to evolve 64-bit strings, which are used to explore creative domains. Entries are evaluated based on the originality of the domain, the diversity of the structures evolved, the appeal of the domain and the evolved individuals, and the extent to which the 64 bits are used.

This entry uses the 64 bits as a basis to create an integer string, which in turn is used with the Grammatical Evolution system [5], to evolve parameters for Jenn3d [4], a visualiser of Cayley graphs of finite Coxeter groups. The mapping process is regulated through a context-free grammar, which defines the parameter space to navigate.

The resulting system creates fully-formed images from the start, and adapts to the changing styling preferences of the user. The use of 64 bits limits the range of visualisations achievable, but still allows for an extensive exploration of the domain.

This document describes the approach taken. Section 2 describes the methodology used, including overviews of the Jenn3d software, the evolutionary approach, the encoding used, and the fitness evaluation; Section 3 exhibits some examples generated. Appendix A provides details of the software implementation, and finally Appendix B provides high resolution visualisations of the examples from Section 3.

2 Methodology

2.1 Jenn3d

Our approach is based on previous work [3], in which a rich set of parameters were evolved for the Jenn3d software.

Jenn3d is a visualiser of finite Coxeter groups on 4 elements. These groups can be represented as reflections of Euclidean 4-space, or as reflections of the 3-sphere. It builds Cayley graphs using the Todd-Coxeter algorithm, and visualizes those graphs by embedding them in the 3-sphere, then stereographically projecting the graph from the 3-sphere to Euclidean 3-space, and finally rendering the 3D structure as a 2D picture. Jenn3d renders using OpenGL, and is fast enough to allow the user to rotate and navigate in curved 3D space, and gain an intuition for the geometry of the 3-sphere.

The rich and complex domain of visualisations representable through Jenn3d is defined by the following parameters:

1. the 4×4 Coxeter matrix, as specified by the 6 integers in the upper triangle matrix;
2. a subset of up to 3 of the 4 generators, by which vertices should be fixed (the larger the subset, the fewer vertices in the quotient);
3. a list of group elements to define edges, where each element is written as a string in the generating set; and
4. a list of faces, where each face is written as a pair of generating elements.

Note that this parameter set is partial and redundant in that *(i)* most Coxeter matrices result in an infinite group and the Todd-Coxeter algorithm does not converge; *(ii)* permuting the generators results in the same structure, but with a different initial visualization; *(iii)* the lists of group elements defining edges are really sets, and so are invariant under permutation and duplication; and *(iv)* when defining edges, each group element definition can be written in infinitely many ways modulo group equality, e.g., $r_4r_4r_2r_1r_3 = r_2r_1r_3$, since $r_4r_4 = 1$ is the identity.

This large and complex parameter space of drawings is very difficult to navigate, and is an excellent candidate for exploration using Grammatical Evolution.

2.2 Evolutionary Approach

To explore the Jenn3d parameter space, Grammatical Evolution (GE) [5] was used. GE typically uses a variable-length Genetic Algorithm (GA) [2] to create binary or decimal strings, which choose productions from a given grammar, and thus generate syntactically correct solutions of the search space. A fixed-length string can also be used; however, the search space becomes finite and (somewhat) restricted.

Fig. 1 shows the grammar used. In it both the required and optional parameters for Jenn3d are specified. The Coxeter matrix parameter is drawn from a fixed set, as overly complex matrices are too computationally demanding, but all other parameters are not only optional, but also variable in size.

```

<cmdline> ::= -c <CoxeterMatrix> <StabilizingGenerators> <Edges>
              <Faces> <VertexWeights>
<CoxeterMatrix> ::= <Torus> | <FreePolyhedra> | <FreePolytope>
<Torus> ::= <Int_2_12> 2 2 2 2 <Int_2_12>
<FreePolyhedra> ::= 3 3 2 2 2 2 | 3 4 2 2 2 2 | 3 5 2 2 2 2
                     | 4 3 2 2 2 2 | 5 3 2 2 2 2 | 3 2 3 2 2 2
                     | 3 2 4 2 2 2 | 3 2 5 2 2 2 | 4 2 3 2 2 2
                     | 5 2 3 2 2 2 | 2 2 2 2 3 3 | 2 2 2 2 3 4
                     | 2 2 2 2 3 5 | 2 2 2 2 4 3 | 2 2 2 2 5 3
<FreePolytope> ::= 3 3 3 2 2 2 | 3 3 2 2 3 2 | 3 3 2 2 4 2
                     | 3 3 2 2 5 2 | 3 4 2 2 3 2 | 3 5 2 2 3 2
                     | 4 3 2 2 3 2 | 3 2 3 2 2 3 | 3 2 3 2 2 4
                     | 3 2 3 2 2 5 | 3 2 4 2 2 3 | 4 2 3 2 2 3
                     | 5 2 3 2 2 3 | 2 2 3 2 3 3
<StabilizingGenerators> ::= | -v <Comb0123>
<Edges> ::= | -e <EdgeSet>
<EdgeSet> ::= <Comb0123> | <EdgeSet> <Comb0123>
<Faces> ::= | -f <FaceSet>
<FaceSet> ::= <FourInt0_3> | <FaceSet> <FourInt0_3>
<FourInt0_3> ::= <Int0_3>
                  | <Int0_3><Int0_3>
                  | <Int0_3><Int0_3><Int0_3>
                  | <Int0_3><Int0_3><Int0_3><Int0_3>
<VertexWeights> ::= | -w <Int1_12> <Int1_12> <Int1_12> <Int1_12>
<Int0_3> ::= 0 | 1 | 2 | 3
<Int1_12> ::= 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12
<Int2_12> ::= 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12
<Int3_12> ::= 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12
<Comb0123> ::= 0 | 1 | 2 | 3
                | 01 | 02 | 03 | 12 | 13 | 23
                | 012 | 013 | 023 | 123 | 0123

```

Figure 1: Grammar used to navigate the Jenn3d parameter space.

2.3 Encoding

64 bit binary strings are evolved using the search algorithm provided for the competition [1]. These are then transformed into integer strings. As each of these integers is used to choose productions associated to a *non-terminal* symbol defined in the grammar, the number of bits required to represent each inte-

ger is dependent on that grammar. An analysis of the grammar used (Fig. 1) shows that the symbol with the highest number of associated productions is `<FreePolyhedra>`, with 15 productions; therefore a minimum of 4 bits are required to encode each integer. The 64 bit strings are thus transformed into 16 integer strings.

Note that this introduces biases in the exploration of the phenotype space. For the `<FreePolyhedra>` symbol, for example, all productions have a 1/15 chance of being chosen, apart from the first production (`<FreePolyhedra> → 3 3 2 2 2 2`), which will have a 2/15 choice of being selected; this is due to the fact that each integer has a range of [0, 15], and GE’s usage of the modulus operator to map integers to the choice of productions. GE usually deals with these biases by using large amounts of redundancy, through the use of many bits per integer (typically 32 or even 64, in systems directly manipulating integers).

Also note that not all 2^{64} possible binary strings generate unique individuals. Some do not generate valid phenotypes at all, as the mapping process might not terminate; others generate the same choices in the grammar, and hence the same phenotypes; and finally some combinations of the leftmost bits will generate shorter mappings, which will not use some of the rightmost bits.

Even so, the number of possible combinations is still very large, and the functionality of each bit can vary, depending on the production choices of the bits preceding it. Furthermore, from a visualisation point of view, the search space is potentially infinite, as the user can select from a multitude of rotation and zoom combinations for each visualisation.

2.3.1 Efficient Exploitation

The original study [3] used a custom GA and extensions in GE, to introduce crossover markers through the grammar. This allowed parameter sets to be exchanged between individuals, thus increasing the visual relationship between parents and offspring. As the search algorithm supplied for the competition does not recognise these extensions, the user experience is somewhat limited, and the system acts more like an exploration tool.

Also note that the provided Java framework [1] also uses a Monte-Carlo (MC) algorithm [6] as a search procedure. This however is incompatible with the representation used, as deep searches (i.e. explorations of the rightmost bits in the bit string) can potentially always generate the same visualisation; this is particularly true for bit strings where the mapping process does not use the whole genotype string. As such, the MC algorithm was not used; more details are provided in Appendix A.

2.4 Fitness Evaluation

As the objective of the system is to evolve attractive and personalised visualisations, it is ran in an interactive manner. Each correctly generated individual is exposed to the user, to receive a fitness score. This allows the individual to

directly interact with the 3D-visualisation, have a better understanding of the generated structure, and achieve his/her preferred projection.

Fig. 2 shows the Jenn3D interface, extended so that a scoring process is present. Ideally, the evolutionary process proceeds in an endless manner; every time a fitness score is attributed to a structure, a new one is presented immediately after. If the user instead chooses to *exit* the application, the evolutionary process terminates. The full range of exploration tools in Jenn3d is available for each presented structure; this includes options to save the evolved parameters, and/or export a high-resolution image of the current visualisation.

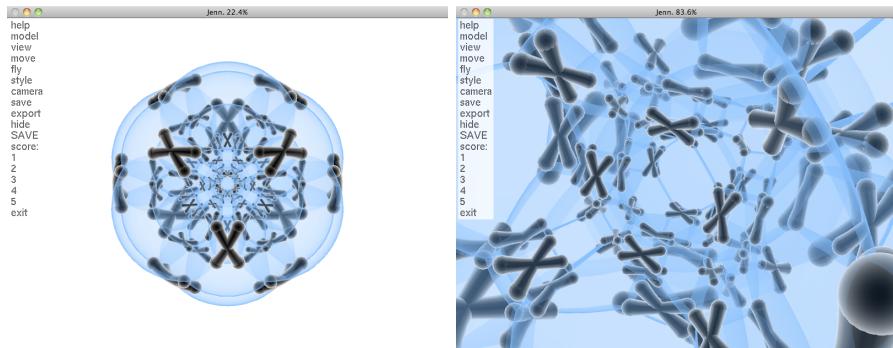


Figure 2: The Jenn3d interface, along with the extensions encoded. An example image is shown in the left; the same image, when zoomed in and rotated (right), can produce a dramatically different visualisation.

2.4.1 Fitness and Typicality

Table 1 shows the range of possible fitness and typicality scores. Due to the complexity of the Todd-Coxeter algorithm, some visualisations are impossible to generate, causing the Jenn3d software to crash¹; as these might be close to correct (and potentially attractive) visualisations, fitness and typicality scores of 0.5 are automatically attributed.

Ideally, the user has a large control of the evolutionary process, through the fitness score. A fitness score of 1.0, for example, guarantees that the current visualisation is replaced by a random one in the next generation, whereas a score of 5.0 guarantees that the visualisation is passed unchanged to the next generation, thus potentially being used as a seed for alternative, similar visualisations (as per the scoring process proposed previously [3]). The GA provided in the

¹This can be frustrating for the user; a smart way to deal with this problem (in a Mac environment) is to leave the crash report window open, as only a single such window can be open at any given time.

Table 1: Fitness and Typicality scores, and events generating those scores; fitness values 1-5 are user supplied

Event	Fitness	Typicality
Unsuccessful GE mapping	0.0	0
Non-convergence of Tedd-Coxeter algorithm	0.5	0.5
Rejected visualisation	1.0	1.0
Poor visualisation score	2.0	1.0
Average visualisation score	3.0	1.0
High visualisation score	4.0	1.0
Visualisation remains in population unchanged	5.0	1.0

Java framework of the competition does not comply with this scoring system, which limits to some extent the user experience of the system.

3 Examples

Table 2 shows example projections, achieved through evolution, along with their hexadecimal, integer and mapped strings. The specific visualisations were chosen to the taste of the authors. Higher resolution images are shown in Appendix B, Fig. 3.

References

- [1] Cameron Browne. World in a word 64-bit design challenge. <http://www.cameronius.com/research/cec/>, June 2013.
- [2] John H. Holland. *Adaptation in Natural and Artificial Systems*. University of Michigan Press, 1975.
- [3] Miguel Nicolau and Dan Costeloe. Using grammatical evolution to parameterise interactive 3d image generation. In Cecilia Di Chio et al., editor, *Applications of Evolutionary Computation - EvoApplications 2011: EvoCOMNET, EvoFIN, EvoHOT, EvoMUSART, EvoSTIM, and EvoTRANSLOG, Torino, Italy, April 27-29, 2011, Proceedings, Part II*, volume 6625 of *Lecture Notes in Computer Science*, pages 374–383. Springer, 2011.
- [4] Fritz Obermeyer. Jenn3d for visualizing coxeter polytopes. <http://jenn3d.org>, June 2010.
- [5] Michael O'Neill and Conor Ryan. *Grammatical Evolution - Evolutionary Automatic Programming in an Arbitrary Language*, volume 4 of *Genetic Programming*. Kluwer Academic, 2003.

Table 2: Parameters evolved for the examples shown in Fig. 3

	Hex: 0x9134db20eb2d6f07 Genotype: 9 1 3 4 13 11 2 0 14 11 2 13 6 15 0 7 Phenotype: '-c 3 2 2 2 2 5 -e 0 0123 -f 23'
	Hex: 0x53090ffede18f1b8 Genotype: 5 3 0 9 0 15 15 14 13 14 1 8 15 1 11 8 Phenotype: '-c 3 3 2 2 5 2 -e 0 -f 21'
	Hex: 0xc3298ff8d8003028 Genotype: 12 3 2 9 8 15 15 8 13 8 0 0 3 0 2 8 Phenotype: '-c 5 2 2 2 2 4 -v 13 -e 123 13'
	Hex: 0xc3090fe89c10f098 Genotype: 12 3 0 9 0 15 14 8 9 12 1 0 15 0 9 8 Phenotype: '-c 5 2 2 2 2 2 -v 0 -e 13 -f 03'
	Hex: 0xdb410f50dc59f1ba Genotype: 13 11 4 1 0 15 5 0 13 12 5 9 15 1 11 10 Phenotype: '-c 2 2 2 2 3 4 -e 0 -f 01 -w 4 2 12 11'
	Hex: 0xd1410f70dc59f0ac Genotype: 13 1 4 1 0 15 7 0 13 12 5 9 15 0 10 12 Phenotype: '-c 3 4 2 2 2 2 -e 0 -f 01 -w 4 1 11 1'
	Hex: 0xdb400f50dc50f13a Genotype: 13 11 4 0 0 15 5 0 13 12 5 0 15 1 3 10 Phenotype: '-c 2 2 2 2 3 4 -w 6 1 2 1'
	Hex: 0xd2410ef0d859f4be Genotype: 13 2 4 1 0 14 15 0 13 8 5 9 15 4 11 14 Phenotype: '-c 3 5 2 2 2 2 -e 0123 -f 01 -w 4 5 12 3'

- [6] Richard S. Sutton and Andrew G. Barto. *Reinforcement Learning: An Introduction*. MIT Press, 1998.

Appendix A: Software details

The Jenn3d software is written in C++, as is the GE mapping library used. In order to integrate it in the Java framework provided for the competition, the implementation of the *Individual* (abstract) class in `Jenn.java` uses a system call to the C++ framework. This call is made in the `measureQuality()` method. The java code waits for an exit value from the system call and then reads from the file `fitness.txt` (created by the C++ code). The typicality value is then derived from the fitness value (as shown in Table 1). Image generation is performed automatically by the C++ code when the user assigns a fitness score and the generated file is stored in the `GEMapJenn` directory. This simplifies the operation of the `export()` method in `Jenn.java` since all that is required is a file-copy operation. The PNG files from the `GEMapJenn` folder are (deliberately) not deleted so that the user may still wish to peruse discarded individuals that never made it to the final population. However, considering that each exported individual could take approximately 1MB of disk space, it may be necessary for the user to clean up these files after prolonged use of the system.

Inside the `GEMapJenn` implementation, a design decision was made to export the viewpoint of the individual at the point of fitness allocation (for example after the user has had a chance to explore the image via zooming, rotating or panning) as opposed to the default state of the freshly initialised individual.

It is important to note that in addition to the implementation of a new `domain` package, some minor changes to other code in the framework (to facilitate interactive evolution) were also necessary. The superclass `Search.java` was modified to include a new field, `userQuit` and the main loops in subclasses `SearchGA.java` and `SearchMC.java` were modified to incorporate this value.

An ant buildfile has been supplied to aid with building and executing the code. To do this, call `ant run` from the `JennWord` directory. Note that before this can be executed, the C++ source code of the combined Jenn3d+GE mapper must be compiled first. To do so, run `make (make -f makefile.mac` in a Mac environment) in the `GEMapJenn` directory. In a Linux environment, installations of the `libpng` and `freeglut3` libraries are required, whereas in a Mac a recent installation of XCode is required (the software has not been tested in a Windows environment).

When the user executes `ant run`, the framework class `World` is run. In the original framework version, this class called out to `SearchMC` and `SearchGA`, running MonteCarlo and Genetic Algorithm searches respectively. In the version provided here, the `SearchMC` code has been commented out, so the default execution launches straight into a GA. The reason for doing this is to showcase the beauty and complexity of the GA-generated images from the very outset. Images produced by the `SearchMC` code tend to be very similar to each other, or the genotypes fail to produce valid mappings at all. Considering that this is an interactive system, it is important to resist the (almost inevitable) onset of user-fatigue by all means necessary. To re-enable the `SearchMC` functionality, it will be necessary to uncomment the code in the `init()` method of the `World` class.

Appendix B: Example Individuals

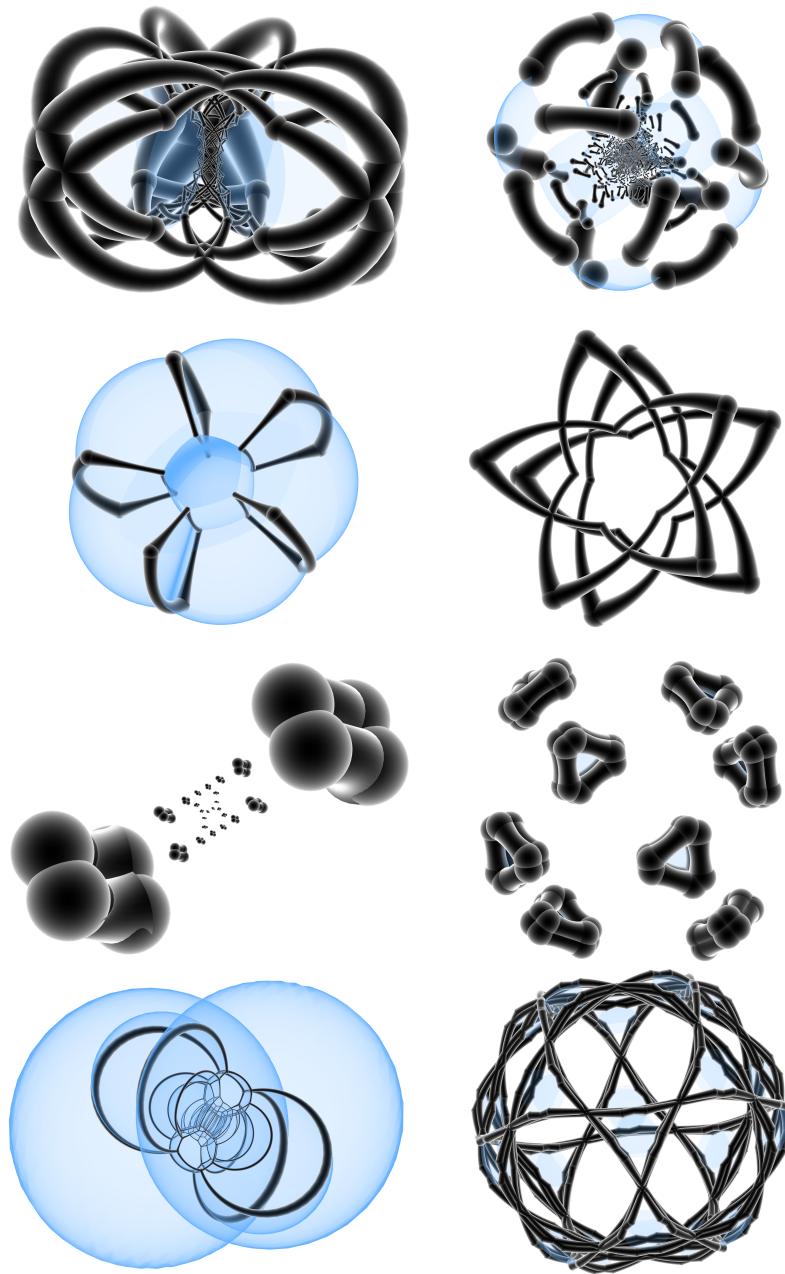


Figure 3: Mapping examples.