Home Assignment - 1

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1 Introduction

2 Theory

Let $\mathbf{Y}_k$ represent the n known observations of spatial data and $\mathbf{Y}_u$ represent the m data points that needs to be estimated. For ordinary Kriging ($\mu = \mathbf{1}\beta$), the optimal predictions are given by

$$\begin{aligned}\hat{\mathbf{Y}}_u &= \mathbf{1}_u\hat{\beta} + \Sigma_{uk}\Sigma_{kk}^{-1}\left(\mathbf{Y}_k - \mathbf{1}_k\hat{\beta}\right)\\ \hat{\beta} &= \left(\mathbf{1}_k^T\Sigma_{kk}^{-1}\mathbf{1}_k\right)^{-1}\mathbf{1}_k^T\Sigma_{kk}^{-1}\mathbf{Y}_k,\end{aligned}$$

where

$$\begin{bmatrix}\mathbf{Y}_k\\ \mathbf{Y}_u\end{bmatrix} \in \mathbf{N}\left(\begin{bmatrix}\mathbf{1}_k\beta\\ \mathbf{1}_u\beta\end{bmatrix}, \begin{bmatrix}\Sigma_{kk} & \Sigma_{ku}\\ \Sigma_{uk} & \Sigma_{uu}\end{bmatrix}\right).$$

Firstly, we show that the predictions are linear in the observations, i.e., $\hat{\mathbf{Y}}_u = \boldsymbol{\lambda}^T\mathbf{Y}_k$ for some $\boldsymbol{\lambda}$. We may re-write $\hat{\beta}$ as

$$\begin{aligned}\hat{\beta} &= \boldsymbol{\gamma}^T\mathbf{Y}_k\\ \boldsymbol{\gamma} &= \left(\left(\mathbf{1}_k^T\Sigma_{kk}^{-1}\mathbf{1}_k\right)^{-1}\mathbf{1}_k^T\Sigma_{kk}^{-1}\right)^T.\end{aligned}$$

We note that the vector $\boldsymbol{\gamma}^T$ is of dimensions $1 \times n$, and therefore, $\hat{\beta}$ is a scalar. Substituting the above in the expression for $\hat{\mathbf{Y}}_u$, we obtain

$$\begin{aligned}\hat{\mathbf{Y}}_u &= \mathbf{1}_u\boldsymbol{\gamma}^T\mathbf{Y}_k + \Sigma_{uk}\Sigma_{kk}^{-1}\mathbf{Y}_k - \Sigma_{uk}\Sigma_{kk}^{-1}\mathbf{1}_k\boldsymbol{\gamma}^T\mathbf{Y}_k\\ &= \left(\mathbf{1}_u\boldsymbol{\gamma}^T + \Sigma_{uk}\Sigma_{kk}^{-1} - \Sigma_{uk}\Sigma_{kk}^{-1}\mathbf{1}_k\boldsymbol{\gamma}^T\right)\mathbf{Y}_k\\ &= \boldsymbol{\lambda}^T\mathbf{Y}_k,\end{aligned}$$

where $\boldsymbol{\lambda} = \left(\mathbf{1}_u\boldsymbol{\gamma}^T + \Sigma_{uk}\Sigma_{kk}^{-1} - \Sigma_{uk}\Sigma_{kk}^{-1}\mathbf{1}_k\boldsymbol{\gamma}^T\right)^T$.

Secondly, we show that the predictions are unbiased.

3 Conclusions