

**Course Name:** Open Channel  
**Hydraulics**  
**Couse Code:** CIE 401  
**Level:** 4  
**Department:** Civil Engineering  
**Term No:** 2<sup>st</sup> term



**Final-term Exam**  
**Date:** 17/6/2023  
**Time:** 3.0 hour.  
**No of pages:** 2  
**No of questions:** 4  
**Total Mark:** 60 Marks

**Answer all questions and assume any massing data**

**Question 1 (10 Marks) (C2: a2; C12: b1, b2)**

**Write True (✓) or False (X) for the following statements:**

1. The non-uniform flow may be gradually varied, as in a hydraulic jump, or rapidly as in a backwater curve. ( )
2. In the uniform flow,  $S_o = S_w = S_f$ . ( )
3. If the bed slope ( $S_o$ ) is flatter than the critical slope ( $S_c$ ) for that discharge, the bed slope is called mild slope (M). ( )
4. It is discovered that the velocity is extremely near to the mean velocity at a depth of 0.6y from the free surface. ( )
5. Stilling basins are used to dissipate of excess energy in the hydraulic structures. ( )
6. For efficient dissipation of energy, the Froude number of the incoming flow,  $FN_1$  should be between 4.5-9.0. in this case, a good stable jump is formed. ( )
7. For a most economical rectangular channel, the hydraulic mean depth, is greater than to the depth of flow. ( )
8. Hydraulic turbines extract energy of flowing water and coupled with generators convert it to electric power. ( )
9. The flow in an open channel is uniform when the mean velocity (as well as other flow characteristics) from one section to another is not constant. ( )
10. A mild-sloped channel is followed by a steep-sloped channel. The profiles of gradually varied flow in the channel are  $M_1, S_2$  ( )

**Question 2 (12 marks) (C2: a2; C12: b1, b2)**

**Complete the missing data**

1. If the critical depth at a section of a rectangular channel is 1.5 m. The specific energy at that section is ..... (3 marks)
2. If a hydraulic jump occurs in a rectangular, horizontal, frictionless channel. The pre-jump depth for a discharge per unit width of  $2 \text{ m}^3/\text{s}/\text{m}$  and the energy loss 1 m is ..... (3 marks)
3. Flow happens at a critical depth of 0.5 m in a rectangular channel of 4 m width. The value of discharge is ..... (3 marks)
4. A centrifugal pump is required to lift water to a total head 40 m at the rate of 50 L/s. The power required for the pump, if the overall efficiency is 62% is..... (3 marks)

**Question 3 (18 marks) (C12: b1, b2)**

1. A trapezoidal channel is 10 m wide, side slopes 1:1, and carries water at a rate of  $100 \text{ m}^3/\text{s}$ . if the water depth is 1.5 m. (i) Whether a hydraulic jump will formed, determine: (ii) the sequent depths, and (iii) the power lost through the jump in  $H_p$ . (9 marks)

2. A brick lined trapezoidal canal has side slopes 1.5 horizontal and 1 vertical. It is required to carry a discharge of  $15 \text{ m}^3/\text{s}$ . If the average velocity of flow not exceed 1 m/s,

find: (a) the minimum perimeter for minimum amount of lining, (b) Bed slope assuming Manning's  $n = 0.015$ . **(9 marks)**

**Question 4                    (20 marks)                    (C2: a2; C12: b1, b2)**

(1) A rectangular channel is 10 m wide and carries 15 m<sup>3</sup>/s at normal depth of 1.2 m. a weir is constructed across the channel which rises the water depth to 2.5 m. If the bed slope is 1 in 2000, calculate the distance of the point where the depth is 1.8 m using the approximate method. Take Manning's  $n = 0.012$ . **(10 marks)**

(2) A long wide channel has a slope of 1:1000, a Manning's  $n$  of 0.015 and a discharge of 5 m<sup>3</sup>/s per meter width.

(i) Calculate the normal depth **(2 marks)**

(ii) Calculate the critical depth. **(2 marks)**

(iii) In a region of the channel the bed is raised by a height of 0.5 m over a length sufficient for the flow to be parallel to the bed over this length. Determine the depths upstream, downstream and over the raised bed., ignoring any friction losses. Sketch the flow, including gradually varied upstream and downstream. **(3 marks)**

(iv) In the same channel, the bed is lowered by 0.5 m from its original level. Again, determine the depths upstream, downstream and over the raised bed., ignoring any friction losses. Sketch the flow. **(3 marks)**

**With my best wishes**

**Assoc. Prof. /Dr. Mohamed El-Sayed Gabr**

## Model answer

### **Question 1 (10 Marks) (C2: a2; C12: b1, b2)**

**Write True (✓) or False (X) for the following statements:**

1. The non-uniform flow may be gradually varied, as in a hydraulic jump, or rapidly as in a backwater curve. (X)
2. In the uniform flow,  $S_o = S_w = S_f$ . (✓)
3. If the bed slope ( $S_o$ ) is flatter than the critical slope ( $S_c$ ) for that discharge, the bed slope is called mild slope (M). (✓)
4. It is discovered that the velocity is extremely near to the mean velocity at a depth of  $0.6y$  from the free surface. (✓)
5. Stilling basins are used to dissipate of excess energy in the hydraulic structures. (✓)
6. For efficient dissipation of energy, the Froude number of the incoming flow,  $FN_1$  should be between 4.5-9.0. in this case, a good stable jump is formed (✓)
7. For a most economical rectangular channel, the hydraulic mean depth, is greater than to the depth of flow. (X)
8. Hydraulic pumps extract energy of flowing water and coupled with generators convert it to electric power. (X)
9. The flow in an open channel is uniform when the mean velocity (as well as other flow characteristics) from one section to another is not constant. (X)
10. A mild-sloped channel is followed by a steep-sloped channel. The profiles of gradually varied flow in the channel are  $M_1, S_2$  (X)

### **Question 2 Complete the missing data (12 marks)**

1. If the critical depth at a section of a rectangular channel is 1.5 m. The specific energy at that section is 2.25 m

**Solution**

$$E_c = 1.5 \quad y_c = 1.5 * 1.5 = 2.25 \text{ m.}$$

2. A hydraulic jump occurs in a rectangular, horizontal, frictionless channel. What would be the pre-jump depth if the discharge per unit width is  $2 \text{ m}^3/\text{s}/\text{m}$  and the energy loss is 1 m?

**Solution**

**The correct option is B 0.3 m**

$$2q^2/g = y_1 y_2 (y_1 + y_2) \dots \text{(i)}$$

$$H_L = (y_2 - y_1)^3 / 4y_1 y_2 \dots \text{(ii)}$$

**Given,**

$$H_L = 1 \text{ m}$$

$$(y_2 - y_1)^3 = 4y_1 y_2$$

Put,  $y_2 = my_1$  in equation (ii),

$$y_1 = 4 \text{ m} (m-1)^3$$

Similarly, put

$y_2 = my_1$  in equation (i),

we get

$$m(m+1)y_1^3 = 2 \times 2^2 / 9.81 = 0.8155$$

$$m(m+1)\{4m(m-1)^3\}^3 = 0.8155$$

$$m(m+1)(m^3)/(m-1)^9 = 0.815543$$

by trial and error,  $m = 5.09$

$$y_1 = 0.3 \text{ m}$$

$$y_2 = 1.527 \text{ m}$$

**3. Flow happens at a critical depth of 0.5 m in a rectangular channel of 4 m width. The value of discharge is 4.4 m<sup>3</sup>/s**

Solution

$$\text{Discharge} = 4.4 \text{ m}^3/\text{s}$$

(d) For a rectangular channel, critical depth is given by

$$yc^3 = q^2/g$$

$$\Rightarrow q^2 = yc^3/g$$

$$\Rightarrow q^2 = (0.5)^3 \times 9.81$$

$$\Rightarrow q = 1.107 \text{ m}^2/\text{s}$$

But discharge,

$$Q = q \times B = 1.107 \times 4$$

$$= 4.4 \text{ m}^3/\text{s}$$

4. A centrifugal pump is required to lift water to a total head 40 m at the rate of 50 L/s. The power required for the pump, if the overall efficiency is 62% is.....

$$H_m = \text{manometric head of water} = 40 \text{ m}$$

$$Q = \text{discharge of pump} = 50 \text{ L/s} = 0.05 \text{ m}^3/\text{s}$$

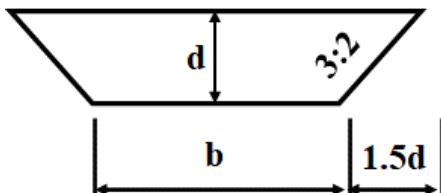
$$\eta_o = \text{Overall efficiency of the pump} = 62\% = 0.62.$$

$$\gamma = \text{Unit weight of water} = 1000 \text{ kg/m}^3$$

$$P(\text{h.p.}) = \frac{\gamma Q H_m}{75 \eta_o} = \frac{1000 \times 0.05 \times 40}{75 \times 0.62} = 43 \text{ Hp.}$$

### Question 3 (20 Marks)

1. A brick lined trapezoidal canal has side slopes 1.5 horizontal and 1 vertical. It is required to carry a discharge of  $15 \text{ m}^3/\text{s}$ . If the average velocity of flow not exceed  $1 \text{ m/s}$ , find: (a) the minimum perimeter for minimum amount of lining, (b) Bed slope assuming Manning's  $n = 0.015$ . (5 Marks)



Given side slope,  $m = 1.5$

Discharge,  $Q = 15 \text{ m}^3/\text{s}$

$V = 1 \text{ m/s}$

$$\therefore \text{Aera, } A = Q/V = 15 \text{ m}^2$$

Manning's  $n = 0.015$

Wetted perimeter, Let:

$P$  = Wetted perimeter of the canal,

$b$  = the bed width,

$d$  = depth of water,

$$\therefore \text{Aera, } A = d(b+1.5d) = 15 \quad (1)$$

For minimum amount of lining,

the section has to treat as a most

economical one (best section).

Therefore, half of the top = sloping side

$$\frac{b+2*1.5 d}{2} = d\sqrt{m^2 + 1}$$

$$= d\sqrt{1.5^2 + 1} = 1.8 d$$

$$b + 3d = 2*1.8 d = 3.6 d \quad \therefore b = 0.6 d$$

Substituting the value of  $b$  in Equation (1),

$$d(0.6 d+1.5d) = 15$$

$$15 = 2.1 d^2 \quad \therefore d = 2.66 \text{ m.}$$

$$b = 0.6 * 2.66 = 1.6 \text{ m.}$$

Now wetted perimeter, P

$$P = b + 2d\sqrt{m^2 + 1} = 1.6 + 2 * 2.66\sqrt{1.5^2 + 1} = 11.2 \text{ m.}$$

Bed slope

Let, S = bed slope of the canal

We know that the case of the most economical trapezoidal section, the hydraulic mean depth =  $d/2 = 2.66/2 = 1.33 \text{ m.}$

Using Manning's formula,  $V = \frac{1}{n} R_h^{2/3} S_e^{1/2} = 1 = \frac{1}{0.015} 1.33^{3/2} S_e^{1/2}$

$$S = 1/6500.$$

**2.** A trapezoidal channel is 10 m wide, side slopes 1:1, and carries water at a rate of 100 m<sup>3</sup>/s. if the water depth is 1.5 m. (i) Whether a hydraulic jump will form, determine: (ii) the sequent depths, and (iii) the power lost through the jump in Hp.

**Solution (7 marks)**

$$i. \quad A_1 = 1.5 (10 + 1 \times 1.5) = 17.25 \text{ m}^2$$

$$V_1 = Q/A_1 = 100/17.25 = 5.797 \text{ m/s}$$

$$T_1 = 10 + 2 \times 1.5 = 13 \text{ m}$$

$$y_{h1} = A_1/T = 17.25/13 = 1.327 \text{ m} \quad F_{N_1} = \frac{V_1}{\sqrt{gy_{h1}}} = \frac{5.797}{\sqrt{9.81 \times 1.327}} = 1.61 > 1$$

The flow is supercritical, a hydraulic jump is possible.

$$F_1 = F_2$$

$$\frac{Q^2}{gA_1} + A_1 \bar{y}_1 = \frac{Q^2}{gA_2} + A_2 \bar{y}_2$$

$$F_1 = \frac{100^2}{9.81(17.25)} + \frac{1.5}{2}(1.5 \times 10) + 2 \times \frac{1.5}{3}(0.5 \times 1.5 \times 1(1.5)) = 71.47 \text{ t.}$$

$$F_2 = \frac{Q^2}{gA_2} + A_2 \bar{y}_2$$

$$F_2 = \frac{100^2}{9.81(y_2)(10+1 \times y_2)} + \frac{y_2}{2}(y_2 \times 10) + 2 \times \frac{y_2}{3}(0.5 \times y_2 \times my_2) = 71.47 \text{ t.}$$

$$y_2 = 2.637 \text{ m}$$

$$h_l = E_1 - E_2$$

$$h_l = \left( y_1 + \frac{Q^2}{2gA_1^2} \right) - \left( y_2 + \frac{Q^2}{2gA_2^2} \right)$$

$$h_l = \left( 1.5 + \frac{100^2}{2(9.81)(17.25^2)} \right) - \left( 2.637 + \frac{100^2}{2(9.81)(33.324^2)} \right) = 0.117 \text{ m}$$

$$\begin{aligned} \text{Power lost through the jump (HP)} &= \frac{\gamma Q h_l}{75} \\ &= \frac{1000 (100)(0.117)}{75} = 156 \text{ HP} \end{aligned}$$

#### Question 4: (20 Marks)

1. A rectangular channel is 10 m wide and carries 15 m<sup>3</sup>/s at normal depth of 1.2 m. a weir is constructed across the channel which rises the water depth to 2.5 m. if the bed slope is 1 in 2000, calculate the distance of the point where the depth is 1.8 m using the approximate method. Take Manning's n = 0.012. (10 Marks)

#### Solution:

Get the average water depth  $y_m = (2.5+1.8)/2 = 2.15 \text{ m}$ .

Get average velocity  $V_m = 15/(2.15*10) = 0.7 \text{ m/s}$ .

$R_m = (2.15*10)/(10+2*2.15) = 1.503 \text{ m}$ .

From Manning's formula

$$V = \frac{1}{n} R^{2/3} S_E^{1/2} = 0.7 = \frac{1}{0.012} (1.503)^{2/3} S_{fm}^{1/2}$$

$$S_{fm} = 4.1 \times 10^{-5}$$

$$\frac{dy}{dx} = \frac{S_o - S_E}{1 - \frac{Q^2 T}{g A^3}}$$

$$\left( \frac{dy}{dx} \right)_m = \frac{S_o - S_E}{1 - \frac{Q^2 T}{g A^3}} = \frac{0.0005 - 0.000041}{1 - \frac{15^2 \times 10}{9.81(10 * 2.15)^3}} = 0.000469$$

$$L = \frac{h_1 - h_2}{\left( \frac{dy}{dx} \right)_m} = \frac{2.5 - 1.8}{0.000469} = 1492.5 \text{ m}$$

2. A long wide channel has a slope of 1:1000, a Manning's n of 0.015 and a discharge of 5 m<sup>3</sup>/s per meter width.

- (i) Calculate the normal depth (2 marks)
- (ii) Calculate the critical depth. (2 marks)
- (iii) In a region of the channel the bed is raised by a height of 0.5 m over a length sufficient for the flow to be parallel to the bed over this length. Determine the depths upstream, downstream and over the raised bed., ignoring any friction losses. Sketch the flow, including gradually varied upstream and downstream. (3 marks)
- (iv) In the same channel, the bed is lowered by 0.5 m from its original level. Again, determine the depths upstream, downstream and over the raised bed., ignoring any friction losses. Sketch the flow. (3 marks)

## Solution

### Normal depth computations

$$S = 0.001$$

$$n = 0.015 \text{ m}^{-1/3} \text{ s}$$

$$q = 5 \text{ m}^2 \text{ s}^{-1}$$

Discharge per unit width:

$$q = Vh, \quad \text{where} \quad V = \frac{1}{n} R_h^{2/3} S^{1/2} \quad (\text{Manning}), \quad R_h = h \quad (\text{"wide" channel})$$

$$\Rightarrow q = \frac{1}{n} h^{2/3} S^{1/2} h$$

$$\Rightarrow q = \frac{h^{5/3} \sqrt{S}}{n}$$

$$\Rightarrow h_n = \left( \frac{nq}{\sqrt{S}} \right)^{3/5} = \left( \frac{0.015 \times 5}{\sqrt{0.001}} \right)^{3/5} = 1.679 \text{ m}$$

**Answer:** normal depth = 1.68 m.

(c) Critical depth:

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{5^2}{9.81}\right)^{1/3} = 1.366 \text{ m}$$

**Answer:** critical depth = 1.37 m.

(d) To determine the type of behaviour over the raised bed, compare the total head under critical conditions (the minimum energy necessary to get over the weir at this flow rate) with that available in the approach flow.

#### Critical

$$h_c = 1.366 \text{ m}$$

$$E_c = \frac{3}{2}h_c = 2.049 \text{ m}$$

$$z_{weir} = 0.5 \text{ m}$$

$$H_c = z_{weir} + E_c = 2.549 \text{ m}$$

#### Approach Flow

Because the channel is described as “long” it will have sufficient fetch to develop normal flow; hence the approach-flow head is that for the normal depth ( $h = 1.679 \text{ m}$ ):

$$\begin{aligned} H_a &= E_a = h_n + \frac{V_n^2}{2g} \\ &= h_n + \frac{q^2}{2gh_n^2} \\ &= 1.679 + \frac{5^2}{2 \times 9.81 \times 1.679^2} \\ &= 2.131 \text{ m} \end{aligned}$$

At the normal depth the available head ( $H_a$ ) is less than the minimum required to get over the weir ( $H_c$ ). Hence the water depth must increase upstream (“backing up”), to raise the head immediately upstream of the weir. Thus:

- critical conditions *do* occur;
- the total head through the device is the critical head ( $H = H_c = 2.549 \text{ m}$ ).

Over the weir there is a critical flow transition, so the depth here is critical:  $h = h_c = 1.366 \text{ m}$ .

Just up- or downstream,

$$\begin{aligned} H &= E = h + \frac{V^2}{2g} \quad \text{where} \quad V = \frac{q}{h} \\ \Rightarrow H &= h + \frac{q^2}{2gh^2} \end{aligned}$$

$$\Rightarrow 2.549 = h + \frac{1.274}{h^2}$$

Upstream, rearrange for the deep, subcritical, solution:

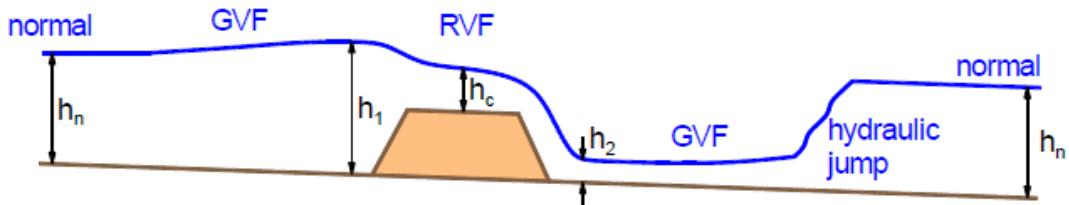
$$h = 2.549 - \frac{1.274}{h^2}$$

Iteration (from, e.g.,  $h = 2.549$ ) gives  $h = 2.310$  m.

Downstream, rearrange for the shallow, supercritical solution:

$$h = \sqrt{\frac{1.274}{2.549 - h}}$$

Iteration (from, e.g.,  $h = 0$ ) gives  $h = 0.8715$  m.



**Answer:** depths upstream, over, downstream of the weir: 2.31 m, 1.37 m, 0.871 m.

- (e) The flow does not require additional energy to pass a depressed section; hence, the total head throughout is that supplied by the approach flow ( $H = H_a = 2.131$  m) and the flow remains subcritical. The depths just upstream and downstream of the lowered section are those in the approach flow; i.e. normal depth.

As bed height  $z_b$  decreases, specific energy  $E$  must increase to maintain the same total head. In the lowered section:

$$\begin{aligned} H &= z_b + E \\ \Rightarrow 2.131 &= -0.5 + E \\ \Rightarrow E &= 2.631 \text{ m} \end{aligned}$$

Then

$$E = h + \frac{V^2}{2g} \quad \text{where} \quad V = \frac{q}{h}$$

$$\Rightarrow E = h + \frac{q^2}{2gh^2}$$

$$\Rightarrow 2.631 = h + \frac{1.274}{h^2}$$

As we require the subcritical solution, rearrange as

$$h = 2.631 - \frac{1.274}{h^2}$$

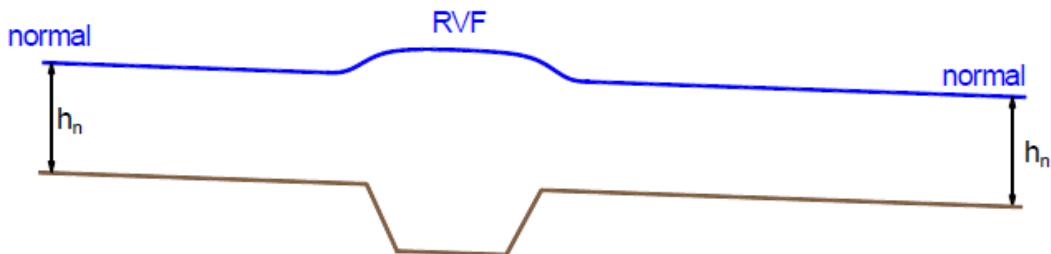
Iteration (from, e.g.,  $h = 2.631$ ) gives  $h = 2.412$  m.

(Note that this is the depth of the water column. The actual surface level here is

$$z_s = -0.5 + h = 1.912 \text{ m}$$

so the overall water level also rises in this section.)

so the overall water level also rises in this section.)



**Answer:** depths upstream, within, downstream of the lowered section: 1.68 m, 2.41 m, 1.68 m.